IEEE 1057 Jitter Test of Waveform Recorders

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***Abstract* — The jitter test of analog to digital converters is traditionally carried out with one of the methods recommended in the IEEE Standard for Digitizing Waveform Recorders, std. 1057. Here we study the uncertainty of one of those methods and point out the bias inherent to the estimator recommended for measuring the ADC jitter and suggest an alternate estimator. Expressions are also presented for the determination of the precision of a given estimate from the number of samples used, the standard deviation of the additive noise present in the test setup, the jitter standard deviation and the stimulus signal parameters. In addition, an expression for the computation of the minimum number of samples required to guarantee a given bound on the estimation uncertainty is presented which is useful in optimizing the test duration.**

***Index Terms* — Analog to Digital Converter, Test, Jitter, Phase noise.**

I. INTRODUCTION

THE jitter, or aperture uncertainty, in analog to digital converters (ADCs), is a random variation in the instant of sampling. This ADC parameter is of special importance in ADCs used in digital communication receivers where the decision between which symbols were transmitted is intimately related to the instant where the input signal is sampled [1-2]. In radio receivers, the noise level, and therefore the effective Number of Bits (ENOB) are not only dependent on the quantization noise. Jitter present in receiver ADC clock is one of the main causes of loss of performance in wireless communications [3-4]. The effects of ADC clock jitter on the system Signal-to-Noise ratio in waveform recorders are discussed in [5] while an improved jitter measurement method has been proposed in [6]. Many jitter estimators are proposed in prior work [7-10]. This paper, however, focuses on the jitter tests methods proposed in IEEE standard 1057, both the 1994 version [11] and the 2007 version currently in balloting [12]. This IEEE standard suggests three different methods for jitter estimation. One of those is appropriate for the use in systems where the clock signal is available externally (section 4.9.2.3 in [11] and 12.2.3 in [12]). The other two can be used more generally (section 4.9.2.1/4.9.2.2 in [11] and 12.2.1/12.2.2 in [12]). These three methods only permit the estimation of an upper bound on the amount of jitter present since the result obtained also includes other non idealities like ADC pattern errors, amplitude noise, quantization noise and harmonic distortion. The methods in sections 4.9.2.1 and 4.9.2.2 of [11] (12.2.1 and 12.2.2 in [12]) were compared by using a low bandwidth (100 Hz) seismic data recorder as the measurement system. In active marine seismology, the quality of data is directly related to the acquisition timing. The results of jitter estimation of this system are reported and discussed in [13]. These results have shown that the method suggested in section 4.9.2.2 of [11] (12.2.2 in [12]) is the only one appropriate when the amount of amplitude or quantization noise present is significant since it does not include their contribution when estimating jitter. This is the method we are going to study here. The analysis carried out will focus on the statistical properties of the estimated value of jitter standard deviation. We will not consider at present the effect that harmonic distortion has on the estimator and which can be significant. Work is being carried out on this area and will be subject of a future publication.

In section II we describe the test method and in section III we start analyzing the estimator statistics by computing its bias and concluding that the estimator suggested in [11-12] for this test method is biased. As a consequence we propose a new estimator in section IV. We then proceed to the precision analysis of both estimators in section V. In section VI we present the experimental results that validate the theoretical study presented. Finally we derive an expression for the computation of the minimum number of samples required to guarantee a certain bound on the estimation uncertainty (section VII). In VIII we sum up the results achieved and highlight future work that needs to be done to fully understand the uncertainty contributions of the jitter measurement method studied. This paper presents the first results obtained to achieve that goal.

II. JITTER TEST

Test 4.9.2.2 of [11] (12.2.2 in [12]) is based on the fact that the presence of jitter in the sampling instant translates into an

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increase in the amplitude noise of the sampled voltage which depends on the slope of the input signal. The jitter test consists in applying a low frequency (f_a) sine wave to the ADC input,

$$ya(t) = C + A\cos(2\pi f_a t + \varphi), \tag{1}$$

where C, A and φ represent the sine wave offset, amplitude and initial phase respectively. After that, a given number of samples (M) are acquired whose voltage, after quantization, will be:

$$za_i = Q \cdot round\left\{\frac{C + A\cos\left[2\pi f_a\left(t_i + \delta_i\right) + \varphi\right] + n_i}{Q}\right\}, \tag{2}$$

where Q represents the ADC quantization step. Inevitably those samples will be affected by amplitude noise (n) and jitter (δ). Representing the effect of the quantizer by an additive term (q) we can write (2) as

$$za_i = C + A\cos\left[2\pi f_a\left(t_i + \delta_i\right) + \varphi\right] + n_i + q_i\,. \tag{3}$$

This assumes that the quantization error is independent of the stimulus signal. This assumption is valid only if the characteristic function of the stimulus signal is “band-limited”, that is, if it is null outside an interval of length 2π/Q around 0 [16]. In the case of a sine wave, the characteristic function has infinite bandwidth:

$$\Phi_y(u) = J_0\left(\frac{2\pi A}{Q}u\right). \tag{4}$$

The higher is A in relation to Q, the higher will be the constant multiplying u and more concentrated around 0 the characteristic function will be. As a consequence, if A is high enough we can consider the characteristic function as “band-limited” and consequently consider that the quantization error is uniform and independent of the signal.

The sine wave that best fits the acquired samples, in a least squares error sense, is determined [11]. From the fitted sine wave parameters $\left(\widehat{C}_a,\ \widehat{A}_a \text{ and } \widehat{\varphi}_a\right)$ the ideal value of the sampled voltage can be computed:

$$\widehat{ya}_i = \widehat{C}_a + \widehat{A}_a\cos\left(2\pi f_a t_i + \widehat{\varphi}_a\right). \tag{5}$$

From here we compute the mean square difference between the ideal input voltage and the voltage of the actual sample:

$$mse_a = \frac{1}{M}\sum_{i=1}^{M}\left(za_i - \widehat{ya}_i\right)^2\,. \tag{6}$$

Then, another signal with a higher frequency (f_b), is applied to the ADC (yb), the same number of samples is acquired (zb) and the mean square error between the acquired samples and the fitted sine wave ($\widehat{yb}$) is computed:

$$mse_b = \frac{1}{M}\sum_{i=1}^{M}\left(zb_i - \widehat{yb}_i\right)^2\,. \tag{7}$$

Finally the ADC jitter standard deviation is estimated using

$$\hat{\sigma}_t = \frac{\sqrt{mse_b - mse_a}}{\sqrt{2}\pi f_b A}\,. \tag{8}$$

Note that in the published version of the IEEE 1057 standard [11] there is a typo in eq. 109. It should read f_2 instead of f in the denominator. In this paper we use the indexes “a” and “b” to represent the two different frequencies, instead of “1” and “2”.

