Implementation of Single and Coupled Microstrip Lines in APLAC

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Abstract

The transmission line model for the single and coupled microstrip line components implemented in APLAC are presented. A detailed description of the algorithm used to calculate the characteristic impedance and the effective permittivity is given for the single microstrip, the symmetrical and asymmetrical pair of coupled microstrip lines, and the n symmetrical coupled microstrip lines. The effects due to shielding, and conductor and dielectric loss are also accounted for.

Indexing terms: microstrip line, symmetrical coupled microstrip lines, asymmetrical pair of coupled microstrip lines, microwave device, transmission line.

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1 Introduction

The objective of this report is to provide a comprehensive documentation on the symmetric and asymmetric coupled microstrip line models implemented in APLAC, an analog circuit simulator [16]. Since the models for the symmetrical and asymmetrical coupled microstrip lines use equations governing the single microstrip line, the models for the single microstrip line, proposed by Hammerstad and Jensen [4], and Jansen and Kirschning [7], are also discussed here. The discussion on the single microstrip does not cover the APLAC microstrip model entirely and excludes the effects of a microstrip short-circuited at one end and the case when a ground-plate is placed alongside the microstrip (coplanar waveguide case). Also, microstrip configurations like the step, gap, bend, taper, T- and X-junctions are excluded from this discussion.

A complete description on the syntax and use of these components is given in [16]. The following discussion describes the closed form empirical equations and numerical solutions used to model the said components. All equations here have been taken from the literature, and no attempt is made to derive them.

The symmetrical and asymmetrical pair of coupled microstrip lines had earlier been implemented in APLAC but did not function as expected. The basic problem was that the coupled microstrip line components in pre-6.25 versions did not show correct behaviour as conductor thickness was increased from the ideal zero-thickness case. As a result, a thorough investigation into the problem was begun which led to the complete reconstruction of the model for the symmetric coupled pair of microstrip lines, and is now based on the model proposed by Kirschning and Jansen [9]. The model for the asymmetric pair of coupled microstrip lines is based on the model proposed by Sellberg [14] and effects due to dispersion in this model are according to Tripathi [15]. The model for symmetric $n$ coupled transmission lines is based on [13]. Some of the changes made, especially those affecting the functions governing the effects due to finite thickness, affect the model for the single microstrip line. All the equations required to construct these models are reproduced in this report.
2 Transmission line model for microstrip components

The physical dimensions of lumped elements may be larger than the wavelength of the propagating microwave voltage and so they cannot be used to model circuit elements in the microwave frequency region that ranges from 300 MHz to 30 GHz. Instead, the transmission line (figure 1), with distributed elements, is used for this purpose.

Dispersion results when the frequency components of a voltage pulse propagating along the transmission line propagate with different phase velocities, so distorting the pulse. Dielectric and conductor losses grow rapidly with increasing frequency so also causing distortion. These phenomena should be accounted for when designing microwave circuits.

This section begins with a brief review of the relations governing the ideal transmission line required to get an understanding of the microstrip component models described later.

2.1 Ideal transmission line

$Z_0$, the characteristic impedance, $\tau$, the propagation delay that depends on the length of the line, and $\alpha$, the attenuation constant, are the parameters used to model the transmission line. The transmission line is considered to consist of a large number of infinitely short segments of lumped elements of length $dz$, with a series impedance

$$Z_s = R + j\omega L,$$

where $R$ and $L$ are the distributed resistance and inductance, respectively, and a shunt admittance

$$Y_p = G + j\omega C,$$

where $G$ and $C$ are the distributed capacitance, respectively, distributed along the entire line. One such segment is shown in figure 1.

A pair of first-order differential equations, called the telegrapher’s or telegraphist’s equations, are used to describe the transmission line mathematically:

$$\begin{align*}
\frac{\partial U(z)}{\partial z} &= -Z_s I(z) \\
\frac{\partial I(z)}{\partial z} &= -Y_p U(z).
\end{align*}$$

(1)

$U$ and $I$ are the voltage and current, respectively, of a differential segment of the transmission line as shown in figure 1(b), and $z$ denotes distance along the line. The
telegrapher’s equations can be combined to give the wave equations
\[
\frac{\partial^2 U(z)}{\partial z^2} = \gamma^2 U(z)
\]
\[
\frac{\partial^2 I(z)}{\partial z^2} = \gamma^2 I(z),
\]
where the propagation constant $\gamma$ (1/m) is
\[
\gamma = \sqrt{Z_s Y_p} = \alpha + j \beta.
\]

The real part $\alpha$ of the propagation constant is called the attenuation constant (Np/m) and the imaginary part $\beta$ (rad/m) is the phase constant, though neither $\alpha$ nor $\beta$ are really constants but are complicated functions of angular frequency $\omega$. The solution of the wave equations can be thought to consist of two waves propagating in opposite directions. The impedance of the line is the ratio of the voltage to the current in a given direction. For an infinitely long line, the impedance is independent of $z$ and is called the characteristic impedance of the line; it can be written as
\[
Z_0 = \frac{Z_s}{\gamma} = \frac{\gamma}{Y_p} = \sqrt{\frac{Z_s}{Y_p}}.
\]

The phase velocity of the propagating wave is obtained from
\[
v = \lambda f = \frac{\omega}{\beta}.
\]

For a lossless line, $R = G = 0$ and so also $\alpha = 0$. This means that
\[
Z_s = j \omega L \quad \quad Y_p = j \omega C
\]
\[
Z_0 = \sqrt{\frac{L}{C}} \quad \text{(real)}
\]
\[
\gamma = j \beta = j \omega \sqrt{LC}
\]
\[ v = \sqrt{\frac{1}{LC}}. \]  

This approximation is reasonable for frequencies of up to about 4 GHz since losses are small in this range.

The *quasistatic approximation*, where the phase velocity \( v = \frac{1}{\sqrt{\mu \varepsilon}} \), is also very useful up to the range of about 4 GHz, beyond which effects due to dispersion have to be accounted for. Here \( \mu \) is the *magnetic permeability* and \( \varepsilon \) the *permittivity* of the medium in which the transmission line conductors are placed.

The microstrip models described below are all frequency-domain models.

## 2.2 Microstrip line

The microstrip line is composed of a thin conducting metal strip of width \( w \), thickness \( t \) and length \( l \) placed on a nonmagnetic dielectric substrate that is in turn placed on a conducting metal ground-plane. The substrate has thickness \( h \) and relative permittivity \( \varepsilon_r \). The structure is shown in figure 2. Above the conducting strip is a second dielectric material, air. In addition, the structure may have a cover plate at a height \( h_2 \) from the substrate surface so introducing effects due to shielding.

### 2.2.1 Static approximation of the characteristic impedance and effective permittivity including effects due to shielding

The microstrip line static model gives accurate results for low frequencies (up to about 4 GHz). It uses physical dimensions of the microstrip line as parameters from which static i.e. zero frequency, impedance and effective permittivity are calculated. The equations here are from [4] and equations governing shielding effects are taken from [11].

The physical dimensions of the microstrip are normalized with respect to the substrate height \( h \). The following symbols are used to denote the normalized dimensions:

\[ u = \frac{w}{h}, \quad t_h = \frac{t}{h}, \quad h_{2h} = \frac{h_2}{h}. \]  

---

Figure 2: Cross-section of a microstrip line.
The model first calculates the impedance of the microstrip line in a homogeneous medium. This means that the substrate material and the material above the conductor are the same. The equations here use air, whose relative permittivity $\varepsilon_r = 1$, as the homogeneous material. The impedance is

$$Z_{0\infty}(0, u) = \frac{\eta_0}{2\pi} \ln \left( \frac{f(u)}{u} + \sqrt{1 + \left(\frac{2}{u}\right)^2} \right),$$

(8)

where

$$f(u) = 6 + (2\pi - 6) \exp \left[ -\left(\frac{30.666}{u}\right)^{0.7528} \right]$$

and $\eta_0 = 376.73$ $\Omega$ is the wave or intrinsic impedance of free space. The subscript 0 indicates a homogeneous air dielectric and the argument 0 refers to zero frequency, i.e. static values. Subscript $\infty$ indicates that the cover height $h_2 = \infty$, and so equation (8) does not include effects due to shielding.

Shielding effects are accounted for by subtracting a correction term $\Delta Z_{0h_2}(0, u)$ from $Z_{0\infty}(0, u)$ [11]. Hence the effective impedance with shielding accounted for is

$$Z_0(0) = Z_{0\infty}(0, u) - \Delta Z_{0h_2}(0, u),$$

(9)

and the correction term is given by

$$\Delta Z_{0h_2}(0, u) = PQ$$

(10)

$$P = 270 \left[ 1 - \tanh \left( 1.192 + 0.706 \sqrt{1 + \frac{h_2}{\eta_0}} - \frac{1.389}{1 + h_2} \right) \right]$$

$$Q = 1.0109 - \text{artanh} \left( \frac{0.012u + 0.177u^2 - 0.027u^3}{(1 + h_2)^2} \right).$$

APLAC is coded in the C programming language which does not have a built-in \text{artanh} function. For convenience the function is defined below:

$$\text{artanh}(x) = \frac{1}{2} \ln \frac{1 + x}{1 - x}. $$

In order to account for effects due to shielding, the development of the concept of the filling factor $q$ was necessary. According to March [11]

$$q = (q_\infty - q_c)q_c,$$

(11)

where $q_c$ is the correction for a finite cover height $h_2$ given by

$$q_c = \tanh \left( 1.043 + 0.121h_2 - \frac{1.164}{h_2} \right),$$

(12)
$q_t$ is the shielding correction due to finite conductor thickness given by

$$q_t = \frac{2 \ln 2}{\pi} \cdot \frac{t_b}{\sqrt{u}}, \quad (13)$$

and $q_\infty$ is the filling factor for an open microstrip (infinite cover height, $h_2 = \infty$) with zero conductor thickness given by

$$q_\infty = \left(1 + \frac{10}{u}\right)^{-a(u) b(\epsilon_r)}, \quad (14)$$

$$a(u) = 1 + \frac{1}{49} \ln \frac{u^4 + \left(\frac{u}{52}\right)^2}{u^4 + 0.432} + \frac{1}{18.7} \ln \left[1 + \left(\frac{u}{18.1}\right)^3\right]$$

$$b(\epsilon_r) = 0.564 \left(\frac{\epsilon_r - 0.9}{\epsilon_r + 3}\right)^{0.053}.$$  

The effective relative permittivity is, thus, given by

$$\epsilon_{\text{eff}}(0, u) = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} q. \quad (15)$$

Again the zero in the argument means zero frequency indicating a static effective relative permittivity.

