Project Risk Analysis and Management Exercises (Part II, Chapters 6, 7)

Chapter II.6

Exercise 1
For the decision tree in Figure 1, assume Chance Events E and F are independent.

   a) Draw the appropriate decision tree and calculate the EVPI for Chance Event E only.
   b) Draw the appropriate decision tree and calculate the EVPI for Chance Event F only.
   c) Draw the appropriate decision tree and calculate the EVPI for both Chance Events E and F; that is, perfect information for both E and F is available before a decision is made.
   d) Draw the influence diagram that corresponds to the decision tree for Figure 1. How would this influence diagram be changed in order to answer parts a), b) and c).
   e) Is it necessary to assume that the events represented in these exercise are independent? What other assumption could be made?

Exercise 2
The claim was made in chapter II.6 that information always has a positive value. What do you think of this? Can you imagine any situation in which you would prefer not to have some unknown information revealed?

   a) Suppose you have just visited your physician because of a pain in your abdomen. The doctor has indicated some concern and has ordered some tests whose results the two of you ate expecting in a few days. A positive test result will suggest that you may have a life-
threatening disease, but even if the test is positive, the doctor would want to confirm it with further tests. Would you want the doctor to tell you the outcome of the test? Why or why not?

b) Suppose you are selling your house. Real-estate transaction laws require that you disclose what you know about significant structural defects. Although you know of no such defects, a buyer insists on having a qualified engineer inspect the house. Would you want to know the outcome of the inspection? Why or why not?

c) Suppose you are negotiating to purchase an office building. You have a lot of experience negotiating commercial real-estate deals, and your agent has explained this to the seller, who is relatively new to the business. As a result, you expect to do very well in this negotiation. Because of the unique circumstance of the building, your agent has suggested obtaining an appraisal of the property by a local expert. You know exactly how you would use this information; it would provide an upper bound on what you are willing to pay. This fact is also clear to the seller, who will know whether you obtained the appraisal but will only find out the appraised value if you elect to reveal it. Would you obtain the appraisal? Why or why not?

**Exercise 3**

Consider another oil-wildcatting problem. You have mineral rights on a piece of land that you believe to have oil underground. There is only a 10% chance that you will strike oil if you drill, but the payoff is $200,000. It cost $10,000 to drill. The alternative is not to drill at all, in which case your profit is zero.

a) Draw a decision tree to represent your problem. Should you drill?

b) Draw an influence diagram to represent your problem. How could you use the influence diagram to find EVPI?

c) Calculate EVPI. Use either the decision tree or the influence diagram.

d) Before you drill you might consult a geologist who can assess the promise of the price of land. She can tell you whether your prospects are “good” or “poor”. But she is not a perfect predictor. If there is no oil, the conditional probability is 0.95 that she will say prospects are good. If there is no oil, the conditional probability is 0.85 that she will say poor. Draw a decision tree that includes the “Consult Geologist” alternative. Be careful to calculate the appropriate probabilities to include in the decision tree. Finally, calculate the EVII for the geologist. If she charges $7000, what should you do?
Exercise 4
One of the principles that arises from a decision-analysis approach to valuing information is worthless if no possible informational outcome will change the decision. For example, suppose that you are considering whether to make a particular investment. You are tempted to hire a consultant recommended by your Uncle Jake (who just went bankrupt last year) to help you analyse the decision. If, however, you think carefully about the things that the consultant might say and conclude that you would (or not would) make the investment regardless of the consultant’s recommendation, then you should not hire the consultant. This principle makes perfectly good sense in the light of our approach; do not pay for information that cannot possibly change your mind.

In the medical area, however, it is standard practice for physicians to order extensive batteries of test for patients. Although different kinds of patients may be subjected to different overall sets of tests, it is nevertheless the case that many of these tests provide information that is worthless in a decision-analysis sense; the doctor’s prescription would be the same regardless of the outcome of a particular test.

Questions

1. As a patient, would you be willing to pay for such tests? Why or why not?
2. What incentives do you think the doctor might have for ordering such tests, assuming he realizes that his prescription would not change?
3. How do his incentives compare to yours?

Chapter II.7

Exercise 1
We have not given specific definition of risk. How would you define it? Give examples of lotteries that vary in riskiness in terms of your definition of risk.

Exercise 2
Explain in your own words the idea of a certainty equivalent.
Exercise 3
Suppose a decision maker has the utility function shown in table 1. An investment opportunity has EMV $1236 and expected utility 0.93. Find the certainty equivalent for this investment and the risk premium.

<table>
<thead>
<tr>
<th>Wealth</th>
<th>Utility value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2500</td>
<td>1.5</td>
</tr>
<tr>
<td>1500</td>
<td>1.24</td>
</tr>
<tr>
<td>1000</td>
<td>0.93</td>
</tr>
<tr>
<td>600</td>
<td>0.65</td>
</tr>
<tr>
<td>400</td>
<td>0.47</td>
</tr>
<tr>
<td>0</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Exercise 4
A decision maker’s assessed risk tolerance is $1210. Assume that this individual’s preferences can be modelled with an exponential utility function.

a) Find $U(1000)$, $U(800)$, $U(0)$ and $U(-1250)$.

b) Find the expected utility for an investment that has the following payoff distribution:
   \[ P(1000)=0.33 \]
   \[ P(800)=0.21 \]
   \[ P(0)=0.33 \]
   \[ P(-1250)=0.13 \]

c) Find the exact certainty equivalent for the investment and the risk premium.

d) Find the approximate certainty equivalent using the expected value and variance of the payoffs.
e) Another investment possibility has expected value $2400 and standard deviation $300. Find the approximate certainty equivalent for this investment.

