V - Other modulations for wireless communications*

1 Multicarrier modulation

All the modulations discussed until now have a single carrier (SC). Herein we are going to analyze modulations characterized by using more than one carrier. The basic idea is to divide the transmitted bitstream into many different substreams and send them over many different subchannels using carriers with different frequencies. Typically the subchannels are orthogonal under ideal propagation conditions, in which case multicarrier modulation is referred to as *orthogonal frequency division multiplexing* (OFDM). The data rate on each of the subchannels is much less than the total data rate and the corresponding subchannel bandwidth is much less than the total system bandwidth [1].

An appealing feature of OFDM is the simplicity of the receiver design due to the efficiency with which OFDM can cope with the effects of frequency-selective multipath channels. Besides, a key advantage of using OFDM is that modulation and demodulation can be made in the discrete domain by using a discrete Fourier transform (DFT) or the fast Fourier transform (FFT). OFDM has been adopted for wireless local area networks (LANs) such as HIPERLAN/2 and IEEE 802.11; it has been also adopted in Europe for Digital Audio Broadcasting (DAB) and Digital Video Broadcasting (DVB).

The popularity of OFDM stems from its ability to transform the wideband frequency-selective channel to a set of parallel flat-fading narrowband channels, which substantially simplifies the channel equalization problem. Assume that the information-bearing symbols are to be transmitted at the rate of R_s symbols/s over a multipath channel. The duration of each symbol is therefore $T_s = 1/R_s$. If the delay spread, $\tau_{\max}$, of the channel is larger than about 10% of the symbol duration, then the received signal may suffer from significant intersymbol interference (ISI), which can drastically increase the symbol error probability unless countermeasures are undertaken. Such channel is said to be frequency-selective. There are two main approaches to cope with such channels. The first approach is to use a single-carrier modulation with an equalizer at the receiver

*FDNunes, IST 2021.

to compensate for the ISI, which spans $\lceil \tau_{\max}/T_s \rceil$ symbols. The implementation of the equalizer may be very challenging for channels with large delay spreads and high data rates. The second approach is based on multicarrier modulation, such as OFDM [2].

In an OFDM system the available bandwidth is divided into a large number of subchannels, over each of which the wireless channel fading can be considered flat. The original data stream of rate R_s is split into N parallel data streams, each of rate R_s/N. The symbol duration T for these parallel data streams is therefore increased by a factor of N; that is, $T = NT_s$. Each of the data streams modulates a carrier with a different frequency and the resulting signals are transmitted simultaneously. In the receiver, the ISI in each channel (stream) is reduced to $\lceil \tau_{\max}/(NT_s) \rceil$. Thus, an advantage of OFDM is that, for frequency-selective fading channels, the OFDM symbols are less affected by channel fades then the single-carrier transmitted ones. While many symbols might be lost in a single carrier system in the presence of a channel fade, the symbols of an OFDM system can still be correctly detected since only a small fraction of each symbol may be affected by the fade.

1.1 IDFT/DFT implementation of OFDM system

The complex envelope of an OFDM signal is given by [3], [4]

$$z(t) = A \sum_{k=-\infty}^{\infty} b(t - kT; \mathbf{x}_k) \tag{1}$$

with

$$b(t - kT; \mathbf{x}_k) = g(t - kT) \sum_{n=0}^{N-1} x_{k_n} \exp\left[j \frac{2\pi n (t - kT)}{T} \right], \quad kT \leq t < (k+1)T \tag{2}$$

where k is the block (time) index, N is the block length (number of subcarriers), $\mathbf{x}_k = \{x_{k_0}, x_{k_1}, \ldots, x_{k_{N-1}}\}$ is the data symbol block at epoch k and $g(t)$ is the signaling pulse.

The data symbols $\mathbf{x}_{k_n}$ are often chosen from a QAM or PSK constellation, although any 2-D signal constellation can be used.

