Matrix Converters: The Direct Control Approach Using Sliding Mode

S. Ferreira Pinto, J. Fernando Silva

Abstract – This paper presents a new approach to solve the direct control problem of matrix converters with input LC filters using the sliding mode control technique. Sliding mode controllers can be tailored to consider the dynamics of the converter and its associated LC filter and, together with the space vector representation technique, can determine the switching from one bidirectional switch to another, being appropriate to the non-linear on-off behavior of the matrix converter power semiconductors. Moreover, this switching occurs just in time, guaranteeing fast response times and precise control actions, ensuring that the output voltages and the input currents track their references, thus allowing the input power factor regulation. This regulation, can be made independent of the input filter parameters, which is useful in applications requiring unity input power factor, such as ac drives, or applications needing variable and accurate input power factor regulation, usually related to power quality enhancement. The simulations and obtained experimental results with a laboratory prototype show that the designed sliding mode controllers ensure the direct control of matrix converters output variables over a wide range of output frequencies (0.01Hz to 300Hz) and guaranteeing a leading or lagging input power factor regulation.

Keywords: Matrix Converters, Space Vector, Sliding Mode Control.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>C</td>
<td>input filter capacitor</td>
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<td>C</td>
<td>Concordia transformation matrix</td>
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<td>D</td>
<td>Blondel-Park transformation matrix</td>
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<td>E_A, E_B, E_C</td>
<td>voltages of the matrix converter RLE load</td>
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<td>e_iq</td>
<td>input current error in dq coordinates</td>
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<td>e_α, e_β</td>
<td>output voltages error in αβ coordinates</td>
</tr>
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<td>i_a, i_b, i_c</td>
<td>matrix converter input currents</td>
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<tr>
<td>i_α, i_β, i_c</td>
<td>matrix converter output currents</td>
</tr>
<tr>
<td>i_d, i_q</td>
<td>matrix converter input currents in dq coordinates</td>
</tr>
<tr>
<td>I_m, i_b, i_c</td>
<td>mains currents</td>
</tr>
<tr>
<td>i_d, i_q</td>
<td>mains currents in dq coordinates</td>
</tr>
<tr>
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<td>input filter inductors currents</td>
</tr>
<tr>
<td>i_d, i_q</td>
<td>input filter inductors currents in dq coordinates</td>
</tr>
<tr>
<td>i_α, i_β</td>
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<tr>
<td>i_d, i_q</td>
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<tr>
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<tr>
<td>k_iq</td>
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<tr>
<td>k_α, k_β</td>
<td>gains of the output voltages sliding surfaces</td>
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<td>l</td>
<td>input filter inductor</td>
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<td>input active power</td>
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<td>Q_m</td>
<td>input reactive power</td>
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<td>input filter resistance</td>
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<td>bidirectional switch</td>
</tr>
<tr>
<td>S_α, S_β</td>
<td>output voltages sliding surfaces</td>
</tr>
<tr>
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<td>switching period</td>
</tr>
<tr>
<td>v_a, v_b, v_c</td>
<td>matrix converter input phase voltages</td>
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<tr>
<td>v_A, v_B, v_C</td>
<td>matrix converter output phase voltages</td>
</tr>
<tr>
<td>v_ab, v_bc, v_ca</td>
<td>input filter capacitors voltages</td>
</tr>
<tr>
<td>v_AB, v_BC, v_CA</td>
<td>matrix converter output line voltages</td>
</tr>
<tr>
<td>v_d, v_q</td>
<td>input filter capacitors voltages in dq coordinates</td>
</tr>
<tr>
<td>v_α, v_β</td>
<td>input filter capacitors voltages in αβ coordinates</td>
</tr>
<tr>
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I. Introduction

Matrix converters, also known as all silicon AC-AC power converters, contain an array of bi-directional semiconductor switches, connecting all the input-voltage lines with all the output voltages. Bi-directional switches are associations of power semiconductors consisting of a pair of devices with turn-off capability, usually IGBTs, in either a common collector or a common emitter back-to-back arrangement, each one connected with an anti parallel diode (Fig. 1) [1], [2], [3]. Nevertheless, these diodes may be avoided if reverse blocking IGBTs [4] are used.