The two frequencies used should be as distinct as allowed by the system bandwidth in order to have as distinct values of mse_a and mse_b as possible.

III. BIAS OF THE IEEE JITTER TEST ESTIMATOR

In this section we are going to compute the bias of estimator (8). To achieve this we first determine the bias of the computed mean square errors mse_a and mse_b. From (6) we can write

$$E\left\{mse_a\right\} = \frac{1}{M}\sum_{i=1}^{M}E\left\{\left(za_i - \widehat{ya}_i\right)^2\right\}. \tag{9}$$

In this paper we will consider that the error in the estimation of the sine wave parameters is negligible and thus we will substitute $\widehat{ya}$ by ya which is given by (1). Introducing (3) and (5) into (9) and making $\hat{A} = A$, $\hat{C} = C$ and $\hat{\varphi} = \varphi$ leads to

$$E\left\{mse_a\right\} = \frac{1}{M}\sum_{i=1}^{M}E\left\{\begin{bmatrix}A\cos\left[2\pi f_a\left(t_i + \delta_i\right) + \varphi\right] + \\ + n_i + q_i - A\cos\left(2\pi f_a t_i + \varphi\right)\end{bmatrix}^2\right\}. \tag{10}$$

Using the trigonometric relation

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)\,, \tag{11}$$

we can write

$$E\left\{mse_a\right\} = \\ = \frac{1}{M}\sum_{i=1}^{M}E\left\{\begin{bmatrix}n_i + q_i - \\ -A\sin\left(2\pi f_a\delta_i\right)\sin\left(2\pi f_a t_i + \varphi\right) - \\ -A\left[1 - \cos\left(2\pi f_a\delta_i\right)\right]\cos\left(2\pi f_a t_i + \varphi\right)\end{bmatrix}^2\right\}. \tag{12}$$

This expression can be simplified in the situations where the amount of jitter is small when compared with the sampling period. In those cases we can use the fact that

$$\cos(a) \approx 1 \quad \text{and} \quad \sin(a) \approx a \quad \text{for} \quad |a| \ll 1\,. \tag{13}$$

This assumption is not valid in all situations as is the case, for instance, of high frequency sampling oscilloscopes [14]. Here, however, we will consider only the situation where (13) is valid. From (12) we have

$$E\left\{mse_a\right\} \approx \frac{1}{M}\sum_{i=1}^{M}E\left\{\left[n_i + q_i - 2\pi f_a\delta_i A\sin\left(2\pi f_a t_i + \varphi\right)\right]^2\right\}. \tag{14}$$

which can be written also as

$$E\left\{mse_a\right\} \approx \frac{1}{M}\sum_{i=1}^{M} E\left\{\begin{array}{l} n_i^2+q_i^2+ \\ +\left(2\pi f_a\delta_i A\right)^2\sin^2\left(2\pi f_a t_i+\varphi\right)- \\ -2\left(n_i+q_i\right)\left(2\pi f_a\delta_i A\right)\sin\left(2\pi f_a t_i+\varphi\right)\end{array}\right\} \tag{15}$$

If we take into account that n, q and δ are independent and have zero mean.

Both additive noise and jitter are considered normally distributed random variables in this study with standard deviations σ_n^2 and σ_t^2 respectively. The quantization error can be considered an uniform random variable in an interval of length Q, if the conditions in [16] are satisfied. In that case its standard deviation is given by $Q/\sqrt{12}$. We can thus write (15) as

$$E\left\{mse_a\right\} \approx \sigma_n^2+\frac{Q^2}{12}+\left(2\pi f_a\sigma_t A\right)^2\frac{1}{M}\sum_{i=1}^{M}\sin^2\left(2\pi f_a t_i+\varphi\right). \tag{16}$$

Again we will make another simplifying assumption. In this case we will consider that the acquisition of the input signal is carried out during an integer number of periods (*J*), that is, the signal frequency, sampling frequency (f_s) and number of samples satisfy

$$\frac{f_a}{f_s}=\frac{J}{M}\ ,\ J\in\mathbb{N} \text{ and } J \text{ not multiple of } \frac{M}{2}. \tag{17}$$

In that case the summation in (16) is:

$$\sum_{i=1}^{M}\sin^2\left(2\pi f_a t_i+\varphi\right)=\frac{M}{2}. \tag{18}$$

Note that the sampling instants are given by $t_i = i/f_s$. The assumption is reasonable because we can choose whatever values we want for those frequencies and the number of samples. In practice, however, due to instrument inaccuracies, the actual value of those frequencies may not be exactly the values chosen and which satisfy (17) but are close enough considering typical frequency errors smaller than 100 ppm. If a non integer number of periods is acquired a bias will affect the estimator. In this work, however, we will not consider this scenario.