If a rectangular signaling pulse is used

$$g(t) = \Pi \left(\frac{t - T/2}{T} \right) \tag{3}$$

then the $\Delta f = 1/T$ frequency separation of the subcarriers ensures that the signals of the different subchannels are orthogonal regardless of the block of transmitted symbols. In fact, the inner product of the signals of subchannel i and n is

$$\int_{kT}^{(k+1)T} g^2(t-kT)x_{k_i}x_{k_n}^* \exp\left[j\frac{2\pi(i-n)(t-kT)}{T}\right] dt$$

$$= x_{k_i}x_{k_n}^* \int_0^T \exp\left[j\frac{2\pi\,(i-n)\,\lambda}{T}\right] d\lambda = 0 \tag{4}$$

The orthogonality allows the signals of different subchannels to be separated out by the demodulator at the receiver.

Let $f_n = n/T$, $n = 0, 1, \ldots, N-1$, be the frequency of each subcarrier in baseband, as shown in Fig. 1.

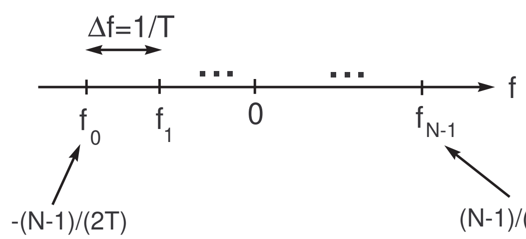

Figure 1: Distribution of the subcarrier frequencies in baseband

The spectrum of signal $b(t; \mathbf{x}_k)$ is

$$\begin{aligned} B(f;\mathbf{x}_k) &= \mathcal{F}\{b(t;\mathbf{x}_k)\} = \int_{kT}^{(k+1)T} \exp(-j2\pi ft) \sum_{n=0}^{N-1} x_{k_n} \exp[j2\pi f_n(t-kT)]\, dt \\ &= T\exp[-j\pi fT(1+2k)] \sum_{n=0}^{N-1} (-1)^n x_{k_n} \mathrm{sinc}[(f-f_n)T] \end{aligned} \tag{5}$$

The magnitudes of the spectra of the normalized subchannels, $|x_{k_n}||\mathrm{sinc}[(f-f_n)T]|$, are plotted in Fig. 2. Notice that, although the subchannels are mutually orthogonal, their spectra overlap, allowing OFDM to have a good spectral efficiency.

Assume for simplicity that $A = 1$ in (1). Sampling $z(t)$ in the interval $kT \le t < (k+1)T$ with sampling rate $f_s = N/T = 1/T_s$, the transmitted signal may be written in a column vector as

$$\mathbf{S}_k = [z(kT), z(kT+T_s), z(kT+2T_s), \ldots, z(kT+(N-1)T_s)]^T \tag{6}$$

where

$$\begin{aligned} s_{k_i} \equiv z(kT + iT_s) &= \sum_{n=0}^{N-1} x_{k_n} \exp\left(j\frac{2\pi i\, nT_s}{T}\right) \\ &= \sum_{n=0}^{N-1} x_{k_n} \exp\left(j\frac{2\pi i\, n}{N}\right), \quad i = 0, \ldots, N-1 \end{aligned} \tag{7}$$

which is the inverse discrete Fourier transform (IDFT) of the vector [5]

$$\mathbf{X}_k = [x_{k_0}\, x_{k_1} \ldots x_{k_{N-1}}]^T \tag{8}$$

Matricially we have $\mathbf{S}_k = \mathbf{W}\mathbf{X}_k$, where $\mathbf{W}$ is the $(N \times N)$ matrix with elements

$$W(i, n) = \exp\left(j\frac{2\pi i\, n}{N}\right) \tag{9}$$

Since $\mathbf{W}$ is non-singular, $\mathbf{W}^{-1}$ exists and its elements are given by

$$W^{-1}(i, n) = \frac{1}{N} \exp\left(-j\frac{2\pi i\, n}{N}\right) \tag{10}$$

and vector $\mathbf{X}_k$ can be recovered at the receiver by means of $\mathbf{X}_k = \mathbf{W}^{-1}\mathbf{S}_k$, which is the discrete Fourier transform (DFT) of $\mathbf{S}_k$ [4]

$$x_{k_n} = \frac{1}{N} \sum_{i=0}^{N-1} s_{k_i} \exp\left(-j\frac{2\pi i\, n}{N}\right) \tag{11}$$

