MAP30#4

Question 1 — (X-bar,S^2) joint scheme for μ and σ^2

```
n = 5;
μ0 = 1050;
σ0 = 25^2;
incontrolARLindiv = 200.;

distmu = NormalDistribution[0, 1];
γmu = Quantile[distmu, 1 - 1/(2 * incontrolARLindiv)]
ξmu[δ_, θ_] = 1 - (CDF[distmu, (γmu - δ)/θ] - CDF[distmu, (-γmu - δ)/θ]);
ARLmu[δ_, θ_] = 1 / ξmu[δ, θ];
ARLmu[0, 1]

distsigma = ChiSquareDistribution[n - 1];
γsigma = Quantile[distsigma, 1 - 1/incontrolARLindiv]
ξsigma[θ_] = 1 - CDF[distsigma, γsigma/θ^2];
ARLsigma[θ_] = 1/ξsigma[θ];
ARLsigma[1]

ξmusigma[δ_, θ_] = ξmu[δ, θ] + ξsigma[θ] - ξmu[δ, θ] × ξsigma[θ];
ARLmusigma[δ_, θ_] = 1/ξmusigma[δ, θ];
ARLmusigma[0, 1]
ARLmusigma[δ_, θ_] = (ARLmu[δ, θ] * ARLsigma[θ])/(ARLmu[δ, θ] + ARLsigma[θ] - 1);
ARLmusigma[0, 1]

shiftsigma = 1.1;
ARLmu[0, shiftsigma]
ARLsigma[shiftsigma]

(ξmu[0, shiftsigma] × (1 - ξsigma[shiftsigma]))/ξmusigma[0, shiftsigma]
(1/ARLmu[0, shiftsigma] × (1 - 1/ARLsigma[shiftsigma]))/(1/ARLmusigma[0, shiftsigma])
(1/ARLmu[0, shiftsigma] × (1 - 1/ARLsigma[shiftsigma])) /
 (1/ARLmu[0, shiftsigma] + 1/ARLsigma[shiftsigma] -
   1/ARLmu[0, shiftsigma] * 1/ARLsigma[shiftsigma])
(1/93.3245 * (1 - 1/65.0272))/(1/93.3245 + 1/65.0272 - 1/93.3245 × 1/65.0272)
```

```
2.80703

200.

14.8603

200.
```

```
100.251

100.251

93.3245

65.0272

0.406905

0.406905

0.406905

0.406905
```

Question 2 — Single sampling plan for attributes with RECTIFYING INSPECTION

```
Clear[Evaluate[Context[] <> "*"]];
p1 = 0.005; (* AQL *)
α = 1 - 0.95; (* producer's risk *)
p2 = 0.175; (* LTPD *)
β = 0.15; (* consumer's risk *)

Q[c_, x_] = Quantile[ChiSquareDistribution[2 × (c + 1)], x];
r[c_] = N[Q[c, 1 - β], 5] / N[Q[c, α], 5];
i = 0;
While[r[i] > p2/p1, Print["Do not use acceptance number c=", i, " because r(c)=",
   N[Q[i, 1 - β], 5], "/", N[Q[i, α], 5] , "=", r[i], ">p2/p1=", p2/p1];
 i++]
Print["Use the acceptance number c=", i, " because r(c)=",
 N[Q[i, 1 - β], 5], "/", N[Q[i, α], 5] , "=", r[i], "≤p2/p1=", p2/p1]
samplesize[c_] = Ceiling[Q[i, 1 - β] / (2 × p2)];
If[Ceiling[Q[i, 1 - β] / (2 × p2)] ≤ Floor[Q[i, α] / (2 × p1)],
 Print["Use the sample size n=", samplesize[i], "."],
 Print["Houston, we have a problem!"]]
```

```
Do not use acceptance number c=0 because r(c)=3.79424/0.102587=36.9857>p2/p1=35.

Use the acceptance number c=1 because r(c)=6.74488/0.710723=9.49017≤p2/p1=35.

Use the sample size n=20.
```

```
(* Unnecessary verification *)
ntot = 1000; (* lot size *)
exactdist[p_] = HypergeometricDistribution[samplesize[i], Round[ntot × p], ntot];
If[CDF[exactdist[p1], i] ≥ 1 - α && CDF[exactdist[p2], i] ≤ β,
 Print["The single sampling plan for attributes
    complies with the producer's and consumer's risk points."],
 Print["The single sampling plan for attributes does not comply
    with the producer's or the consumer's risk points."]]
```

```
The single sampling plan for attributes
  complies with the producer's and consumer's risk points.
```

```
(* Average Outgoing Quality (AOQ) or percentage of defective due to rectifying inspection in a
 single sampling plan and using the binomial approximation to the acceptance probability *)
```