Matrix converters present the advantage of not needing an intermediate energy storage link, but its absence implies input/output variable coupling, increasing the difficulty to define adequate control strategies. The first PWM (Pulse Width Modulation) techniques were mainly concerned with the output voltage control, neglecting the waveform quality of the input currents (sometimes presenting high harmonic contents). Only in 1980 Alesina and Venturini introduced the high frequency PWM approach [5], [6] enabling low harmonic contents for both output voltages and input currents. Since then, other control techniques [7], [8] such as Space Vector Modulation (SVM) [9], [10], [11] have been studied. SVM is based on the complex plane representation, as vectors, of all the voltages or currents resulting from all the matrix converter switching combinations. This approach allows a better selection of the required voltage and current vectors, simplifying control algorithms and providing maximum voltage transfer ratio without the need to add third harmonic modulator components [12].

This paper presents a direct control approach using both the space vector representation and the sliding mode control technique [13]. This technique allows on-line regulation of output voltages and input power factor. Also, it guarantees the on-line compensation of the displacement factor introduced by the input filter, a subject not addressed in previous publications.

The matrix converter model, considering the input filter, will be presented in Section II. Based on the derived model, the sliding mode controllers will be designed in Section III. Simulation and experimental results will be shown in Section IV and the conclusions will be presented in section V.

II. Matrix Converter Model

II.1. Ideal Matrix Model

Ideal three phase matrix converters may be considered as an array of nine bi-directional switches allowing the connection of each one of the three input phases to any one of the three output phases (Fig. 1). Representing each \( S_{kj} \) \( k, j \in \{1, 2, 3\} \) switch as a variable with two possible states: \( S_{kj}=1 \) if the switch is closed (ON) and \( S_{kj}=0 \) if it is open (OFF), the nine matrix converter switches may be represented as a 3x3 matrix (1), allowing the establishment of algebraic relations between input and output variables.

However, in order to guarantee that the input phases, connected to the mains, are never short-circuited and that the inductive output phases are never opened, the sum of all the \( S_{kj} \) belonging to each one of matrix \( S \) rows must always be equal to 1 (1). As a result of these constraints, there are only 27 possible switching combinations [12].

\[
S = \begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}
\sum_{j=1}^{3} S_{kj} = 1 \quad k, j \in \{1, 2, 3\} \quad (1)
\]

According to figure 1, the output phase voltages \( v_A, v_B, v_C \) are related to the input phase voltages \( v_a, v_b, v_c \) (2), depending on the states of matrix \( S \) switches. Also, the input phase currents \( i_a, i_b, i_c \) are related to the output phase currents \( i_A, i_B, i_C \) (3), depending on the transpose of matrix \( S \).

\[
[v_A, v_B, v_C]^T = SV_a, v_b, v_c \quad (2)
\]
\[
[i_A, i_B, i_C]^T = S^T[i_a, i_b, i_c] \quad (3)
\]

To use the State-Space Vector technique, it is necessary to represent the output voltages and input currents, which result from the 27 switching combinations, as vectors in the \( \alpha \beta \) plane (table I), using the Concordia transformation \([X_{\alpha, \beta, 0}]^T = C^T[X_{a,b,c}]^T\), with \( C \) defined in (4).
According to their amplitude and phase characteristics, these vectors can be grouped in three different categories: I) vectors with fixed amplitude and time varying phase (in general are not used in the SVM approaches); II) vectors with fixed phase and time varying amplitude; III) zero amplitude vectors.

Usually, matrix converter modulation techniques are obtained considering the ideal matrix converter described in this section. However, neglecting the input filter in the derivation of the modulation indexes originates a displacement factor between the input voltages and their currents [8]. To avoid this undesirable displacement factor introduced by the input filter, the matrix converter dynamic model should be obtained considering the nine bi-directional power switches (1), the three phase input filter and the output load.

The sliding mode controllers design, here presented, is then based on this switched state space model. Together with the definition of adequate strategies to choose the space vectors, the input filter displacement factor can be made zero, guaranteeing the simultaneous tracking of the desired output voltages and input currents.

