How to make a Quantum Computer

- There are four local forces in the Universe
  - Electromagnetic force
  - Gravitational force
  - Strong nuclear force
  - Weak nuclear force

- Nonlocal force
  - Quantum collapse during measurement
Information is represented in a quantum computer by qubits

By teleportation we can transfer quantum information inside a quantum computer or indeed between quantum computers

No teleportation theorem

- A classical information channel can not transmit quantum information
  - Remember: no cloning theorem?
  - Quantum states can not be copied!
- A quantum state cannot be determined via a single measurement
- Once converted to classical information, quantum information cannot be recovered
How to teleport a Qubit

- Alice wants to teleport a qubit to Bob
- A qubit may represent a superposition
  - Alice can not make any measurement
- A qubit corresponds to a particle
  - Alice wants that Bob receives an exact replica of the particle

The state of a particle corresponds to a qubit which represents a simple 2-state quantum system

The state of particle can be written as representing the spin state
Our quantum teleportation scheme requires Alice and Bob to share a maximally entangled state beforehand, for instance one of the four Bell states.

The four Bell states form an orthonormal basis:
- also called the Bell operator basis
- Particle $A$ and $B$

\[ |\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B) \]
\[ |\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B) \]
\[ |\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B) \]
\[ |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B) \]

Alice takes one of the particles in the pair, and Bob keeps the other one.
The subscripts $A$ and $B$ in the entangled state refer to Alice's or Bob's particle.
We will assume that Alice and Bob share the entangled state $|\Phi^+\rangle$.
Alice has two particles, the one she wants to teleport $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and $A$ one of the entangled pair

Bob has one particle, $B$

In the total system, the state of these three particles is given by

$|\psi\rangle \otimes |\Phi^+\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$

$= \frac{\alpha}{\sqrt{2}}(|000\rangle + |011\rangle) + \frac{\beta}{\sqrt{2}}(|100\rangle + |111\rangle)$

$= \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\beta}{\sqrt{2}}|100\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|111\rangle$

Alice will then make a partial measurement in the Bell basis on the two qubits in her possession
The Bell basis describes four orthogonal states
By the measurement she will couple her both particles
She will indicate which, of the four orthogonal states was present

**Bell basis** describes four orthogonal states

\[
\begin{align*}
|\phi^+\rangle &= \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \\
|\phi^-\rangle &= \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle) \\
|\psi^+\rangle &= \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle) \\
|\psi^-\rangle &= \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)
\end{align*}
\]
The three particle state shown above thus becomes the following four-term superposition in the new basis:

\[
\frac{1}{2}(|\Phi^+\rangle \otimes (\alpha|0\rangle + \beta|1\rangle) + |\Phi^-\rangle \otimes (\alpha|0\rangle - \beta|1\rangle) + |\Psi^+\rangle \otimes (\beta|0\rangle + \alpha|1\rangle) + |\Psi^-\rangle \otimes (-\beta|0\rangle + \alpha|1\rangle)).
\]

The actual teleportation starts when Alice measures her two qubits in the Bell basis.

We have done a change of basis on Alice's part of the system into the orthogonal basis (Bell state)

- No operation has been performed and the three particles are still in the same state.
Alice's two particles are now entangled to each other, in one of the four Bell states.

The entanglement originally shared between Alice's and Bob's is now broken. Bob's particle takes on one of the four superposition states shown above.

Note how Bob's qubit is now in a state that resembles the state to be teleported:

- $|\Phi^+\rangle \otimes (\alpha|0\rangle + \beta|1\rangle)$
- $|\Phi^-\rangle \otimes (\alpha|0\rangle - \beta|1\rangle)$
- $|\Psi^+\rangle \otimes (\beta|0\rangle + \alpha|1\rangle)$
- $|\Psi^-\rangle \otimes (-\beta|0\rangle + \alpha|1\rangle)$

Bob's particle takes on one of the four superposition states shown above.

The four possible states for Bob's qubit are unitary images of the state to be teleported.

Alice now has complete knowledge of the state of the three particles; the result of her Bell measurement tells her which of the four states the system is in.

- She simply has to send her results to Bob through a classical channel.
- Two classical bits can communicate which of the four results she obtained.
After Bob receives the message from Alice, he will know which of the four states his particle is in.

Using this information, he performs a unitary operation on his particle to transform it to the desired state:

\[ \alpha |0\rangle + \beta |1\rangle \]

If Alice indicates her result is \( \Phi^+ \), Bob knows his qubit is already in the desired state and does nothing.