Using (18) we can write (16) as

$$E\left\{mse_a\right\} \approx \sigma_n^2+\frac{Q^2}{12}+2\left(\pi f_a\sigma_t A\right)^2. \tag{19}$$

The same reasoning can be applied to the samples acquired with the high frequency sine wave:

$$E\left\{mse_b\right\} \approx \sigma_n^2+\frac{Q^2}{12}+2\left(\pi f_b\sigma_t A\right)^2. \tag{20}$$

We are now ready to compute the expected value of the estimator (8). To first approximation the expected value of the square root of a variable is equal to the square root of its expected value [15, pp. 113]. We thus have

$$E\left\{\hat{\sigma}_t\right\} \approx \frac{\sqrt{E\left\{mse_b\right\}-E\left\{mse_a\right\}}}{\sqrt{2}\pi f_b A}. \tag{21}$$

Using (19) and (20) leads to

$$E\left\{\hat{\sigma}_t\right\} \approx \frac{\sqrt{2\left(\pi f_b\sigma_t A\right)^2-2\left(\pi f_a\sigma_t A\right)^2}}{\sqrt{2}\pi f_b A} \tag{22}$$

Which, after further simplification leads to

$$E\left\{\hat{\sigma}_t\right\} \approx \sigma_t\sqrt{1-\frac{f_a^2}{f_b^2}}. \tag{23}$$

By observing equation (23) we conclude that estimator (8) is biased since $E\left\{\hat{\sigma}_t\right\} \neq \sigma_t$. To minimize the estimation error one should have a low value of f_a and a high value of f_b as possible.

IV. NEW ESTIMATOR PROPOSED

As concluded in the previous section, the estimator (8) which is the one recommended in method 4.9.2.2 of IEEE standard 1057 [11] (12.2.2 of [12]) to estimate jitter in Waveform Digitizers and ADCs in general, is biased. The expected value of this estimator is given by (23). Using this information we suggest a new estimator for that test:

$$\hat{\sigma}_t = \frac{\sqrt{mse_b-mse_a}}{\sqrt{2}\pi A\sqrt{f_b^2-f_a^2}}. \tag{24}$$

If frequencies f_a and f_b can be properly chosen, that is, if we can have $f_b \gg f_a$, then, in practice, the difference between using (24) instead of (8) is negligible. We maintain however that there is no reason to use a biased expression when an unbiased one is available which equally easy to use.

From (24) and using (19) and (20), we have

$$E\left\{\hat{\sigma}_t\right\} = \frac{\sqrt{E\left\{mse_b\right\}-E\left\{mse_a\right\}}}{\sqrt{2}\pi A\sqrt{f_b^2-f_a^2}} = \sigma_t. \tag{25}$$

which proves that estimator (24) is unbiased. Again note that this is so because we are not considering the eventual presence of harmonic distortion in the stimulus signal or caused by the waveform recorder nor that the samples acquisition may have been carried out over a non integer number of periods due to mismatch in stimulus signal and sampling clock frequencies.

To validate the results obtained so far about the bias of the estimator, we used a Monte Carlo procedure. The test was repeated 1000 times on a simulated ADC having jitter and amplitude noise. The conditions of the test are presented in Tab. 1.

Tab. 1 – Settings used for the Monte Carlo simulation.

Setting	*Value*
Sine Wave Amplitude (A)	10 V
Sine Wave Offset (C)	0
Low Sine Wave Frequency (f_a)	100 kHz
High Sine Wave Frequency (f_b)	1 MHz
ADC Quantization Step (Q)	1 μV
Number of Acquired Samples (M)	1000
Sampling Frequency (f_s)	100 MHz

Injected Additive Noise (σ_n)	50 mV
Injected Jitter (σ_t)	0 to 10 ns
Number of Repetitions (R)	1000
Confidence Level (ν)	99.9 %

The expected value of the jitter estimates obtained was computed and its difference to the actual jitter standard deviation is represented in Fig. 1. The vertical bars translate the uncertainty of the expected value due to a finite number of repetitions [15, pp. 248].

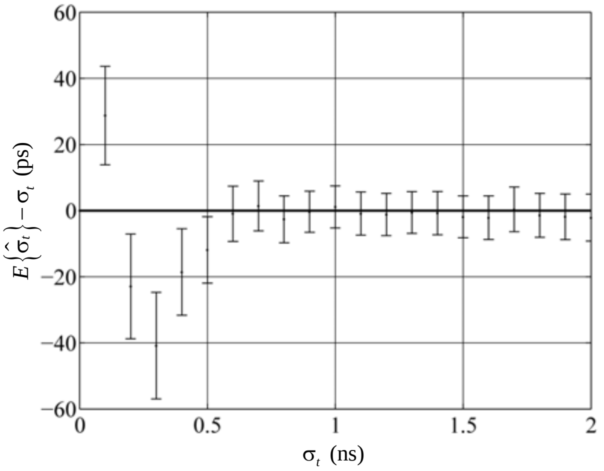

Fig. 1 – Representation of the error of the expected value of the jitter estimation as a function of the actual standard deviation of jitter. The solid line represents the values given by (25) and the vertical bars represent the result of Monte Carlo simulations of the jitter test method.

It can be clearly seen that for values of jitter standard deviation greater than 0.5 ns the result confirm that the estimator (24) is unbiased. The divergence observed for smaller values of jitter standard deviation are due to the simplification that the expected value of the square root of a variable is really not exactly the same as the square root of its expected value. As a consequence, we should state more accurately that the estimator is unbiased as long as the jitter present is not such, given the test frequencies used, that it leads to a small difference in value between the measured mean square errors at the two test frequencies.

V. UNCERTAINTY OF THE JITTER TESTS

In this section we will focus on the uncertainty of the estimators (8) and (24). The precision of the estimates is related to the standard deviation of the random variable $\hat{\sigma}_t$. To compute it we will start by computing the variance of mse_a and mse_b. From (6) and considering the different samples uncorrelated, we have:

$$VAR\left\{mse_a\right\} = \frac{1}{M^2}\sum_{i=1}^{M} VAR\left\{\left(za_i - \widehat{ya}_i\right)^2\right\}. \tag{26}$$

Using the same reasoning as in section III we can write

$$VAR\left\{mse_a\right\} = \\ = \frac{1}{M^2}\sum_{i=1}^{M} VAR\left\{\begin{array}{l} n_i^2 + q_i^2 + \\ +\left(2\pi f_a \delta_i A\right)^2 \sin^2\left(2\pi f_a t_i + \varphi\right) - \\ -2\left(n_i + q_i\right)\left(2\pi f_a \delta_i A\right)\sin\left(2\pi f_a t_i + \varphi\right) \end{array}\right\}. \tag{27}$$

