



Database System Concepts

Algoritmos de Update em Árvores B⁺

Departamento de Engenharia Informática
Instituto Superior Técnico

1st Semester
2009/2010



Outline

Database
System
Concepts

Insertion

Deletion

1 Insertion

2 Deletion



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Database
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Insertion

Deletion

1 Insertion

2 Deletion



Procedure *Insert*

```
procedure insert(value  $K$ , pointer  $P$ )
    Find the leaf node  $L$  that should contain  $K$ 
    if  $L$  has less than  $n - 1$  values then
        insert-in-leaf( $L, K, P$ )
    else begin
        Create node  $L'$ 
        Copy  $L.P_1 \dots L.K_{n-1}$  to  $T$  (an auxiliary node)
        insert-in-leaf( $T, K, P$ )
        Set  $L'.P_n = L.P_n$ ; Set  $L.P_n = L'$ 
        Erase  $L.P_1 \dots L.K_{n-1}$  from  $L$ 
        Copy  $T.P_1 \dots T.K_{\lceil n/2 \rceil}$  to  $L.P_1 \dots$ 
        Copy  $T.P_{\lceil n/2 \rceil + 1} \dots T.K_n$  to  $L'.P_1 \dots$ 
        Let  $K'$  be the smallest key in  $L'$ 
        insert-in-parent( $L, K', L'$ )
    end
```



Procedure *Insert-in-leaf*

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```
procedure insert-in-leaf(node  $L$ , value  $K$ , pointer  $P$ )  
  if  $K < L.K_1$  then  
    Insert  $P, K$  into  $L$  before  $L.P_1$   
  else begin  
    Let  $K_i$  be the least value in  $L$  that is less than  $K$   
    Insert  $P, K$  into  $L$  after  $T.K_i$   
  end
```

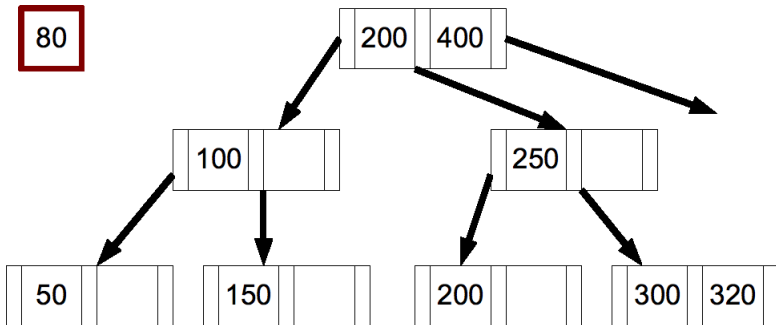


Procedure *Insert-in-parent*