In order to get the characteristic impedance of the microstrip line placed on a dielectric with relative permittivity $\epsilon_r$ we need to divide the impedance of the microstrip line in a homogeneous material by the square root of the effective relative permittivity, i.e.

$$Z_L(0) = \frac{Z_0(0)}{\sqrt{\epsilon_{\text{eff}}(0, u)}}. \quad (16)$$

The above equations are accurate to within 0.5 percent in the range $\epsilon_r < 60$, $0.01 \leq u \leq 60$ and $h_2 t > 1$. The accuracy of the impedance and effective relative permittivity without shielding is better than 0.2 percent in the range $\epsilon_r \leq 128$ and $0.01 \leq u \leq 100$.

### 2.2.2 Effect of finite strip thickness

The effect of finite conductor thickness $t$ is calculated according to the method proposed in [4]. In the first step, a normalized width correction term accounting for the finite strip thickness is calculated for a homogeneous microstrip structure from

$$\Delta u_{t,1} = \frac{t}{\pi} \ln \left(1 + \frac{4e \tanh^2 \sqrt{6.517u}}{t}\right) \quad (17)$$
from which the resulting effective width is obtained as

\[ u_{t,1} = u + \Delta u_{t,1} \]  

\hspace{1cm} (18)

e in the expression for the correction term is the Neperian base 2.71828. This effective width \( u_{t,1} \) is then used in the place of \( u \) in equation (8) to calculate the resulting impedance \( Z_{0\infty}(0, u_{t,1}) \) of the homogeneous microstrip structure.

In the next step, the effective width for the structure having a substrate with a relative permittivity \( \epsilon_r \) is determined using

\[ u_{t,\epsilon_r} = u + \frac{\Delta u_{t,1}}{2} \left( 1 + \frac{1}{\cosh \sqrt{\frac{1}{\epsilon_r}} - 1} \right) \]  

\hspace{1cm} (19)

The effective relative permittivity of this structure is then determined using the expression

\[ \epsilon_{eff,t}(0) = \epsilon_{eff}(0, u_{t,\epsilon_r}) \left[ \frac{Z_{0\infty}(0, u_{t,1})}{Z_{0\infty}(0, u_{t,\epsilon_r})} \right]^2 \]  

\hspace{1cm} (20)

where \( \epsilon_{eff}(0, u_{t,\epsilon_r}) \) is the effective relative permittivity obtained from equation (15) using \( u_{t,\epsilon_r} \) instead of \( u \), as indicated by the argument \( u_{t,\epsilon_r} \), and \( Z_{0\infty}(0, u_{t,1}) \) and \( Z_{0\infty}(0, u_{t,\epsilon_r}) \) are the impedances similarly obtained from equation (8) using \( u_{t,1} \) and \( u_{t,\epsilon_r} \), respectively.

Finally, the characteristic impedance of the microstrip structure having finite conductor thickness is calculated from

\[ Z_{0\infty,t}(0) = \frac{Z_{0\infty}(0, u_{t,\epsilon_r})}{\sqrt{\epsilon_{eff,t}(0)}} \]  

\hspace{1cm} (21)

### 2.2.3 Dispersion

Both the characteristic impedance and the effective permittivity of the microstrip line change with frequency due to dispersion. The effective relative permittivity increases with frequency and asymptotically approaches \( \epsilon_r \). Dispersion causes a small increase in the characteristic impedance of the microstrip line for high frequencies. The effects due to dispersion should be taken into account for high frequencies (above about 4 GHz). In the equations below \( \epsilon_{eff}(0) \) is either \( \epsilon_{eff}(0, u) \) or \( \epsilon_{eff,t}(0) \), depending on whether the effect of finite strip thickness is ignored or not.

The dispersion model proposed in [4] is used in **LEVEL 1** calculations in APLAC. The effective permittivity is calculated from

\[ \epsilon_{eff}(f) = \epsilon_r - \frac{\epsilon_r - \epsilon_{eff}(0)}{1 + G \left( \frac{f}{f_p} \right)^2} \]  

\hspace{1cm} (22)

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\( f_p \) is an approximation of the cut-off frequency of the first *transverse electromagnetic* (TEM) mode and is given by

\[
f_p = \frac{Z_L(0)}{2\mu_0 h},
\]

where \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \) is the permeability of vacuum, and \( G \) is an empirically determined factor that is sufficiently accurate for all substrates in use, given by

\[
G = \frac{\pi^2}{12} \frac{\epsilon_r - 1}{\epsilon_{\text{eff}}(0)} \sqrt{\frac{2\pi Z_L(0)}{\eta_0}}.
\]

Argument \( f \) indicates a frequency dependence.

The LEVEL 2 dispersion model is based on [7], and is a sequence of expressions that are easily programmable. The expressions for the effective permittivity are [6]

\[
\epsilon_{\text{eff}}(f) = \epsilon_r - \epsilon_r - \epsilon_{\text{eff}}(0) \frac{1}{1 + P(f)}
\]

(23)

\[
P(f) = P_1 P_2 [(0.1844 + P_3 P_4) f_n]^{1.5763}
\]

\[
P_1 = 0.27488 + u \left[ 0.6315 + \frac{0.525}{(1 + 0.0157 f_n)^{20}} \right] - 0.065683 \cdot \exp(-8.7513 u)
\]

\[
P_2 = 0.33622 \left[ 1 - \exp(-0.03442 \epsilon_r) \right]
\]

\[
P_3 = 0.0363 \cdot \exp(-4.6u) \left\{ 1 - \exp \left[ - \left( \frac{f_n}{38.7} \right)^{4.97} \right] \right\}
\]

\[
P_4 = 1 + 2.751 \left\{ 1 - \exp \left[ - \left( \frac{\epsilon_r}{15.916} \right)^8 \right] \right\},
\]

where \( f_n \text{ GHz-mm} \) is the frequency normalized with respect to the substrate height,

\[
f_n = \frac{f h}{10^6}.
\]

The effective characteristic impedance due to dispersion is [7]

\[
Z_L(f) = Z_L(0) \left( \frac{R_{13}}{R_{14}} \right) R_{17}
\]

(24)

\[
R_1 = 0.03891 \epsilon_r^{1.4}
\]

\[
R_2 = 0.267 u^7
\]

\[
R_3 = 4.766 \cdot \exp \left( -3.228 u^{0.641} \right)
\]
\[ R_4 = 0.016 + (0.0514 \epsilon_r)^{1.524} \]

\[ R_5 = \left( \frac{f_n}{28.843} \right)^{12} \]

\[ R_6 = 22.2u^{1.92} \]

\[ R_7 = 1.206 - 0.3144 \cdot \exp(-R_4) \left[ 1 - \exp(-R_2) \right] \]

\[ R_8 = 1 + 1.1275 \left\{ 1 - \exp \left[ -0.004625 R_3 \epsilon_r^{1.674} \left( \frac{f_n}{18.365} \right)^{2.745} \right] \right\} \]

\[ R_9 = 5.086 R_4 \frac{R_5}{0.3838 + 0.386 R_4} \frac{\exp(-R_6)}{1 + 1.2992 R_5} \frac{(\epsilon_r - 1)^6}{1 + 10(\epsilon_r - 1)^6} \]

\[ R_{10} = 0.00044 \epsilon_r^{2.136} + 0.0184 \]

\[ R_{11} = \left( \frac{f_n}{19.47} \right)^6 \frac{1}{1 + 0.0962 \left( \frac{f_n}{19.47} \right)^6} \]

\[ R_{12} = \frac{1}{1 + 0.00245 u^2} \]

\[ R_{13} = 0.9408 \epsilon_{\text{eff}}(f_n)^{R_8} - 0.9603 \]

\[ R_{14} = (0.9408 - R_9) \epsilon_{\text{eff}}(0)^{R_8} - 0.9603 \]

\[ R_{15} = 0.707 R_{10} \left( \frac{f_n}{12.3} \right)^{1.097} \]

\[ R_{16} = 1 + 0.0503 \epsilon_r^{2} R_{11} \left\{ 1 - \exp \left[ - \left( \frac{u}{15} \right)^6 \right] \right\} \]

\[ R_{17} = R_7 \left[ 1 - 1.1241 \frac{R_{12}}{R_{16}} \exp \left( -0.026 f_n^{1.15656} - R_{15} \right) \right]. \]

In the above expressions the terms \( R_1, R_2 \) and \( R_8 \) should be restricted to numerical values less than or equal to 20 in order to prevent problems due to overflow in the computer.
2.2.4 Attenuation loss

Losses due to dissipation in a microstrip structure are made up of dielectric losses and conductor losses. The total loss or attenuation is given by the attenuation coefficient in dB/m by

\[ \alpha = \alpha_c + \alpha_d, \]  

(25)

where \( \alpha_c \) and \( \alpha_d \) are the attenuation coefficients due to conductor and dielectric losses respectively.