Taking into account that n and δ are normal random variables with standard deviations of σ_n and σ_t respectively and q is uniformly distributed between –Q/2 and Q/2, we have

$$VAR\left\{n_i^2\right\} = 2\sigma_n^4,\ VAR\left\{q_i^2\right\} = \frac{Q^4}{180} \text{ and } VAR\left\{\delta_i^2\right\} = 2\sigma_t^4 . \tag{28}$$

Using (28), we can write (27) as:

$$VAR\left\{mse_a\right\} = \frac{2\sigma_n^4}{M} + \frac{Q^4}{180 \cdot M} + \\ + 2\sigma_t^4\left(2\pi f_a A\right)^4 \frac{1}{M^2}\sum_{i=1}^{M}\sin^4\left(2\pi f_a t_i + \varphi\right) + \\ + 4\left(\sigma_n^2 + \frac{Q^2}{12}\right)\sigma_t^2\left(2\pi f_a A\right)^2 \frac{1}{M^2}\sum_{i=1}^{M}\sin^2\left(2\pi f_a t_i + \varphi\right) . \tag{29}$$

Again we will consider that the acquisition of the input signal is carried out during an integer number of periods as was done in eq. (16). The first summation in (29) is, in those conditions (eq. (17)), given by

$$\frac{1}{M}\sum_{i=1}^{M}\sin^4\left(2\pi f_a t_i + \varphi\right) = \frac{3}{8}. \tag{30}$$

Introducing (18) and (30) into (29) leads to

$$VAR\left\{mse_a\right\} = \frac{2\sigma_n^4}{M} + \frac{Q^4}{180M} + 2\sigma_t^4\left(2\pi f_a A\right)^4 \frac{3}{8M} + \\ + 4\left(\sigma_n^2 + \frac{Q^2}{12}\right)\sigma_t^2\left(2\pi f_a A\right)^2 \frac{1}{2M} . \tag{31}$$

The same reasoning can be applied to compute the variance of mse_b which will lead to an expression similar to (31) with f_b in place of f_a:

$$VAR\left\{mse_b\right\} = \frac{2\sigma_n^4}{M} + \frac{Q^4}{180M} + 2\sigma_t^4\left(2\pi f_b A\right)^4 \frac{3}{8M} + \\ + 4\left(\sigma_n^2 + \frac{Q^2}{12}\right)\sigma_t^2\left(2\pi f_b A\right)^2 \frac{1}{2M} . \tag{32}$$

The following step is to compute the variance of $\hat{\sigma}_t^{\,2}$ from the variances of mse_a and mse_b. Using (24) we can write

$$\hat{\sigma}_t^{\,2} = \frac{mse_b - mse_a}{\left(\sqrt{2}\pi A\right)^2\left(f_b^2 - f_a^2\right)}. \tag{33}$$

Since the random variables mse_a and mse_b are independent, we can write

$$VAR\left\{\hat{\sigma}_t^{\,2}\right\} = \frac{VAR\left\{mse_b\right\} + VAR\left\{mse_a\right\}}{\left(\sqrt{2}\pi A\right)^4\left(f_b^2 - f_a^2\right)^2}. \tag{34}$$

Inserting (31) and (32) leads to

$$VAR\left\{\hat{\sigma}_t^{\,2}\right\} = \frac{4\sigma_n^4 + \dfrac{Q^4}{90} + \dfrac{3}{4}\sigma_t^4\left(2\pi A\right)^4\left(f_a^4 + f_b^4\right)}{\left(\sqrt{2}\pi A\right)^4\left(f_b^2 - f_a^2\right)^2 M} + \frac{2\left(\sigma_n^2 + \dfrac{Q^2}{12}\right)\sigma_t^2\left(2\pi A\right)^2\left(f_a^2 + f_b^2\right)}{\left(\sqrt{2}\pi A\right)^4\left(f_b^2 - f_a^2\right)^2 M} . \quad (35)$$

Finally we are going to compute the variance of $\hat{\sigma}_t$ from the variance of $\hat{\sigma}_t^{\,2}$ using [15, pp. 113]:

$$\sigma_y^2 \simeq \left|g'\left(\mu_x\right)\right|^2 \sigma_x^2 \quad \text{where} \quad y = g\left(x\right) . \quad (36)$$

Note that μ_x and σ_x are the mean and variance of x respectively. In our case $y = \sqrt{x}$ and thus

$$g'\left(\mu_x\right) = \frac{1}{2\sqrt{\mu_x}} . \quad (37)$$

Combining (35) with (36) and (37) where $x = \hat{\sigma}_t^{\,2}$ and $y = \hat{\sigma}_t$, and using (25), leads to

$$VAR\left\{\hat{\sigma}_t\right\} \simeq \frac{\sigma_n^4 + \dfrac{Q^4}{90} + \dfrac{3}{16}\sigma_t^4\left(2\pi A\right)^4\left(f_a^4 + f_b^4\right)}{\left(\sqrt{2}\pi A\right)^4\left(f_b^2 - f_a^2\right)^2 M\sigma_t^2} + \frac{\dfrac{1}{2}\left(\sigma_n^2 + \dfrac{Q^2}{12}\right)\sigma_t^2\left(2\pi A\right)^2\left(f_a^2 + f_b^2\right)}{\left(\sqrt{2}\pi A\right)^4\left(f_b^2 - f_a^2\right)^2 M\sigma_t^2} . \quad (38)$$

which can be written as

$$VAR\left\{\hat{\sigma}_t\right\} \simeq \frac{1}{M\left(\sqrt{2}\pi A\right)^4\left(f_b^2 - f_a^2\right)^2}\frac{\sigma_n^4 + \dfrac{Q^4}{90}}{\sigma_t^2} + \frac{12}{16M}\frac{\left(f_a^4 + f_b^4\right)}{\left(f_b^2 - f_a^2\right)^2}\sigma_t^2 + \frac{1}{2M\pi^2 A^2}\frac{\left(f_a^2 + f_b^2\right)}{\left(f_b^2 - f_a^2\right)^2}\left(\sigma_n^2 + \frac{Q^2}{12}\right) . \quad (39)$$