### II.2. Matrix Converter Model with Input Filter and Output Load

Assuming that the source is a balanced sinusoidal three-phase voltage supply with frequency \( \omega_a \), the switched state space model equations (5) of the converter, with input filter and load (Fig. 1) are obtained in abc coordinates:

\[
\begin{align*}
\frac{di_a}{dt} &= \frac{1}{3l}v_{bc} - \frac{2}{3l}v_{ca} + \frac{1}{3}v_{i_a} \\
\frac{di_b}{dt} &= -\frac{2}{3l}v_{bc} - \frac{1}{3l}v_{ca} + \frac{1}{3}v_{i_b} \\
\frac{di_c}{dt} &= -\frac{2}{3l}v_{bc} + \frac{2}{3l}v_{ca} + \frac{1}{3}v_{i_c} \\
\frac{dv_{bc}}{dt} &= \frac{1}{3C}i_a - \frac{2}{3C}i_b - \frac{1}{3C}v_{bc} + \frac{1}{3C}v_{i_a} + \frac{2}{3C}v_{i_b} - \frac{1}{3C}v_{i_c} - \frac{1}{3C}v_{bc} \\
\frac{dv_{ca}}{dt} &= -\frac{2}{3C}v_{bc} - \frac{1}{3C}v_{i_a} - \frac{1}{3C}v_{ca} - \frac{2}{3C}v_{i_b} - \frac{1}{3C}v_{i_c} - \frac{2}{3C}v_{bc} + \frac{1}{3C}v_{i_a} + \frac{2}{3C}v_{i_b} \\
\frac{dv_{ab}}{dt} &= -\frac{2}{3C}v_{bc} + \frac{2}{3C}v_{ca} + \frac{1}{3C}v_{bc} - \frac{1}{3C}v_{i_a} + \frac{2}{3C}v_{i_b} + \frac{1}{3C}v_{i_c} + \frac{1}{3C}v_{bc} - \frac{1}{3C}v_{i_a} + \frac{2}{3C}v_{i_b} + \frac{1}{3C}v_{i_c} + \frac{1}{3C}v_{bc} \\
\frac{di_A}{dt} &= -\frac{R}{L}i_A - \frac{1}{3L}v_{bc} - \frac{2}{3L}v_{ca} - \frac{1}{3L}E_A \\
\frac{di_B}{dt} &= -\frac{R}{L}i_B + \frac{2}{3L}v_{bc} - \frac{1}{3L}v_{ca} + \frac{1}{3L}E_B
\end{align*}
\]

In this model, the dependence on the nine \( S_{ij} \) switches is not represented directly. However, the \( v_{bc}, v_{ca} \) output voltages are related to the matrix converter input voltages according to (2) and the matrix converter \( i_a, i_b \) input currents are related to the \( i_A, i_B \) output currents according to (3).

As state-space vectors are conveniently represented in the \( \alpha \beta \) complex plane, the system model equations (5) in abc coordinates, should be transformed to \( \alpha \beta \) coordinates as well, using the Concordia transformation (4).

![Fig. 1. Three-phase matrix converter with input and output filters.](image-url)
<table>
<thead>
<tr>
<th>( v_{ab} )</th>
<th>( v_{bc} )</th>
<th>( v_{ca} )</th>
<th>( i_a )</th>
<th>( i_b )</th>
<th>( i_c )</th>
<th>( v_a )</th>
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<tr>
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<td>( v_{ab} )</td>
<td>( v_{bc} )</td>
<td>( v_{ca} )</td>
<td>( i_a )</td>
<td>( i_b )</td>
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<td>2</td>
<td>( v_{ab} )</td>
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<td>( v_{ca} )</td>
<td>( i_a )</td>
<td>( i_b )</td>
<td>( i_c )</td>
<td>( v_a )</td>
<td>( \delta_a )</td>
<td>( I_a )</td>
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</table>

In the \( \alpha \beta \) coordinate system, the matrix converter output voltages \( v_{ca} \) and \( v_{c\beta} \) (6a) are related to the voltages applied to the input filter capacitors \( v_{ca} \) and \( v_{c\beta} \), where \( \rho_{i_{ca}} \), \( \rho_{i_{c\beta}} \), \( \rho_{j_{ca}} \), and \( \rho_{j_{c\beta}} \), given in (6b), are the equivalent output voltage control variables, functions of the ON/OFF state of the nine \( S_{ij} \) switches [14].

Using these output voltage (6b) and input current (7b) equivalent control variables, the switched state space model (8) for the whole system, in \( \alpha \beta \) coordinates, is obtained:
Even though the input current controllers may be designed in $\alpha\beta$ coordinates [15], using the previously defined equations, there may be some advantages if a new coordinate system, synchronous with the input voltages, is used [16]. In fact, the aim of the input current controllers is to guarantee that they have a nearly sinusoidal shape, allowing a leading or lagging power factor.

