Variance of the Cumulative Histogram of ADCs Due to Frequency Errors

Francisco André Corrêa Alegria and António Manuel da Cruz Serra

Abstract—**The variance in the number of counts of the cumulative histogram, used for the characterization of analog-to-digital converters (ADCs) with the Histogram Method, is calculated without any restrictions regarding the magnitude of the frequency errors on the stimulus and sampling signals, number of periods of the stimulus signal, and number of samples. The formulation adopted allows a graphical interpretation of the problem that helps future developments still needed in this particular subject. The exact knowledge of this variance allows for a more efficient test of ADCs and a more precise determination of the uncertainty of the test result. Numerical simulation and experimental results that validate the theory are shown.**

Index Terms—**Analog-to-digital conversion, analog-to-digital converter (ADC) test, frequency error, histogram method.**

I. Introduction

THE HISTOGRAM method [1] is a tool widely used for the characterization of analog-to-digital converters (ADCs). A signal with a known amplitude probability density function is used to stimulate the converter. Several samples are acquired at a frequency f_s and the cumulative histogram is computed. The cumulative histogram for code k is the number of samples whose digital conversion is equal to or lower than output code k. The converter transition levels and code bin widths are determined by comparing the number of counts experimentally obtained with the number expected from an ideal converter.

To guarantee that all codes have an equal opportunity of being stimulated, the number of samples must be acquired during an integer number of periods of the input signal. Denoting by M the number of samples acquired and by D the number of signal periods, the aforementioned frequencies must satisfy the following relation [3]:

$$\rho = \frac{f}{f_s} = \frac{D}{M}. \tag{1}$$

Besides acquiring the samples during an integer number of periods, it is also necessary for their phases to be evenly distributed. To achieve that, the numbers D and M must be mutually prime [3].

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The initial random phase of the signal in each record of acquired samples will make the number of counts in the cumulative histogram a random variable. The results of the histogram method will thus be random with a probability density function that can be considered normal. By computing the variance of that distribution, an uncertainty interval for the test results may be calculated.

In practice, the referred frequencies do not verify (1) exactly, causing the sample phases to not be uniformly distributed, as it is desirable. This contributes to an increase in the variance of the results. Due to the nonlinear relationship between the number of counts of the cumulative histogram and the transition voltages and sample phases, it is difficult to obtain an analytical expression for the variance of the number of counts. What has been done in the past is to determine an expression for the error of the frequency relation $\Delta\rho$ that guarantees a certain maximum value for the variance of the number of counts of the cumulative histogram [6]. What needs to be done yet is to determine a similar expression for the variance of the number of counts of the histogram.

The analytical approach taken in [6] is not easily extrapolated for other situations, so, in this work, we present a different formulation accompanied by a graphical interpretation that will help, in the future, to determine a limit for the frequency ratio that guarantees a maximum for the variance of the number of counts of the histogram.

In [8]–[10], we have presented previous developments of this work; however, here, a new formulation is used that better illustrates the ideas we wish to transmit, allowing the reader an easier understanding of our work.

II. Preliminaries

The value of the variance of the counts in the cumulative histogram is used to determine the total number of samples that must be acquired to guarantee that the results of the INL, obtained by the histogram test, have an uncertainty smaller than a given chosen value. The expression traditionally used for this determination is the one developed by Jerome Blair [2] and later adopted by the IEEE 1057-1994 standard [3]

$$R = \left(\frac{2^{N-1}K_u}{B}\right)^2 \frac{a\pi}{M}\left(1.1\frac{\sigma^*}{V} + 0.2\frac{a\pi}{M}\right) \tag{2}$$

where B is the maximum allowed uncertainty in LSB, K_u is the confidence level, N in the number of bits, a is the amount of overdrive, σ * is the input-equivalent noise standard deviation, V is the full-scale voltage, and R is the number of records to be acquired. Each record has M samples. The value 0.2 present in (2) was computed by averaging the variance over all cumulative histogram bins for a certain value of frequency error. We believe, in accordance with [4], that the worst-case value should be used instead of the average value. We propose in this paper that equation (2) should be used to estimate the total number of samples required, but the value 0.2 should be replaced by the maximum value of variance, for the range of values of ρ determined by the stimulus signal and sampling clock frequency errors, as determined in the following section.