$$\text{AOQ}[n\_, c\_, p\_] = \frac{(n_{tot} - n) \times p \times \text{CDF}[\text{BinomialDistribution}[n, p], c]}{n_{tot}};$$

```
midp = 0.1;
AOQ[samplesize[i], i, midp]
(* Associated relative reduction of the percentage of defective *)
```

$$\left(1 - \frac{\text{AOQ}[\text{samplesize}[i], i, \text{midp}]}{\text{midp}}\right) \times 100$$

```
(* Results using the tables *)
(1000 - 20) * midp * 0.3917 / 1000
(1 - % / midp) * 100
```

```
0.0383912

61.6088

0.0383866

61.6134
```

Question 3 — Single sampling plan for variables with KNOWN STANDARD DEVIATION

```
p1 = 0.015; (* AQL *)
α = 0.05; (* producer's risk *)
p2 = 0.065; (* LTPD *)
β = 0.1; (* consumer's risk *)

gdist = NormalDistribution[0, 1];
Φ[x_] := CDF[gdist, x];
Ω[x_] := Quantile[gdist, x];
```

$$\left(\frac{\Omega[1-\alpha]-\Omega[\beta]}{\Omega[p_2]-\Omega[p_1]}\right)^2;$$

$$n_\sigma = \text{Ceiling}\left[\left(\frac{\Omega[1-\alpha]-\Omega[\beta]}{\Omega[p_2]-\Omega[p_1]}\right)^2\right];$$

$$k_\sigma = \frac{\Omega[p_2]\times\Omega[1-\alpha]-\Omega[p_1]\times\Omega[\beta]}{\Omega[\beta]-\Omega[1-\alpha]};$$

$$\text{PVar}[n\_, p\_] = \Phi\left[\sqrt{n}\times(-k_\sigma-\Omega[p])\right];$$

```
i = nσ;
While[PVar[i, p1] < 1 - α || PVar[i, p2] > β,
 Print["Do not use sample size nσ=", i, " because Pa[p1]=",
  PVar[i, p1], "<", 1 - α , " or Pa[p2]=", PVar[i, p2], ">", β];
 i++]

Print["Use sample size nσ=", i, " and acceptance constant ", kσ,
 " because Pa[p1]=", PVar[i, p1], "≥", 1 - α , " and Pa[p2]=", PVar[i, p2], "≤", β]
nσ = i;
```

```
Use sample size nσ=20 and acceptance constant
 1.80138 because Pa[p1]=0.95042≥0.95 and Pa[p2]=0.0994428≤0.1
```

```
(* The 2nd condition fails if we use the tables *)
```

$\text{Round}\left[\sqrt{20} * (-1.80138 + 2.1701), 0.01\right];$

$\text{Round}\left[\sqrt{20} * (-1.80138 + 1.5141), 0.01\right];$

```
 (* Thus nσ+1 *)
```

$\text{Round}\left[\sqrt{n_\sigma + 1} * (-1.80138 + 2.1701), 0.01\right]$

0.9545

$\text{Round}\left[\sqrt{n_\sigma + 1} * (-1.80138 + 1.5141), 0.01\right]$

1 - 0.9066

```
1.69

0.9545

-1.32

0.0934
```

```
U = 3;
sigma = 0.1;
dist = NormalDistribution[U - 2 * sigma, sigma];
data = RandomVariate[dist, nσ + 1];
Mean[data]
(U - Mean[data]) / sigma
If[(U - Mean[data]) / sigma ≥ kσ, Print["We should accept the lot."],
 Print["We should reject the lot."]]
```

```
2.78012

2.19882

We should accept the lot.
```