Applying the Blondel-Park transformation to the matrix converter switched state-space model (8), with $D^T$ defined in (9) and neglecting ripples, all the input variables become time invariant, allowing the sliding mode controller design, as well as the choice of the most adequate state space vectors.

$$D = \begin{bmatrix} \cos(\Theta) & -\sin(\Theta) \\ \sin(\Theta) & \cos(\Theta) \end{bmatrix} \quad \Theta = \omega_0 t \quad (9)$$

In the Blondel-Park reference frame (9), the switched state space model for the whole matrix converter is (10).

$$\begin{align*}
\frac{dv_{up}}{dt} &= \frac{1}{2\sqrt{3}C}i_{up} + \frac{1}{2C}i_{ip} - \frac{1}{3Cr}v_{up} + \frac{1}{2\sqrt{3}Cr}i_{up} \\
&\quad - \frac{\rho_{iga}}{\sqrt{3}} - \frac{\rho_{igg}}{\sqrt{3}} - \frac{\rho_{igg}}{\sqrt{3}} - \frac{\rho_{igg}}{\sqrt{3}} + \frac{1}{2Cr}i_{up} + \frac{1}{2Cr}i_{ip} \\
\frac{di_{up}}{dt} &= -\frac{R}{L}i_{up} + \frac{1}{L}E_a + \\
\frac{di_{ip}}{dt} &= -\frac{R}{L}i_{ip} + \frac{1}{L}E_b - \\
\frac{di_{up}}{dt} &= -\frac{R}{L}i_{up} + \frac{1}{L}E_c - \\
\frac{di_{ip}}{dt} &= -\frac{R}{L}i_{ip} + \frac{1}{L}E_d - \\
\frac{di_{q}}{dt} &= -\frac{R}{L}i_{q} + \frac{1}{L}E_e + \\
\frac{di_{r}}{dt} &= -\frac{R}{L}i_{r} + \frac{1}{L}E_f + \\
\end{align*}$$

Using the first two equations of this state-space model, the input currents $i_d$ and $i_q$ are calculated according to (11).

$$\begin{align*}
i_d &= i_d + \frac{1}{r} \frac{di_d}{dt} - \omega_0 i_d \\
i_q &= i_q + \frac{1}{r} \frac{di_q}{dt} + \omega_0 i_q \\
i_d &= \frac{1}{2r}v_c - \frac{1}{2\sqrt{3}r}v_e + \frac{1}{r}v_d \\
i_q &= i_q + \frac{1}{2r}v_c - \frac{1}{2\sqrt{3}r}v_e + \frac{1}{r}v_q \\
\end{align*}$$

The sliding mode input current controller design will be designed based on these equations.

### III. Sliding Mode Controllers Design

Matrix converters, like most power electronic converters, are variable structure systems, as a result of the ON/OFF high frequency switching of their power semiconductors. Sliding mode control techniques present special interest for these systems, as they can use this variable structure capability, to successfully solve the direct control problem, guaranteeing the choice of the most appropriate Space Vectors, and ensuring that the adequate semiconductors switch in time. As a result, the controlled system slides along a predefined surface. Constrained to that surface, the
system order is reduced and the robustness can be
equipped by an adequate sliding surface.
However, the design of these controllers for matrix
converters and the choice of the most appropriate Space
Vectors represent a tough challenge, as a result of the
coupling between the input and output variables. In fact,
according to table I, the output voltage vectors depend
on the input (mains) voltages and the input current
vectors depend on the output currents, assumed nearly
sinusoidal, but dependent on the matrix output voltages.
To overcome these problems, it will be necessary: a)
to guarantee the adequate control of the output variables
in order to use the current Space Vectors as defined
in table I; b) to consider the input/output power
constraint.