```
procedure insert-in-parent(node  $N$ , value  $K'$ , node  $N'$ )  
  if  $N$  is the root then begin  
    Create a new node  $R = (N, K', N')$   
    Make  $R$  the new root; return  
  end  
  Let  $P = \text{parent}(N)$   
  if  $P$  has less than  $n$  pointers then  
    Insert  $K', N'$  into  $P$ , after  $N$   
  else begin  
    Copy  $P$  to  $T$   
    Insert  $K', N'$  into  $T$ , after  $N$   
    Erase all from  $P$ ; Create node  $P'$   
    Copy  $T.P_1 \dots T.P_{\lceil n/2 \rceil}$  to  $P$   
    Let  $K'' = T.K_{\lceil n/2 \rceil}$   
    Copy  $T.P_{\lceil n/2 \rceil + 1} \dots T.P_{n+1}$  to  $P'$   
    insert-in-parent( $P, K'', P'$ )  
  end
```



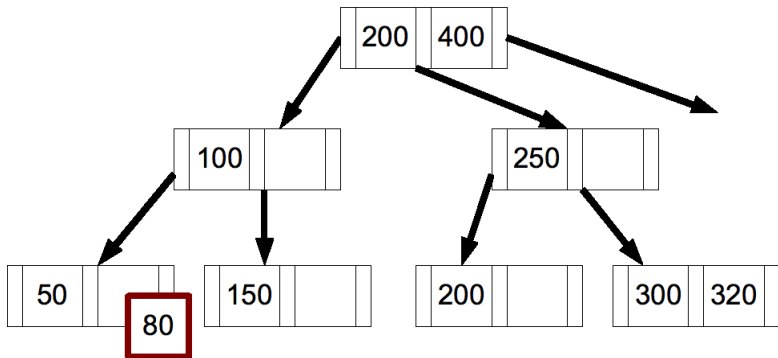
Insertion Example



Find the leaf node L that should contain K



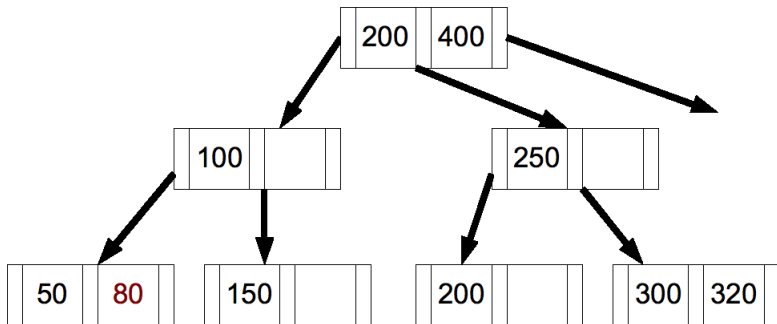
Insertion Example



Find the leaf node L that should contain K



Insertion Example



if L has less than $n - 1$ values **then**
insert-in-leaf(L, K, P)

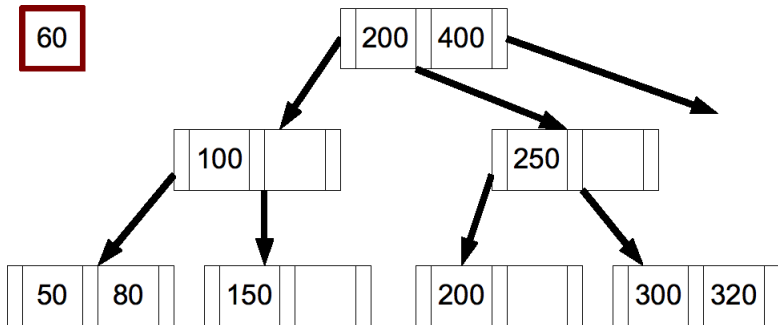


