Aerodynamics

Turbulent Flow

Mean velocity profile, U

1. Linear profile in the linear (laminar) sub-layer, $y^+ < 5$

$$U^+ = y^+$$

2. Semi-logarithmic profile in wall (log) law,
   $y^+ > 30 - 50, \quad y < 0{,}1 - 0{,}2\delta$

$$U^+ = \frac{1}{\kappa}\ln\left(y^+\right) + C \qquad k = 0{,}41 \;\; C \cong 5{,}2$$

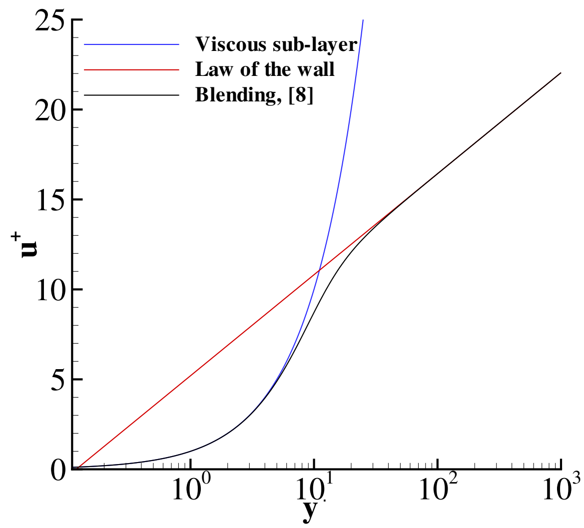
3. Continuous shift from 1 to 2 in the buffer-layer

Aerodynamics

Turbulent Flow

Mean velocity profile, U

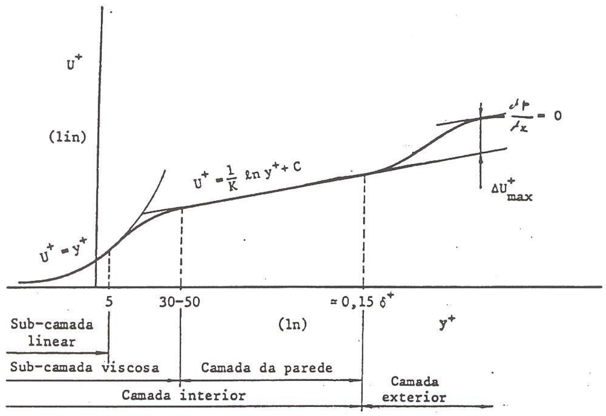
3. Continuous shift from 1 to 2 in the buffer-layer



Turbulent Flow

Mean velocity profile, U

4. Outer part of the profile where the velocity tends to U_e^+ defined from the difference to the log-law



$$U_e^+ = \frac{U_e}{u_\tau} = \sqrt{\frac{2}{C_f}}$$

Turbulent Flow

Mean velocity profile, U

- Wake component

$$\Delta U^+ = U^+ - \left(\frac{1}{\kappa}\ln\left(y^+\right) + C\right) = \frac{\Pi}{\kappa} w\left(\frac{y}{\delta'}\right)$$

- Coles (empirical) wake profile

$$w\left(\frac{y}{\delta'}\right) = 1 - \cos\left(\pi \frac{y}{\delta'}\right)$$

- *y=δ'* is the location where the maximun difference to the log law occurs

Aerodynamics

Turbulent Flow
Mean velocity profile, U

- Wake component magnitude/intensity

$$\Delta U^+_{max} = 2\frac{\Pi}{\kappa}$$

- Mean velocity profile outside the viscous sub-layer

$$U^+ = \frac{1}{\kappa}\ln\left(y^+\right) + C + \frac{\Pi}{\kappa}\left(1 - \cos\left(\pi \frac{y}{\delta'}\right)\right)$$

Aerodynamics

Turbulent Flow
Self-preserved flows

- von Kármán's integral equation

$$\frac{d}{dx}\left(\rho U_e^2 \theta\right) = \tau_w + \delta^* \frac{dP}{dx}$$

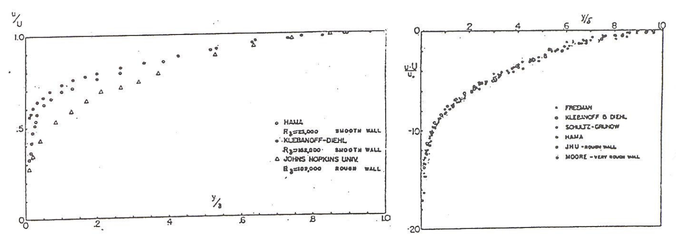
- History/memory parameter

$$\beta = \frac{\delta^*}{\tau_w}\frac{dP}{dx}$$

- Self-preserved flow, β=*constante*

Turbulent Flow

Self-preserved flows



- Clauser's equilibrium parameter

$$G = \sqrt{\frac{2}{C_f}}\frac{H-1}{H} = \frac{\int_0^h \left[\frac{U_e - U}{u_\tau}\right]^2 dy}{\int_0^h \left[\frac{U_e - U}{u_\tau}\right] dy}$$

