The Histogram Test of ADCs with Sinusoidal Stimulus is Unbiased by Phase Noise

Francisco André Corrêa Alegria, *Member, IEEE,* António Manuel da Cruz Serra, *Senior Member, IEEE*

Abstract – ***This paper is intended to show that the presence of normally distributed phase noise or jitter in the test setup or in the ADC itself does not cause a bias in the Sinusoidal Histogram Test estimation of the transfer function of an ADC. The analytical proof presented demonstrates that there is no need to use extra overdrive when stimulating the ADC as is the case with amplitude noise.***

Index Terms – ***Analog to digital converter, histogram test, jitter, phase noise, bias.***

I. INTRODUCTION

The goal of the Histogram Test or Code Density Test [1] of Analog to Digital Converters (ADC) is to estimate its transfer function. The transfer function of an ADC is the relation between the input voltage and output code and is used to determine which was the value of the voltage at the ADC input, at the sampling instant, from the digital word (code) obtained. The transfer function of an ADC is fully defined by its transition voltages or, alternatively, by one of the transition voltages and the code bin widths.

The knowledge of the actual transition voltages of an ADC allows the quantification of converter gain and offset error, the integral and differential non-linearity (INL and DNL respectively), which give an important indication about the performance of the ADC and which are generally used when choosing one for a given application. Another advantage of knowing the ADC transfer function is the possibility of correcting its non ideal behavior which would allow the use of a converter that otherwise would not be suitable for the purpose intended.

The Histogram Test consists in stimulating the ADC with a sinusoidal signal and acquiring asynchronously a given number of samples. The result of the analog to digital conversion of those samples is then used to build an histogram which is the number of samples having each of the possible ADC output codes. By comparing this number with the number expected of an ideal ADC, the actual transition voltages can be estimated.

There are several factors that can have an influence on the accuracy of the results. They are, just to mention a few, frequency error of the stimulus signal, frequency error of the sampling clock, additive white noise, sampling instant jitter and phase noise in both the stimulus signal and the clock signal which controls the sampling. In this paper only the effect of jitter and phase noise is considered. Phase noise is the random variation of the phase of a periodic signal which can be caused by different mechanisms which depend on the oscillator architecture. In the case of a resonator-based (feedback loop) oscillator, for instance, they can be amplitude noise or flicker noise in the amplifier, flicker frequency or frequency random walk on the resonator or even noise in the oscillator output buffer. In the case of signal sources using Phased Locked Loop (PLL), other noise sources are present. Phase noise can affect the result of the ADC test by existing in the stimulus signal or in the sampling clock used. Jitter, on the other hand, is a fluctuation of the sampling instant of the ADC and can be due, among other effects, to the non idealities in the sample and hold circuit or variable delays in the signal path between clock oscillator and the sampling circuit due to electromagnetic interference, crosstalk or conductor imperfections. Since in the Histogram Test a periodic signal is being sampled, the effect of jitter is to change the actual phase of the samples acquired and as such produces the same effects as phase noise in the stimulus signal itself. Taking into account this and to simplify the presentation both effect will from now on be designated by "Phase Noise".

Phase noise and additive noise can have several sources which can be deterministic, like harmonic distortion, or random like thermal noise. In this paper we focus on the effects of thermal noise only since it is always present (at ambient temperatures) and one of the main contributions to the total noise. Future work should be done taking into account other noise sources.

Contrary to the effect of additive noise, which causes an error in the estimated transition voltages besides an increase in uncertainty [2-6], the presence of phase noise does not affect the accuracy of the results, causing only an increase in the uncertainty. This result, which will be demonstrated here, is important not only in the quantification of the quality of the ADC test results but also is choosing the test parameters. It has been demonstrated in the past that amplitude noise (additive noise) causes a bias

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in the estimation of the ADC transition voltages which are located in the extremes of the transfer function. Those regions correspond to the peaks of the sinusoidal signal used to stimulate the ADCs. To minimize this effect, overdrive is generally used. This technique consists in using a stimulus signal with amplitude greater than what would be strictly necessary to stimulate all codes of the ADC. This achieves a reduction in the error caused by amplitude noise. A similar concern is also legitimate regarding phase noise. Does it cause an error in the test estimates and if so does using extra overdrive has the same effect as with additive noise? The answer given here is that it does not cause a bias in the estimation and thus no extra overdrive is necessary in the presence of phase noise. This conclusion will be fully justified with analytical proof and with experimental results of ADC testing where additive and phase noise have been purposely added to the test setup.