Insertion Example

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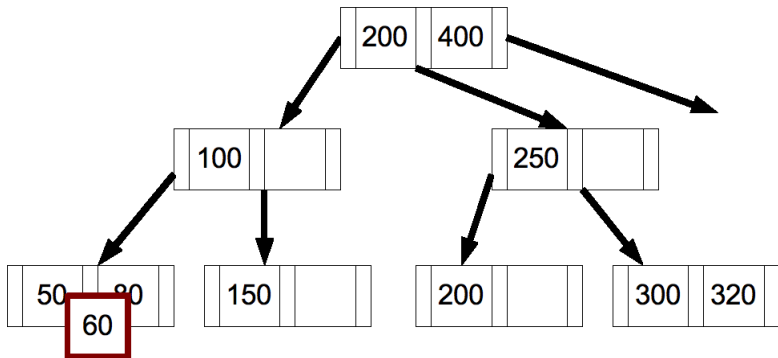
Deletion



Find the leaf node L that should contain K



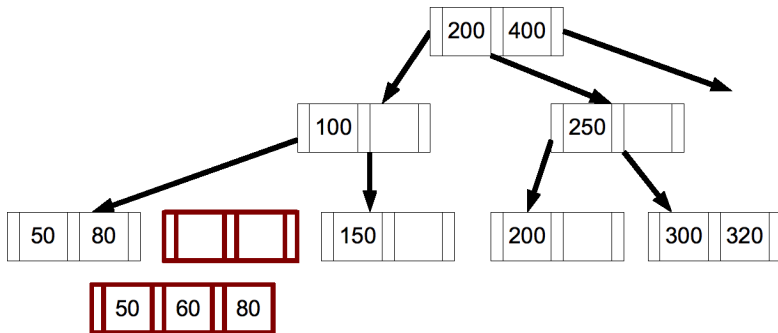
Insertion Example



Find the leaf node L that should contain K



Insertion Example



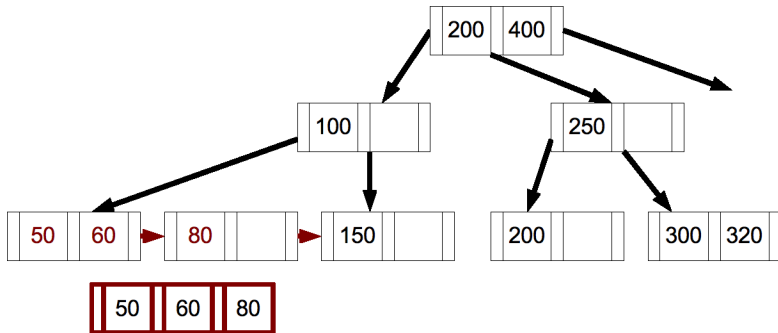
Create node L'

Copy $L.P_1 \dots L.K_{n-1}$ to T (an auxiliary node)

insert-in-leaf(T, K, P)



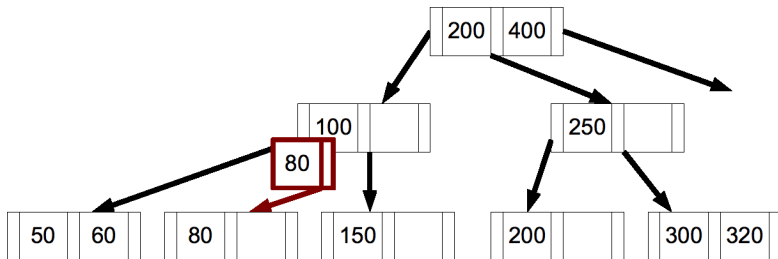
Insertion Example



Set $L'.P_n = L.P_n$; Set $L.P_n = L'$
Erase $L.P_1 \dots L.K_{n-1}$ from L
Copy $T.P_1 \dots T.K_{\lceil n/2 \rceil}$ to $L.P_1 \dots$
Copy $T.P_{\lceil n/2 \rceil + 1} \dots T.K_n$ to $L'.P_1 \dots$



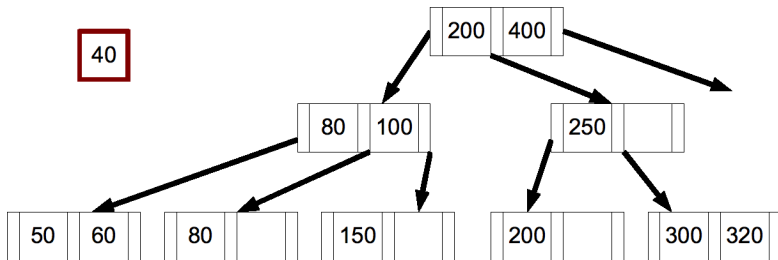
Insertion Example



Let K' be the smallest key in L'
 $\text{insert-in-parent}(L, K', L')$



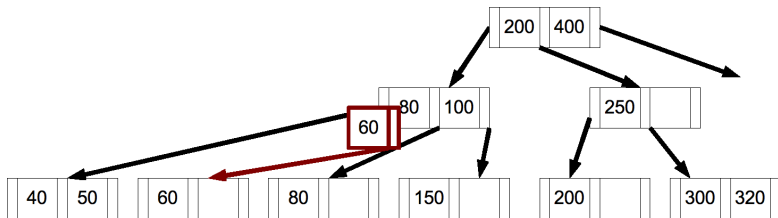
Insertion Example



Find the leaf node L that should contain K



Insertion Example



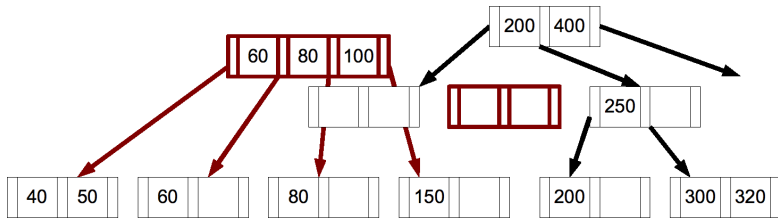