- *G* is constant for zero pressure gradient and sufficiently large Reynolds number

Turbulent Flow

Simplified forms of the mean velocity profile

- Power-law profile

$$\frac{U}{U_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$$

- Integral parameters

$$\frac{\delta^*}{\delta} = \frac{1}{n+1} \quad \frac{\theta}{\delta} = \frac{n}{(n+1)(n+2)} \quad H = 1 + \frac{2}{n}$$

Turbulent Flow

Simplified forms of the mean velocity profile

- Power-law profile $\frac{U}{U_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$

1. It does not safisty the linear sub-layer

2. It does not satisfy the wall (log) law

3. $\frac{\partial U}{\partial y} \neq 0$ at $y = \delta$

4. $\frac{\partial U}{\partial y} = \infty$ at $y = 0$

Aerodynamics

Turbulent Flow

Simplified forms of the mean velocity profile

- Power-law profile $\frac{U}{U_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$

- From experimental data:

$$\frac{dP}{dx} = 0 \rightarrow n \approx 7$$

$$\frac{dP}{dx} < 0 \rightarrow n \approx 7 - 10$$

$$\frac{dP}{dx} > 0 \rightarrow n \approx 3 - 7$$

Turbulent Flow

Zero pressure gradient (flat plate) boundary-layer

- Integration of von Kármán's integral equation

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U_e^2}$$

- Power-law profile

$$\frac{U}{U_e} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

- τ_w from a friction law for fully-developed flow in pipes

$$\lambda = \frac{4\tau_w}{1/2\rho U_{med}^2} = 0{,}3164 R_e^{-1/4} \qquad R_e = \frac{U_{med} D}{\nu}$$

$$U_{med} = 0{,}8 U_{\max} \quad \delta \approx R \quad U_e \approx U_{\max}$$

Aerodynamics

Turbulent Flow

Zero pressure gradient (flat plate) boundary-layer

- Shear-stress at the wall

$$\frac{\tau_w}{\rho U_e^2} = 0{,}0225\left(\frac{\nu}{U_e \delta}\right)^{1/4}$$

- von Kármán's integral equation with δ as the dependent variable

$$\frac{7}{72}\frac{d\delta}{dx} = 0{,}0225\left(\frac{\nu}{U_e \delta}\right)^{1/4}$$

Turbulent Flow

Zero pressure gradient (flat plate) boundary-layer

- Assuming turbulent flow from *x=0* (*δ=0*)

$$\frac{\delta}{x} = 0{,}37 R_{e_x}^{-1/5} \qquad \frac{\delta^*}{x} = 0{,}046 R_{e_x}^{-1/5}$$

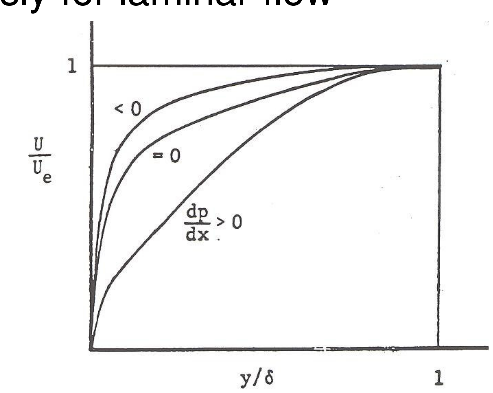
$$\frac{\theta}{x} = 0{,}036 R_{e_x}^{-1/5} \qquad H = 1{,}29$$

$$C_f = 0{,}0576 R_{e_x}^{-1/5} \qquad C_D = 0{,}072 R_{e_x}^{-1/5}$$

Turbulent Flow

Effect of the pressure gradient

- Qualitatively, the effect is similar to that discussed previously for laminar flow

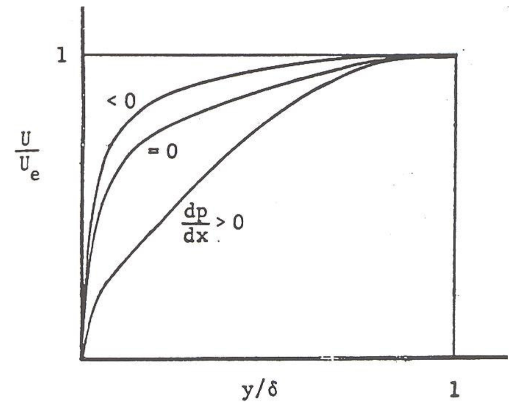


- H decreases for favourable pressure gradient and it increases for adverse pressure gradient