The technical content of this paper is an extension and improvement of the paper presented at IEEE Instrumentation and Technology Conference (IMTC) in 2006 [9]. It contains a more detailed explanation of the analytical derivations presented including new figures to illustrate the procedures involved. It also reports experimental results fully validating the conclusion reached which were not available at the time of the IMTC conference. These additions are intended to increase the rigor of the mathematical derivations as well as the ease of understanding and the confidence in the conclusions.

In the next section (II) the theoretical framework used, and which was already published by the authors elsewhere [7] is reviewed, the aspects concerning the phase noise are highlighted. The following section (III) is used to demonstrate that the mean of the transition voltages is unbiased (no error is introduced). In section IV experimental results are reported that validate the conclusion presented throughout the paper. Finally some conclusions are extracted about the work presented (section V).

II. THEORETICAL FRAMEWORK

In the past [7] the authors developed a theoretical framework that allows the study of the influence of:

- Phase Noise and Jitter;
- Additive Noise;
- Frequency error of the stimulus signal;
- Frequency error of the sampling clock;
- Unknown initial sampling phase due to asynchronous sampling;

in the mean and variance of the transition voltages and code bin widths of an ADC estimated with the Traditional Histogram Test. This framework was an improvement over existing works since it took into account all the mentioned effects at the same time instead of analyzing the influence of each one separately and adding the results. This framework is now used here to study specifically the influence of phase noise.

The stimulus signal traditionally used is a sine wave because it is the shape that can be created with less distortion using currently available commercial equipment. The value of the sinusoidal stimulus signal in the sampling instant of sample j can be written as

$$x_j = d - A\cdot\cos\left(2\pi f\cdot t_j + \theta + \varphi\right) \tag{1}$$

where d and A are the stimulus signal offset and amplitude. The values of the sampled voltages (v_j) are equal to the value of the stimulus signal in the sampling instant (x_j) plus the input-equivalent wideband noise (n_v).

$$v_j = n_v + d - A\cdot\cos\left(2\pi f\cdot t_j + \theta + \varphi\right) \tag{2}$$

To simplify the computations, some normalizations are made here. Let u_j be the normalized sample voltage.

$$u_j = \frac{v_j - d}{A} = \frac{n_v}{A} - \cos\left(\gamma_j + \theta\right) = n - \cos\left(\gamma_j + \theta\right) \tag{3}$$

where n is the normalized input-equivalent wideband noise and γ_j is the sample phase in the absence of phase noise:

$$\gamma_j = 2\pi f\cdot t_j + \varphi\,. \tag{4}$$

The normalized value of the sinusoidal stimulus signal in the sampling instant of sample j is defined as

$$y_j = -\cos\left(\gamma_j + \theta\right). \tag{5}$$

Here we consider the presence of phase noise with a normal distribution with zero mean and standard deviation σ_θ,. Its probability density function (pdf) is

$$f_\theta\left(x\right) = \frac{1}{\sigma_\theta\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_\theta^2}}\,. \tag{6}$$

The pdf of y_j, determined in [7] is

$$f_{y_j}\left(y\,|\,\gamma_j\right) = \frac{1}{\sqrt{1-y^2}} \sum_{m=-\infty}^{\infty} \left[\begin{array}{l} f_\theta\left(\operatorname{acos}\left(-y\right) - \gamma_j + 2\pi m\right) + \\ + f_\theta\left(-\operatorname{acos}\left(-y\right) - \gamma_j + 2\pi m\right) \end{array}\right]. \tag{7}$$

Note that a constant phase error will not affect the results since the sampling is done asynchronously with the stimulus signal and thus the initial sampling phase is not known. For this reason there is no loss of generality when considering a phase noise distribution with a null mean.

In Fig. 1 and Fig. 2 this pdf is represented for a value of sample phase, γ_j, of 0.5π rad and 0.6π rad respectively. These values were chosen because they better illustrate this function and particularly its dependence on γ_j . Comparing the two it can be seen that the location of the maximum changes which is expected since it occurs at $y=\cos(\gamma_j)$.