Equation (38) allows the computation of the variance of the estimated jitter standard deviation obtained with the proposed estimator (24). In the approximation $f_b \gg f_a$ the proposes estimator is equal to the IEEE 1057 estimator, eq. (8), and its variance is, from (38),

$$VAR\left\{\hat{\sigma}_t\right\} \underset{f_b \gg f_a}{\simeq} \frac{\sigma_n^4 + \dfrac{Q^4}{90}}{M\left(\sqrt{2}\pi A f_b\right)^4}\frac{1}{\sigma_t^2} + \frac{1}{M\left(\sqrt{2}\pi A f_b\right)^2}\left(\sigma_n^2 + \frac{Q^2}{12}\right) + \frac{3}{4M}\sigma_t^2 . \quad (40)$$

In Fig. 2 the result of Monte Carlo Simulations for the determination of the estimator standard deviation is presented. Again, 1000 repetitions were used. The result is depicted by the vertical bars which represent the confidence intervals [15, pp. 253]. It clearly supports the claim that the standard deviation of the jitter estimation obtained with (24) can be computed using the expression given in (39).

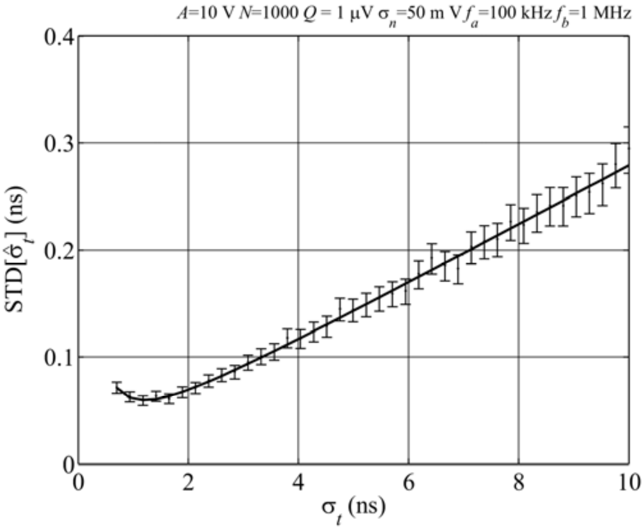

Fig. 2 – Representation of the standard deviation of the jitter estimation as a function of the actual standard deviation of the jitter. The solid line represents the values given by (39) and the vertical bars represent the result of Monte Carlo simulations of the jitter test method.

The conditions of the test are the same as those used for the validation of the estimator expected value (Tab. 1).

For very low values of jitter standard deviation, the approximations made using (36) cease to be valid because the jitter is not enough to always make the mean square error measured at high frequency (*mse*$_b$), higher than the its value when measured at low frequency (*mse*$_a$). We can empirically consider a threshold on the value of jitter for this situation, the minimum of the estimator standard deviation. This value can be obtained by calculating the derivative of (39) with respect to σ_t and equating it to 0. The result obtained is

$$\sigma_{t\min} = \sqrt[4]{\frac{\dfrac{Q^4}{90} + \sigma_n^4}{3\pi^4 A^4\left(f_b^4 - f_a^4\right)}} . \quad (41)$$

If we encounter an application where *mse*$_b$ is smaller that *mse*$_a$, we should increase the frequency difference $f_b^4 - f_a^4$ which will increase the value of *mse*$_b$ in relation to *mse*$_a$. This corresponds to pushing the minimum of the estimator standard deviation (Fig. 2) to the left (decrease in the value given by (41)).

VI. EXPERIMENTAL VALIDATION

In order to validate the results obtained so far, namely, that the estimator (24) is unbiased and that the standard uncertainty of that estimator can be computed using (38), we measured jitter in an actual ADC using the method under study.

Since we are interested in studying the statistical properties of the jitter estimator we need a setup where we can control the amount of jitter present. ADC jitter is a deterministic or random delay between the ideal sampling instants and the actual sampling instant (here we are just considering normally distributed random jitter). To be able to carry on our study we have to be able to accurately control the jitter present. It is not practical to have different ADCs with different values of jitter to test. We can, however, mimic the effect of jitter in the ADC with jitter in the transition time of the clock signal which controls the sampling. The two jitters are equivalent and, in fact, when we are measuring the ADC jitter with the test recommended in the IEEE standard, we are actually measuring both jitters (and also the stimulus signal phase noise). Here we are going to inject the desired amount of jitter in our test setup by controlling the phase noise of the clock signal produced by a Tektronix arbitrary function generator. The clock signal used was a square wave phase modulated by Gaussian noise generated by an Agilent function generator. In this way we can inject different amounts of jitter by controlling the power of the Gaussian noise produced.

The ADC under test is one embedded in a National Instruments data acquisition board, model NI 6023. This board was plugged into a PCI slot of a personal computer which is used to program the data acquisition board, store the acquired samples, control the 4 instruments used and obtain the jitter estimate using eq. (24). The test setup is depicted in Fig. 3. This data acquisition board used was chosen because it has an external input that can be used to connect a clock signal for the timing of the analog to digital conversions. Note that this is required because we want to inject jitter in our test setup for the purpose of studying the measurement method. For those just wanting to measure jitter, that is not necessary. That is why this method, although not the best in separating the different sources of jitter present in a test setup, is appropriate when measuring jitter on waveform recorders and oscilloscopes that generally do not have the capability of using an external clock.

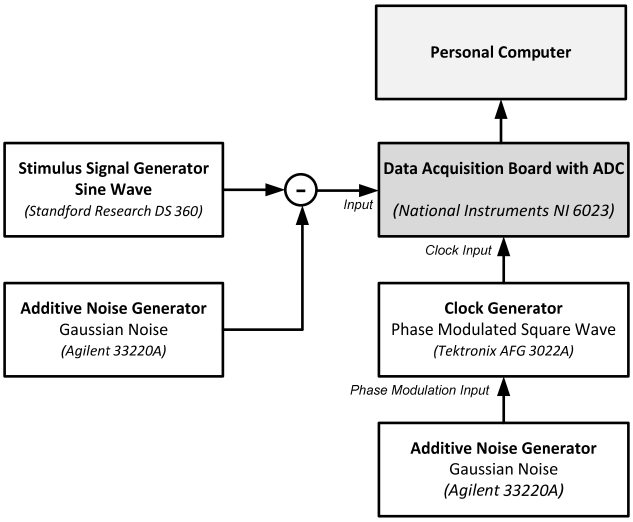

Fig. 3 – Test Bench. The personal computer controls all instruments through GPIB or USB interfaces. The combination of the sinusoidal stimulus signal and the normally distributed noise is carried out inside the data acquisition board through the use of one of its differential inputs.