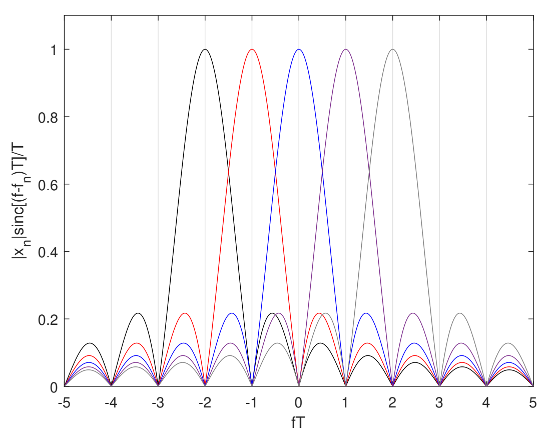

Figure 2: Magnitude spectra $|B(f; \mathbf{x}_k)|$ of the subchannels for $N = 5$ with $|x_{k_n}| = 1$

The use of IDFT/DFT eliminates the presence of a bank of subcarrier oscillators at the transmitter and receiver. Besides, by selecting the number of subcarriers as the power of two, we can replace the DFT by the fast Fourier transform (FFT) leading to a significant reduction of the computational effort.

1.2 Cyclic prefix

Each subchannel has a symbol duration T which is chosen sufficiently large to remove most ISI (also known as interblock interference, IBI) that might be introduced by the channel (for those values of T the channel fading for each subchannel is considered flat). While the removal of most IBI generally yields good performance, it is possible to remove all IBI when the maximum delay spread of the channel is known. This can be done by either (i) adding a guard interval between block transmissions equal to the maximum channel delay spread, in which case a block does not interfere with subsequent blocks, or by (ii) adding a *cyclic prefix* after the IDFT (as shown in Fig. 3), which is later removed in the receiver [1]. If we employ a guard interval with no signal transmission we can perfectly eliminate ISI, but a sudden change of waveform contains higher spectral components that result in inter-subcarrier interference [4]. The use of a cyclic prefix avoids this problem.

Assume that the channel delay spread has a maximum value $\tau_{\max}$. Thus, the channel delay spread lasts a maximum of $\mu \geq f_s \tau_{\max}$ samples for the sampling rate f_s. The cyclic prefix $\{s_{k_{N-\mu}}, \ldots, s_{k_{N-1}}\}$ consists of the last μ values of the $\mathbf{S}_k$ vector. These μ samples are appended to the beginning of each block of samples, yielding the new input to the D/A converter (DAC), i.e., the sequence $\{s_{k_{N-\mu}}, \ldots, s_{k_{N-1}}, s_{k_0}, s_{k_1}, \ldots, s_{k_{N-1}}\}$. This means that the cyclic prefix increases the number of samples per block from N to $N + \mu$. When a block is received, the first μ samples are assumed to be corrupted by ISI from the previous block. We remove the cyclic prefix by discarding those samples in the received vector $\widetilde{\mathbf{R}}_k$ (see Fig. 4). This vector is ideally a replica of $\mathbf{S}_k$ but is affected by channel noise and distortion due to the channel characteristics.

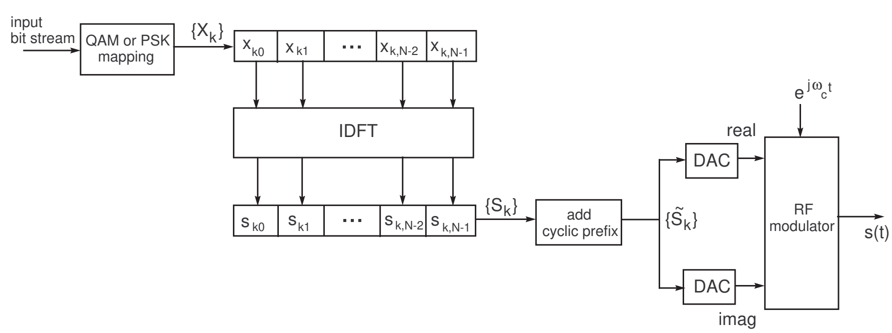

Figure 3: Implementation of OFDM transmitter with IFFT

Considering that the introduction of the cyclic prefix should not modify the symbol transmission rate $r_s = 1/T_s$, this means that the block duration T should be kept unchanged. The sampling rate without the cyclic prefix is $f_s = N/T$ (corresponding to N samples per block) and with the cyclic prefix it is increased to