III. Variance of the Cumulative Histogram

Let us consider a sinusoidal stimulus signal with amplitude A , offset d , period T and phase φ

$$v = d - A \cdot \cos(2\pi \cdot f \cdot t + \varphi). \tag{3}$$

A. Normalized Variables

Numbering the samples from 0 to $M-1$ and considering that the first sample ($j = 0$) occurs at $t = 0$, the sampling instants are defined by $t_j = j \cdot T_s$ where T_s is the sampling interval. The phase of each sample is given by the term inside the brackets in (3): $2\pi \cdot (f/f_s) \cdot j + \varphi$. To ease the computations presented later, we chose to introduce a normalized phase value, z_j, ranging from 0 to M

$$z_j = M \left\langle \rho \cdot j + \frac{\varphi + \pi}{2\pi} \right\rangle \tag{4}$$

where $\langle\rangle$ represents the fractional part of its argument, that is, $\langle x \rangle = x - \lfloor x \rfloor$, where $\lfloor x \rfloor$ is the highest integer equal to or less than x. The factor π added to the initial stimulus signal phase φ was chosen to make the normalized phase of the first sample z_0 equal to $M/2$ for a null value of φ.

The same is done for the initial phase of the stimulus signal φ

$$\tau = M \left\langle \frac{\varphi + \pi}{2\pi} \right\rangle \tag{5}$$

leading to normalized values of the initial phase τ from 0 to M.

The voltage of the samples, considering the normalized phases z_j, is thus given, from (3), by

$$v_j = d - A \cdot \cos\left(\frac{2\pi}{M} z_j - \pi\right). \tag{6}$$

A sample belongs to class k of the cumulative histogram if its voltage is equal to or less than the transition voltage $T[k+1]$

$$v_j \leq T[k+1] \Leftrightarrow d - A \cdot \cos\left(\frac{2\pi}{M} z_j - \pi\right) \leq T[k+1]. \tag{7}$$

This can be written as

$$-a\cos\left(\frac{d - T[k+1]}{A}\right) \leq \frac{2\pi}{M} z_j - \pi \leq a\cos\left(\frac{d - T[k+1]}{A}\right). \tag{8}$$

The variable β_k is given by

$$\beta_k = \frac{M}{\pi} a\cos\left(\frac{d - T[k]}{A}\right). \tag{9}$$

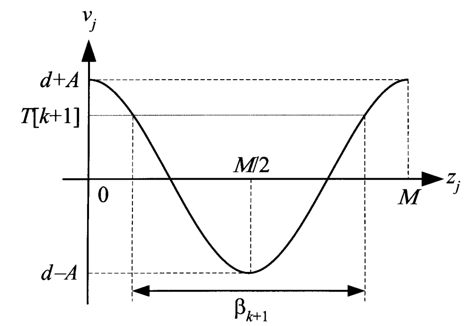

Fig. 1. Representation of the stimulus signal and phase interval β_{k+1} of the samples that belong to bin k of the cumulative histogram.

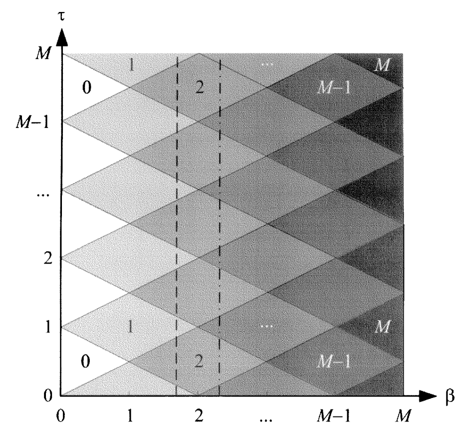

Fig. 2. Contour representation of the number of counts in bin k of the cumulative histogram for different values of initial stimulus signal phase (τ) and transition voltage (β) when there are no frequency errors.