This amounts to the trivial unitary operation, the identity operator.
If the message indicates $|\psi^\rangle$, Bob would send his qubit through the unitary gate given by the Pauli matrix

$$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

to recover the state

If Alice's message corresponds to $|\psi^\rangle$, Bob applies the operator

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

to his qubit
Finally, for the remaining case, the appropriate transformation is given by

\[
\sigma_3\sigma_1 = i\sigma_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.
\]

**Computer Simulation**

- Quantum communication channel is established

![Diagram of quantum circuit](image)
The unknown state is entangled with the quantum communication channel (XOR, R operator)

- for unknown state $a|0\rangle + b|1\rangle$ we get

$$\text{unknownState} = 0.5 \ket{0} + 0.866025 \ket{1}$$

$$0.25 \ket{0, 0, 0} + 0.433013 \ket{0, 0, 1} + 0.433013 \ket{0, 1, 0} + 0.25 \ket{0, 1, 1} - 0.25 \ket{1, 0, 0} + 0.433013 \ket{1, 0, 1} + 0.433013 \ket{1, 1, 0} - 0.25 \ket{1, 1, 1}$$

- Next Alice measures the bits on lines 1 and 2

$$0.866025 \ket{1, 1, 0} - 0.5 \ket{1, 1, 1}$$

- Bit of line 1 and 2 are both one
All possible values could be:

\[
{\{1, 0\}, -0.5 \kappa[1, 0, 0] + 0.866025 \kappa[1, 0, 1]} \\
{\{1, 1\}, 0.866025 \kappa[1, 1, 0] - 0.5 \kappa[1, 1, 1]} \\
{\{0, 1\}, 0.866025 \kappa[0, 1, 0] + 0.5 \kappa[0, 1, 1]}
\]

Bob receives Alice's 2-bit classical message \(\{1, 1\}\) and immediately converts those bits to corresponding kets for input into the quantum circuit:

- \(0.5 \kappa[1, 1, 0] + 0.866025 \kappa[1, 1, 1]\)

\[
\begin{array}{c}
\text{0.5 ket}[0] + 0.866025 \text{ ket}[1] \\
\end{array}
\]
No-communication theorem

- Instantaneous transfer of information between two observers is impossible
- Shared entanglement alone cannot be used to transmit quantum information
- Otherwise one could transfer information backward in time

Any real quantum computer is going to incur kinds of errors caused by myriad physical processes such as decoherence, cosmic radiation, and spontaneous emission
- Difficulties in maintaining a state
- Preserving entangled particles until they are needed for quantum teleportation
Mach-Zehnder interferometer is a particularly simple device for demonstrating interference by division of amplitude.

A light beam is first split into two parts by a beam splitter and then recombined by a second beam splitter.

**Mach-Zehnder Interferometer**
- Only one photon is emitted
- Several experiments are repeated
- The path the photon chooses ↑ or → is represented by superposition
- The half mirror $H$ acts like a Hadamard operator
  \[
  H|\rightarrow\rangle = \frac{1}{\sqrt{2}}|\rightarrow\rangle + \frac{1}{\sqrt{2}}|\uparrow\rangle
  \]
  \[
  H\left(\frac{1}{\sqrt{2}}|\rightarrow\rangle + \frac{1}{\sqrt{2}}|\uparrow\rangle\right) = |\rightarrow\rangle
  \]
Mach-Zehnder Interferometer

Many candidates how to build a quantum computer

The large number of candidates shows explicitly that the topic, in spite of rapid progress, is still in its infancy

- D-Wave Systems Inc. claims to be the world’s first — and only — provider of quantum computing systems designed to run commercial applications

  - However, since D-Wave Systems has not released the full details many experts in the field have expressed skepticism
- 2001, IBM Test-Tube Quantum Computer
  - (Isaac Chuang and Costantino Yannoni)
  - Seven-qubit quantum computer that solved factorization of the number 15 using the Shor's Algorithm

- Custom-designed molecules in a test tube representing 7 qubits
Heteropolymer-Based

- Heteropolymer-Based Quantum Computers
- Idea behind the heteropolymer computer is to use a linear array of atoms as memory cells
- Each atom can be either in an excited or grounded state
- This gives the basis for a binary arithmetic

Software consists of a sequence of laser pulses of particular frequencies that induce transitions of particular frequencies that induce transitions in certain atoms of the polymer
A molecular digital computer that relies on transitions among energy levels in atoms to switch states.

Each atom has three energy levels:

- State 0 is the ground state and represents bit 0.
- State 1 is a metastable state and represents bit 1.
- State 2 is a rapidly decaying exited state either to 0 or 1.
Ion Trap-Based

- The Cirac-Zoller scheme uses a linear array of trapped ions as the basis for quantum memory register
  - The trapping is arranged by electromagnetic fields, logical states of the qubits encoded in the energy states of the individual ions and the vibration states between the ions

Each ion is considered as a 2-state system containing a ground state and excited state
- The ions are arranged in a linear array such that each ion can be irradiated with light from a laser
- Laser pulses have the effect of exciting specific transitions in specific ions allowing the array to be placed in arbitrary superposed states
NMR-Based

- Adapt Nuclear Magnetic Resonance techniques to accomplish the basic operations of a quantum computer
- Consists of a test-tube sized sample of some liquid, with each molecule of this liquid acting as an independent quantum memory register

- We would not measure the observables of a register
- Measure the ensemble average of all the nuclear spins in the sample