III.1. Design of the Three Phase Output Voltage
Controller
The output voltages of matrix converters are switched
discontinuous variables, as they depend directly on the
control variables (6a). Therefore, if high enough
switching frequencies are considered (much higher than
the input and output matrix converter fundamental
frequencies), it is possible to assume that, in each
switching period $T_s$, the output voltage average values
$\bar{v}_{oa}$ and $\bar{v}_{ob}$ (12) are nearly equal to their reference
average values $\bar{v}_{oa,ref}$ and $\bar{v}_{ob,ref}$:

$$\frac{1}{T_s} \int_{nT_i}^{(n+1)T_i} v_{oa} \, dt = \frac{1}{T_s} \int_{nT_i}^{(n+1)T_i} v_{oa,ref} \, dt$$

$$\frac{1}{T_s} \int_{nT_i}^{(n+1)T_i} v_{ob} \, dt = \frac{1}{T_s} \int_{nT_i}^{(n+1)T_i} v_{ob,ref} \, dt$$  \hspace{1cm} (12)

If the output voltage error variables $e_a = v_{oa,ref} - v_{oa}$
and $e_b = v_{ob,ref} - v_{ob}$ are defined, the sliding surfaces
$S(e_a, t)$ $S(e_b, t)$ (13), with $k_o > 0$ and $k_o > 0$, that result
from (12) are derived [17]. These sliding surfaces depend
only on the output voltage errors and not on the system parameters, therefore ensuring robustness.

$$S(e_a, t) = \frac{k_o}{T} \int_0^{T_s} (v_{oa,ref} - v_{oa}) \, dt = 0$$
$$S(e_b, t) = \frac{k_o}{T} \int_0^{T_s} (v_{ob,ref} - v_{ob}) \, dt = 0$$  \hspace{1cm} (13)

The derivatives (14) of these sliding surfaces can be
obtained directly from (13), expressed as a function of
the output voltage errors [17].

$$\dot{S}(e_a, t) = k_o (v_{oa,ref} - v_{oa})$$
$$\dot{S}(e_b, t) = k_o (v_{ob,ref} - v_{ob})$$  \hspace{1cm} (14)

In order to guarantee that the system slides along the
surfaces (13), it is necessary to guarantee that the state
trajectories near the sliding surfaces verify the stability
conditions (15) [17].

$$S(e_a, t) \dot{S}(e_a, t) < 0$$
$$S(e_b, t) \dot{S}(e_b, t) < 0$$  \hspace{1cm} (15)

Based on the stability condition (15), the criteria used
to choose the state space vectors are:
a) If $S/e_a > 0$ then $\dot{S}(e_a) < 0$. Therefore, it
should be chosen a vector capable of increasing the
output voltage $v_{oa}$.
b) If $S/e_a < 0$ then $\dot{S}(e_a) > 0$. Therefore, it
should be chosen a vector capable of decreasing the
output voltage $v_{oa}$.
c) If $S/e_a = 0$ it should be chosen a vector which
does not significantly change the output voltage $v_{oa}$
or $v_{ob}$ component.

The same procedure is applied to the $\beta$ component
error, giving a total of nine possible vector choices.
Since a total of 27 vectors are available, the choice of the
most adequate space vectors must be done based not
only on the output voltage controllers but also
considering the input current controllers, obtained in the
following section.

III.2. Design of the Three Phase Input Current
Controller
The control of the input displacement factor considers
that the converter is conservative. If the high frequency
harmonics, as well as the converter losses are neglected,
the input power ($P_{in}$) equals the output power (16).
Input and output powers are written as functions of the
input and output voltages and currents (in the chosen $dq$
rotating frame the input voltage $v_{iq}$ is equal to zero).

$$P_{in} = v_{iq}(t)i_{iq}(t) = \frac{1}{3} \left( \frac{\sqrt{3}}{2} v_{oa}(t) + \frac{1}{2} v_{ob}(t) \right) i_{iq}(t) +$$
$$+ \frac{1}{3} \left( -\frac{1}{2} v_{oa}(t) + \frac{\sqrt{3}}{2} v_{ob}(t) \right) i_{iq}(t)$$  \hspace{1cm} (16)

According to (16), the choice of one output voltage
vector automatically defines the instantaneous value of
the input \(i_d(t)\) current. This represents a clear advantage of using this coordinate system, instead of the \(\alpha\beta\) coordinate system. As a result, in \(dq\) coordinates, the input displacement factor should be controlled acting directly on the input \(i_q(t)\) current, which results in the input reactive power (17) control.

\[
Q_{in} = v_d(t)i_q(t) \tag{17}
\]

To determine the sliding surface associated to the current control, the mains \(i_q(t)\) current and its first derivative (18) are expressed as a function of the system state variables and based on the state space model equations determined in (10) and (11).