A very low distortion sine wave generator from Stanford Research Systems was used to generate the signal used to stimulate the ADC. We also added a given amount of additive noise to the ADC input to mimic the presence of additive noise in the ADC. Using another Agilent function generator we added Gaussian noise to the sine wave by making use of the differential input of the data acquisition board.

We implemented the IEEE jitter test in National Instruments LabView. The application developed completely automates the test study, from test parameters calculation, to instrument control, data gathering and processing, graphical representation of results and Monte Carlo analysis.

Before carrying out the tests, there were two constants that had to be determined: the ratio between generated additive noise and the voltage noise present in the ADC (K_v), and the ratio between the additive noise generated and the amount of jitter present in the clock signal (K_t).

In the first case we adjusted the function generator to a given value of noise rms voltage and measured the standard deviation of noise in the ADC using the method described in IEEE 1057 for the estimation of random noise using sine fitting. Five measurements were made and linear regression was used to arrive at $K_v = 0.2344$ (correlation of 0.9994). This test was carried out with the internal clock of the data acquisition board set with a frequency of 100 kHz and a sine wave stimulus signal frequency of 25 kHz. The test was repeated for lower values of sine wave frequency but the value obtained for K_v was the same showing that the combined values of jitter present in the ADC and phase noise of the internal clock were negligible. The value obtained is close to the expected one that can be computed if we take into account the specification for the bandwidth of the National Instruments NI6023 data acquisition board, which is 500 kHz, and the Agilent AG33220A function generator specification for the bandwidth of noise, which is 9 MHz. These specifications and other relevant ones are presented in Tab. 2. By taking the square root of the ratio between these two numbers, we get

$$\sqrt{\frac{500}{9000}} = 0.2357 \tag{42}$$

which is very close to the measured value of 0.2344. The difference can be explained by measurement uncertainty and by considering that the noise bandwidth of 9 MHz specified by Agilent for the AG3322A is just an approximated value and that the 500 kHz bandwidth for the data acquisition board is the small signal bandwidth and not the equivalent noise bandwidth.

Tab. 2 – Manufacturer specifications for the instruments and ADC used.

Specification	***Value***
Stimulus Signal Generator (Stanford Research DS 360)	
Sine Wave Amplitude Accuracy	1 %
Sine Wave Offset Accuracy	1% + 25 mV
Sine Wave Frequency Accuracy	25 ppm+4 mHz
Total Harmonic Distortion	-93 dB
Clock Generator (Tektronix AFG 3022)	
Frequency Accuracy	1 ppm
Jitter	500 ps
External Phase Modulation Bandwidth	25 kHz
Degrees of Phase Modulation per control voltage unit	Not specified
Additive/Phase Noise Generator (Agilent AG 33220A)	
Noise Bandwidth	≈ 9 MHz
Amplitude Accuracy	1 % / 1 mVpp
Analog/Digital Converter (National Instruments 6023)	
Number of Bits	12
Number of Most Significant Bits Used	8
Integral Non Linearity (INL)	1.5 LSB
Differential Non Linearity (DNL)	1 LSB
Gain Error	0.02 %
Offset Error	0.5 mV
Bandwidth (small signal)	500 kHz
Additive Noise	0.1 LSB
Jitter	Not specified

To compute the second constant (K_t), we produced different values of additive noise rms voltage (5 points from 100 mV to 1 V), with the generator connected to the external modulation input of the clock generator and used a digital phosphor oscilloscope from Tektronix to measure the amount of jitter present in the clock (30000 transition measurements carried out for each of the 5 points). The linear regression gave a value of K_t = 266.82 ns/V (correlation of 1.0000). We can not determine an expected value for this constant from the instruments specifications because the modulation constant (º/V) of the Tektronix is not supplied in the specification sheets.

We also used this oscilloscope to draw the histogram of the measured values and by visual inspection concluded that it had a good Gaussian distribution. We tried to use the Tektronix AFG3022 arbitrary function generator to produce the additive noise instead of the Agilent AG33220A but it showed a poor statistical distribution of the noise voltages.

The statistical properties of the estimator were measured by repeating the jitter measurement in the same conditions a given number of times (R) and computing the average and standard deviation of the different values obtained. The results were compared with the theoretical ones given by (25) and (39). We carried out this analysis in several different conditions by varying the following parameters:

- Stimulus signal frequency;
- Stimulus signal amplitude;
- Sampling frequency;
- Quantization step;
- Additive noise standard deviation;
- Jitter standard deviation;
- Number of samples acquired.

In all cases where the jitter standard deviation was not too small, the experimental results were in agreement with the theoretical ones within confidence intervals obtained for a confidence level of 99.9%. Here we present the results for one set of those conditions that we judge were the most illustrative and representative of actual conditions. The values used can be found in Tab. 3.

Tab. 3 – Experimental Setup Settings.

Setting	***Value***
Sine Wave Amplitude (A)	4 V
Sine Wave Offset (C)	0
Low Sine Wave Frequency (f_a)	2.478 kHz
High Sine Wave Frequency (f_b)	24.9878 kHz
ADC Full Scale (FS)	5 V
ADC Quantization Step (Q)	39.0625 mV
Number of Acquired Samples (M)	8192
Sampling Frequency (f_s)	100 kHz
Injected Additive Noise (σ_n)	20 mV
Injected Clock Phase Noise (σ_t)	0 to 1.72 º
Number of Repetitions (R)	200
Confidence Level (ν)	99.9 %

The results obtained for the error of the estimation (difference between expected value and actual value) are depicted in Fig. 4 for a range of injected jitter from 0 to 300 ns (values of jitter standard deviation). We can see that all confidence intervals (vertical bars), with the exception of the first one (for the absence of jitter), are around 0 (theoretical value) which shows that the estimator is unbiased in those conditions.