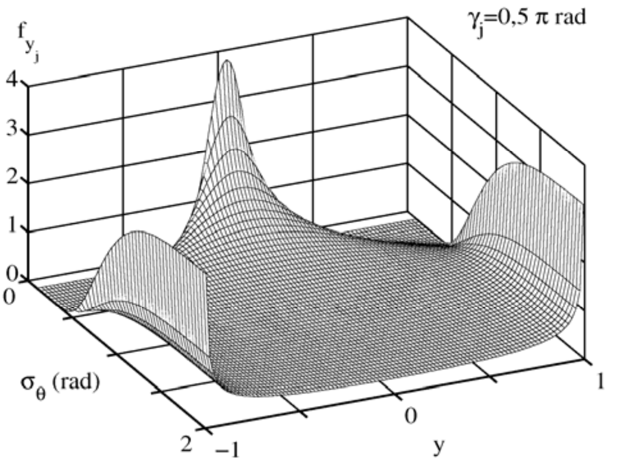

Fig. 1 Representation of the pdf of the value of the stimulus signal in the sampling instant for a sample phase $\gamma_j = 0.5\pi$ rad as a function of the standard deviation of the phase noise (σ_θ).Note that the case of null phase noise standard deviation ($\sigma_\theta = 0$) is not represented because it would imply an infinite value for the p.d.f. for y=0.

Considering also that the input-equivalent noise is normally distributed, with a null mean and a standard deviation σ_n, the pdf of the sample voltages (u_j) can be determined by convolving the pdf of the noise (f_n) with the pdf of the value of the stimulus signal in the sampling instant (f_{y_j}) since these variables are independent (the ideal stimulus signal is uncorrelated with thermal noise present in the test setup).

$$f_{u_j}\left(u \mid \gamma_j\right) = f_n * f_{y_j}\left(u\right) \tag{8}$$

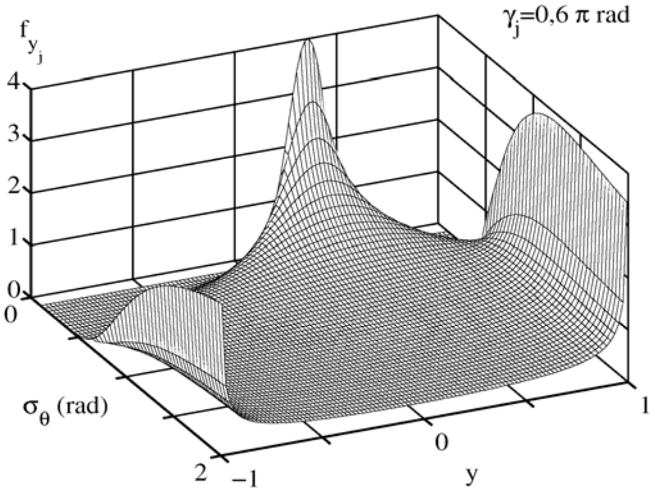

Fig. 2 Representation of the p.d.f. of the value of the stimulus signal in the sampling instant for a sample phase $\gamma_j = 0.6\pi$ rad as a function of the standard deviation of the phase noise (σ_θ).Note that the case of null phase noise standard deviation ($\sigma_\theta = 0$) is not represented because it would imply an infinite value for the p.d.f. for y=0.

Using (7) leads to

$$f_{u_j}(u \mid \gamma_j) = \int_{-1}^{1} \frac{f_n\left(u-y\right)}{\sqrt{1-y^2}} \cdot \sum_{i=-\infty}^{\infty} \left[\begin{array}{l} f_\theta\left(\arccos\left(-y\right) - \gamma_j + 2\cdot\pi\cdot i\right) + \\ + f_\theta\left(-\arccos\left(-y\right) - \gamma_j + 2\cdot\pi\cdot i\right) \end{array} \right] \cdot \mathrm{dy} \tag{9}$$

Substituting y by $-\cos(x)$ in the integral allows this expression to be simplified

$$f_{u_j}(u \mid \gamma_j) = \int_{0}^{\pi} f_n\left(u + \cos\left(x\right)\right) \sum_{i=-\infty}^{\infty} \left[\begin{array}{l} f_\theta\left(x - \gamma_j + 2\pi i\right) + \\ + f_\theta\left(-x - \gamma_j + 2\pi i\right) \end{array} \right] \mathrm{d}x \tag{10}$$

Equation (10) is plotted in Fig. 3 for different values of normalized sample voltage, u, and phase noise standard deviation, σ_θ.

