

1. PRIMITIVAS

(1) Possíveis primitivas:

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|--------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| 1) $\frac{1}{30}(10x^3 + 24x^{5/2} + 15x^2)$ | 25) $\frac{e^x}{2}(\sen x - \cos x)$ |
| 2) $-\frac{3}{4}(1-x)^{4/3}$ | 26) $\sqrt{1-x^2} + x \arcsen x$ |
| 3) $\frac{\sen^5 x}{5}$ | 27) $\frac{2}{135}(1+3x)^{3/2}(9x-2)$ |
| 4) $\ln \ln x $ | 28) $-2x + 2 \operatorname{arctg}(x) + x \ln(1+x^2)$ |
| 5) e^{e^x} | 29) $-e^{\cos x}$ |
| 6) $\frac{x}{2} - \frac{\sen(2x)}{4}$ | 30) $e^{\sen^2 x}$ |
| 7) $-\frac{1}{x}$ | 31) $2x - 2x \ln(x) + x \ln^2(x)$ |
| 8) $-\frac{1}{2} \ln(1+e^{-x^2})$ | 32) $\ln \left \frac{2-x}{1-x} \right $ |
| 9) $-\ln \cos x $ | 33) $\frac{1}{2} \ln \left \frac{1-x}{1+x} \right $ |
| 10) $-\frac{1}{\operatorname{arctg} x}$ | 34) $\frac{2}{\sqrt{3}} \operatorname{arctg} \left(\frac{1+2x}{\sqrt{3}} \right)$ |
| 11) $\frac{2^x}{\ln 2}$ | 35) $\frac{1}{3} \operatorname{arctg}(x^3)$ |
| 12) $-x + \tan x$ | 36) $\frac{\operatorname{arctg}(x)}{2} - \frac{1}{2} \ln 1-x + \frac{1}{4} \ln(1+x^2)$ |
| 13) $e \sen(e^x)$ | 37) $\frac{1}{x} + x + \frac{x^2}{2} + \ln 1-x $ |
| 14) $-\frac{1}{2} \sqrt{1-x^4}$ | 38) $-\frac{2}{x-1} + x + \ln x-1 $ |
| 15) $\frac{\arcsen(x^2)}{2}$ | 39) $-\ln \left \frac{1+\sqrt{1+2x}}{1-\sqrt{1+2x}} \right $ |
| 16) $\sqrt{x^2+2x+3}$ | 40) $-\frac{3}{x^{10/3}} + 6x^{1/6} + 2\sqrt{x} - 6 \ln(1+x^{1/6})$ |
| 17) $-\frac{1}{2 \sen^2(x)}$ | 41) $-\frac{1}{2(1+e^x)} - \ln \left \frac{1+e^x}{1-e^x} \right $ |
| 18) $\arcsen x + (1-x^2)^{1/2}$ | 42) $-\frac{1}{1+x} + \frac{1}{2} \ln \left \frac{1-x}{1+x} \right $ |
| 19) $\frac{ x x}{2}$ | 43) $\frac{1-4x}{2(x-1)^2}$ |
| 20) $\operatorname{arctg}(e^x)$ | 44) $\arcsen(\ln x) - \sqrt{1-\ln^2 x}$ |
| 21) $\frac{1}{\pi} \ln(\cosh(\pi x))$ | 45) $\frac{1}{2} x \sqrt{1-x^2} + \arcsen x$ |
| 22) $\frac{\cosh(x^2)}{2}$ | 46) $-\frac{x}{2} + \frac{\operatorname{arctg} x}{2} + \frac{1}{2} x^2 \operatorname{arctg} x$ |
| 23) $-\frac{1}{4x^2} - \frac{\ln(x)}{2x^2}$ | 47) $2x \cos(x) + \sen(x) + (x^2-2) \sen x$ |
| 24) $-\frac{1}{2} x \cos(\ln x) + \frac{1}{2} x \sen(\ln x)$ | 48) $8 \ln \sen x - \frac{1}{2} \ln(1+\sen^2 x)$ |

49) $\frac{3}{2} \operatorname{arctg}(x^{2/3})$

51) $\frac{1}{5}(2x + \ln |2 \cos(x) + \operatorname{sen}(x)|)$

50) $\frac{1}{2} \operatorname{arctg}\left(\frac{\operatorname{sen} x}{2}\right)$

(2) Em $]-\infty, 0[$, $f - g$ é constante. O mesmo acontece em $]0, +\infty[$. Os valores em cada um dos intervalos não têm de ser iguais. (Simplificando as expressões, verifique que $f - g = \pm 1$)

(3) Mostre que a substituição $t = \tan(x/2)$ é tal que

$$\cos x = \frac{t^2 - 1}{t^2 + 1}, \quad \operatorname{sen} x = \frac{2t}{t^2 + 1}.$$

(5a)

$$-x + \frac{2 \operatorname{sen}(x/2)}{\cos(x/2) - \operatorname{sen}(x/2)}$$

(5b)

$$-\ln |\cos(x/2)| + \ln |\cos(x/2) + \operatorname{sen}(x/2)|$$

(6a)

$$y(x) = \frac{1}{8} \ln \left(\frac{1 + 4x^2}{5} \right) + 2$$

(6b)

$$y(x) = -\sqrt{x^2 - 5}$$

(6c)

$$y(x) = \ln(x^2 + e)$$

(6d)

$$y(x) = -\frac{1}{\sqrt{3 - 2\sqrt{1 + x^2}}}$$

(7)

$$y(x) = 1 - Ae^{-x}, \quad A \in \mathbb{R}$$

(8)

$$y(x) = \frac{Ae^x}{1 + Ae^x}, \quad A \in \mathbb{R}$$

2. INTEGRAÇÃO

(1) a) F b) V c) F

(3a) $-\cos(x^2)$

(3b) $2xe^{-x^4} - e^{-x^2}$

(3c) $\int_1^{x^2} \cos(\sqrt{t}) dt + 2x^2 \cos(\sqrt{x})$

(5a) $1/4$

(5b) $2/3$

(6a) $-\ln 2$

(6b) $1/2$

$$(6c) \pi^2/32$$

$$(6d) -1$$

$$(6e) \ln(2)/4$$

$$(6f) \arctg(\sqrt{6}) - \arctg(\sqrt{2})$$

$$(6g) (\pi + 2)/2$$

$$(6h) \frac{1}{2} \ln(3/2)$$

$$(6i) (\pi - \ln 4)/8$$

$$(8a) 44/3$$

$$(8b) 1/12$$

$$(9i) 1/2$$

$$(9ii) 21/2$$

$$(9iii) (1 - \ln 2)/2$$