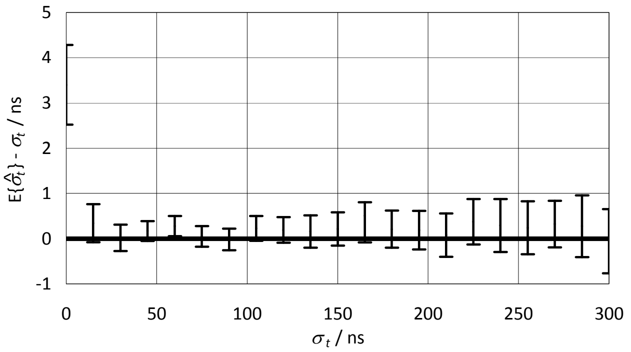

Fig. 4 – Representation of experimental results of the difference between the expected value of the jitter estimation and the injected jitter, as a function of the actual standard deviation of the jitter. The solid line represents the theoretical values, which are 0 in this case (unbiased estimator). The vertical bars represent the confidence intervals of the result of the Monte Carlo simulations of the jitter test method (99.9 % confidence level).

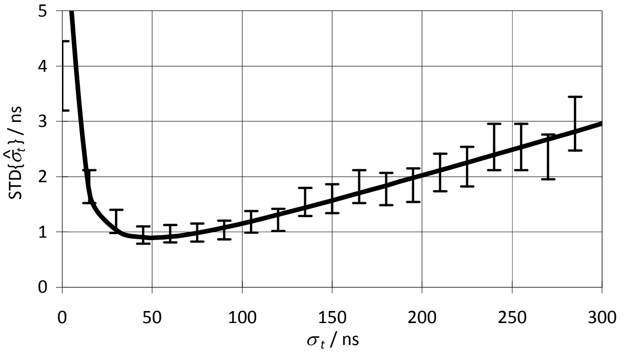

Fig. 5 – Representation of experimental results of the standard deviation of the jitter estimation as a function of the actual standard deviation of the jitter. The solid line represents the values given by (39). The vertical bars represent the confidence intervals of the result of the Monte Carlo simulations of the jitter test method (99.9 % confidence level).

The confidence interval of the first point is not around zero because the estimator (24) is biased when the amount of jitter is small because in some cases the value of mse_a will be higher that the value of mse_b. In those cases we can not take the square root. The same happens in Fig. 5 were we depict the standard deviation of the estimated jitter. We can see that the confidence intervals are all around the theoretical value given by (39) and depicted as a solid line.

These results validate the assumptions made in the conditions that were used in this paper (Tab. 3).

VII. Minimum Number of Samples Required

One of the important considerations when performing the jitter test is to know how many samples should be acquired. There is a compromise to be made between the test time and the estimation uncertainty. The higher the number of samples acquired (M), the lower will be the test result uncertainty, as can be seen in (38), but longer the test will take to complete, which, in the case of production line testing of ADCs, is critical.

Using the statistics calculated for the value of the jitter estimation we can determine a confidence interval inside which the true value of the measured jitter standard deviation is with a certain confidence level [17],

$$\hat{\sigma}_t - K_\nu\sqrt{VAR\{\hat{\sigma}_t\}} \le \sigma_t \le \hat{\sigma}_t + K_\nu\sqrt{VAR\{\hat{\sigma}_t\}}\,, \tag{43}$$

where K_ν is the coverage factor corresponding to a certain confidence level ν and which depends on the statistical distribution of the estimator.

In the case in study, the square of the estimator $\hat{\sigma}_t^2$ has a distribution which tends to a normal one as the number of samples tends to infinite. This is demonstrated by the Central Limit Theorem [15] applied to variables mse_a and mse_a which are the summation of a large number of random variables (eq. (6) and (7)).

The estimator (24) will thus have a statistical distribution that is the distribution of a variable which is the square root of a randomly distributed variable. Its probability density function can be obtained by [15, pp. 96]

$$f_{\hat{\sigma}_t}(y) = 2y f_{\hat{\sigma}_t^2}\left(y^2\right)U(y), \tag{44}$$

where $U(y)$ is 1 for positive y and 0 otherwise and

$$f_{\hat{\sigma}_t^2}(y) = \frac{1}{\sqrt{2\pi}\sigma_{\hat{\sigma}_t^2}} e^{-\frac{\left(y-\mu_{\hat{\sigma}_t^2}\right)^2}{2\sigma^2_{\hat{\sigma}_t^2}}} \tag{45}$$

is the Gaussian probability function. The probability density function of the jitter estimator is thus

$$f_{\hat{\sigma}_t}(y) = \frac{2yU(y)}{\sqrt{2\pi}\sigma_{\hat{\sigma}_t^2}} e^{-\frac{\left(y^2-\mu_{\hat{\sigma}_t^2}\right)^2}{2\sigma^2_{\hat{\sigma}_t^2}}}\,. \tag{46}$$

To compute the coverage factor we need the cumulative distribution function (cdf), $F(x)$, which by definition is

$$F(x) = \int_{-\infty}^{x} f(y)\,dy\,. \tag{47}$$

The cdf of the jitter estimator is thus

$$F_{\hat{\sigma}_t}(x) = \int_0^x \frac{2y}{\sqrt{2\pi}\sigma_{\hat{\sigma}_t^2}} e^{-\frac{\left(y^2-\mu_{\hat{\sigma}_t^2}\right)^2}{2\sigma^2_{\hat{\sigma}_t^2}}}\,dy\,. \tag{48}$$