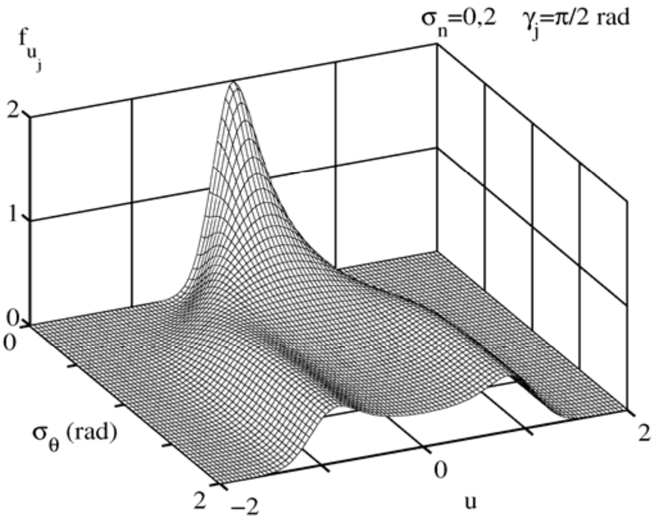

Fig. 3 Representation of the p.d.f. of the value of the sampled voltage, u, for a sample phase $\gamma_j = 0.5\pi$ rad and additive noise with 20% the stimulus signal amplitude ($\sigma_n = 0.2$) as a function of the standard deviation of the phase noise (σ_θ).

The probability that a sample j belongs to a class k of the cumulative histogram (p_k) is equal to the probability that the normalized sample voltage is lower than or equal to the normalized transition voltage $U[k+1]$ which, by definition, is the value of the probability distribution function of the sample voltage evaluated at $U[k+1]$:

$$p_k\left(\gamma_j\right) = F_{u_j}\left(U\left[k+1\right] \mid \gamma_j\right) = \int_{-\infty}^{U[k+1]} f_{u_j}\left(u \mid \gamma_j\right) \cdot du \tag{11}$$

In [7] the following expression was derived for the mean of the number of counts of the cumulative histogram:

$$\mu_{c_k} = \frac{M}{2\pi} \int_{-\pi}^{\pi} p_k\left(\varphi\right) d\varphi \tag{12}$$

The mean of the number of count of the cumulative histogram will later on be used to determine the mean of the estimated transition voltages and code bin widths.

III. ACCURACY OF THE TEST RESULTS

A. Mean of the number of counts of the cumulative histogram

The mean of the number of counts of the cumulative histogram, given by (12) can be written, using (11), as

$$\mu_{c_k} = \frac{M}{2\pi} \int_{-\pi}^{\pi} \int_{-\infty}^{U[k+1]} f_{u_j}\left(u \mid \varphi\right) \cdot du \cdot d\varphi \, . \tag{13}$$

Inserting (8) leads to

$$\mu_{c_k} = \frac{M}{2\pi}\int_{-\pi}^{\pi}\int_{-\infty}^{U[k+1]}\int_{-\infty}^{\infty} f_n\left(u-y\right) f_{y_j}\left(y\,|\,\varphi\right)\cdot dy\cdot du\cdot d\varphi\,. \quad (14)$$

The integral in u of the p.d.f. of the additive noise (f_n) is, by definition, its distribution function (F_n):

$$\mu_{c_k} = M\int_{-\infty}^{\infty} F_n\left(U\left[k+1\right]-y\right)\left[\frac{1}{2\pi}\int_{-\pi}^{\pi} f_{y_j}\left(y\,|\,\varphi\right)d\varphi\right]\cdot dy\;.(15)$$

where

$$F_n\left(x\right) = \frac{1}{2}+\frac{1}{2}erf\left(\frac{x}{\sqrt{2}\sigma_n}\right) \quad (16)$$

is the propability distribution function of a zero mean normal random variable with standard deviation σ_n.