The dielectric loss in dB/m is given by [2]

\[ \alpha_d = \frac{20\pi f}{\ln 10 c_0} \frac{\epsilon_r \epsilon_{\text{eff}}(0) - 1}{\epsilon_r - 1} \tan \delta_d, \]  

(26)

where \( c_0 \) is the velocity of light, \( \tan \delta_d \) is the loss tangent of the dielectric material and \( \epsilon_{\text{eff}}(0) \) is calculated from equation (15). In APLAC the loss tangent \( \tan \delta_d \) is given by the user using the identifier \text{TAND} [16].

Conductor loss in dB/m is derived from [4] and is given by

\[ \alpha_c = \frac{20\pi f}{\ln 10 c_0} \frac{\sqrt{\epsilon_{\text{eff}}(0)}}{Q_c}. \]  

(27)

The factor \( Q_c \) is the strip inductive quality factor and is approximated by

\[ Q_c = \frac{\pi Z_0(0) hf u}{R_s c_0 K}, \]  

(28)

where \( Z_0(0) \) is the static impedance of the microstrip line in a homogeneous medium from equation (8), \( R_s \) is the surface resistance (due to the skin effect) and \( K \) is the current distribution factor. \( R_s \) is an increasing function of surface roughness \( \Delta \),

\[ R_s(\Delta) = R_s(0) \left\{ 1 + \frac{2}{\pi} \arctan \left[ 1.4 \left( \frac{\Delta}{\delta} \right)^2 \right] \right\}, \]  

(29)

\( R_s(0) \) is the skin resistance for a smooth surface given by

\[ R_s(0) = \frac{1}{\sigma \delta}, \]  

(30)

and \( \sigma \) and \( \delta \) are the conductivity and skin depth, respectively, of the conductor. The argument zero in equation (30) indicates a smooth surface, \( \Delta = 0 \), and not frequency as before. The skin depth, defined as the depth below the conductor surface at which the current density decreases to \( 1/e \) of its maximum value at the conductor surface, is obtained from

\[ \delta = \frac{1}{\sqrt{\pi \mu_0 f \sigma}}. \]  

(31)
The current distribution factor $K$ is

$$K = \exp \left[ -1.2 \left( \frac{Z_0(0)}{\eta_0} \right)^{0.7} \right]. \quad (32)$$

The conductor loss ($\alpha_c$) calculations above are valid for a minimum conductor thickness of \( t \approx 3\delta \).

The dielectric loss $\alpha_d$ is usually very small compared to the conductor loss $\alpha_c$.

### 2.2.5 Effective parameters

The phase velocity of the wave propagating in the microstrip line is

$$v_p = \frac{c_0}{\sqrt{\varepsilon_{\text{eff}}(f)}}, \quad (33)$$

where $c_0$ is the velocity of light. The propagation delay is easily found once the phase velocity is known. The capacitance and inductance per unit length can be obtained by manipulating equations (4) and (6) as

$$C = \frac{1}{v_p Z_L(0)} \quad (34)$$

$$L = \frac{Z_L(0)}{v_p}. \quad (35)$$

For low frequencies the attenuation constant $\alpha$ is given by [5]

$$\alpha = \frac{R}{2Z_L(0)} \text{ Np/m}. \quad (36)$$

Since $\alpha$ is known in dB/m, the series resistance per unit length is obtained as

$$R = \frac{\ln 10}{10} \alpha_c Z_L(0). \quad (36)$$

Similarly, the shunt conductance per unit length is obtained from

$$G = \frac{\ln 10}{10} \frac{\alpha_d}{Z_L(0)}. \quad (37)$$
2.2.6 Open-ended microstrip line

The open-ended microstrip line is modeled by a small extension $\Delta l$ in the length of the strip that is added to the physical length. Two models have been implemented in APLAC. The model proposed by Hammerstad [3] is used in LEVEL 1 simulation and the more accurate and elaborate model proposed by Kirschning, Jansen and Koster [10] [5] in LEVEL 2.

Hammerstad’s model is given by

$$\frac{\Delta l}{H} = 0.102 \frac{u + 0.106}{u + 0.264} \left\{ 1.166 + \frac{1}{\epsilon_r} \left[ 0.9 + \ln (u + 2.475) \right] \right\}.$$  \hspace{1cm} (38)

Kirschning et al. have presented a sequence of equations to model the extension in length. The equations are

$$\frac{\Delta l}{H} = ACE/D$$  \hspace{1cm} (39)

$$A = 0.434907 \frac{(\epsilon_{eff}(0)^{0.81} + 0.26) (u^{0.8544} + 0.236)}{(\epsilon_{eff}(0)^{0.81} + 1.89) (u^{0.8544} + 0.87)}$$

$$B = 1 + \frac{u^{0.371}}{2.358 \epsilon_r + 1}$$

$$C = 1 + \frac{0.5274}{\epsilon_{eff}(0)^{0.9236}} \arctan \left( 0.084u^{1.9413/B} \right)$$

$$D = 1 + 0.0377 \{ 6 - 5 \cdot \exp \left[ 0.036(1 - \epsilon_r) \right] \} + \arctan \left( 0.067u^{1.456} \right)$$

$$E = 1 - 0.218 \cdot \exp(-7.5u).$$

2.3 Coupled transmission lines

The model for $n$ coupled lossless transmission lines shown in figure 3 consists of $n$ uncoupled lines and coupling transformation network banks at the input and the output ports. It is based on the assumption that the TEM mode is the only mode of propagation in the lines. The voltages and currents on a lossless $n$-line system are given by the generalized telegrapher’s equations [13]

$$\begin{bmatrix} \varepsilon(z,t) \\ \iota(z,t) \end{bmatrix} = - \begin{bmatrix} 0 & L \\ C & 0 \end{bmatrix} \begin{bmatrix} \varepsilon(z,t) \\ \iota(z,t) \end{bmatrix},$$  \hspace{1cm} (40)

where vectors $\varepsilon(z,t)$ and $\iota(z,t)$ denote the voltages and currents, respectively, and $z$ and $t$ denote distance and time, respectively. Superscripts $z$ and $t$ denote differentiation with respect to space and time, respectively. $L$ and $C$ are the inductance
and capacitance matrices whose elements represent self and mutual parameters per unit length of the lines. Matrix $C$ is symmetric and is given by

$$C = \begin{bmatrix} C_{1,1} & -C_{1,2} & \cdots & -C_{1,n} \\ \vdots & \vdots & \cdots & \vdots \\ -C_{n,1} & -C_{n,2} & \cdots & C_{n,n} \end{bmatrix}, \quad (41)$$

where the diagonal

$$C_{i,i} = C_{i,0} + \sum_{j=1, j\neq i}^{n} C_{i,j}, \quad (42)$$

$C_{i,0}$ is the capacitance per unit length of line $i$ with respect to ground and $C_{i,j}$ is the capacitance per unit length between line $i$ and line $j$.

For structures of interest, with either a single or multilayered dielectric medium whose magnetic properties are the same as those of free space,

$$L = L_0 = \mu_0 \varepsilon_0 C_0^{-1}, \quad (43)$$

where $C_0$ is the capacitance matrix of the same set of transmission lines with the dielectric replaced by air. In other words, the inductance matrix $L$ is calculated by determining the capacitance matrix for the set of transmission lines with the dielectric medium replaced by air.
In order to map the voltage vector \( \mathbf{v} \) onto \( \hat{\mathbf{v}} \) and the current vector \( \mathbf{i} \) onto \( \hat{\mathbf{i}} \) the following is done:

\[
\mathbf{v} = M_V \hat{\mathbf{v}}
\]

\[
\mathbf{i} = M_I \hat{\mathbf{i}},
\]

where \( M_V \) and \( M_I \) are the right eigenvector matrices of the \( LC \) and \( CL \) matrices respectively. Due to the structure of the physical problem, matrices \( LC \) and \( CL \) have the same eigenvalues and

\[
M_I^{-1} = M_V^\top,
\]

where superscript \( \top \) means the transpose operation of the matrix. As a result the inductance and capacitance matrices map as follows:

\[
\hat{L} = M_V^{-1} L [M_V^{-1}]^\top
\]

\[
\hat{C} = M_V^\top C M_V.
\] (44)

Thus, in order to compute the transformation network and the parameters for the system of \( n \) coupled transmission lines, the matrix of right eigenvectors of matrix \( LC \) (or \( CL \)) needs to be computed. For a non-symmetric matrix, the eigenvector matrix is not easily computable, if at all.

### 2.4 Solution to the eigenvector problem of symmetric \( n \) coupled transmission lines

The discussion proceeds with the case for symmetric coupled transmission lines where it is assumed that identical lines are equally spaced. It is also assumed that only the mutual capacitances between adjacent lines are considered, which results in an \( n \)th order symmetric tridiagonal capacitance matrix

\[
C = \begin{bmatrix}
C_{1,1} & -C_{1,2} & 0 & 0 & \cdots & 0 \\
-C_{1,2} & C_{1,1} & -C_{1,2} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & & \vdots \\
0 & \cdots & 0 & -C_{1,2} & C_{1,1} & -C_{1,2} \\
0 & \cdots & 0 & 0 & -C_{1,2} & C_{1,1}
\end{bmatrix}
\] (45)

The eigenvector matrix of a symmetric matrix is easily computed using the method suggested in [13]. The first step is to compute the eigenvalues of the \( n \)-order matrix \( T \) defined by

\[
T_{ij} = 1 \quad \text{,} \quad |i - j| = 1,
\]

\[
T_{ij} = 0 \quad \text{,} \quad \text{otherwise,}
\]

\[
i, j = 1, \cdots, n
\] (46)
whose characteristic polynomial $\phi(\mu)$ is given by the recursive expression

$$
\phi_k(\mu) = \mu \phi_{k-1}(\mu) - \phi_{k-2}(\mu), \quad k = 2, \ldots, n
$$

(47)

$$
\phi_0(\mu) = 1
$$

$$
\phi_1(\mu) = \mu.
$$

The eigenvalues $\mu$ are the solutions to the equation $\phi_n(\mu) = 0$ and are given by

$$
\mu_i = -2 \cos \frac{i\pi}{n+1}, \quad i = 1, \ldots, n.
$$

(48)

This step is dependent on the number of lines only and so needs to be calculated only once.