\[
\begin{align*}
\theta &= i_q \\
\frac{di_q}{dt} &= -\omega i_q - \frac{1}{3Cr} i_q + \\
&\quad + \left( -\frac{1}{6V_sCr} \omega v_{qd} + \frac{1}{6Cr^2} \omega v_{qd} \right) v_q + \\
&\quad + \frac{1}{2\sqrt{3l}} v_{id} - \frac{1}{2l} v_q + \frac{1}{3Cr} \rho_{qd} i_{qo} + \rho_{iq} i_{qo} + \\
&\quad + \frac{1}{r} \frac{dv_q}{dt} - \frac{1}{3Cr} v_q + \frac{1}{l} v_q
\end{align*} \tag{18}
\]

The first derivative of the input \(i_q(t)\) current (18) depends directly on the control variables \(\rho_{qo}, \rho_{iq}\). Therefore, the sliding surface \(S_q(e_{qo}, t)\) [17] is defined to be one order lower (zero order function), linearly dependent on the input current error \(e_q = i_{qref} - i_q\) (system order reduction) and is calculated according to (19).

As the sliding surface (19) [17] is defined as a linear combination of the input current error (a Routh-Hurwitz polynomial), and the system (18) is in the canonical form, it guarantees system stability. Also, as (19) is not dependent on the filter parameters, it ensures robustness.

\[
S_q(e_{qo}, t) = k_q \left[ i_{qref} - i_q \right] \tag{19}
\]

The sliding function (19) for a switched converter cannot equal zero for all values of time \(t\). However, if the stability condition (20) is verified, the system will reach sliding mode and the input \(i_q(t)\) current will track its reference within a certain current ripple:

\[
S_q(e_{qo}, t) S_q(e_{qo}, t) < 0 \tag{20}
\]

The criteria used to choose the state space vectors are established based on the stability condition (20) and can be stated as follows:

a) If \(S_q(e_{qo}, t) > 0\), a vector should be chosen, capable of guaranteeing that the sliding surface value will decrease (imposing a negative sliding surface derivative). This will be accomplished if the chosen vector presents \(i_q > 0\), and is capable of decreasing the \(i_q\) current component.

b) If \(S_q(e_{qo}, t) < 0\), it should be chosen a vector capable of guaranteeing that the sliding surface value will increase (imposing a positive sliding surface derivative). This will be accomplished if the chosen vector presents \(i_q < 0\), and is capable of decreasing the \(i_q\) current component.

### III.3. State-Space Vector Selection

The state-space vectors have to be chosen in order to verify the sliding mode equations (13) (15) and (19) (20).

According to the criteria established to choose the adequate output voltages state-space vectors, the sliding mode is reached only when the vectors applied to the converter have the desired amplitude and direction:

a) The vectors of group I (Table 1) have high fixed amplitude and should be able to guarantee the stability condition. However, as they rotate in the \(\alpha\beta\) plane, they may not have the correct direction when necessary (there are only six vectors). Besides, they are not easy to locate as it will be necessary to consider at least 12 sectors for the mains voltages (Fig. 2), in order to know their approximate location.

b) The vectors of group II have variable amplitude and may not always guarantee the stability condition (15). For that reason, at each time instant, only the twelve highest amplitude voltage vectors, which must be able to guarantee the stability condition (15), should be chosen to control the output variables.

c) The null vectors of group III guarantee the stability condition when both sliding surface values are nearly zero, \(S_d(e_{qo})=0\) and \(S_q(e_{qo})=0\).

As a result of condition a), in order to simplify the control system, the six vectors of group I will not be used. The remaining 15 vectors (the twelve highest amplitude vectors of group II and the three null vectors of group III) are always in seven different locations in the \(\alpha\beta\) plane (Fig. 2) and should be chosen according to the previously defined criteria.
Fig. 2. a) Input voltages and their corresponding sector; b) Output voltage state-space vectors represented according to the input voltages location: six rotating vectors \{1g, 2g, 3g, 4g, 5g, 6g\} eighteen amplitude varying vectors \{±1, ±2, ±3, ±4, ±5, ±6, ±7, ±8, ±9\} and three coincident zero vectors \(Z = za, zb\ \text{or} \ zc\).