The second term in square brackets is the total probability density function of y_j, the stimulus signal voltage at the sampling instant when the initial sampling phase, φ, is uniformly distributed in a 2π interval as is the case with asynchronous sampling:

$$f_{y_j}\left(y\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f_{y_j}\left(y\,|\,\varphi\right)d\varphi \quad (17)$$

Using (7) leads to

$$f_{y_j}\left(y\right) = \frac{\displaystyle\int_{-\pi}^{\pi}\sum_{m=-\infty}^{\infty}\begin{bmatrix} f_\theta\left(\text{acos}\left(-y\right)-\varphi+2\pi m\right)+ \\ +f_\theta\left(.\text{acos}\left(-y\right)-\varphi+2\pi m\right)\end{bmatrix}d\varphi}{2\pi\sqrt{1-y^2}}, |\,y\,|<1 \quad (18)$$

Careful inspection of the term in square brackets in (18) leads to the conclusion that the integral of the summation is equal to a single integral from –∞ to ∞:

$$f_{y_j}\left(y\right) = \frac{\displaystyle\int_{-\infty}^{\infty}\begin{bmatrix} f_\theta\left(\text{acos}\left(-y\right)-\varphi\right)+ \\ +f_\theta\left(-\text{acos}\left(-y\right)-\varphi\right)\end{bmatrix}\cdot d\varphi}{2\pi\sqrt{1-y^2}}\;,\;\; \left|y\right|<1. \quad (19)$$

Considering that each one of the fractions inside the square brackets is the p.d.f of a Gaussian distribution with mean acos(–y) and standard deviation σ_θ, the integral from –∞ to ∞ has the value 1:

$$f_{y_j}\left(y\right) = \frac{1}{\pi\sqrt{1-y^2}}\;,\;\; |\,y\,|<1\,. \quad (20)$$

Introducing in (16) leads to

$$\mu_{c_k} = \frac{M}{\pi}\int_{-1}^{1}\left[\frac{F_n\left(U\left[k+1\right]-y\right)}{\sqrt{1-y^2}}\right]\cdot dy\;. \quad (21)$$

Substituting y by –cos(x) leads to

$$\mu_{c_k} = \frac{M}{\pi}\int_{0}^{\pi}\left[F_n\left(U\left[k+1\right]+\cos\left(x\right)\right)\right]\cdot dx\,, \quad (22)$$

which proves that the mean of the number of counts of the cumulative histogram, does not depend on the phase noise present (σ_θ), but only on the additive noise (σ_n).

In Fig. 4 the mean of the number of counts of the cumulative histogram can be seen as a function of the additive noise standard deviation and of the transition voltage.

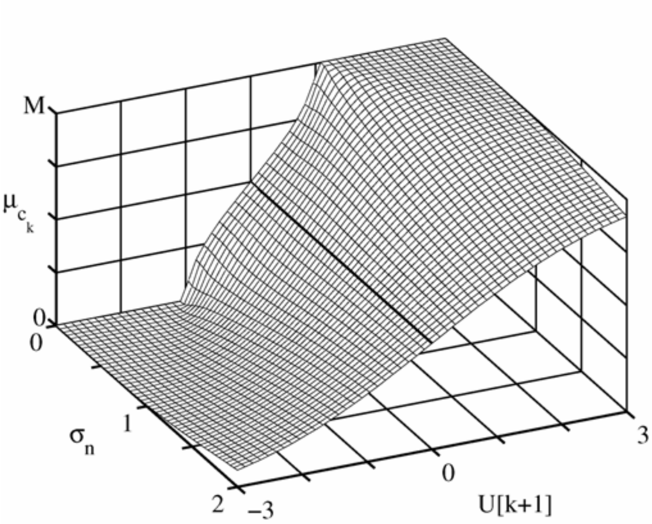

Fig. 4 Representation of the mean of the number of counts of the cumulative histogram as a function of the standard deviation of the additive noise (σ_n) and the normalized transition voltage $U[k+1]$.

B. *Mean of the transition voltages estimation*

The transition voltages are estimated from the cumulative histogram using [2]

$$T_{est}\left[k+1\right] = C - A\cos\left(\frac{\pi}{M}c_k\right). \quad (23)$$

The mean of the estimator (23) is approximately given by [8, pp. 113]:

$$\mu_{T_{est}[k+1]} = C - A\cos\left(\frac{\pi}{M}\mu_{c_k}\right)+\frac{\pi^2}{M^2}A\cos\left(\frac{\pi}{M}\mu_{c_k}\right)\frac{\sigma_{c_k}^2}{2}+\ldots \quad (24)$$

.