The next step is to compute the matrix $M$ of right eigenvectors of $T$ from

$$
M_{i,j} = \frac{\phi_{i-1}[\mu_j(T)]}{\gamma_j}, \quad i, j = 1, \ldots, n
$$

(49)

$$
\gamma_j^2 = \sum_{i=1}^{n} \{\phi_{i-1}[\mu_j(T)]\}^2.
$$

Now with the eigenvector matrix computed, the voltages or currents for each dependent source in figure 4 may be computed from

$$
v_i(z,t) = \sum_{j=1}^{n} M_{i,j} \hat{v}_j(z,t) - \hat{v}_i(z,t)
$$

$$
i_i(z,t) = \sum_{j=1}^{n} M_{i,j} \hat{i}_j(z,t) - \hat{i}_i(z,t),
$$

(50)

where $z = 0$ or $z = l$, $l$ being the length of the transmission line.

Figure 4: Complete model for the $i$th transmission line segment.

The following sections describe methods used to compute the $L$ and $C$ matrices, from which the impedance matrix $\tilde{Z}$ may be computed.
2.5 Impedance of symmetric \( n \) coupled microstrip lines

The symmetric \( n \) coupled microstrip line structure is made up of \( n \) identical metal conductors of width \( w \), thickness \( t \), length \( l \), and with spacing \( s \) between them placed on a nonmagnetic dielectric of height \( h \) that is in turn placed on a metal ground plane.

In addition to the earlier assumptions, it is assumed that the only significant coupling capacitances are between adjacent strips and all other side effects are considered negligible. Hence, only the parameters for a symmetrical pair of coupled microstrip lines need to be calculated. The static even- and odd-mode impedances are first calculated from equations (69) and (70) in section 2.6, from which the impedance matrix for the system of \( n \) coupled lines is then computed. The circuit parameters (per unit length) of the symmetric pair of coupled microstrip lines are shown in figure 5.

\[\hat{L} \quad \hat{L}_M \quad \hat{C} \quad \hat{C}\]

\[\hat{C} \quad \hat{C}\]

Figure 5: Circuit constants per unit length for a pair of symmetric coupled microstrips.

The following expressions for the phase velocities of both the even and odd modes and consequently the equivalent inductances and capacitances of the pair of coupled lines are from [5]. The phase velocity is given by

\[v_{p,m} = \frac{c_0}{\sqrt{\varepsilon_{\text{eff},m}(0)}}, \quad (51)\]

where subscript \( m \) is either \( e \) or \( o \) for the even and odd modes, respectively, and \( c_0 \approx 3 \cdot 10^8 \text{ m/s} \) is the velocity of light. The equivalent strip-to-ground and mutual capacitances of the structure (see figure 5) are calculated from

\[\hat{C} = \frac{1}{v_{p,e}Z_{L,e}(0)}, \quad (52)\]

\[\hat{C}_M = \frac{1}{2} \left( \frac{1}{v_{p,o}Z_{L,o}(0)} - \frac{1}{v_{p,e}Z_{L,e}(0)} \right). \quad (53)\]
$Z_{L,e}(0)$ and $Z_{L,o}(0)$ are the even- and odd-mode static characteristic impedances of a symmetric pair of coupled microstrip lines and their significance is explained in section 2.6.

The capacitance matrix for a symmetric pair of coupled lines is of the form

$$
C = \begin{bmatrix}
\hat{C} + \hat{C}_M & -\hat{C}_M \\
-\hat{C}_M & \hat{C} + \hat{C}_M
\end{bmatrix}.
$$

(54)

For the system of $n$ coupled lines with couplings between adjacent lines only the capacitance matrix is of the form

$$
C = \begin{bmatrix}
\hat{C} + \hat{C}_M & -\hat{C}_M & 0 & 0 & 0 & \ldots & 0 \\
-\hat{C}_M & \hat{C} + 2\hat{C}_M & -\hat{C}_M & 0 & 0 & \ldots & 0 \\
0 & -\hat{C}_M & \hat{C} + 2\hat{C}_M & -\hat{C}_M & 0 & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & -\hat{C}_M & \hat{C} + 2\hat{C}_M & -\hat{C}_M & 0 \\
0 & \cdots & 0 & 0 & -\hat{C}_M & \hat{C} + 2\hat{C}_M & -\hat{C}_M \\
0 & \cdots & 0 & 0 & 0 & -\hat{C}_M & \hat{C} + \hat{C}_M
\end{bmatrix}.
$$

(55)

It has been observed from analysis of practical circuits that more accurate results are obtained when the matrix diagonal is obtained from

$$
\hat{C}_1 = \hat{C} + \frac{2n}{2 + n} \hat{C}_M
$$

(56)

for an $n$ strip structure instead of $\hat{C}_1 = \hat{C} + 2\hat{C}_M$ (see equation (42)). As the number of strips increase the factor approaches the value two, which implies coupling only between two adjacent strips.

The equivalent strip inductance $\hat{L}$ and the mutual inductance $\hat{L}_M$ between the two strip inductances are given by

$$
\hat{L} = \frac{1}{2} \left( \frac{Z_{L,e}(0)}{v_{p,e}} + \frac{Z_{L,o}(0)}{v_{p,o}} \right)
$$

(57)

$$
\hat{L}_M = \frac{1}{2} \left( \frac{Z_{L,e}(0)}{v_{p,e}} - \frac{Z_{L,o}(0)}{v_{p,o}} \right).
$$

(58)

The impedance $\hat{Z}_L$ and the propagation delay $\hat{\tau}$ for each line $i$ are then calculated using the relations

$$
\hat{Z}_L(0) = \sqrt{\frac{\hat{L} + \mu_i(T)\hat{L}_M}{\hat{C}_1 - \mu_i(T)\hat{C}_M}}
$$

(59)

$$
\hat{\tau} = \sqrt{(\hat{L} + \mu_i(T)\hat{L}_M)(\hat{C}_1 - \mu_i(T)\hat{C}_M)}.
$$

(60)
2.6 Symmetric pair of coupled microstrip lines

A symmetric pair of coupled microstrip lines is composed of two parallel microstrip lines of equal width $w$ situated close together with a spacing $s$ and of length $l$ mounted on a nonmagnetic dielectric substrate of thickness $h$ and with a relative permittivity $\epsilon_r$. This structure is, in turn, mounted on a metallized common ground-plane as shown in figure 6. The structure may have a cover plate mounted at a height $h_2$ above the surface of the substrate that affects its effective impedance if sufficiently close.

![Figure 6: Cross-section of a symmetric coupled microstrip structure.](image)

The symmetric coupled microstrip line structure implemented in APLAC is called Mclin, and accepts more than two strips to model a coupled structure [16].

2.6.1 Static parameters including effects due to shielding

The model first calculates the even- and odd-mode effective permittivities and impedances of the structure. Even-mode parameters are obtained by applying a voltage of equal magnitude and phase to both conductors. This produces an electric field that has components only tangential to the axis of symmetry that divides the structure such that one conductor lies on either side of the axis. Therefore, the axis of symmetry can be replaced by a magnetic wall, whose electric field has its normal component equal to zero and magnetic field its tangential component equal to zero ($E_{\text{norm}} = 0, H_{\text{tan}} = 0$). The circuit may thus be divided into two halves, each half having a characteristic impedance $Z_{L,e}$ and effective permittivity $\epsilon_{\text{eff,e}}$. Odd-mode parameters are obtained by applying a voltage of equal magnitude but with opposite phase to the two conductors. This results in an electric field that has only components normal to the axis of symmetry. So, the axis of symmetry may now be replaced by an electric wall with the tangential component of its electric field and normal component of its magnetic field equal to zero ($E_{\text{tan}} = 0, H_{\text{norm}} = 0$). Again the circuit may be divided into two halves, with each half having a characteristic impedance $Z_{L,o}$ and effective permittivity $\epsilon_{\text{eff,o}}$. The wave propagating on the coupled line pair is described by the superposition of the even and odd modes [5].

The expressions that follow describe the effective dielectric constants and the power-current characteristic impedances of a pair of coupled microstrip lines as
shown in figure 6, and are discussed in detail in [9]. The physical dimensions of the structure have again been normalized with respect to the substrate height as

\[ u = \frac{w}{h}, \quad g = \frac{s}{h}, \quad t_h = \frac{t}{h}, \quad h_{2h} = \frac{h_2}{h}. \]  

(61)

The results are accurate to within 1-percent in the range

\[ 0.1 \leq u \leq 10, \quad 0.1 \leq g \leq 10, \quad 1 \leq \varepsilon_r \leq 18, \]  

(62)

where \( \varepsilon_r \) the relative permittivity of the substrate material.

The even-mode static (frequency = 0) effective permittivity is calculated according to Hammerstad and Jensen [4] and is given by

\[ \varepsilon_{\text{eff},e}(0) = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} q, \]  

(63)

where \( q \) is the filling factor from equation (11),

\[ q_\infty = \left( 1 + \frac{10}{\nu} \right) ^{-a_o(u, \varepsilon_r) - b_o(u, \varepsilon_r)} \]  

(64)

\[ \nu = \frac{u(20 + g^2)}{10 + g^2} + g \cdot \exp(-g) \]

\[ a_o(u, \varepsilon_r) = 0.7287 \left( \varepsilon_{\text{eff}}(0) - \frac{\varepsilon_r}{3} \right) ^{0.053} \]

\[ b_o(\varepsilon_r) = 0.564 \left( \frac{\varepsilon_r - 0.9}{\varepsilon_r + 3} \right) ^{0.053} \]

and

\[ q_{e,e} = \tanh \left( 1.626 + 0.107 h_{2h} - \frac{1.733}{\sqrt{h_{2h}}} \right) \]  

for \( h_{2h} \leq 39 \)

\[ q_{e,e} = 1 \]  

for \( h_{2h} > 39 \).