Create node L'

...

$\text{insert-in-parent}(L, K', L')$



Insertion Example



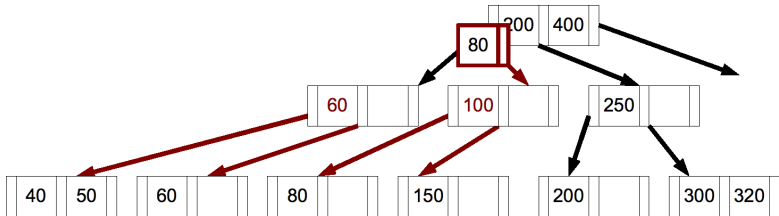
Let $P = \text{parent}(L)$

Copy P to T

Insert K', L' into T , after L

Erase all from P ; Create node P'

Insertion Example



Copy $T.P_1 \dots T.P_{\lceil n/2 \rceil}$ to P

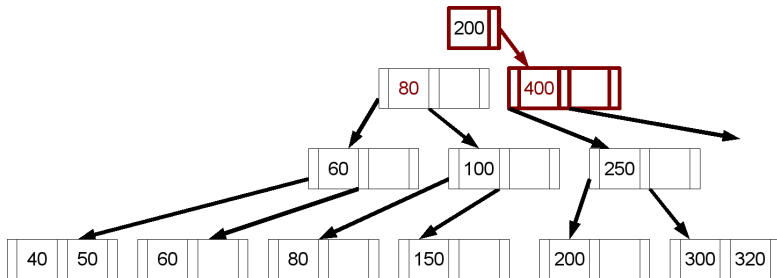
Let $K'' = T.K_{\lceil n/2 \rceil}$

Copy $T.P_{\lceil n/2 \rceil + 1} \dots T.P_{n+1}$ to P'

insert-in-parent(P, K'', P')



Insertion Example



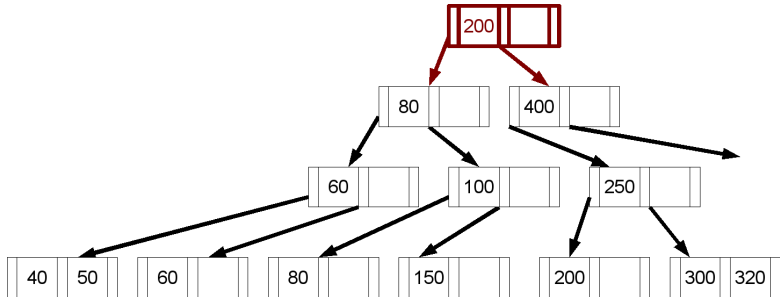
Let $P = \text{parent}(L)$

...

$\text{insert-in-parent}(P, K'', P')$



Insertion Example



if N is the root **then begin**

 Create a new node $R = (N, K', N')$

 Make R the new root; **return**



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Procedure *Delete*

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```
procedure delete(value  $V$ , pointer  $P$ )  
    Find the leaf node  $L$  that contains  $V$ ,  $P$   
    delete-entry( $L$ ,  $V$ ,  $P$ )
```



