MEEC

Controlo em Espaço de Estados

2020/2021

Exam – 30 June 2020

Duration 3 hours

Consultation not allowed

Grades: P1a)2 b)1c)1d)1 **P2**a)2 b)2 c)1 d)1 **P3** a)1 b)2 c)1 **P4**a)1 b)1 **P5**a)2 b)1

P1. Consider the autonomous (without input) state model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

a) Using the modal decomposition, find the time response when the initial condition is

$$x^a(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

b) Repeat with the initial condition

$$x^b(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

c) Compute the state transition matrix (exponential matrix)

d) Find $x(0)$ such that $x(1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

P2. For the system

$$\dot{x} = Ax + bu, \quad y = Cx,$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

a) Say, justifying using the corresppondING matrices, if this state realization is controllable and/or observable.

b) Compute the gains (components of K) of the state feedback law $u(t) = -Kx(t)$ that place the poles of the controlled system at -2 e -3.

c) Design na asymptotic observer that places the observer poles at -5 e -6.

d) Compute the denominator of the controller (set of observer+state estimates feedback.

P3. Consider

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -(x_1 + x_1^2)$$

a) Find all the equilibrium points.

b) Obtain the linearized system around each of the equilibrium points of the system. For each of them, say, justifying, what can you conclude about the stability of the equilibrium point of the nonlinear system from the corresponding linearization.

c) For the point, or points, where you can take conclusions on the stability of the nonlinear system from its linearization, do a qualitative sketch of the phase portrait around it (or them). You can't use the signs of the derivative

Help: Jacobian matrix

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

P4. Consider

$$\dot{x} = ax + u,$$

Where the input u and the state x are scalars and a is na unknown, but constant, parameter. To control this system we a $u(t) = -Kx(t)$ where

$$K = \hat{a} - a_m,$$

a_m being a given parameter and $\hat{a}$ the estimate of a, adjusted such that

$$V(x, \tilde{a}) = \frac{1}{2}(x^2 + \tilde{a}^2)$$

Is a Lyapunov function, and where $\tilde{a} = \hat{a} - a$.

a) Find an adaptation rule for $\hat{a}$ that ensures that V is a Lyapunov function. What can you say about the stability on the basis of the Lyapunov theorem?

b) In relation to the adaptation law that you obtained in a), whet can you say? about stability using the invariant set theorem? Justify.

P5. Consider the control law that minimizes

$$J(u) = \frac{1}{2}\int_0^{\infty}[(r - x(t))^2 + \rho u^2(t)]\, dt \qquad (11)$$

for the scalar system

$$\dot{x} = x + u \qquad (12)$$

where r is a constant reference that we want x to track, and ρ is a positive parameter. Define the tracking error in percentage as

$$\frac{r-\bar{x}}{r} \times 100, \qquad (13)$$

where $\bar{x}$ is the equilibrium value of x.

a) Using Pontryagin's Principle, compute the optimal control.

b) Find a condition in ρ that ensures that, in steady-state, the tracking error in percentage is smaller or equal than 5%.

Help

$$\frac{dx}{dt} = f(x,u) \qquad x(0) = x_0 \qquad J(u) = \Psi(x(T)) + \int_0^T L(x,u)dt$$

$$-\left(\frac{d\lambda}{dt}\right)' = \lambda'(t) f_x(x(t),u(t)) + L_x(x(t),u(t)) \qquad \lambda'(T) = \Psi_x(x(T))$$

$$H(\lambda,x,u) = \lambda' f(x,u) + L(x,u)$$

$$f_x = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \qquad L_x = \begin{bmatrix} \frac{\partial L}{\partial x_1} & \frac{\partial L}{\partial x_2} \end{bmatrix} \qquad \Psi_x = \begin{bmatrix} \frac{\partial \Psi}{\partial x_1} & \frac{\partial \Psi}{\partial x_2} \end{bmatrix}$$