Kirschning and Jansen [9] remodeled Hammerstad and Jensen’s equations for the odd-mode static effective permittivity, so improving accuracy, to get

\[ \varepsilon_{\text{eff},o}(0) = \left( \frac{\varepsilon_r + 1}{2} + a_o(u, \varepsilon_r) - \varepsilon_{\text{eff}}(0) \right) q + \varepsilon_{\text{eff}}(0) \]  

(66)

\[ q_\infty = \exp \left( -c_o \cdot g^{d_o} \right) \]  

(67)

\[ a_o(u, \varepsilon_r) = 0.7287 \left( \varepsilon_{\text{eff}}(0) - \frac{\varepsilon_r + 1}{2} \right) \left[ 1 - \exp(-0.179 u) \right] \]
\[ b_o(\varepsilon_r) = \frac{0.747\varepsilon_r}{0.15 + \varepsilon_r} \]

\[ c_o = b_o(\varepsilon_r) - (b_o(\varepsilon_r) - 0.207) \cdot \exp(-0.414u) \]

\[ d_o = 0.593 + 0.694 \cdot \exp(-0.562u) . \]

The odd-mode shielding correction term is

\[ q_{c,o} = \tanh \left( \frac{9.575}{1 - h_{2h}} - 2.965 + 1.68h_{2h} - 0.311h_{2h}^2 \right) \]

for \( h_{2h} < 7 \)

\[ = 1 \]

for \( h_{2h} \geq 7 . \)

The quantity \( \varepsilon_{\text{eff}}(0) \) is the effective permittivity of a single microstrip of width \( w \) and zero conductor thickness and is calculated using equation (15). The subscripts e and o refer to the even and odd modes, respectively, and the argument 0 implies static parameters. In both, the even- and odd-mode cases, \( q_{t} \) is obtained from equation (13). The correction terms for finite cover height, \( q_{c,e} \) and \( q_{c,o} \), are from [11].

The next step is to calculate the even- and odd-mode characteristic impedances of the coupled microstrip pair. This is done using the equations presented in [9]. For the even mode, the static characteristic impedance is

\[ Z_{L,e}(0) = \frac{Z_L(0)\sqrt{\varepsilon_{\text{eff}}(0)}}{1 - \frac{Z_L(0)}{\eta_0} \sqrt{\varepsilon_{\text{eff}}(0)} Q_4} , \]

where \( Z_L(0) \) and \( \varepsilon_{\text{eff}}(0) \) are the static characteristic impedance and effective permittivity, respectively, of a single microstrip of width \( w \) and zero conductor thickness, \( \eta_0 \) is the intrinsic impedance of free space and \( Q_4 \) is given by

\[ Q_4 = \frac{2Q_1}{Q_2} \exp(-g)u^{Q_3} + (2 - \exp(-g))u^{-Q_3} \]

\[ Q_3 = 0.1975 + \left[ 16.6 + \left( \frac{8.4}{g} \right)^6 \right]^{-0.387} + \frac{1}{241} \ln \left[ \frac{g^{10}}{1 + \left( \frac{g}{3.4} \right)^{10}} \right] \]

\[ Q_2 = 1 + 0.7519g + 0.189g^{2.31} \]

\[ Q_1 = 0.8695u^{0.194} . \]
Similarly the odd-mode characteristic impedance is given by

\[ Z_{L,o}(0) = \frac{Z_L(0) \sqrt{\varepsilon_{\text{eff}}(0)}}{1 - \frac{Z_L(0)}{\eta_0} \sqrt{\varepsilon_{\text{eff}}(0)} Q_{10}} \]

where

\[ Q_{10} = \frac{Q_2 Q_4 - Q_5 \cdot \exp \left( \ln(u) Q_6 u^{-Q_6} \right)}{Q_2} \]

\[ Q_9 = \ln(Q_7) \left( Q_8 + \frac{1}{16.5} \right) \]

\[ Q_8 = \exp \left( -6.5 - 0.95 \ln(g) - \left( \frac{g}{0.15} \right)^5 \right) \]

\[ Q_7 = \frac{10 + 190g^2}{1 + 82.3g^3} \]

\[ Q_6 = 0.2305 + \frac{1}{281.3} \ln \left[ \frac{g^{10}}{1 + \left( \frac{g}{5.8} \right)^{10}} \right] + \frac{1}{5.1} \ln \left( 1 + 0.598g^{1.154} \right) \]

\[ Q_5 = 1.794 + 1.14 \ln \left( 1 + \frac{0.638}{g + 0.517g^{2.43}} \right) \]

Effects due to shielding are, again, accounted for using the expressions in [11].

The correction due to shielding in the even- and odd-mode characteristic impedances of a symmetric pair of coupled microstrip lines in a homogeneous air dielectric (\( \varepsilon_r = 1 \)) is calculated from equation (9). The correction term \( \Delta Z_0(0) \) is calculated from a different set of expressions for the even and odd modes. For the even mode, the correction is

\[ \Delta Z_{0,e}(0) = f_e(u, h_{2h}) \cdot g_e(g, h_{2h}) \]

\[ f_e(u, h_{2h}) = 1 - \text{artanh} \left[ A + (B + Cu)u \right] \]

\[ A = \frac{-4.351}{(1 + h_{2h})^{1.842}} \]

\[ B = \frac{6.639}{(1 + h_{2h})^{1.861}} \]
\[ C = \frac{-2.291}{(1 + h_{2h})^{1.90}} \]
\[ g_o(g, h_{2h}) = 270 \left[ 1 - \tanh \left( D + E \sqrt{1 + h_{2h}} - \frac{F}{1 + h_{2h}} \right) \right] \]
\[ D = \frac{0.747}{\sin \left( \frac{\pi}{2} x \right)} \]
\[ E = 0.725 \sin \left( \frac{\pi}{2} y \right) \]
\[ \log_{10} F = 0.11 - 0.0947 g \]
\[ \log_{10} x = 0.103 g - 0.159 \]
\[ \log_{10} y = 0.0492 g - 0.073 \]

and for the odd mode the correction is
\[ \Delta Z_{0,o}(0) = f_o(u, h_{2h}) \cdot g_o(g, h_{2h}) \]  
\[ f_o(u, h_{2h}) = u^J \]
\[ J = \tanh \left( \frac{(1 + h_{2h})^{1.585}}{6} \right) \]
\[ g_o(g, h_{2h}) = 270 \left[ 1 - \tanh \left( G + K \sqrt{1 + h_{2h}} - \frac{L}{1 + h_{2h}} \right) \right] \]
\[ G = 2.178 - 0.796 g \]
\[ K = \begin{cases} \log_{10}(20.492 g^{0.174}) & g > 0.858 \\ 1.30 & g \leq 0.858 \end{cases} \]
\[ L = \begin{cases} 2.51 g^{-0.462} & g > 0.873 \\ 2.674 & g \leq 0.873 \end{cases} \]

In order to get the corrections for the impedances of the coupled lines on a dielectric substrate with relative permittivity \( \epsilon_r \), the corrections obtained for the air dielectric need to be divided by the square root of the respective effective permittivity, i.e.
\[ \Delta Z_{L,e}(0) = \frac{\Delta Z_{L,e}(0)}{\sqrt{\epsilon_{eff,e}(0)}} \]  
\[ \Delta Z_{L,o}(0) = \frac{\Delta Z_{L,o}(0)}{\sqrt{\epsilon_{eff,o}(0)}} \]

As in section 2.2, the subscript L in the above equations implies a dielectric with relative permittivity \( \epsilon_r \).
2.6.2 Correction due to finite strip thickness

For the coupled microstrip lines the correction required due to finite strip thickness is according to Jansen [6]. The effective widths for the even and odd modes are calculated from

\[ u_{t,e} = u + \Delta u \left[ 1 - \frac{1}{2} \cdot \exp \left( \frac{-0.69\Delta u}{\Delta t} \right) \right] \]
\[ u_{t,o} = u + \Delta u \left[ 1 - \frac{1}{2} \cdot \exp \left( \frac{-0.69\Delta u}{\Delta t} \right) \right] + \Delta t \]  \hspace{1cm} (75)
\[ \Delta t = \frac{t_h}{g\epsilon_r}, \]

where the subscript \( t \) refers to finite strip thickness and subscripts \( e \) and \( o \) refer to the even and odd modes respectively. The above equations are valid for technologically meaningful geometries and \( s > 2t \). \( \Delta u \) in equation (75) is the normalized width correction for the case of a single microstrip.

The normalized width correction of Hammerstad and Jensen for the single strip discussed earlier is not used here as it tends to exaggerate the effect of finite strip thickness [9]. Instead, the width correction for the single strip \( \Delta u \) is taken from [1] and is given by

\[ \Delta u = \frac{1.25}{\pi} t_h \left[ 1 + \ln \left( \frac{\frac{2}{t_h} + \frac{4\pi u - 2}{t_h}}{1 + \exp \left[ -100 \left( u - \frac{1}{2\pi} \right) \right]} \right) \right] \] \hspace{1cm} (76)

The normalized effective even- and odd-mode widths from equation (75) are used to calculate the effective permittivities and impedances from equations (63), (69) and (71), and (66), (70) and (72), respectively.