Procedure *delete-entry*

```
procedure delete-entry(node  $L$ , value  $V$ , pointer  $P$ )
  delete  $V, P$  from  $L$ 
  if  $L$  is root and has only 1 child then begin
    delete  $L$ 
    make the child the new root
  return
end
if  $L$  has too few pointers/values then
begin
  Let  $L'$  be the previous or next child of  $parent(L)$ 
  Let  $V'$  be the value between  $L$  and  $L'$  in  $parent(L)$ 
  if entries in  $L$  and  $L'$  fit in one node then
    coalesce-nodes( $L, V', L'$ )
  else
    redistribute-nodes( $L, V', L'$ )
end
```



Procedure *coalesce-nodes*

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```
procedure coalesce-nodes(node  $L$ , value  $V'$ , node  $L'$ )  
  if  $L$  is a leaf then  
    begin  
      Append all  $(K_i, P_i)$  pairs in  $L$  to  $L'$   
      Set  $L'.P_n$  to  $L.P_n$ , if needed  
    end  
  else  
    Append  $V'$  and all pointers/values in  $L$  to  $L'$   
    delete-entry(parent( $L$ ),  $V'$ ,  $L$ )  
    delete node  $L$ 
```

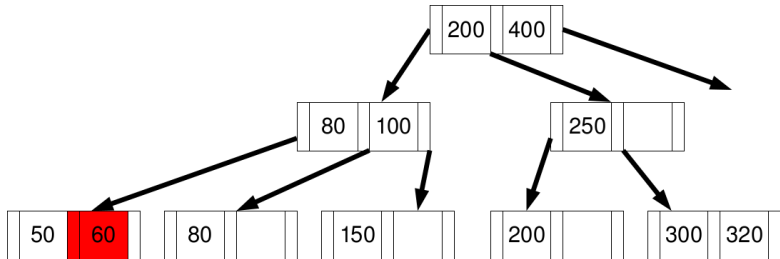



Procedure *redistribute-nodes*

