DM DEPARTAMENTO DE MATEMÁTICA TÉCNICO LISBOA

Reliability and Quality Control

LMAC, MMA

2nd. Semester – 2023/2024
2024/05/21 — 17:10
MAP30 #4

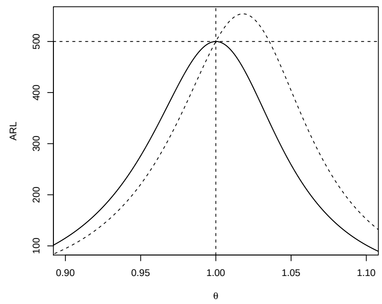
Duration: **30** minutes

- Write your number and name below.
- Add your answers to this and the following page.
- Please justify all your answers.
- This test has ONE PAGE and TWO QUESTIONS. The total of points is 5.0.

Number: **Name**:

1. The high-voltage output of a certain power supply used in a copy machine is assumed to have a normal distribution with nominal mean and standard deviation μ_0 and σ_0, respectively. Samples of n power supply units have been inspected every hour.

(a) A statistician suggested an ARL-unbiased EWMA chart for monitoring the process variance σ^2. **(1.0)**
i) Identify the control statistic of this chart, when the initial value is equal to $\ln(\sigma_0^2)$.
ii) The ARL profiles of two EWMA charts for σ^2 can be found in the plot on the right.
Which profile corresponds to the ARL-unbiased EWMA chart? Why?



- **Control statistic of the ARL-unbiased EWMA chart for the variance of normal output**
$$W_N = \begin{cases} \ln(\sigma_0^2), & N = 0 \\ (1-\lambda)\, W_{N-1} + \lambda \ln(S_N^2), & N \in \mathbb{N}, \end{cases}$$
where $\lambda \in (0,1]$ and S_N^2 is the variance of the Nth random sample.

- **Identifying the requested ARL profile**
The solid line corresponds to the ARL profile of the ARL-unbiased EWMA chart for σ^2 because the ARL curve attains a maximum at $\theta = 1$. This chart offers a more balanced protection against both increases and decreases in the process variance than the ARL-biased EWMA chart for σ^2 whose ARL profile is associated with the dashed line.

(b) Briefly describe the obtention of the PMS_{III}, the probability of a misleading signal of Type III, of a joint EWMA scheme for μ and σ^2. **(1.5)**

- **Joint scheme for μ and σ^2**
Without loss of generality, this joint scheme comprises two upper one-sided EWMA charts, one for μ and another one for σ^2.

- **Probability of a misleading signal of Type III**
Let $RL_\mu(\delta,\theta)$ (resp. $RL_\sigma(\theta)$) denote the RL of the EWMA chart for μ (resp. σ), where $\delta = \sqrt{n}\,(\mu-\mu_0)/\sigma$ ($\delta \geq 0$) and $\theta = \sigma/\sigma_0$ ($\theta \geq 1$) are the magnitudes of the shits in the process mean and standard deviation. Then the probability of a misleading signal of Type III is equal to
$$\begin{aligned} PMS_{III}(\theta) &= P[RL_\sigma(\theta) > RL_\mu(0,\theta)] \\ &= \sum_{i=1}^{+\infty} P_{RL_\mu(0,\theta)}(i) \times \bar{F}_{RL_\sigma(\theta)}(i) \\ &= \sum_{i=1}^{+\infty} \left[\bar{F}_{RL_\mu(0,\theta)}(i-1) - \bar{F}_{RL_\mu(0,\theta)}(i)\right] \times \bar{F}_{RL_\sigma(\theta)}(i), \quad \theta > 1, \end{aligned}$$
according to subsection 10.4.6 (Lemma 10.38).

- **Obtaining $PMS_{III}(\theta)$**
  Following the lecture notes, we can obtain an approximation to the $PMS_{III}(\theta)$ by replacing the survival functions in the formula of this probability by the Markovian approximations found in Proposition 10.34 and described in detail in Subsection 10.4.3.

2. Suppose that a vendor ships components in lots of size $N = 1000$. A single-sampling plan for attributes with rectifying inspection is being used with $(n, c) = (10, 0)$.

(a) Find the level p of lot quality that will be accepted approximately 34.8678% of the time. **(1.0)**
Hint: Use the binomial approximation.

- **Single sampling plan (for attributes)**
  $N = 5000$ (lot size), $n = 10$ (sample size) $c = 0$ (acceptance number)
- **Auxiliary r.v. and its approximate distribution**
  $D =$ number of nonconforming components in the sample $\stackrel{a}{\sim}$ binomial(n, p)
- **Obtaining the requested level p of lot quality**

$$\begin{aligned} p \quad : \quad & P(D \leq c) \simeq 0.348678 \\ & F_{binomial(n=10,p)}(0) \simeq 0.348678 \\ & (1-p)^{10} \simeq 0.348678 \\ & p \simeq 1 - 0.348678^{1/10} \simeq 0.10. \end{aligned}$$

[Alternatively, we could consult the tables of the c.d.f. of the binomial distribution with $n = 10$ and find the value of p satisfying $F_{binomial(n,p)}(c) \simeq 0.3487 \simeq 0.348678$, we obtain $p \simeq 0.10$.]

(b) Calculate the AOQ and ATI when the lot contains 10% of defective items. Comment on these results and on the impact of rectifying inspection. **(1.5)**

- **Probability of lot acceptance**
  $P_a(p) = P(D \leq c) \simeq F_{binomial(n,p)}(c)$
- **Requested average outgoing quality**

$$\begin{aligned} AOQ(p = 0.1) \quad & = \quad \frac{p\,(N-n)\,P_a(p)}{N} \\ & \stackrel{n=10,c=0,etc.}{\simeq} \quad \frac{0.1 \times (1000-10) \times 0.348678}{1000} \\ & \simeq \quad 0.03451912. \end{aligned}$$

- **Requested average total inspection**

$$\begin{aligned} ATI(p = 0.1) \quad & = \quad n\,P_a(p) + (N-n)\,[1 - P_a(p)] \\ & \simeq \quad 10 \times 0.348678 + 1000 \times (1 - 0.348678) \\ & \simeq \quad 654.8088. \end{aligned}$$

- **Comments**
  Since the associated relative reduction of the fraction nonconforming is

$$\begin{aligned} \left[1 - \frac{AOQ(p)}{p}\right] \times 100\% \quad & \simeq \quad \left(1 - \frac{0.03451912}{0.1}\right) \times 100\% \\ & \simeq \quad 65.48088\% \end{aligned}$$

we can add that the adoption of RECTIFYING INSPECTION really pays off, when $p = 10\%$. However, this comes with a cost — the expected number of inspected items increases from the fixed sample size of $n = 10$ items to $648.2956 > N/2 = 500$, almost 65 fold if we adopt rectifying inspection; indeed, the associated relative increase in the expected number of inspected items equals

$$\begin{aligned} \left[1 \frac{ATI(p)}{n} - 1\right] \times 100\% \quad & \simeq \quad \left(1 - \frac{654.8088}{10}\right) \times 100\% \\ & \simeq \quad 6448.088\%. \end{aligned}$$