Considering the high number of samples traditionally acquired in the Histogram Test it is safe to neglect the last term represented in (24) resulting in:

$$\mu_{T_{est}[k+1]} = C - A\cos\left(\frac{\pi}{M}\mu_{c_k}\right). \quad (25)$$

In Fig. 5 the mean of the normalized transition voltage estimation

$$\mu_{U_{est}[k+1]} = \frac{\mu_{T_{est}[k+1]}-C}{A} = -\cos\left(\frac{\pi}{M}\mu_{c_k}\right), \quad (26)$$

is plotted as a function of additive noise standard deviation and normalized transition voltage.

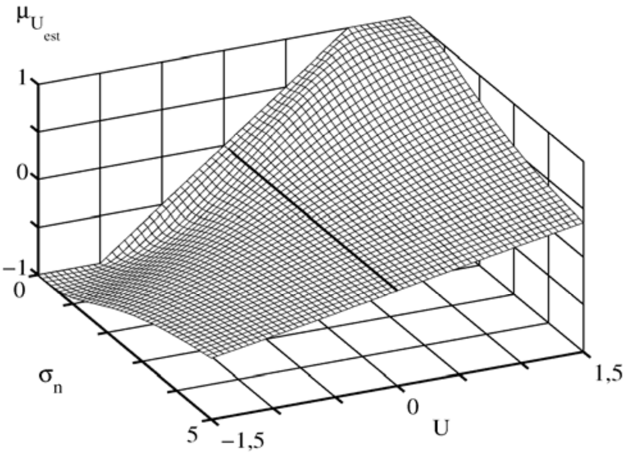

Fig. 5 Representation of the mean of the normalized transition voltages estimation as a function of the standard deviation of the additive noise (σ_n) and the normalized transition voltage $U[k+1]$.

Since the mean of the number of counts of the cumulative histogram is independent on the presence of phase noise, as demonstrated previously, the mean of the estimated transition voltages will also be unaffected by that kind of noise and thus (23) is an unbiased estimator with respect to the phase noise.

Note that since the code bin widths are estimated by subtracting the values of the estimates of two consecutive transitions voltages, their estimation is also unaffected by phase noise.

The same goes for the ADC gain and offset error, which are estimated from the transition voltages and thus are not biased by the presence of phase noise.

IV. MONTE CARLO SIMULATION

To validate the proof given in this paper, about the influence of phase noise in the estimates of transition voltages and code bin widths, the behavior of an ideal ADC was simulated in a computer.

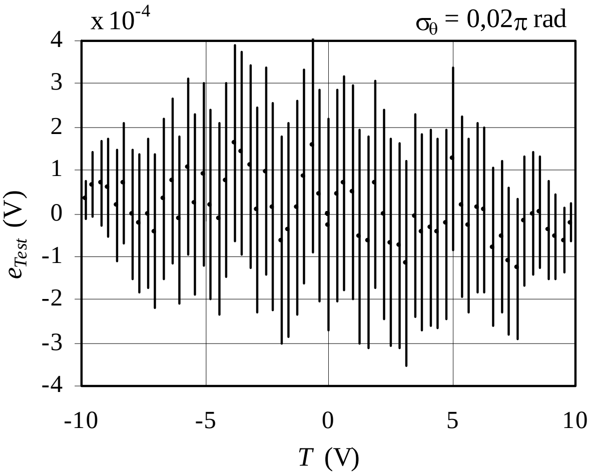

Fig. 6. Error of the estimated transition voltages using the Histogram Method and a simulated ideal ADC affected by phase noise.

The simulated ideal ADC having 6 bits and a full scale voltage of 10 V was tested with the Histogram Method using 10 000 samples and a sinusoidal stimulus signal with a 10 V amplitude.

The error in the estimation of the transition voltages was determined by subtracting the average of the estimated transition voltages from its ideal value. The number of points used to compute the mean was 200 000. The result for a simulated phase noise of 0.02π rad is plotted in Fig. 8. It can be seen from the 99.9% confidence intervals represented with vertical bars that the assumption of null error is validated.