```
procedure redistribute-nodes(node  $L$ , value  $V'$ , node  $L'$ )  
  if  $L'$  is a predecessor of  $L$  then begin  
    if  $L$  is a non-leaf node then begin  
      Let  $L'.P_m$  be the last pointer in  $L'$   
      Remove  $L'.K_{m-1}, L'.P_m$  from  $L'$   
      Insert  $L'.P_m, V'$  as the first pointer/value in  $L$   
      Replace  $V'$  in  $parent(L)$  by  $L'K_{m-1}$   
    end  
    else begin  
      Let  $L'.P_m, L'.K_m$  be the last pointer/value pair in  $L'$   
      Remove  $L'.P_m, L'.K_m$  from  $L'$   
      Insert  $L'.P_m, L'.K_m$  as the first pointer/value in  $L$   
      Replace  $V'$  in  $parent(L)$  by  $L'K_m$   
    end  
  end  
else ... perform the symmetric case ...
```



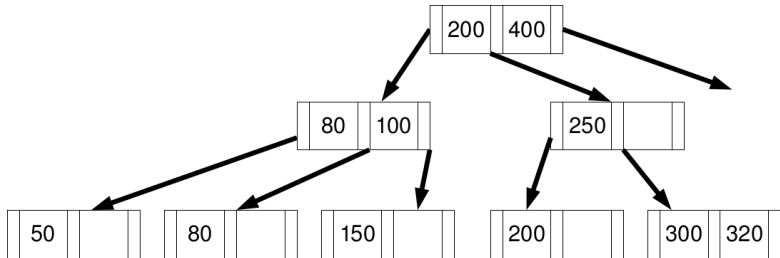
Deletion Example



Find the leaf node L that contains V, P



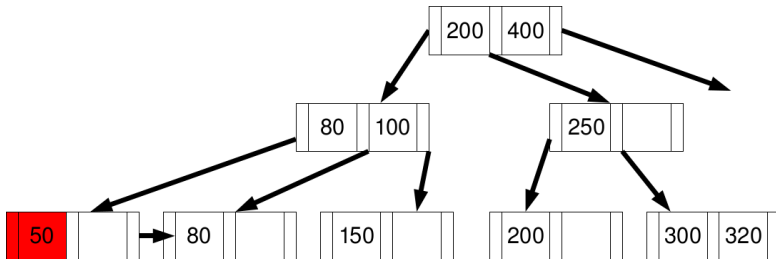
Deletion Example



delete V, P from L