The samples that belong to cumulative histogram class k are those whose normalized phase satisfies

$$\frac{M}{2} - \frac{\beta_{k+1}}{2} \leq z_j \leq \frac{M}{2} + \frac{\beta_{k+1}}{2}. \tag{10}$$

This interval is represented in Fig. 1.

B. Absence of Frequency Errors

Given the initial stimulus signal phase, through its normalized value τ and the phase interval width β_{k+1}, related to transition voltage $T[k+1]$, it is possible to know the number of samples, c_k, from the M acquired, that belong to bin k of the cumulative histogram as can be seen in Fig. 2 in the case where there are no frequency errors.

The uniform pattern observed is due to the fact that there are no frequency errors, and, thus, the sample normalized phases are evenly spaced between 0 and M . The diagonal lines observed correspond to the boundaries of the phase interval given by the equal signs in expression (10)

$$z_j = \frac{M}{2} \pm \frac{\beta_{k+1}}{2}, \qquad j = 0, 1, \ldots, M-1. \tag{11}$$

Introducing (4), we have

$$M \left\langle \rho \cdot j + \frac{\varphi + \pi}{2\pi} + \frac{1}{2} \right\rangle = \frac{M}{2} \pm \frac{\beta_{k+1}}{2}, \qquad j = 0, 1, \ldots, M-1, \tag{12}$$

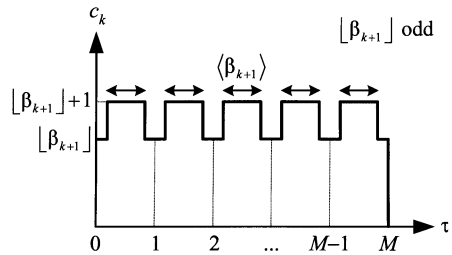

Fig. 3. Representation of the number of counts in bin k of the cumulative histogram as a function of the initial stimulus signal phase for odd values of $\lfloor\beta_{k+1}\rfloor$.

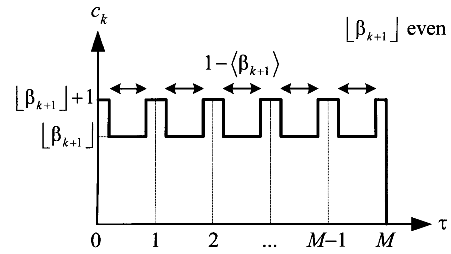

Fig. 4. Representation of the number of counts in bin k of the cumulative histogram as a function of the initial stimulus signal phase for even values of $\lfloor\beta_{k+1}\rfloor$.

that is

$$\rho \cdot j + \frac{\varphi+\pi}{2\pi} + \frac{1}{2} = \frac{1}{2} \pm \frac{\beta_{k+1}}{2M} + I, \quad j = 0, 1, \ldots, M-1 \text{ and } I \text{ integer.} \tag{13}$$

This expression is equivalent to

$$\left\langle \frac{\varphi+\pi}{2\pi} \right\rangle = \left\langle -\rho \cdot j \pm \frac{\beta_{k+1}}{2M} \right\rangle, \quad j = 0, 1, \ldots, M-1. \tag{14}$$

Introducing expression (5) leads to

$$\tau = M \left\langle -\rho \cdot j \pm \frac{\beta_{k+1}}{2M} \right\rangle \qquad j = 0, 1, \ldots, M-1, \tag{15}$$

which represents M upward (plus signal) and M downward (minus signal) diagonal lines with a slope of $\pm 1/2$.

The intersection of two of those diagonal lines, for instance, j_1 and j_2 , occurs for values of β given by

$$\beta_{intersect} = M \langle \pm\rho (j_1 - j_2) \rangle, \quad j_1 \text{ and } j_2 = 0, 1, \ldots, M-1. \tag{16}$$

In the absence of frequency errors, that is, when the sampling frequency and the stimulus signal frequency satisfy (1), $\rho = D/M$, and the values of β given by (16)

$$\beta_{intersect} = M \left\langle \pm \frac{D}{M} (j_1 - j_2) \right\rangle, \quad j_1 \text{ and } j_2 = 0, 1, \ldots, M-1 \tag{17}$$

are integer numbers from 0 to M.