Turbulent Flow

Effect of the pressure gradient

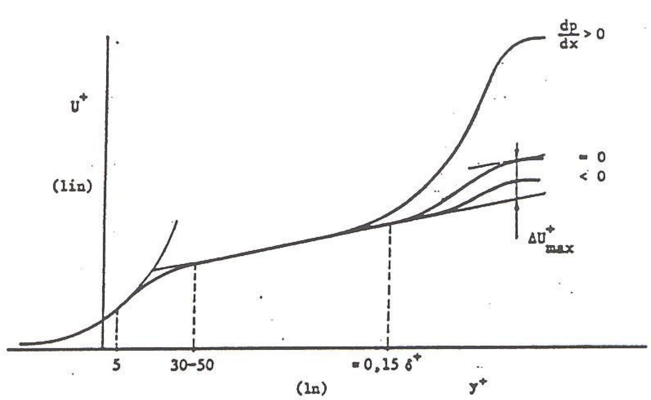
- Entrainment is strongly dependent on the gradients of the mean velocity profile in the outer region of the boundary-layer (production of turbulence kinetic energy is proportional to the gradients of the mean velocity)



Turbulent Flow

Effect of the pressure gradient

- Effect on the wall (log) law



- Validity of wall (log) law at flow separation is extremely doubtful

Aerodynamics

Turbulent Flow

Effect of the pressure gradient

- Turbulent flow is significantly more resistent to flow separation than laminar flow.

1. Velocity close to the wall is larger than in laminar flow
2. Diffusion is significantly higher than in laminer flow (separation depends on the ratio between the pressure force and diffusion)

Aerodynamics

Turbulent Flow

Head's method

- von Kármán's integral equation

$$\frac{d\theta}{dx}+\theta\frac{H+2}{U_e}\frac{dU_e}{dx}=\frac{C_f}{2}$$

- Entrainment velocity, V_E

$$V_E=\frac{d}{dx}\int_0^{\delta}U\,dy=\frac{d}{dx}\left[U_e\left(\delta-\delta^*\right)\right]$$

- Shape parameter, H_1

$$H_1=\frac{\delta-\delta^*}{\theta}$$

Turbulent Flow
Head's method

- Additional equation proposed

$$\frac{d}{dx}\left(U_e\theta H_1\right)=U_e F\left(H_1\right) \qquad H_1=G(H)$$

- Experimental fit to determine $F(H_1)$ and $G(H)$

$$F\left(H_1\right)=0{,}0306\left(H_1-3\right)^{-0{,}6169}$$

$$H_1=G(H)=\begin{cases}0{,}8234\left(H-1{,}1\right)^{-1{,}287}+3{,}3 & \Leftarrow H\le 1{,}6\\ 1{,}5501(H-0{,}6778)^{-3{,}064}+3{,}3 & \Leftarrow H>1{,}6\end{cases}$$

Turbulent Flow
Head's method

- Ludwieg-Tillman correlation to determine C_f

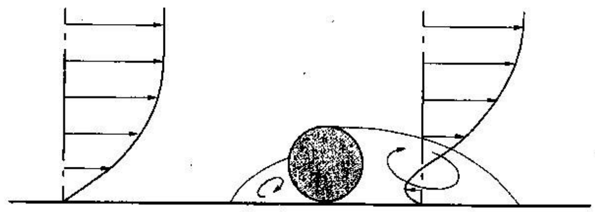
$$C_f=0{,}246\times10^{-0{,}678H}\times R_{e_\theta}^{-0{,}268}$$

- Flow separation predicted for $H\approx 2.4-2.8$

- Numerical solution with a Runge-Kutta scheme

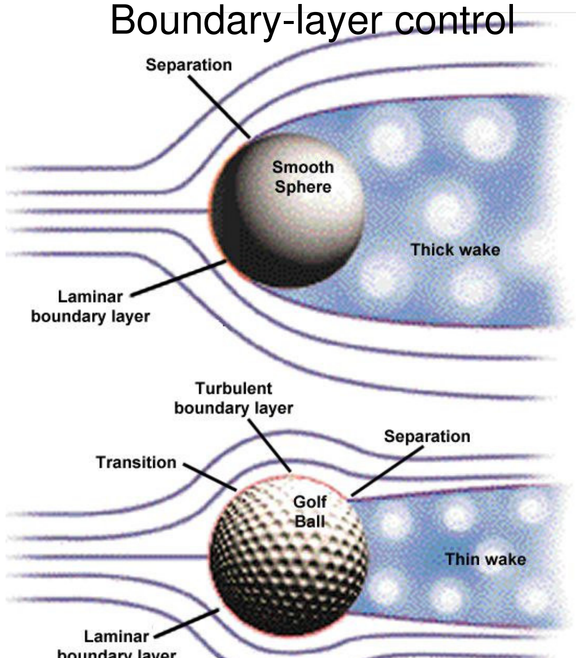
Turbulent Flow
Boundary-layer control

• Forced transition: roughness or trip wire
The objective is to delay or avoid flow separation