The results for the estimated DNL, computed with

$$DNL_{est}\left[k\right]=\frac{G\cdot W_{est}\left[k\right]-Q}{Q}\,, \tag{27}$$

where Q is the ideal quantization step, G is the ADC gain and W is the step width given by

$$W_{est}\left[k\right]=T_{est}\left[k+1\right]-T_{est}\left[k\right]\,, \tag{28}$$

is plotted in Fig. 9 which also satisfies the affirmation that the phase noise does not cause a bias in the estimated DNL.

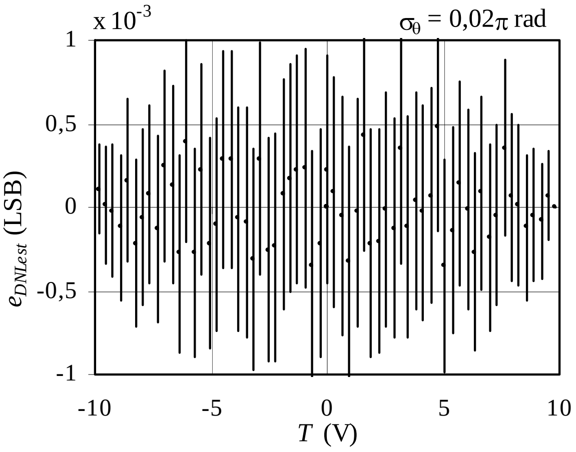

Fig. 7. Error of the DNL using the Histogram Method and a simulated ideal ADC affected by phase noise.

V. EXPERIMENTAL VALIDATION

To validate the proof given here about the influence of phase noise in the estimates of transition voltages of an ADC, several tests were carried out to estimate the error and precision of the Histogram Method results, namely the INL.

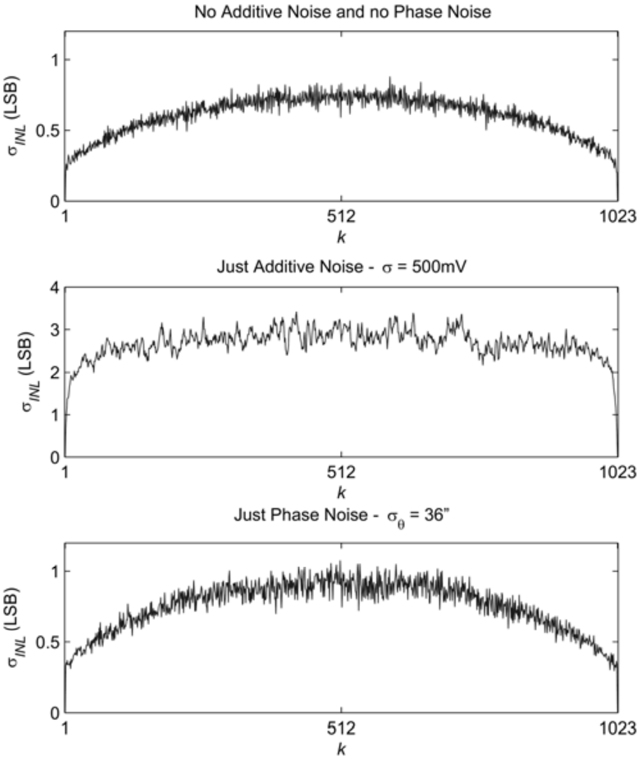

Fig. 8. Representation of the standard deviation of the estimated INL in the absence of noise (top) and in the presence of additive noise (middle) and phase noise (bottom).

The ADC tested was part of data acquisition board (DAQ) for a personal computer. The board used, a National Instruments NI6222 has differential inputs and can use for sampling an external clock signal. The test setup used consisted of three function generators. The first one was a Stanford Research DS360 used to produce the sinusoidal stimulus signal due to its low distortion. Another generator, an Agilent 33220A, was used to produce additive white Gaussian noise (AWGN) with a 20 MHz bandwidth. One of the differential input of the DAQ was used to add the sinusoidal signal with the AWGN. The control of the RMS value of the generated additive noise permitted to inject in the test setup different levels of additive noise. The third and final function generator, again an Agilent 33220A, was used to produce a clock signal of 200 kHz with rectangular shape from 0 to 5V. This function generator has the possibility to modulate the phase of the produced signal with white Gaussian noise. Controlling the RMS values of this noise it was possible to control the amount of phase noise in the clock signal in order to have different jitter values in the test setup. Three sets of tests were carried out. The first one with no additive noise or phase noise. The Histogram Test was carried out 100 times and the mean and standard deviation of the estimated terminal based INL was computed. The results obtained are depicted in the top plots of Fig. 8 and Fig. 9. The ADC was tested in its 10V range. The 16-bit ADC has a stated INL and DNL lower than 0.5 LSB. In order to make it possible to control the test conditions, namely the amount of additive noise and phase noise, only the 10 most significant bits were used. Taking into account the stated INL and DNL, the 10-bit ADC can thus be considered ideal. The stimulus signal used had an amplitude of 10.2814 V and a frequency of 1982.7 Hz. The Histogram Test was carried out with 1000 records of 807 samples each.