From Fig. 2, it can be seen that the representation of the number of counts c_k of the cumulative histogram, as a function of τ, has one of two shapes depending on $\lfloor\beta_{k+1}\rfloor$ being odd (Fig. 3) or even (Fig. 4). The values of β in these two figures are signaled by the two vertical lines in Fig. 2 (dashed line for $\lfloor\beta_{k+1}\rfloor$ odd and dash-dotted line for $\lfloor\beta_{k+1}\rfloor$ even).

In the case where the beginning of the samples acquisition is not controlled, the variable τ can be considered random with uniform distribution [6] in an interval of length M. The mean of c_k is then given by

$$\mu_{c_k} = \frac{1}{M} \int_0^M c_k(\tau) \cdot d\tau \tag{18}$$

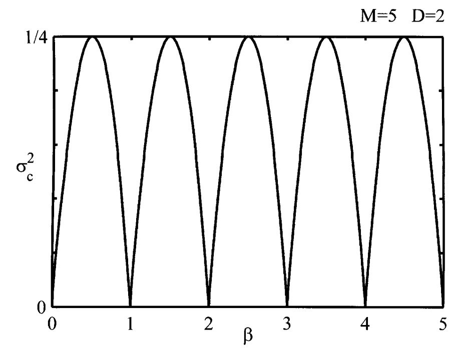

Fig. 5. Representation of the variance of the number of counts of the cumulative histogram as a function of the phase interval width β, when there are no frequency errors.

and can be easily computed from Fig. 3 and Fig. 4

$$\begin{aligned}\mu_{c_k} &= \frac{1}{M} \left[M \left(\lfloor\beta_{k+1}\rfloor + 1 - \lfloor\beta_{k+1}\rfloor \right) \langle\beta_{k+1}\rangle + M \lfloor\beta_{k+1}\rfloor \right] \\ &= \beta_{k+1}.\end{aligned} \tag{19}$$

The second moment is given by

$$m_{2c_k} = \frac{1}{M} \int_0^M c_k^2(\tau) \cdot d\tau \tag{20}$$

and can also be easily computed from Fig. 3 and Fig. 4

$$\begin{aligned}m_{2c_k} &= \frac{1}{M} \Big[M \left((\lfloor\beta_{k+1}\rfloor + 1)^2 - \lfloor\beta_{k+1}\rfloor^2 \right) \\ &\qquad \cdot \langle\beta_{k+1}\rangle + M \lfloor\beta_{k+1}\rfloor^2 \Big] \\ &= \langle\beta_{k+1}\rangle - \langle\beta_{k+1}\rangle^2 + \beta_{k+1}^2.\end{aligned} \tag{21}$$

Using (19) and (21), the variance of the number of counts of the cumulative histogram in the absence of frequency errors is a function of the width β_{k+1} of the phase interval

$$\sigma_{c_k}^2 = m_{2c_k} - \mu_{c_k}^2 = \langle\beta_{k+1}\rangle - \langle\beta_{k+1}\rangle^2 \tag{22}$$

which is represented in Fig. 5.

It can be seen that the representation of the variance as a function of the phase interval width is composed of M parabolic arcs. The variance is 0 for integer values of β and has a maximum of 1/4.

C. Presence of Frequency Errors

When there are frequency errors, the uniform pattern observed in Fig. 2 becomes distorted as can be seen in Fig. 6.

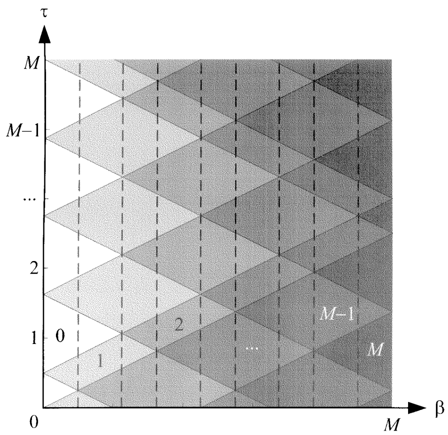

Fig. 6. Contour representation of the number of counts in bin k of the cumulative histogram for different values of initial stimulus signal phase (τ) and transition voltage (β) when there are frequency errors.