Fig. 3. a) Output currents and their corresponding sector; b) Input current state-space vectors represented according to the output currents location: eighteen amplitude varying vectors \{±1, ±2, ±3, ±4, ±5, ±6, ±7, ±8, ±9\} and three coincident zero vectors \(Z = za, zb\ \text{or} \ zc\). The \(dq\) axis is represented considering that the input voltages are located in sector \(V_i\) (the grey zone is the \(d\) axis location).
Therefore, if two comparators with three levels each ( \( C_d(t) \in \{-1,0,1\} \), \( C_p(t) \in \{-1,0,1\} \)) are used to quantize the deviations of (13) from zero, nine output voltage error combinations are obtained, allowing the choice of only 9 of the 15 available vectors.

The redundancies that arise from the existence of these 15 vectors and only nine possible error combinations will be used to control the input power factor. In fact, if a two level comparator \( C_i(t) \) is used to quantize the deviations of the sliding function \( S_{i_q}(e_{i_q}, t) \) result (19) from zero, eighteen error combinations (nine output voltage error combinations and two input current error combinations, \( 9 \times 2 = 18 \)) (table 2), will be defined, enabling the selection of the previously referred 15 vectors.

To choose the adequate input current vector, in the chosen dq frame, it is further necessary:

a) To know the location of the output currents, as the input currents depend on the output currents location (table I, Fig. 3);

b) To synchronize the dq frame with the input voltages.

As in the chosen frame (synchronous with \( v_{i_u} \) input voltage) the dq axis location (Fig. 3) depends on the \( v_{i_u} \) input voltage location, the sign of the input current vector \( i_{i_q} \) component can be determined knowing the location of the input voltages and the location of the output currents (Fig. 3).

Analyzing Fig. 3, it can be concluded that the redundant vectors chosen to control the output voltages have similar effect on the \( i_{i_q} \) (t) current, but opposite effects on the \( i_{i_d} \) (t) current, as expected.

As an example, if the output voltage errors are lower than zero, \( C_d(t) = -1 \) and \( C_p(t) = -1 \), it should be chosen a vector capable of decreasing the output \( v_{o_a} \) and \( v_{o_p} \) voltages and, in order to guarantee the stability condition (15), that vector should have high negative \( v_{o_a} \) and \( v_{o_p} \) components. Supposing that the input voltages are located in sector \( V_i \) 2 (Fig. 2. a, b), one of the vectors \(-3 \) or \(-2 \) should be chosen to control the output voltages. According to Fig. 3, both these vectors have \( i_{i_q} \) current components with the same sign but \( i_{i_d} \) current components with opposite sign. As a consequence, if the output currents are located in sectors \( I_i \) 12, 1 (Fig. 3. a, b) and the input current error is \( C_i(t) = 1 \), vector \(+3 \) should be chosen. However, for the same combination of output voltage and input current errors and input voltages located in sector \( V_i \) 2, 3 (Fig. 3. a, b) and output currents located in sector \( I_i \) 6 (Fig. 3. a, b), the vector \(-2 \) should be chosen instead (table II).

For another combination of output voltage and input current errors, if \( C_d(t) = 1 \) and \( C_p(t) = 0 \), the chosen output voltage space vector should increase the output \( v_{o_a} \) voltage and keep the output \( v_{o_p} \) voltage nearly unchanged. This means that, to guarantee the stability condition (15), that vector should have a high positive \( v_{o_a} \) component and a nearly zero \( v_{o_p} \) component - the chosen vector should be on the \( \alpha \) axis. As these constraints are not verified by any of the available vectors, it should be chosen one of the nearest vectors. For example, if the input voltages are located in sectors \( V_i \) 2, 3 (Fig. 2. a, b) one of the vectors \(-3 \) or \(-5 \), \(+3 \) should be chosen. If the output currents are located in sectors \( I_i \) 12, 1 (Fig. 3. a, b) and the input current error is \( C_i(t) = 1 \), vector \(+2 \) or \(-5 \) should be chosen. However, for the same combination of output voltage and input current errors and input voltages located in sector \( V_i \) 2, 3 (Fig. 3. a, b) but output currents located in sector \( I_i \) 6, 7 (Fig. 3. a, b), the vectors \(-3 \) or \(+6 \) should be chosen instead (table II).

When both the output voltage errors are zero \( C_d(t) = 0 \) and \( C_p(t) = 0 \), the chosen output voltage space vector should keep the output \( v_{o_a} \) and \( v_{o_p} \) voltages nearly unchanged. This means that, to guarantee the stability condition (15), that vector should have nearly zero \( v_{o_a} \) and \( v_{o_p} \) components. In this case, one of the three zero vectors should be chosen, keeping the input \( i_{i_q} \) current nearly unchanged.