2.6.3 Dispersion

The dispersion equations by Kirschning and Jansen [9] are elaborate and their accuracy in the range of applicability (equation (62)) is better than 2.5 percent up to a normalized frequency \( f_n = 20 \). The accuracy is better than 1.5 percent for \( \epsilon_r \leq 12.9 \) and \( f_n \leq 15 \). Frequency \( f_n \) (GHz mm) is normalized with respect to substrate thickness as

\[ f_n = \frac{f_h}{10^6}. \] \hspace{1cm} (77)

Experimental evaluation ([18] [12]) shows that this dispersion model is consistent over a wide range of frequency, strip width and dielectric constants.

The even and odd-mode effective permittivities are obtained from

\[ \epsilon_{\text{eff},m}(f) = \epsilon_r - \frac{\epsilon_r - \epsilon_{\text{eff},m}(0)}{1 + F_m(f)}, \] \hspace{1cm} (78)
where the subscript \( m \) refers to mode and may be either \( e \) or \( o \) for the even and odd modes, respectively. As in section 2.2 argument \( f \) implies frequency dependence. For the even mode

\[
F_e(f) = P_1 P_2 \left[ (P_3 P_4 + 0.1844 P_7) f_n \right]^{1.5763}
\]

\[
P_1 = 0.27488 + u \left[ 0.6315 + \frac{0.525}{(1 + 0.0157 f_n)^{20}} \right] - 0.065683 \cdot \exp(-8.7513 u)
\]

\[
P_2 = 0.33622 [1 - \exp(-0.03442 \epsilon_r)]
\]

\[
P_3 = 0.0363 \cdot \exp(-4.6 u) \left\{ 1 - \exp\left[ -\left( \frac{f_n}{38.7} \right)^{4.97} \right] \right\}
\]

\[
P_4 = 1 + 2.751 \left\{ 1 - \exp\left[ -\left( \frac{\epsilon_r}{15.916} \right)^8 \right] \right\}
\]

\[
P_5 = 0.334 \cdot \exp\left[ -3.3 \left( \frac{\epsilon_r}{15} \right)^3 \right] + 0.746
\]

\[
P_6 = P_5 \cdot \exp\left[ -\left( \frac{f_n}{18} \right)^{0.368} \right]
\]

\[
P_7 = 1 + 4.069 P_6 g^{0.479} \cdot \exp\left( -1.347 g^{0.595} - 0.17 g^{2.5} \right)
\]

and for the odd mode

\[
F_o(f) = P_1 P_2 \left[ (P_3 P_4 + 0.1844 f_n P_{15}) \right]^{1.5763}
\]

\[
P_8 = 0.7168 \left( 1 + \frac{1.076}{1 + 0.0576(\epsilon_r - 1)} \right)
\]

\[
P_9 = P_8 - 0.7913 \left\{ 1 - \exp\left[ -\left( \frac{f_n}{20} \right)^{1.424} \right] \right\} \cdot \arctan\left[ 2.481 \left( \frac{\epsilon_r}{8} \right)^{0.946} \right]
\]

\[
P_{10} = 0.242(\epsilon_r - 1)^{0.55}
\]

\[
P_{11} = 0.6366 [\exp(-0.3401 f_n) - 1] \cdot \arctan\left[ 1.263 \left( \frac{\epsilon_r}{3} \right)^{1.629} \right]
\]

\[
P_{12} = P_9 + \frac{1 - P_9}{1 + 1.183 u^{1.376}}
\]

\[
P_{13} = \frac{1.695 P_{10}}{0.414 + 1.605 P_{10}}
\]
\[ P_{14} = 0.8928 + 0.1072 \left\{ 1 - \exp \left[ -0.42 \left( \frac{f_n}{20} \right)^{3.215} \right] \right\} \]
\[ P_{15} = \left| 1 - \frac{0.8928}{P_{14}} (1 + P_{11}) P_{12} \cdot \exp \left( -P_{13} g^{1.092} \right) \right| \cdot \]

A TEM wave propagates in the substrate with a velocity of \( c_0/\sqrt{\epsilon_r} \), where \( c_0 \) is the velocity of propagation of light. The time required for the wave to propagate in the pair of coupled lines in the even- and odd-modes is
\[ \tau_m = \frac{l}{\sqrt{\epsilon_{eff,m} c_0}}. \quad \text{(81)} \]

The even-mode impedance dispersion is obtained from
\[ Z_{L,e}(f) = Z_{L,e}(0) \frac{\left( 0.9408 \epsilon_{eff}(f)^{C_o} - 0.9603 \right)^{Q_o}}{\left((0.9408 - d_e) \epsilon_{eff}(0)^{C_o} - 0.9603\right)^{Q_o}} \quad \text{(82)} \]

\[ C_o = 1 + 1.275 \left\{ 1 - \exp \left[ -0.004625 p_e^{1.674} \left( \frac{f_n}{18.365} \right)^{2.745} \right] \right\} - Q_{12} + Q_{16} - Q_{17} + Q_{18} + Q_{20} \]
\[ d_e = 5.086q_e \frac{r_e}{0.3838 + 0.386q_e} \cdot \frac{\exp(-22.2u^{1.92})}{1 + 1.2992r_e} \cdot \frac{(\epsilon_r - 1)^6}{1 + 10(\epsilon_r - 1)^6} \]
\[ p_e = 4.766 \cdot \exp \left( -3.228u^{0.641} \right) \]
\[ q_e = 0.016 + (0.0514\epsilon_r)^{4.524} \]
\[ r_e = \left( \frac{f_n}{28.843} \right)^{12} \]
\[ Q_{11} = 0.893 \left[ 1 - \frac{0.3}{1 + 0.7(\epsilon_r - 1)} \right] \]
\[ Q_{12} = 2.121 \frac{\left( \frac{f_n}{20} \right)^{4.91}}{1 + Q_{11} \left( \frac{f_n}{20} \right)^{4.91}} \cdot \exp(-2.87g)g^{0.902} \]
\[ Q_{13} = 1 + 0.038 \left( \frac{\epsilon_r}{8} \right)^{5.1} \]
\[ Q_{14} = 1 + 1.203 \left( \frac{\epsilon_r}{15} \right)^4 \frac{4}{1 + \left( \frac{\epsilon_r}{15} \right)^4} \]

\[ Q_{15} = \frac{1.887 \cdot \exp(-1.5g^{0.84}) g^{Q_{14}}}{1 + 0.41 \left( \frac{f_n}{15} \right)^3 \frac{u^{2/Q_{13}}}{0.125 + u^{1.626/Q_{13}}} \} \]

\[ Q_{16} = \left[ 1 + \frac{9}{1 + 0.403(\epsilon_r - 1)^2} \right] Q_{15} \]

\[ Q_{17} = 0.394 \left\{ 1 - \exp \left[ -1.47 \left( \frac{u}{7} \right)^{0.672} \right] \right\} \left\{ 1 - \exp \left[ -4.25 \left( \frac{f_n}{20} \right)^{1.877} \right] \right\} \]

\[ Q_{18} = 0.61 \frac{1 - \exp \left[ -2.13 \left( \frac{u}{8} \right)^{1.593} \right]}{1 + 6.544g^{4.17}} \]

\[ Q_{19} = \frac{0.21g^4}{(1 + 0.18g^{4.9})(1 + 0.1u^2)} \left[ 1 + \left( \frac{f_n}{24} \right)^3 \right] \]

\[ Q_{20} = \left[ 0.09 + \frac{1}{1 + 0.1(\epsilon_r - 1)^{2.7}} \right] Q_{19} \]

\[ Q_{21} = \left| 1 - \frac{42.54g^{0.133} \cdot \exp(-0.812g)u^{2.5}}{1 + 0.033u^{2.5}} \right| \]

Once again, \( \epsilon_{\text{eff}}(f) \) is the effective permittivity of a single microstrip line from equation (15). The quantity \( Q_o \) also refers to the single microstrip and is the exponential term \( R_{17} \) in equation (24). Similarly the odd-mode impedance is

\[ Z_{L,o}(f) = Z_L(f) + \frac{Z_{L,o}(0) \left( \frac{\epsilon_{\text{eff,}o}(f)}{\epsilon_{\text{eff,}o}(0)} \right)^{Q_{22}} - Z_L(f)Q_{23}}{1 + Q_{24} + (0.46g)^{2.2}Q_{25}} \quad (83) \]

\[ Q_{22} = \frac{0.925 \left( \frac{f_n}{Q_{26}} \right)^{1.536}}{1 + 0.3 \left( \frac{f_n}{30} \right)^{1.536}} \]

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\[ Q_{22} = \frac{0.925 \left( \frac{f_n}{Q_{26}} \right)^{1.536}}{1 + 0.3 \left( \frac{f_n}{30} \right)^{1.536}} \]
\[ Q_{23} = 1 + \frac{0.005 f_n Q_{27}}{1 + 0.812 \left( \frac{f_n}{15} \right)^{1.9}} (1 + 0.025 u^2) \]

\[ Q_{24} = 2.506 Q_{28} u^{0.894} \left[ \frac{(1 + 1.3 u) \frac{f_n}{99.25}}{3.575 + u^{0.894}} \right]^{4.29} \]

\[ Q_{25} = \frac{0.3 f_n^2}{10 + f_n^2} \left[ 1 + \frac{2.333 (\epsilon_r - 1)^2}{5 + (\epsilon_r - 1)^2} \right] \]

\[ Q_{26} = 30 - 22.2 \left[ \frac{\left( \frac{\epsilon_r - 1}{13} \right)^{12}}{1 + 3 \left( \frac{\epsilon_r - 1}{13} \right)^{12}} \right] - Q_{29} \]

\[ Q_{27} = 0.4 g^{0.84} \left[ 1 + \frac{2.5 (\epsilon_r - 1)^{1.5}}{5 + (\epsilon_r - 1)^{1.5}} \right] \]

\[ Q_{28} = \frac{0.149 (\epsilon_r - 1)^3}{94.5 + 0.038 (\epsilon_r - 1)^3} \]

\[ Q_{29} = \frac{15.16}{1 + 0.196 (\epsilon_r - 1)^2} \cdot \]

\[ Z_L(f) \] is the frequency-dependent power-current characteristic impedance of a single microstrip with width \( w \) calculated from equation (24).