Given a desired confidence level, we use (48) to find the coverage factor and hence the confidence interval. To simply the calculation of this interval we can use the fact that $F_{\hat{\sigma}_t}(x)$ is approximately equal to the cdf of a normal distribution with mean $\sqrt{\mu_{\hat{\sigma}_t^2}}$ and standard deviation $\sigma_{\hat{\sigma}_t^2}\Big/\left(2\sqrt{\mu_{\hat{\sigma}_t^2}}\right)$. In Fig. 6 we can see the difference between the two given by

$$CDF_{diff}\left(x\right)=\int_{0}^{x}\frac{2y}{\sqrt{2\pi}\sigma_{\widehat{\sigma}_t^2}}e^{-\frac{\left(y^2-\mu_{\widehat{\sigma}_t^2}\right)^2}{2\sigma^2_{\widehat{\sigma}_t^2}}}dy-$$

$$-\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}\frac{\sigma_{\widehat{\sigma}_t^2}}{2\sqrt{\mu_{\widehat{\sigma}_t^2}}}}e^{\frac{-\left(y-\sqrt{\mu_{\widehat{\sigma}_t^2}}\right)^2}{\frac{\sigma^2_{\widehat{\sigma}_t^2}}{\mu_{\widehat{\sigma}_t^2}}}}dy\quad. \tag{49}$$

In this example, and in general, as long as the mean minus 3 times the standard deviation is not close to 0, the difference is small which makes the use of normal distribution percentiles adequate to determine the coverage factor for this estimator. We have, for instance, K_ν = 2.58 for a 99% confidence level.

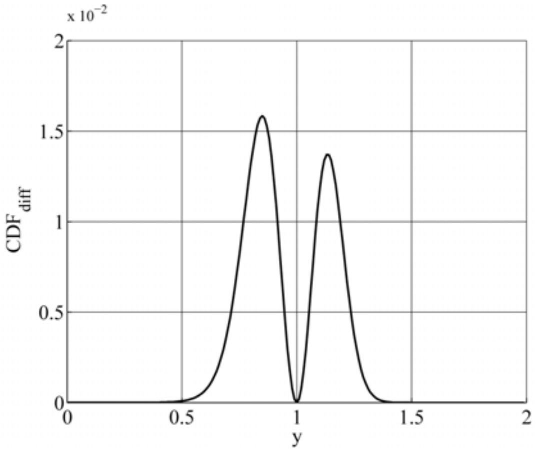

Fig. 6 – Representation of the difference between the cdfs of a normally distributed random variable and the cdf of the square root of a normally distributed variable. In this example the normal distributed variable has mean 1 and standard deviation 0.2.

In Fig. 7 we show the cdf of the estimated jitter computed from experimental values. The test conditions were the same as those used in section VI and the injected value of jitter was 200 ns. A good agreement with the cdf of a Gaussian distribution with the same mean (199.868 ns) and standard deviation (2.236 ns) is observed.

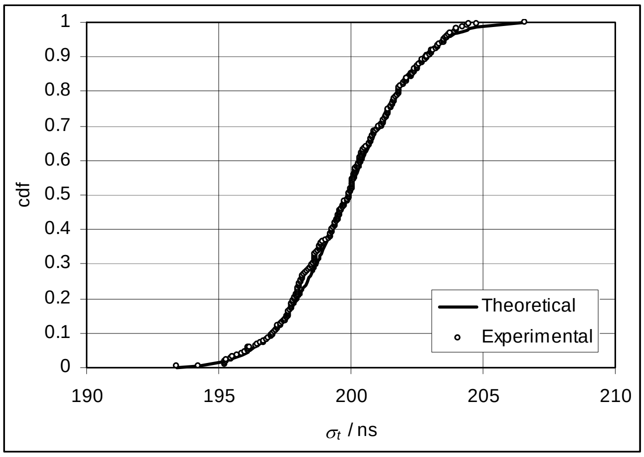

Fig. 7 – Representation of the cdf of the jitter estimator obtained with experimental data. The theoretical cdf of a normal random variable with the same mean and variance is depicted for comparison.

If we wish to have a given desired confidence interval with half-length B_t:

$$\hat{\sigma}_t-B_t\le\sigma_t\le\hat{\sigma}_t+B_t\,, \tag{50}$$

we need to have

$$K_\nu\cdot\sqrt{VAR\left\{\hat{\sigma}_t\right\}}\le B_t\,, \tag{51}$$

Introducing (38) into (51) we can derive an expression for the minimum number of samples required to achieve a certain bound on the estimation uncertainty:

$$M\ge\frac{K_\nu}{B_t^2}\frac{\sigma_n^4+\frac{Q^4}{90}+\frac{3}{16}\sigma_t^4\left(2\pi A\right)^4\left(f_a^4+f_b^4\right)+\frac{1}{2}\left(\sigma_n^2+\frac{Q^2}{12}\right)\sigma_t^2\left(2\pi A\right)^2\left(f_a^2+f_b^2\right)}{\left(\sqrt{2}\pi A\right)^4\left(f_b^2-f_a^2\right)^2\sigma_t^2}\,. \tag{52}$$

This expression is useful in avoiding the acquisition of the wrong number of samples for the application at hand. The use of a value too high will entail longer test duration while the use of a value too low will lead to greater uncertainty than desired.

VIII. CONCLUSIONS

We analyzed one of the test recommended by IEEE in [11-12] for the estimation of the jitter of waveform digitizers and ADCs. We concluded that the estimator suggested is biased if the frequencies used in the test do not satisfy $f_b \gg f_a$ (eq. (23)). We propose a new estimator which is unbiased whatever the value of the frequencies used: eq. (24). We derived an expression for determining the uncertainty of the jitter estimates made with the referred method in the presence of additive noise: eq. (38). Finally we presented an expression (eq. (52)) which is useful for optimizing the test by allowing the tester to know the minimum number of samples required to achieve a desired confidence interval on the estimates.

Several simplifying assumptions were made here which require further work in the future, namely, the study of what happens when:

- samples are acquired during a non integer number of periods of the stimulus signal;
- the amount of jitter is high when compared to the sampling period;
- quantization can not be treated as an error term independent of the stimulus signal.

As stated in the beginning, this work is just the first step in understanding the uncertainty of the jitter measurement method 4.9.2.2 (12.2.2 in the 2007 edition) of the IEEE 1057 standard [11-12]. Further research can be carried out on different uncertainty sources, namely:

- Harmonic distortion;

- Sine fitting parameters uncertainty;
- Stimulus signal and sampling clock frequency error.

ACKNOWLEDGMENT

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