The M^2 intersections between the M upward and the M downward diagonals occur now at $2 \times M$ different values of β . Inserting $\rho = D/M + \Delta\rho$ into (16) gives

$$\beta_{intersect} = M \left\langle \pm \frac{D}{M}(j_1 - j_2) \pm (j_1 - j_2) \cdot \Delta\rho \right\rangle, \quad j_1 \text{ and } j_2 = 0, 1, \ldots, M-1. \tag{23}$$

The solution to this equation, according to [6], is:

$$\beta_{intersect} = \begin{cases} K + m_L \cdot M \cdot \Delta\rho \\ K + m_L \cdot M \cdot \Delta\rho - M^2 \cdot \Delta\rho \end{cases} \quad K = 1, 2, \ldots, M \tag{24}$$

where m_L is given by

$$m_L = M \left\langle \frac{K \cdot M_L}{M} \right\rangle \tag{25}$$

and M_L is the denominator of the element, in the Farey sequence [5], preceding the element D/M.

In relation to the case where there are no frequency errors, it can be seen that each value of β where there was an intersection of two diagonal lines (all integers between 0 and M) gives rise, when there are frequency errors, to two different values of β given by (24). Those borders, in the presence of frequency errors, are represented by vertical dashed lines in Fig. 7.

The representation of the variance as a function of β is now composed of $2M - 1$ parabolic arcs as can be seen in Fig. 7. The vertical lines represent the values of β where there is an intersection between two diagonal lines in Fig. 6.

It can be seen that when there are frequency errors, the variance may exceed the value 1/4, which was the maximum value attained when there were no frequency errors. In [6], an analytical expression was derived which allows the calculation of the maximum error in the frequency ratio ρ which guarantees a maximum variance of 1/4 in the number of counts of the cumulative histogram

$$\Delta\rho \le \frac{1}{2M^2}. \tag{26}$$

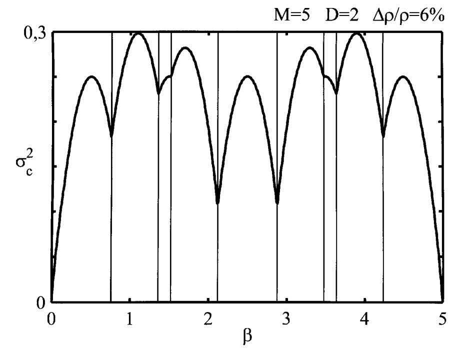

Fig. 7. Representation of the variance of the number of counts of the cumulative histogram as a function of the phase interval width β when there are frequency errors.

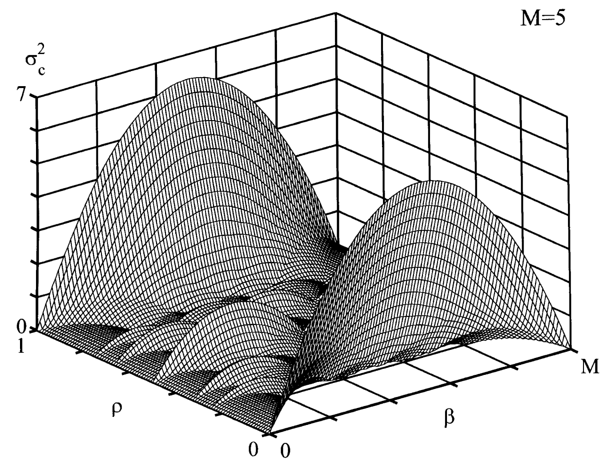

Fig. 8. Representation of σ_c^2 for different values of ρ and βg $(M = 5)$.