This rigorous calculation gives the even- and odd-mode impedances from equations (82) and (83) respectively, the effective relative permittivity from equation (78) and propagation delays from equation (81).

2.6.4 Losses

Losses due to dissipation for the coupled pair of microstrip lines are evaluated in the same way as for a single microstrip line. The dielectric loss \( \alpha_d \) and the conductor loss \( \alpha_c \) are calculated separately for the even and odd modes. The necessary equations are rewritten here for convenience.

The dielectric loss in dB/m is calculated according to [2]

\[ \alpha_{d,m} = \frac{20 \pi f}{\ln 10} \frac{\epsilon_r}{c_0 \sqrt{\epsilon_{eff,m}(0)}} \frac{\epsilon_{eff,m}(0) - 1}{\epsilon_r - 1} \tan \delta_d, \quad (84) \]

where the subscript \( m \) is either \( e \) or \( o \) referring either to the even or the odd mode respectively, \( c_0 \) is the velocity of propagation of light and \( \tan \delta_d \) is the loss tangent of the dielectric material.
Conductor loss in dB/m is from [4] and is given by
\[
\alpha_{c,m} = \frac{20 \pi f \sqrt{\epsilon_{\text{eff},m}(0)}}{\ln 10 c_0 Q_c}. \tag{85}
\]

\(Q_c\), the strip inductive quality factor is
\[
Q_c = \frac{\pi Z_{0,m}(0) h f u}{R_s c_0 K}, \tag{86}
\]
where \(Z_{0,m}(0)\) is the static impedance of the microstrip line pair in a homogeneous medium for the even and odd modes and is obtained by multiplying the static even- and odd-mode impedances by the square root of their respective effective permittivities, i.e. \(Z_{0,m}(0) = Z_{L,m}(0) \sqrt{\epsilon_{\text{eff},m}(0)}\). \(R_s\), the surface resistance which is an increasing function of surface roughness \(\Delta\), is
\[
R_s(\Delta) = R_s(0) \left\{ 1 + \frac{2}{\pi} \arctan \left[ 1.4 \left( \frac{\Delta}{\delta} \right)^2 \right] \right\}, \tag{87}
\]
where \(R_s(0)\) is the skin resistance for a smooth surface given by
\[
R_s(0) = \frac{1}{\sigma \delta}, \tag{88}
\]
and \(\sigma\) and \(\delta\) are the conductivity and skin depth, respectively, of the conductor. The skin depth \(\delta\) is obtained from
\[
\delta = \frac{1}{\sqrt{\pi \mu_0 f \sigma}}. \tag{89}
\]
The current distribution factor \(K\) for the two modes is
\[
K_e = K_o = \exp \left[ -1.2 \left( \frac{Z_{L,e}(0) + Z_{L,o}(0)}{2 \mu_0} \right)^{0.7} \right]. \tag{90}
\]

Again, the conductor loss, \(\alpha_c\), calculations above are valid for a minimum conductor thickness \(t \approx 3\delta\).

The total loss is given by the attenuation coefficient in dB/m by
\[
\alpha = \alpha_c + \alpha_d. \tag{91}
\]

2.7 Asymmetric pair of coupled lines

Like the symmetric pair of coupled lines, the asymmetric pair of coupled microstrip line structure is mounted on a metallized common ground-plane and is composed of two parallel microstrip lines situated close together with a spacing \(s\) and of length \(l\) mounted on a nonmagnetic dielectric substrate of thickness \(h\) and with a relative permittivity \(\epsilon_r\). The widths of the two microstrip lines, however, are not equal and are denoted by \(w_1\) and \(w_2\). The asymmetric coupled line structure is shown in figure 7.

The APLAC model of the asymmetric coupled line structure is called Maclin and may contain more than two strips in its structure. The discussion that follows is limited to the two strip case.
Figure 7: Asymmetric coupled microstrip line structure.

2.7.1 Equivalent capacitances

The Maclin model is based on the model presented in [14] which calculates the equivalent capacitances and inductances per unit length of the structure as shown in figure 8.

Referring to figure 8, $C_{10}$ is the capacitance formed by strip $w_1$ and the ground-plane, $C_{20}$ is the capacitance formed by strip $w_2$ and the ground-plane, $C_{12}$ is the capacitance formed by the two strips, $L_1$ is the inductance of strip $w_1$, $L_2$ is the inductance of strip $w_2$ and $L_{12}$ is the mutual inductance between the two strips. In order to calculate the strip-to-ground and strip-to-strip capacitances of the asymmetrical pair of coupled lines, the strip-to-ground and strip-to-strip capacitances for the symmetrical pair of microstrip coupled lines of widths $w_1$ and $w_2$, spacing $s$ and relative permittivity $\epsilon_r$ need to be known. The inductances are found by calculating the capacitances of the same asymmetrical coupled line structure but with an air dielectric, i.e. $\epsilon_r = 1$, as described in equation (43).

The model described below is accurate to within one percent for the case of a symmetrical pair of coupled microstrips in the range $1 < \epsilon_r < 20$ and

$$(10u)^2 \quad \text{for } u < 0.1$$
\begin{align*}
0.001 + \frac{1}{(200u)^2} \leq g \leq 1 \quad \text{for } 0.1 \leq u \leq 10 . \tag{92} \\
(10/u)^4 \quad \text{for } u > 10 
\end{align*}

As before, the dimensions of the asymmetric microstrip structure are normalized with respect to the substrate height as
\begin{align*}
u_1 = \frac{w_1}{h} \quad u_2 = \frac{w_2}{h} \quad g = \frac{s}{h} . \tag{93} 
\end{align*}

The first step in modelling the asymmetrical pair of coupled lines is to obtain the even- and odd-mode characteristic impedances and effective permittivities for a symmetrical pair of coupled lines for widths \( w_1 \) and \( w_2 \) from equations (69), (70), (63) and (66) for a homogeneous air dielectric as well as for a dielectric substrate with relative permittivity \( \epsilon_r \), thus giving four sets of impedances and permittivities.

The strip-to-ground and strip-to-strip capacitances for each of the four cases of symmetrical lines are then evaluated from [5]

\begin{align*}
C_{0,\text{sym}}(u, g, \epsilon_r) &= \epsilon_0 \eta_0 \frac{\sqrt{\epsilon_{\text{eff},e}(0)}}{Z_{L,e}(0)} \tag{94} \\
C_{12,\text{sym}}(u, g, \epsilon_r) &= \frac{\epsilon_0 \eta_0}{2} \left( \frac{\sqrt{\epsilon_{\text{eff},o}(0)}}{Z_{L,o}(0)} - \frac{\sqrt{\epsilon_{\text{eff},e}(0)}}{Z_{L,e}(0)} \right) . \tag{95} 
\end{align*}

The mutual capacitance \( C_{12} \) for the asymmetrical pair of coupled lines is then determined by averaging the values for two symmetric couplers with normalized widths \( u_1 \) and \( u_2 \) as

\begin{equation}
C_{12}(u_1, g, u_2) = \left( \frac{2}{C_{12,\text{sym}}(u_1, g, \epsilon_r)^{-K_m} + C_{12,\text{sym}}(u_2, g, \epsilon_r)^{-K_m}} \right)^{1/K_m} , \tag{96}
\end{equation}

where \( K_m \) has been obtained by fitting as

\( K_m = 0.95 + 0.33 \ln(u_1 u_2) - 0.4 \ln g \).

The strip-to-ground capacitances are given by

\begin{align*}
C_{10} &= C_{0,\text{sym}}(u_1, g) + [1 - K_s(\epsilon_r)] \left[ C_{12,\text{sym}}(u_1, g, \epsilon_r) - C_{12}(u_1, g, u_2) \right] \tag{97} \\
C_{20} &= C_{0,\text{sym}}(u_2, g) + [1 - K_s(\epsilon_r)] \left[ C_{12,\text{sym}}(u_2, g, \epsilon_r) - C_{12}(u_1, g, u_2) \right] , \tag{98}
\end{align*}

where \( K_s(\epsilon_r) \) is obtained by modelling as

\( K_s(\epsilon_r) = \frac{K_s(1)}{1 + 0.58(1 - 1/\sqrt{\epsilon_r})} \)

\( K_s(1) = [0.21 - 0.023 \ln(u_1 u_2)] \cdot \exp \{-g[1.56 + 0.22 \ln (u_1 u_2)]\} \).
Care must be taken not to confuse the capacitances for the symmetrical and asymmetrical coupled lines. The capacitances in equations (96), (97) and (98) should be evaluated for the dielectric substrate case as well as for the air dielectric case when $\varepsilon_r = 1$. In the latter case, the term $K_s(1)$ should be used instead of $K_s(\varepsilon_r)$ in equations (97) and (98).

The quasi-static line constants per unit length for the coupled microstrips are the elements of the capacitance and inductance matrices below:

$$C = \begin{bmatrix} C_{11} & -C_{12} \\ -C_{12} & C_{22} \end{bmatrix}$$

$$L = \mu_0 \varepsilon_0 \begin{bmatrix} C_{110} & -C_{120} \\ -C_{120} & C_{220} \end{bmatrix}^{-1}.$$  

2.7.2 Dispersion model

The dispersion model used here is according to Tripathi [15]. The frequency-dependent effective parameters are easily obtained from the capacitances calculated earlier.

The capacitances referred to in the following discussion are the capacitances modelling the asymmetrical coupled line pair. Capacitances $C_{11}$, $C_{12}$ and $C_{22}$ are the capacitances for the structure with a dielectric substrate, and capacitances $C_{110}$, $C_{120}$ and $C_{220}$ are the capacitances for the structure with an air substrate, $\varepsilon_r = 1$.