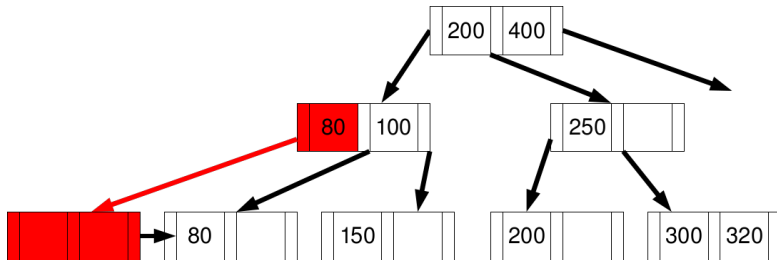
Deletion Example



Find the leaf node L that contains V, P



Deletion Example



delete V, P from L

if L has too few pointers/values **then**

begin

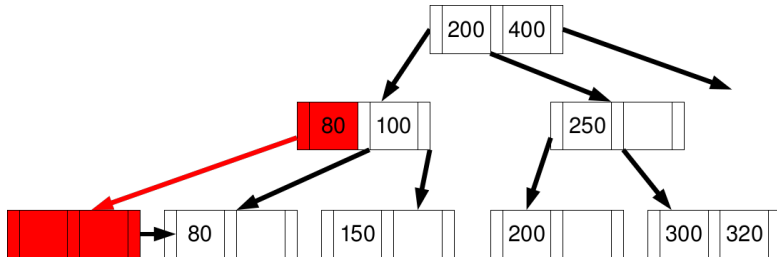
Let L' be the previous or next child of $parent(L)$

Let V' be the value between L and L' in $parent(L)$

if entries in L and L' fit in one node **then**

 coalesce-nodes(L, V', L')

Deletion Example



if L is a leaf **then**

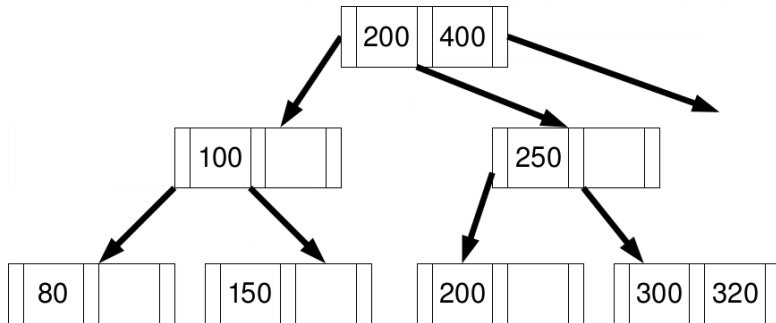
begin

Append all (K_i, P_i) pairs in L to L'

delete-entry($parent(L), V', L$)

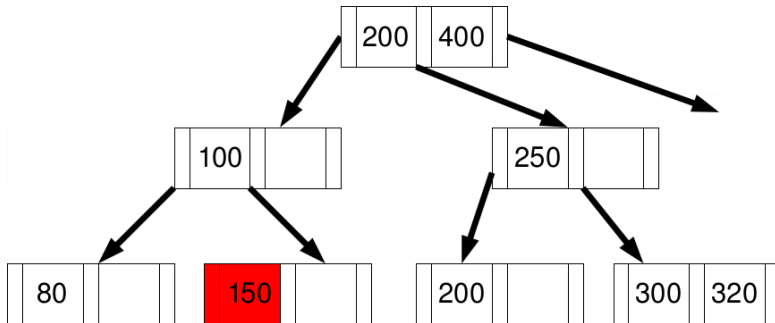


Deletion Example





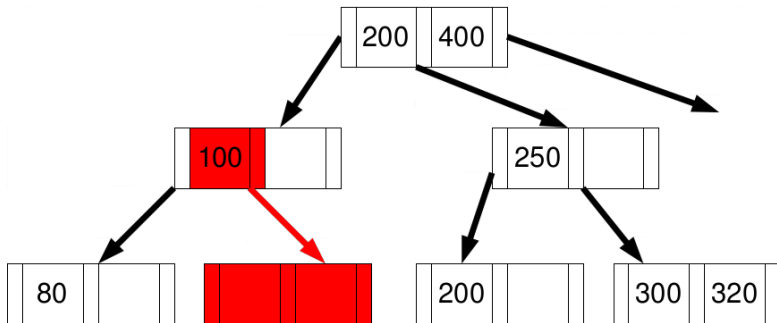
Deletion Example



Find the leaf node L that contains V, P



Deletion Example



...

$\text{coalesce-nodes}(L, V', L')$

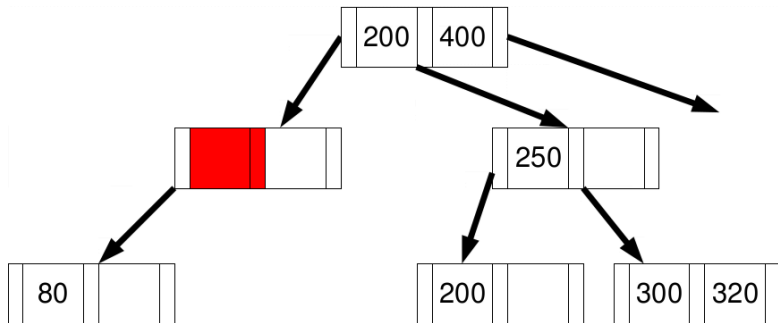
...

$\text{delete-entry}(\text{parent}(L), V', L)$