Looking at Fig. 8 and comparing the bottom and middle plots we observe the expected increase in standard deviation of the estimated INL when a noise with an RMS value of 500 mV was added to the test setup. The same is observed when just phase noise (36º RMS) was added by comparing the top and bottom plots.

Regarding the mean of the estimated INL we can see a bias in the middle plot of Fig. 9 as expected which is due to additive noise since it is not present in the top plot. Note that ideally, for the 10-bit ADC, the INL would be less than 0.01 ($\approx 0.5/2^6$) considering that the real 16-bit ADC as an INL lower than 0.5 LSB.

Finally looking at the bottom plot of Fig. 9 we see no evident bias in spite of the presence of phase noise. This is what is expected since, as was shown previously, phase noise and jitter do not cause an error in the estimation of the histogram test results contrary to what happens with additive noise.

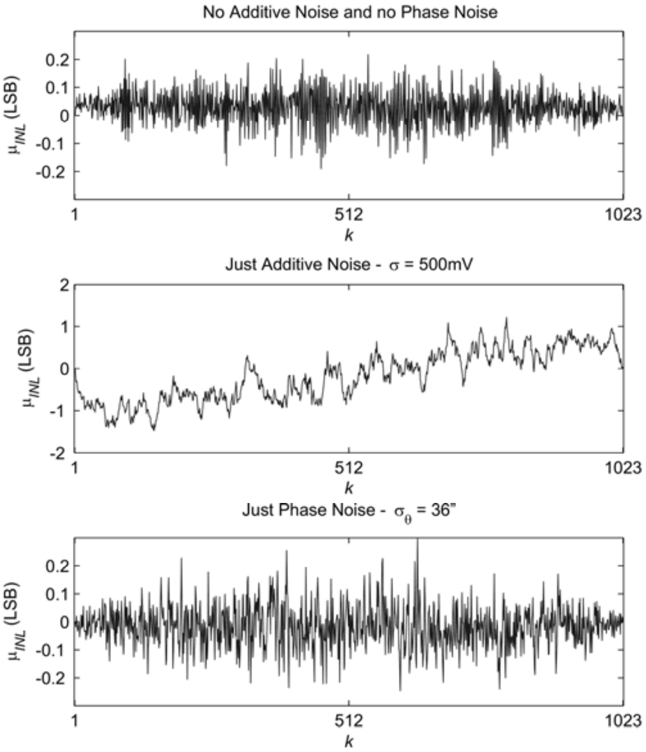

Fig. 9. Representation of the mean value of the estimated INL in the absence of oise (top) and in the presence of additive noise (middle) and phase noise (bottom).

VI. CONCLUSION

In this paper, a theoretical framework for the study of the influence of several factors on the uncertainty of ADC test results obtained with the Histogram Method, was used to show that the presence of phase noise or jitter does not cause a bias on the estimation of the ADC transfer function and its related characteristics: INL, DNL, gain and offset error.

This conclusion is important not only in quantifying the quality of the estimations obtained with the Histogram Method but also when choosing the test parameters by showing that no extra overdrive is needed due the presence of jitter or phase noise in the ADC under test or the test setup itself.

The analysis presented here is a new step in understanding the performance of the Histogram Test Method of ADC testing. It increases the already ample knowledge regarding its behavior in the presence of non idealities like amplitude noise, stimulus signal distortion and frequency error to the case of phase noise in the stimulus signal and sampling clock and jitter in the ADC. The better understanding of the method leads ultimately to a more efficient use in terms of cost and time.

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