This limit can also be obtained with the formulation used here. Consider that a parabolic arc can be expressed with the following expression:

$$\sigma_c^2 = \sigma_m^2 - (\beta - \beta_m)^2 \tag{27}$$

where the parameters β_m and σ_m^2 represent the position and the maximum value of the arc, respectively. Given two known points in the arc (β_1, σ_{c1}^2) and (β_2, σ_{c2}^2), it is possible to determine those parameters using

$$\begin{aligned} \beta_m &= \frac{\sigma_{c1}^2 - \sigma_{c2}^2}{2(\beta_1 - \beta_2)} + \frac{\beta_1 + \beta_2}{2} \\ \sigma_m^2 &= \sigma_{c1}^2 + (\beta_1 - \beta_m)^2. \end{aligned} \tag{28}$$

For the values of β given by (24), the variance is, for $m_L \cdot M \cdot \Delta\rho < 1$

$$\begin{aligned} \sigma_{c1}^2 &= \langle\beta_1\rangle - \langle\beta_1\rangle^2 = m_L \cdot M \cdot \Delta\rho - (m_L \cdot M \cdot \Delta\rho)^2 \\ \sigma_{c2}^2 &= \langle\beta_2\rangle - \langle\beta_2\rangle^2 \\ &= (M - m_L)\, M \cdot \Delta\rho - [(M - m_L)\, M \cdot \Delta\rho]^2. \end{aligned} \tag{29}$$

Combining (24), (28), and (29) gives rise to

$$\begin{aligned} \beta_m &= K + \left(\frac{1}{2M^2} - \Delta\rho\right)(2m_L - M)\, M \\ \sigma_m^2 &= \frac{1}{4} + 2m_L\,(M - m_L)\left(\Delta\rho - \frac{1}{2M^2}\right). \end{aligned} \tag{30}$$

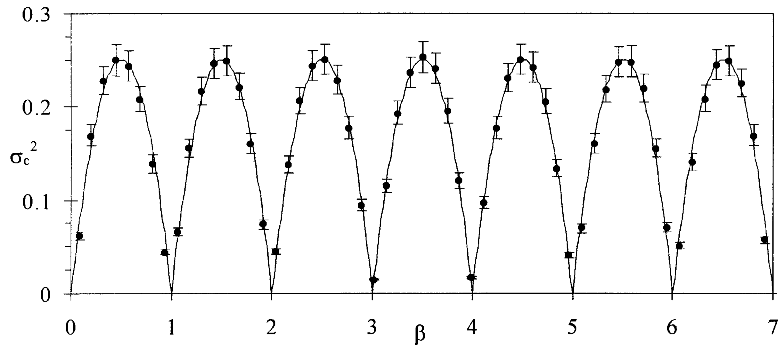

Fig. 9. Representation of the experimental results of the variance of the number of counts of the cumulative histogram (dots) in the case of absence of frequency error ($M = 7$). Vertical bars are used to represent the 99.9% confidence interval. The theoretical value of the variance is represented by a solid line.

Using (30), it is clear that, if $\Delta\rho$ satisfies (26), the variance will never be greater than 1/4.

If the frequencies satisfy this condition, then expression (2) can be used with a factor 0.25 instead of the factor 0.2 proposed in [2].

In Fig. 8, a three-dimensional (3-D) representation of σ_c^2 is presented when ρ varies from 0 to 1 and when β varies from 0 to M. Analyzing Fig. 8, we find that when ρ is a rational number with a denominator of $5(M)$ and a numerator mutually prime with 5, we have a minimum in the value of the variance (ρ: 0.2, 0.4, 0.6, and 0.8). Also, when the value of ρ is one of the elements of the Farey series of order 4 $(M-1)$ we have a local maximum of the variance (ρ: 0.25, 0.33, 0.5, 0.66, and 0.75).

IV. Experimental Results

In order to validate the work presented, two experiments, using an actual ADC, were performed and the results presented here. In those experiments, a PC data acquisition board from Keithley (DAS 1601) was used. The board is based on a 12-bit successive approximation ADC from Burr-Brown (ADS774). The 1-V bipolar range was used, and the samples were acquired at a frequency of 100 kHz (f_s). A function generator from Hewlett-Packard (HP33120A) was used to create the triangular stimulus signal used. This waveform was chosen so that the experimental values would be equally spaced.