delete node L



Deletion Example



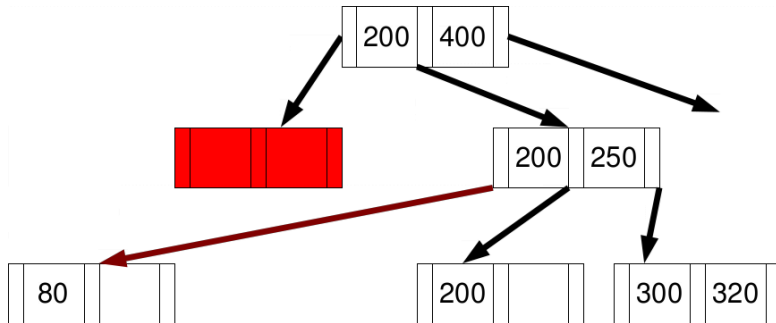
delete V, P from L

...

coalesce-nodes(L, V', L')



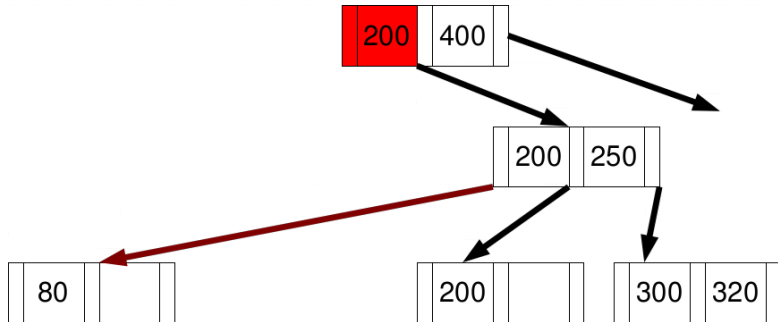
Deletion Example



Append V' and all pointers/values in L to L'



Deletion Example

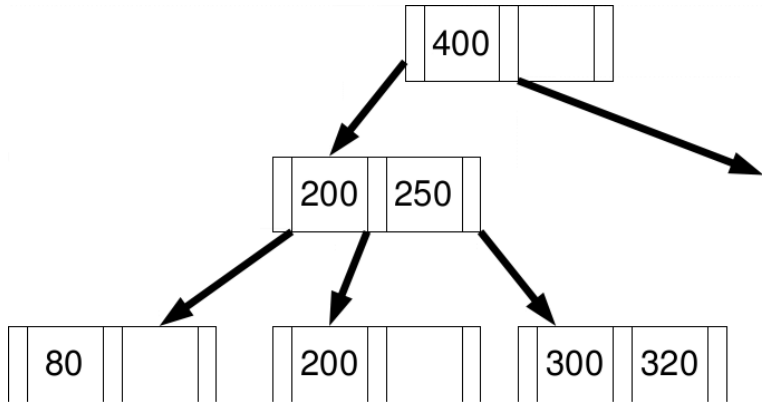


$\text{delete-entry}(\text{parent}(L), V', L)$

delete node L



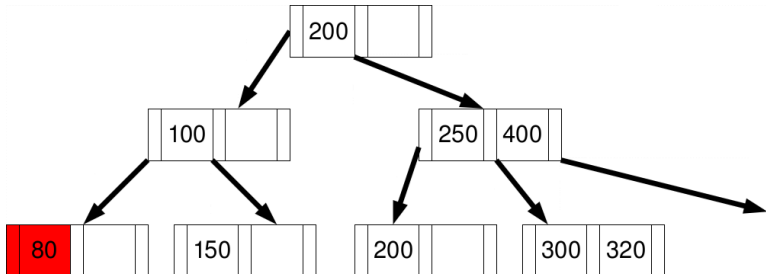
Deletion Example



end



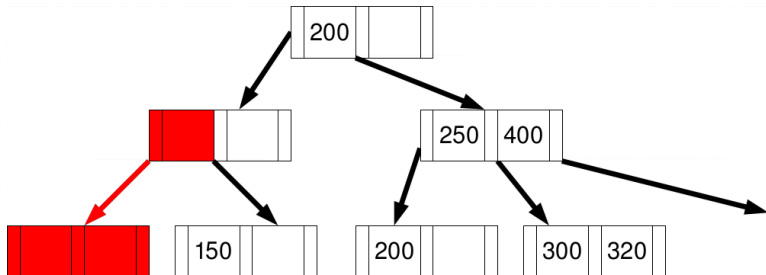
Deletion Example



Find the leaf node L that contains V, P



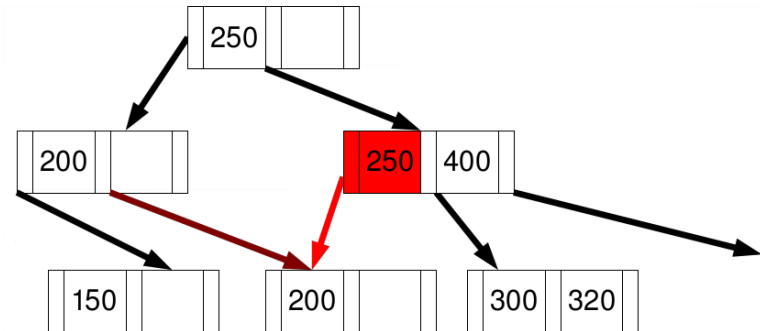
Deletion Example



$\text{redistribute-nodes}(L, V', L')$



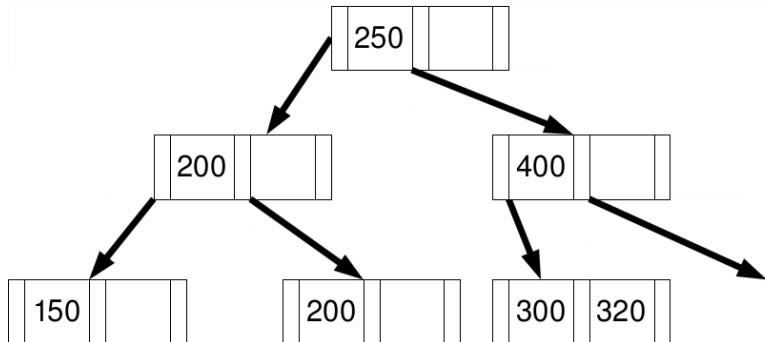
Deletion Example



Let $L'.P_m$ be the first pointer in L'
Insert $V', L'.P_m$ as the last pointer/value in L
Replace V' in $parent(L)$ by $L'.K_{m+1}$



Deletion Example



Remove $L'.K_{m+1}, L'.P_m$ from L'



The End