$$R_{e_{arame}} = \frac{U_e d_{arame}}{\nu} \geq 826$$ Gibbings's criterion

Turbulent Flow
Boundary-layer control

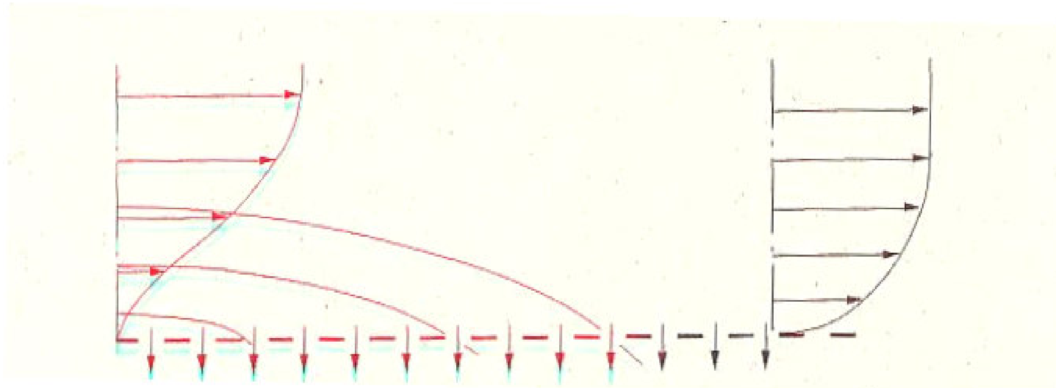


Aerodynamics

Turbulent Flow

Boundary-layer control

- Suction at the wall. Delays (or avoids) flow separation and it also delays transition from laminar to turbulent flow



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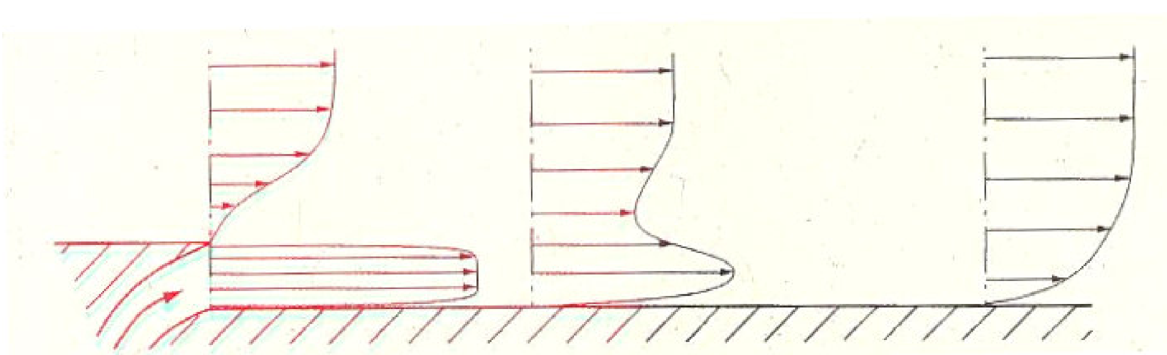
Aerodynamics

Turbulent Flow

Boundary-layer control

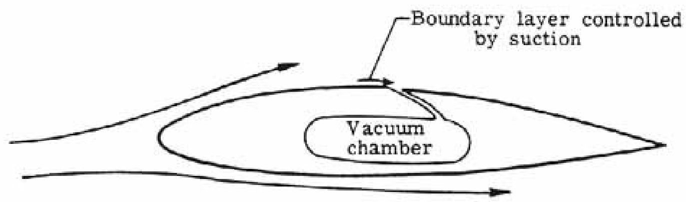
- Blowing. Delays or avoids flow separation, but it favours transition from laminar to turbulent flow



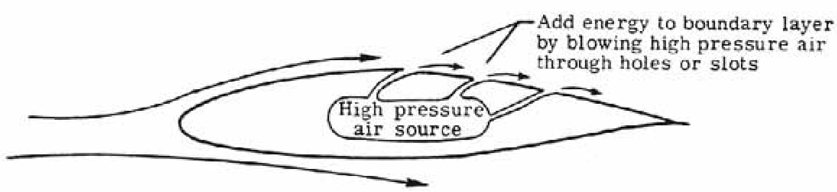
Masters of Mechanical Engineering

Aerodynamics

Turbulent Flow
Boundary-layer control



(a) Suction of boundary layer.



(b) Reenergizing the boundary layer.