A. Absence of Frequency Error

The first experiment was performed to validate the case when there is no frequency error. In actual experimental conditions, there is obviously an error in the frequency of the function generator and the sampling clock. With the equipment used, those errors were limited to 25 ppm. The effect of frequency errors on the variance of the number of counts of the cumulative histogram depends on the number of samples acquired. If one million samples were to be acquired, an error of 25 ppm in the frequencies would have a notable effect in the variance. In the experiment carried out, only seven samples were acquired ($M = 7$), and, thus, the effect of frequency errors can be neglected. A stimulus signal with a 0.9-V amplitude and a frequency of 14 285.714 Hz was used corresponding to

$$\rho = \frac{f}{f_s} = \frac{14\,285.71}{100\,000} \approx \frac{1}{7}. \tag{31}$$

From the 12 bits obtained for each sample, only the six most significant (n_b) were used in order to eliminate the effects of input-equivalent additive noise and phase noise. The ADC can, thus, be considered as having an ideal behavior. For each record (n) of the 5000 (N) acquired, a cumulative histogram was computed ($CH_n\,[k]$). The variance of the number of counts was estimated with

$$s_c^2\,[k] = \frac{1}{N-1}\sum_{n=1}^{N}\left(CH_n\,[k] - \overline{CH\,[k]}\right)^2 \qquad k = 0, 1, \ldots, 2^{n_b}-1 \tag{32}$$

where

$$\overline{CH\,[k]} = \frac{1}{N}\sum_{n=1}^{N} CH_n\,[k], \quad k = 0, 1, \ldots, 2^{n_b}-1. \tag{33}$$

The variance estimated for each class k of the cumulative histogram $(\sigma_c^2\,[k])$ has an associated confidence interval that was determined considering that the variance estimator (32) has a chi-squared distribution [7]

$$\frac{(N-1)\,s_c^2}{\chi^2_{1-\delta/2}\,(N-1)} < \sigma_c^2 < \frac{(N-1)\,s_c^2}{\chi^2_{\delta/2}\,(N-1)}. \tag{34}$$

The experimental results obtained are represented in Fig. 9 (vertical bars) and are in good accordance with the theoretical values (solid line).

The length β of the phase interval corresponding to each class k of the cumulative histogram was computed with

$$\beta\,[k] = \frac{M}{2}\left(\frac{T\,[k+1]-d}{A}+1\right) \qquad k = 0, 2, \ldots, 2^{n_b}-2 \tag{35}$$

where $T[k]$ is the transition voltage, which, in this case, can be considered ideal and can be determined from the full-scale voltage *FS* (1 V in this experiment)

$$T\,[k] = -FS + \left(k - \frac{1}{2}\right)\frac{2FS}{2^{n_b}-1}, \quad k = 1, 2, \ldots, 2^{n_b}-1. \tag{36}$$

Note that there is also an associated error (which is not represented in the figures) in the value of the phase interval length (35) due to inaccuracies in the stimulus signal amplitude (A)

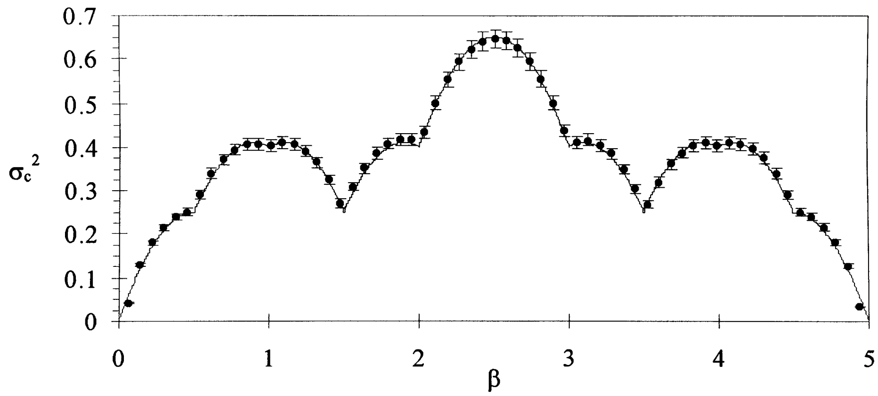

Fig. 10. Representation of the experimental results of the variance of the number of counts of the cumulative histogram (dots) in the case of 50% frequency error ($M = 5$). Vertical bars are used to represent the 99.9% confidence interval. The theoretical value of the variance is represented by a solid line.