With this reasoning, it is possible to obtain the tables for all the input voltages and output currents location.

### Table II: State-space vectors selection, for input voltages located at sectors \( V_i \) 2 and \( V_i \) 3.

<table>
<thead>
<tr>
<th>( I_i )</th>
<th>12,1</th>
<th>12,3</th>
<th>4,5</th>
<th>6,7</th>
<th>8,9</th>
<th>10,11</th>
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<tr>
<td>( C_d(t) )</td>
<td>( C_p(t) )</td>
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The choice of the most adequate state-space vectors, when a direct control approach is used, is not unique. For that reason, the main advantages of this sliding mode based direct control strategy are: a) the use of simple control functions, sliding surfaces (13) and (19), that are not dependent on the system parameters, thus ensuring robustness and guaranteeing that the input currents displacement factor does not depend on the input filter parameters; b) the choice of the most adequate space vectors, that does not impose special conditions.
requirements on the control system processing capability. Other approaches have been proposed, at the expense or an increased computation complexity [15], based on the minimization of the input and output weighted errors and using different control techniques (PD controllers) to control the input ac currents.

IV. Simulation and Experimental Results

In order to evaluate the performance of the direct controlled system, some tests were done, under different operating conditions, and the simulation and experimental results, obtained using the proposed method, were compared with the well known Venturini and Space Vector Modulation strategies.

The controlled matrix converter should be able to guarantee that the output variables follow their references and, at the same time, that the input currents have the desired power factor: unity power factor if the matrix converter is used to control ac drives; and unity or leading/lagging power factor if the converter is used in applications related to power quality enhancement.

The matrix converter prototype was built using three modules from DANFOSS, each one of them with six 1200V 25A IGBTs in a common collector back to back arrangement, and a pair of anti parallel connected diodes. The input filter is \( l=6\,\text{mH}, C=6\,\mu\text{F}, r=25\,\Omega \) and the switching frequency is near 5kHz.

![Image of the circuit](image1)

Fig. 4. Input \( v_1(t) \) voltage and input currents \( i_1(t), i_2(t) \) and \( i_3(t) \). a) Experimental results; b) Simulation results; c) Results obtained using Venturini modulation technique; d) Results obtained using SVM approach.

The steady-state results presented in figures 4 a) b) and 5 show that, using the proposed method, the \( i_2(t) \) input current has a sinusoidal shape and is nearly in phase with the input \( v_1(t) \) voltage. On the contrary, the results obtained using the classic Venturini and SVM modulation methods (Fig. 5 c) d) based on ideal matrix converter models, present a displacement factor between the input \( v_1(t) \) voltage and its corresponding current \( i_2(t) \).

![Image of the circuit](image2)

Fig. 5. Input \( v_1(t) \) voltage and current \( i_2(t) \) using the proposed method (detail of figure 4a).
Also, the controlled matrix converter has a good dynamic response when a step is applied to the \( i_{q} \) current (Fig. 6), guaranteeing its operation with input leading or lagging power factor. As expected, the output current keeps unchanged. This input power factor regulation may have special interest if the controlled matrix converter is used as a Static Var Compensator or as Unified Power Flow Controllers without DC link [18], [19], [20].

Fig. 6. Dynamic response of the direct controlled matrix converter, when a step is applied to the \( i_{q} \) reference current. Input voltage \( v_{a} \), input current \( i_{a} \), input reference current \( i_{arb} \) and output current; a) Experimental results; b) Simulation results.

Fig. 7. Matrix converter output currents: a) Response to a step applied to the output voltages references \( R=3\Omega, L=30mH, E=0 \); b) Response to linear variation in the frequency reference.

The capability of controlling the output voltage amplitude and frequency or phase (Fig. 7 a) b) may also be used in power quality applications or in ac drives control (V/f control).

V. Conclusions

The use of sliding mode controllers in variable structure systems such as matrix converters may present advantages such as easy on-line implementation and a quick and efficient choice of the correct switching combinations, ensuring that the system tracks their references.

The presented results show a good steady-state and dynamic response, allowing the input, as well as the output power factor regulation and the compensation of the displacement factor introduced by the input filter.

This direct control method allows a wide range of output frequencies (0.01Hz to 300Hz) and guarantees a nearly 0.35 leading or lagging input power factor regulation.

References


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