Internal Flow: Heat Transfer Correlations
Fully Developed Flow

• Laminar Flow in a Circular Tube:
  The local Nusselt number is a constant throughout the fully developed region, but its value depends on the surface thermal condition.
  – Uniform Surface Heat Flux \( (q'_w) \):
    \[
    Nu_D = \frac{hD}{k} = 4.36
    \]
  – Uniform Surface Temperature \( (T_s) \):
    \[
    Nu_D = \frac{hD}{k} = 3.66
    \]

• Turbulent Flow in a Circular Tube:
  – For a smooth surface and fully turbulent conditions \((Re_D > 10,000)\), the Dittus – Boelter equation may be used as a first approximation:
    \[
    Nu_D = 0.023 \left( \frac{Re_D}{1000} \right)^{4/5} \Pr^n
    \]
    \[
    \begin{align*}
    n &= 0.3 \quad (T_s < T_m) \\
    n &= 0.4 \quad (T_s > T_m)
    \end{align*}
    \]
  – The effects of wall roughness and transitional flow conditions \((Re_D > 3000)\) may be considered by using the Gnielinski correlation:
    \[
    Nu_D = \frac{(f/8)(Re_D - 1000)\Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}
    \]
Smooth surface:

\[ f = (0.790 \ln \text{Re}_D - 1.64)^{-2} \]

Surface of roughness \( e > 0 \):

\[ f \rightarrow \text{Figure 8.3} \]

• Noncircular Tubes:
  – Use of \textit{hydraulic diameter} as characteristic length:
    \[ D_h \equiv \frac{4A_c}{P} \]
  – Since the local convection coefficient varies around the periphery of a tube, approaching zero at its corners, correlations for the fully developed region are associated with convection coefficients averaged over the periphery of the tube.

  – Laminar Flow:
    The local Nusselt number is a constant whose value (Table 8.1) depends on the surface thermal condition \( T_s \) or \( q_s \) and the duct aspect ratio.

  – Turbulent Flow:
    As a first approximation, the Dittus-Boelter or Gnielinski correlation may be used with the hydraulic diameter, irrespective of the surface thermal condition.
Effect of the Entry Region

- The manner in which the Nusselt decays from inlet to fully developed conditions for laminar flow depends on the nature of thermal and velocity boundary layer development in the entry region, as well as the surface thermal condition.

– Combined Entry Length:
  Thermal and velocity boundary layers develop concurrently from uniform profiles at the inlet.
– Thermal Entry Length:

Velocity profile is fully developed at the inlet, and boundary layer development in the entry region is restricted to thermal effects. Such a condition may also be assumed to be a good approximation for a uniform inlet velocity profile if \( \text{Pr} \gg 1 \). Why?

Average Nusselt Number for Laminar Flow in a Circular Tube with Uniform Surface Temperature:

– Combined Entry Length:

\[
\left[ \frac{\text{Re}_D \text{Pr}}{(L/D)} \right]^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14} > 2: \\
\overline{Nu}_D = 1.86 \left( \frac{\text{Re}_D \text{Pr}}{L/D} \right)^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14}
\]

\[
\left[ \frac{\text{Re}_D \text{Pr}}{(L/D)} \right]^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14} < 2: \\
\overline{Nu}_D = 3.66
\]

– Thermal Entry Length:

\[
\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)\text{Re}_D \text{Pr}}{1 + 0.04[(D/L)\text{Re}_D \text{Pr}]^{2/3}}
\]
Average Nusselt Number for Turbulent Flow in a Circular Tube:

- Effects of entry and surface thermal conditions are less pronounced for turbulent flow and can be neglected.
- For long tubes ($L/D > 60$)
  \[
  \overline{Nu}_D \approx Nu_{D,fd}
  \]
- For short tubes ($L/D < 60$)
  \[
  \frac{\overline{Nu}_D}{Nu_{D,fd}} \approx 1 + \frac{C}{(L/D)^m}
  \]
  \[
  C \approx 1 \quad m \approx 2/3
  \]

• Noncircular Tubes:
- Laminar Flow:
  \[
  \overline{Nu}_{D,\text{nl}}
  \]
  depends strongly on aspect ratio, as well as entry region and surface thermal conditions.
- **Turbulent Flow:**
  As a first approximation, correlations for a circular tube may be used with $D$ replaced by $D_h$.

- When determining $N_{u,D}$ for any tube geometry or flow condition, all properties are to be evaluated at

$$\overline{T}_m \equiv \frac{(T_{m,i} + T_{m,o})}{2}$$

Why do solutions to internal flow problems often require iteration?
The Concentric Tube Annulus

- Fluid flow through region formed by concentric tubes.

- Convection heat transfer may be from or to inner surface of outer tube and outer surface of inner tube.

- Surface thermal conditions may be characterized by uniform temperature \( T_{s,i}, T_{s,o} \) or uniform heat flux \( q_i, q_o \).

- Convection coefficients are associated with each surface, where

\[
q_i = h_i (T_{s,i} - T_m)
\]
\[
q_o = h_o (T_{s,o} - T_m)
\]
• Fully Developed Laminar Flow
  Nusselt numbers depend on $\frac{D_i}{D_o}$ and surface thermal conditions (Tables 8.2, 8.3)

• Fully Developed Turbulent Flow
  Correlations for a circular tube may be used with $D$ replaced by $D_h$.

\[ Nu_i \equiv \frac{h_i D_h}{k} \quad Nu_o \equiv \frac{h_o D_h}{k} \]

\[ D_h = D_o - D_i \]