The following effective static relative permittivities and widths are first calculated:

$$\epsilon_{eff,1}(0) = \frac{C_{11} - C_{12}}{C_{110} - C_{120}}$$

$$\epsilon_{eff,2}(0) = \frac{C_{22} - C_{12}}{C_{220} - C_{120}}$$

$$\epsilon_{eff,3}(0) = \frac{C_{12}}{C_{120}}$$

$$w_{eff,1}(0) = \frac{h(C_{110} - C_{120})}{\varepsilon_0}$$

$$w_{eff,2}(0) = \frac{h(C_{220} - C_{120})}{\varepsilon_0}$$

$$w_{eff,3}(0) = \frac{sC_{120}}{\varepsilon_0}.$$
The next step is to calculate the inflection frequency \( f_p \), that corresponds to the cut-off frequency of the first higher mode of the structure. This is done by finding the lowest-order solution, or, in other words, the smallest zero of

\[
0 = \frac{g}{\sqrt{\varepsilon_{\text{eff},3}(0) \tan(\beta_3 w_{\text{eff},3}(0))}} + \frac{1}{\sqrt{\varepsilon_{\text{eff},1}(0) \tan(\beta_1 w_{\text{eff},1}(0))}} + \frac{1}{\sqrt{\varepsilon_{\text{eff},2}(0) \tan(\beta_2 w_{\text{eff},2}(0))}},
\]

(107)

where

\[
\beta_j = 2\pi \sqrt{\frac{\mu_0}{\mu_0 \varepsilon_0}} \varepsilon_{\text{eff},j}(0), \quad j = 1, 2, 3.
\]

The required zero of equation (107) is found using the Newton-Raphson algorithm [17]. The algorithm is given by

\[
x^{k+1} = x^k - \frac{f(x)}{f'(x)},
\]

(108)

where the superscript \( k \) is the iteration number. The variable \( x \) in this case is the frequency \( f \) and \( f(x) \) is the right hand side of equation (107). The derivative \( f'(x) \) of the right hand side of equation (107) is

\[
\frac{-2\pi}{\sin^2(\beta_3 w_{\text{eff},3}(0))} - \frac{2\pi}{\sin^2(\beta_1 w_{\text{eff},1}(0))} - \frac{2\pi}{\sin^2(\beta_2 w_{\text{eff},2}(0))}.
\]

(109)

Using a suitable initial value for \( f, f_p \), the smallest zero, is found using the iteration routine in equation (108).

The frequency-dependent effective relative permittivity and width are then found from

\[
\varepsilon_{\text{eff},j}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{\text{eff},j}(0)}{1 + G \left( \frac{f}{f_p} \right)^2}, \quad j = 1, 2, 3
\]

(110)

\[
G = 0.6 + 0.009(2\mu_0 h f_p)
\]

(111)

\[
w_{\text{eff},j}(f) = w_j - \frac{w_j - w_{\text{eff},j}(0)}{1 + \left( \frac{f}{f_p} \right)^2}, \quad j = 1, 2, 3.
\]

Substrate height \( h \) is used for \( w_3 \) when evaluating the above equations.

The last step is to calculate the capacitances resulting due to dispersion and are calculated from

\[
C_{11}(f) = \frac{\varepsilon_{\text{eff},1}(f) w_{\text{eff},1}(f)}{h} + \frac{\varepsilon_{\text{eff},3}(f) w_{\text{eff},3}(f)}{s}
\]

(112)

\[
C_{12}(f) = \frac{\varepsilon_{\text{eff},3}(f) w_{\text{eff},3}(f)}{s}
\]

(113)

\[
C_{22}(f) = \frac{\varepsilon_{\text{eff},2}(f) w_{\text{eff},2}(f)}{h} + \frac{\varepsilon_{\text{eff},3}(f) w_{\text{eff},3}(f)}{s}
\]

(114)

The corresponding values for the air dielectric must also be calculated in order to get the required inductance matrix in equation (100).
3 Specific functions implemented in APLAC

This section documents the various functions implemented in APLAC at the ‘C’ level in order to facilitate modifications in the code and, also lists the functions available to users at the input file level [16].

3.1 C-level implementation

The impedance of a single microstrip in a homogeneous air dielectric, including effects due to shielding, is calculated in function \texttt{mlin\_Z01}. If necessary \texttt{mlin\_Z01} calls function \texttt{delta\_u}, which computes the required correction in width, in order to account for finite strip thickness. Function \texttt{mlin\_epse} returns the effective relative permittivity, taking into account finite strip thickness. The characteristic impedance of a microstrip on a dielectric substrate is then calculated in \texttt{mlin\_Z0} by calling the above mentioned functions.

The two methods used to calculate effects due to dispersion have been implemented in \texttt{mlin\_dispersion} (Hammerstad) and \texttt{mlin\_level2} (Kirschning), and may be chosen by the user by specifying \texttt{LEVEL 1} or \texttt{LEVEL 2}, respectively. However, computation of the \texttt{LEVEL 2} effective relative permittivity using the equations given by Kirschning and Jansen [8] is carried out in a separate function \texttt{mlin\_dispersion2}, and is called by \texttt{mlin\_level2} when calculating the corresponding impedance. Equations for the dielectric and conductor losses are in \texttt{mlin\_losses}.

\texttt{Zl\_even\_odd} calculates the static even- and odd-mode impedances and effective permittivities of a pair of coupled microstrip lines, and includes effects due to shielding. Effects due to finite thickness are evaluated in \texttt{mclin\_thick}. However, the effective width correction due to finite strip thickness for the single microstrip that is required in \texttt{mclin\_thick} is now calculated using the equations in [1] in function \texttt{mlin\_thick}. Dispersion is accounted for in \texttt{mlin\_dispersion}.

\texttt{CalcCm} calculates the static strip-to-ground and strip-to-strip capacitances of the asymmetrical pair of coupled microstrips. The static model is implemented in \texttt{AclinValues}. The frequency dependence is calculated in \texttt{Aclin\_Disp}. \texttt{eff\_parf} calculates the effective widths and permittivities from which \texttt{eff\_cap} calculates the frequency-dependent strip-to-ground and strip-to-strip capacitances per unit length of the asymmetrical pair of coupled microstrip lines. The eigenvector matrix is calculated in \texttt{mlayer\_eigenv}.

3.2 Input-file level functions

Several functions are available to the user of APLAC from which various microstrip parameters may be calculated [16]. The ones concerning the microstrip components under discussion are \texttt{Mclin\_Z}, \texttt{Mclin\_Zf}, \texttt{Mlin\_epse}, \texttt{Mlin\_u}, \texttt{Mlin\_w} and \texttt{Mlin\_Z01}.

\texttt{Mclin\_Zf} calculates the characteristic impedance of a symmetrical coupled pair of microstrip lines for given physical specifications. It differs from \texttt{Mclin\_Z} in that
it calculates the characteristic impedance at a given frequency and so accounts for
dispersion effects. Its syntax is

\[
\text{Mclin}\_Zf(rW, rT, rS, rEr, rH, rH2, rF, vrZ)
\]

where \( rW, rT, rS, rEr, rH \) and \( rH2 \) are the physical dimensions of the structure (see
figure 6), and \( rF \) is the frequency at which the impedance is to be calculated. The
function returns vector \( vrZ \) whose four values are

- \( vrZ[0] \), the even-mode impedance,
- \( vrZ[1] \), the odd-mode impedance,
- \( vrZ[2] \), the even-mode effective permittivity and
- \( vrZ[3] \), the odd-mode effective permittivity.

\[\text{Figure 9: Even- and odd-mode impedances and effective permittivities of a sym-}
\text{metric coupler on an FR4 board.}\]

As an example,

\[
\text{Declare VECTOR z REAL 3}
\]

\[
\text{Sweep "Impedance and effective } \epsilon_r \text{"}
\]
\[
+ \text{ LOOP 200 FREQ LOG 1000MEGHz 25GHz}
\]
\[
+ \text{ Y "Z" } \backslash \backslash \text{Omega" 0 75}
\]
\[
+ \text{ Y2 "} \backslash \backslash \text{epsilon_r" } " 0 10
\]
\[
+ \text{ MULTX="G"}
\]
Call mclin_zf(3mm, 20um, 1mm, 4.5, 1.5mm, 20mm, f, z)

Display Y "Z_e" z[0]
+ Y "Z_o" z[1] MARKER=1
+ Y2 "\epsilon_{r,e}" z[2] MARKER=2
+ Y2 "\epsilon_{r,o}" z[3] MARKER=3

EndSweep

plots the even- and odd-mode impedances and effective dielectric-constants as a function of frequency of a symmetric coupler on an FR4 board. The results of the example above are shown in figure 9.
4 Conclusion

All the equations used to model the above mentioned microstrip components in APLAC have been thoroughly documented in this report. In addition, a little background was provided to facilitate the comprehension of the working of the models. However, a detailed background is not given.

The transmission line model for single and coupled microstrips has been presented. It was seen that for the single microstrip line, its characteristic impedance, attenuation constant, and propagation delay need to be computed. The closed form equations required for this purpose were presented. For the coupled microstrip lines, the eigenvector matrix has to be determined in addition to the characteristic impedance, attenuation constant, and propagation delay. Detailed closed form numerical expressions were presented for the symmetrical and asymmetrical pair of coupled lines. It was seen that the \( n \) symmetrical coupled microstrip lines could be modeled by considering one pair of coupled lines at a time.

Although no comparison results have been presented in this report, various APLAC simulations were compared with those using the MDS (Hewlett-Packard) circuit simulator and results were found to be comparable. Good agreement was also found with measurement results presented in the literature.
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