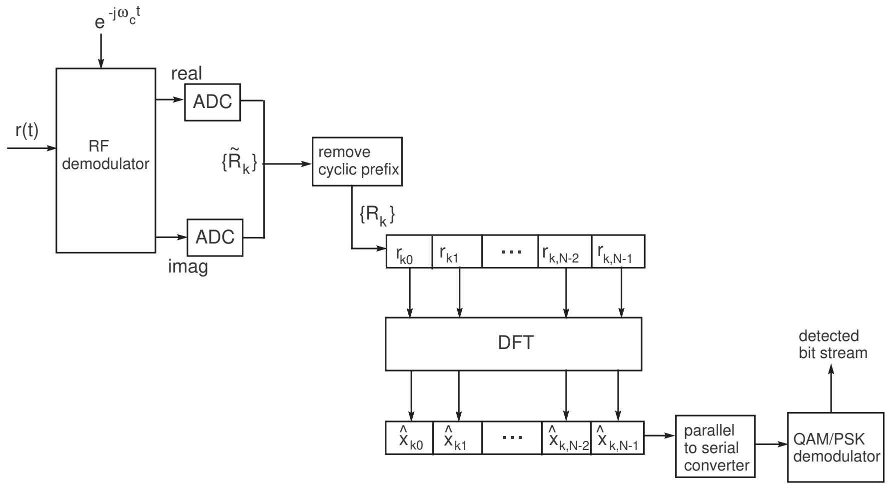

Figure 4: Implementation of OFDM receiver with FFT

$$\widetilde{f}_s = \frac{N}{T - \Delta_g} \tag{12}$$

according to Fig. 5, where Δ_g is the duration of the cyclic prefix guard interval (guard interval). The complex envelope of the transmitted signal in the presence of the cyclic prefix is

$$z(t) = A \sum_{k=-\infty}^{\infty} g(t - kT) \sum_{n=0}^{N-1} x_{k_n} \exp[j2\pi \widetilde{f}_n (t - kT)] \tag{13}$$

with $\widetilde{f}_n = n\Delta f$, where Δf is the separation between the subcarrier frequencies.

Figure 5: OFDM block with and without cyclic prefix

The sampled signal in the block interval $kT \leq t < (k+1)T$ is

$$z(kT + i\Delta t) = A \exp(-j2\pi \widetilde{f}_n kT) \sum_{n=0}^{N-1} x_{k_n} \exp(j2\pi i\, n\Delta t \Delta f) \tag{14}$$

for $i = 0, \ldots, N\lceil 1 + \Delta_g/T \rceil - 1$. Comparing (14) with (7) yields [4]

$$\Delta f = \frac{1}{N\Delta t} = \frac{\widetilde{f}_s}{N} = \frac{1}{T - \Delta_g} \tag{15}$$

which is the new frequency separation between subcarriers.

1.3 OFDM performance

In the OFDM systems the signal is transmitted even in the guard interval although its power is not used for detection. Therefore, in the AWGN channel, the symbol error probability is same as the one of the single-carrier system but we need to take into consideration the power loss associated with the guard interval insertion. This is done by considering that the effective bit SNR is [4]

$$\frac{\widetilde{E}_b}{\mathcal{N}_0} = \frac{T - \Delta_g}{T} \frac{E_b}{\mathcal{N}_0} \tag{16}$$

In the presence of fading, if we can assume that the fading in each subchannel is approximately flat and slow, the formulas obtained in chapter IV for single-carrier modulations can still be utilized, while taking into account the the average bit SNR should be replaced by average effective bit SNR, according to (16).

OFDM is sensitive to carrier frequency offset (CFO) and phase noise; for a large number of subcarriers these imperfections destroy orthogonality and introduce inter-carrier interference which leads to the increase of the symbol error. The CFO is caused by misalignment in subcarrier frequencies or Doppler frequency shift [6], [7]. This sensitivity to frequency offset is often pointed out as a major OFDM disadvantage. It was shown in [6] that, when an offset ΔF exists between the frequencies of the received carrier and the local (receiver) carrier, f_c, the SNR $= E_s/\mathcal{N}_0$ degradation in dB for a symbol rate $r_s = N/T$ is given by

$$D_{dB,MC} \approx \frac{10}{3 \ln 10} \left(\pi N \frac{\Delta F}{r_s}\right)^2 \left(\frac{E_s}{\mathcal{N}_0}\right)_{dB} \quad \text{(dB)} \tag{17}$$

This formula is only accurate for $(\pi N \Delta F / r_s)^2 \ll 3$ [7].