and offset (d) . The accuracy of the amplitude and offset in the case of the function generator used is 1% and 2 mV, respectively.

B. Presence of Frequency Error

Another experiment was performed with a different number of samples $(M = 5)$ and a different stimulus signal frequency: 30 kHz. With this frequency value, the frequency relation ceases to be a fractional number with a denominator equal to the number of samples (M) and a nominator mutually prime with M as in (1). In this case

$$\rho = \frac{f}{f_s} = \frac{30\,000}{100\,000} = 1.5 \cdot \frac{1}{5}. \tag{37}$$

This choice of frequencies corresponds to a 50% frequency error. The results obtained are depicted in Fig. 10, where accordance with the theoretical values is verified.

This experiment was carried out on the $\pm$ 10-V range of the PC data acquisition board using a 10-V triangular stimulus signal. The number of records acquired to estimate the variance was 20 000.

V. Conclusion

The formulation presented here is a useful tool for the study of the variance of the number of counts of the histogram and the cumulative histogram. This work can be used as a foundation for future developments in this area, namely the determination of a limit for the ratio between the stimulus signal frequency and the sampling frequency that guarantees a certain maximum value for the variance of the number of counts of the histogram. Hopefully, it will also assist in the determination of an analytical expression for the variance of the number of counts regardless of the frequency ratio. The goal is to make practical the use of lower cost function generators that have a greater frequency error and to be able to speed up the test by acquiring more samples at the same time (fewer records).

References

[1] J. Doernberg, H.-S. Lee, and D. A. Hodges, "Full-speed testing of A/D converters," *IEEE J. Solid-State Circuits*, vol. SC-19, pp. 820–827, Dec. 1984.

[2] J. Blair, "Histogram measurement of ADC nonlinearities," *IEEE Trans. Instrum. Meas.*, vol. 43, pp. 373–383, June 1994.

[3] IEEE 1057, IEEE standard for digitizing waveform recorders, Dec. 1994.

[4] G. Chiorboli and C. Morandi, "About the number of records to be acquired for histogram testing of A/D converters using synchronous sinewave and clock generators," in *Proc. 4th Workshop ADC Modeling and Testing*, Bordeaux, France, September 9–10, 1999, pp. 182–186.

[5] R. Graham, D. Knuth, and O. Potashnik, *Concrete Mathematics*, 2nd ed. Reading, MA: Addison-Wesley, 1994.

[6] P. Carbone and G. Chiorboli, "ADC sinewave histogram testing with quasichoerent sampling," in *Proc. 17th IEEE IMTC Conf.*,, vol. 1, Baltimore, MD, May 2000, pp. 108–113.

[7] A. Papoulis, *Probability, Random Variables and Stochastic Processes*, 3rd ed. New York: McGraw-Hill, 1991.

[8] F. Corrêa Alegria and A. Cruz Serra, "Influence of frequency errors in the variance of the cumulative histogram," *IEEE Trans. Instrum. Meas.*, vol. 50, pp. 461–464, Apr. 2001.

[9] ——, "Influence of frequency errors in the variance of the cumulative histogram," in *Proc. Conf. Precision Electromagn. Meas.*, Sydney, Australia, May 14–19, 2000, pp. 585–586.

[10] ——, "Variance of the cumulative histogram of ADC's due to frequency errors," in *Proc. IEEE Instrum. Meas. Technol. Conf.*, Budapest, Hungary, May 21–23, 2001, pp. 2021–2026.

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