**Exercise 5**
Many firms evaluate investment projects individually on the basis of expected value and at the same time maintain diversified holdings in order to reduce risk. Does this make sense in light of our discussion of risk attitudes in chapter II.7?

**Exercise 6**
This problem is related to the idea of dominance that we have discussed during the course. Investment D below is said to show “second-order stochastic dominance” over investment C. In this problem, it is up to you to explain why D dominates C.

You are contemplating two alternatives uncertain investments, whose distributions for payoffs are as below.

<table>
<thead>
<tr>
<th>Probabilities</th>
<th>Payoff</th>
<th>Investment C</th>
<th>Investment D</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1/3</td>
<td></td>
<td>¼</td>
</tr>
<tr>
<td>100</td>
<td>1/3</td>
<td>½</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>1/3</td>
<td></td>
<td>¼</td>
</tr>
</tbody>
</table>

a) If your preference function is given by $U(x)=1-e^{-x/100}$, calculate EU for both C and D. Which would you choose?

b) Plot the CDFs for C and D on the same graph. How do they compare? Use the graph to explain intuitively why any risk-averse decision maker would prefer D. (Hint: Think about the concave shape of a risk-averse utility function.)

**Exercise 7**
Utility functions need not relate to dollar values. Here is a problem in which we know little about five abstract outcomes. What is important, however, is that a person who does know what A to E represent should be able to compare the outcomes using the lottery procedures we have studied.

A decision maker faces a risky gamble in which she may obtain one of five outcomes. Label outcomes A, B, C, D and E. A is the most preferred, and E is least preferred. She has made the following three assessments.
• She is indifferent between having C for sure or a lottery in which she wins A with probability 0.5 or E with probability 0.5
• She is indifferent between having B for sure or a lottery in which she wins A with probability 0.4 or C with probability 0.6
• She is indifferent between these two lotteries:
  1. A 50% chance at B and a 50% chance at D
  2. A 50% chance at A and a 50% chance at E
What are U(A), U(B), U(C), U(D) and U(E)?

**Exercise 8**
You have considered insuring a particular item of property (such as an expensive camera, your computer, or your Stradivarius violin), but after considering the risk and the insurance premium quoted, you have no clear preference for either purchasing the insurance or taking the risk. The insurance company then tells you about a new scheme called “probabilistic insurance”. You pay half the above premium but have coverage only in the sense that in the case of a claim there is a probability of one-half that you will be asked to pay the other half of the premium and will be completely covered, or that you will not be covered and will have your premium returned. The insurance company can be relied on to be fair in flipping the coin to determine whether or not you are covered.

  a) Do you consider yourself risk-averse?
  b) Would you purchase probabilistic insurance?
  c) Draw a decision tree for this problem.
  d) Show that a risk-averse individual always should prefer the probabilistic insurance.

(Hint: this is a difficult problem. To solve it you must be sure to consider that you are indifferent between the regular insurance and no insurance. Write out the equation relating these two alternatives and see what it implies. Another strategy is to select a specific utility function – the log utility function \( U(x)=\log(x) \), say – and then find values for the probability of claim, your wealth, the insurance premium, and the value of your piece of property so that the utility of paying the insurance premium is equal to the expected utility of no insurance. Now use these values to calculate the expected utility of the probabilistic insurance. What is the result?)

**Exercise 9**
Buying and selling prices for risky investments obviously are related to certainty equivalents. This problem, however, shows that the prices depend on exactly what is owned in the first place!
Suppose that Peter Brown’s utility for total wealth (A) can be represented by the utility function $U(A) = \ln(A)$. He currently has a $1000 in cash. A business deal of interest to him yields a reward of $100 with probability 0.5 and $0 with probability 0.5.

a) If he owns this business deal in addition to the $1000, what is the smallest amount for which he would sell the deal?

b) Suppose he does not own the deal. What equation must be solved to find the largest amount he would be willing to pay for the deal?

c) For part b, it turns out that the most he would pay is $48.75, Which is not exactly the same as the answer is part a. Can you explain why the amounts are different?

d) Solve your equation in part b to verify the answer ($48.75) given in part c.

Exercise 10
We discussed decreasing and constant risk aversion. Are there other possibilities? Think about this as you work through this problem.

Suppose that a person’s utility function for total wealth is

$$U(A) = 200A - A^2 \text{ for } 0 \leq A \leq 100$$

Where $A$ represents total wealth in thousands of dollars.

a) Graph this preference function. How would you classify this person with regard to her attitude toward risk?

b) If the person’s total assets are currently $10K, should she take a bet in which she will $10K with probability 0.6 and lose $10K with probability 0.4?

c) If the person’s total assets are currently $90K, should she take the bet given in part b?

d) Compare your answers to parts b and c. Does the person’s betting behaviour seem reasonable to you? How could you intuitively explain such behaviour?