Reliability and Quality Control

LMAC, MMA

2nd. Semester – 2022/2023
2023/06/06 — 17:10
MAP30 #4

Duration: **30** minutes

- Write your number and name below.
- Add your answers to this and the following page.
- Please justify all your answers.
- This test has ONE PAGE and TWO QUESTIONS. The total of points is 4.0.

Number: **Name**:

1. Samples of size $n = 5$ of the molecular weight are taken hourly from a chemical process. Admit that this output is independent and the quality characteristic is normally distributed with nominal mean and variance equal to μ_0 and σ_0^2. When the process mean is on-target (i.e., $\delta = \sqrt{n}(\mu - \mu_0)/\sigma_0 = 0$) and the standard deviation has increased 10% (i.e., $\theta = \sigma/\sigma_0 = 1.1$), the standard $\bar{X}-$chart and the upper one-sided S^2-chart have ARL equal to $ARL_\mu(\delta = 0, \theta = 1.1) = 93.3245$ and $ARL_\sigma(\theta = 1.1) = 65.0272$ (resp.). **(1.5)**

Rewrite the formula $PMS_{III}(\theta) = \frac{\xi_\mu(0,\theta)\times[1-\xi_\sigma(\theta)]}{\xi_{\mu,\sigma}(0,\theta)}$ in terms of the ARL of the individual charts for μ and σ^2 and obtain $PMS_{III}(\theta = 1.1)$.

- **Quality characteristic**

  X = molecular weight taken in a chemical process

  $X \sim \text{normal}(\mu, \sigma^2)$, where $\mu = \mu_0 + \delta\sigma_0/\sqrt{n}$ and $\sigma^2 = \theta^2\sigma_0^2$ represent the process mean and variance, respectively.

- **Probabilities of triggering a signal**

  The STANDARD $\bar{X}-$chart and the UPPER ONE-SIDED S^2-chart trigger a signal with probabilities: $\xi_\mu(\delta,\theta)$, $\delta \in \mathbb{R}$; $\xi_\sigma(\theta)$, $\theta \geq 1$.

  Moreover, according to Exercise 10.38, the joint scheme triggers a signal with probability $\xi_{\mu,\sigma}(\delta,\theta) = \xi_\mu(\delta,\theta) + \xi_\sigma(\theta) - \xi_\mu(\delta,\theta) \times \xi_\sigma(\theta)$, $\delta \in \mathbb{R}$, $\theta \geq 1$.

- **Probability of a misleading signal of Type III**

  Taking into account the previous result and the fact the RL of the individual charts are geometrically distributed with parameters equal to the probabilities given above, we get:

$$PMS_{III}(\theta) = \frac{\xi_\mu(0,\theta)\times[1-\xi_\sigma(\theta)]}{\xi_\mu(0,\theta)+\xi_\sigma(\theta)-\xi_\mu(0,\theta)\times\xi_\sigma(\theta)} = \frac{\frac{1}{ARL_\mu(0,\theta)}\times\left[1-\frac{1}{ARL_\sigma(\theta)}\right]}{\frac{1}{ARL_\mu(0,\theta)}+\frac{1}{ARL_\sigma(\theta)}-\frac{1}{ARL_\mu(0,\theta)}\times\frac{1}{ARL_\sigma(\theta)}}. \quad (1)$$

- **Requested PMS of Type III**

  Capitalizing on (1) and since $ARL_\mu(\delta = 0, \theta = 1.1) = 93.3245$ and $ARL_\sigma(\theta = 1.1) = 65.0272$,

$$PMS_{III}(\theta = 1.1) = \frac{\frac{1}{93.3245}\times\left(1-\frac{1}{65.0272}\right)}{\frac{1}{93.3245}+\frac{1}{65.0272}-\frac{1}{93.3245}\times\frac{1}{65.0272}} \simeq 0.406905.$$

2. A data scientist is using a single sampling plan for attributes with $(n,c) = (20,1)$, which complies with the producer's and consumer's risk points $(p_1 = AQL = 0.5\%, 1-\alpha = 0.95)$ and $(p_2 = LTPD = 17.5\%, \beta = 0.15)$. **(1.0)**

Calculate the AOQ when the lot of size $N = 1000$ contains 10% of defective items and rectifying inspection has been adopted. Obtain and comment on the value of the associated relative reduction of the percent defective.

- **Single sampling plan for attributes with rectifying inspection**

  $N = 1000$ (lot size), $n = 20$ (sample size), $c = 1$ (acceptance number)

- **Auxiliary r.v. and its approximate distribution**
  $D =$ number of defective items in the sample $\stackrel{a}{\sim}$ binomial$(20, p)$

- **Probability of lot acceptance**
  $P_a(p) = P(D \leq c) \simeq F_{binomial(n,p)}(c)$

- **Requested average outgoing quality**

$$AOQ(p = 0.1) \stackrel{(13.14)}{=} \frac{p\,(N-n)\,P_a(p)}{N} \stackrel{n=20,c=1,etc.}{=} \frac{0.1 \times (1000-20) \times 0.3917}{1000} \simeq 0.038387$$

- **Requested associated relative reduction of the percent defective**
  Since

$$[1 - AOQ(0.1)/0.1] \times 100\% \simeq (1 - 0.038387/0.1) \times 100\% \simeq 61.6134\%$$

  we can add that the adoption of RECTIFYING INSPECTION really pays off, when $p = 10\%$.

3. Consider a sampling plan by variables with known standard deviation, $n_\sigma = 21$, and $k_\sigma = 1.80138$. **(1.5)**

Verify that it meets the risk points $(p_1, 1-\alpha) = (1.5\%, 0.95)$ and $(p_2, \beta) = (6.5\%, 0.1)$. Make the necessary calculations to determine whether the lot should be accepted when the standard deviation, the upper specification limit, and the sample mean are equal to $\sigma = 0.1$, $U = 3$, and $\bar{x} = 2.78012$.

- **Sampling plan by variables with** KNOWN STANDARD DEVIATION
  $n_\sigma = 21$ (sample size), $k_\sigma = 1.80138$ (acceptance constant), $\sigma = 0.1$ (known standard deviation), $U = 3$ (upper specification limit)

- **Producer's and consumer's risk points**
  $(p_1, 1-\alpha) = (1\%, 0.975)$
  $(p_2, \beta) = (10\%, 0.1)$

- **Requested verification**

$$\begin{aligned}
P_a(p_1) &= \Phi\left(\sqrt{n_\sigma}\,[-k_\sigma - \Phi^{-1}(p_1)]\right) \\
&= \Phi\left(\sqrt{21} \times [-1.80138 - (-2.1701)]\right) \\
&\simeq \Phi(1.69) \\
&\stackrel{table}{=} 0.9545 \\
&\geq 1 - \alpha = 0.95 \\
P_a(p_2) &= \Phi\left(\sqrt{n_\sigma}\,[-k_\sigma - \Phi^{-1}(p_2)]\right) \\
&= \Phi\left(\sqrt{21}\,[-1.80138 - (-1.5141)]\right) \\
&\simeq \Phi(-1.32) \\
&\stackrel{table}{=} 1 - 0.9066 \\
&= 0.0934 \\
&\leq \beta = 0.1.
\end{aligned}$$

  Hence the sampling plan complies with the two given risk points.

- **Checking whether or not the lot should be accepted**
  The lot should be accept iff $Q = \frac{U-\bar{x}}{\sigma} \geq k_\sigma$. For this sample, we have

$$Q = \frac{3 - 2.78012}{0.1} = 2.19882 \geq 1.80138.$$

  Hence, we should accept the lot.