For a single-carrier (SC) modulation the SNR degradation is

$$D_{dB,SC} \approx \frac{10}{3 \ln 10} \left(\pi \frac{\Delta F}{r_s}\right)^2 \tag{18}$$

Thus

$$D_{dB,MC} - D_{dB,SC} \approx \frac{10}{2 \ln 10} \left(\frac{\pi \Delta F}{r_s}\right)^2 \left[N^2 \left(\frac{E_s}{\mathcal{N}_0}\right)_{dB} - 1\right] \tag{19}$$

This result shows that OFDM is orders of magnitude more sensitive to the frequency offset than SC.

Another major problem with OFDM is the relatively high peak-to-peak average power ratio (PAR) that is inherent in the transmitted signal. In general, large signal peaks occur in the transmitted signal when the signals in many of the various subchannels add constructively in phase. Such large signal peaks may result in clipping of the signal voltage in a D/A converter when the OFDM signal is synthesized digitally, and/or it may saturate the power amplifier and, thus, cause intermodulation distortion in the transmitted signal [8].

The choice of the OFDM parameters is a tradeoff between various, often conflicting requirements. The length of the cyclic prefix (CP) is dictated by the delay spread of the channel. Introduction of CP provokes a reduction in bit rate (or wasted bandwidth), as well as an SNR loss; to minimize these inefficiencies, the number of subcarriers, N, should be large. However, a large number of subcarriers leads to high implementation complexity, increased sensitivity to frequency offset and phase noise (since the subcarriers get closer to each other as N increases), and greater peak-to-average power ratios (PAPRs). N is dictated by concerns regarding practical FFT sizes as well as the coherence time of the channel (due, for instance, to the relative transmitter-receiver motion) [2]. OFDM is especially vulnerable to Doppler spread resulting from time variations in the channel transfer function, as is the case in mobile communications. The Doppler spreading destroys the orthogonality of the subcarriers and results in intercarrier interference (ICI) which severely degrade the performance of the OFDM system [8].

In Europe terrestrial digital video broadcasting (DVB-T), with use of OFDM, was standardized by the European Telecommunications Standards Institute (ETSI). The following tables show the specifications of the two modes defined in DVB-T [9] for a 8 MHz bandwidth channel.

Parameter	8K mode	2K mode
Number of subcarriers	6817	1705
Duration T_U	896 μs	224 μs
Carrier spacing $1/T_U$	1116 Hz	4464 Hz
Bandwidth	7.61 MHz	7.71 MHz
Information transmission rate: 4.98 – 31.67 Mbps		

Mode	8K mode				2K mode			
Guard interval Δ_g/T_U	1/4	1/8	1/16	1/32	1/4	1/8	1/16	1/32
Guard interval duration Δ_g (μs)	224	112	56	28	56	28	14	7
Symbol duration $\Delta_g + T_U$ (μs)	1120	1008	952	924	280	252	238	231
FEC (inner code)	Convolutional code ($R = 1/2$, 2/3, 3/4, 5/6, 7/8)							
FEC (inner code)	Reed-Solomon code (204,188)							

The "2K-mode" is suitable for single transmitter operation and for small single frequency networks (SFN) with limited transmitter distances. It provides a Doppler tolerance allowing extremely high speed reception. The "8K-mode" can be used both for single transmitter operation and for small and large SFN networks. It provides a Doppler tolerance allowing high speed reception [9].

2 Spread-spectrum modulations

2.1 multiple access schemes

The signals generated by multiple users may share the same channel without interference if they are orthogonal. Two real signals $x(t)$ and $y(t)$ are orthogonal is they verify

$$\int_{-\infty}^{\infty} x(t)y(t)\,dt = 0$$

This definition is at the basis of the signal time multiplexing (TDMA=time division multiple access). Using the Fourier transform and the integral theorem we obtain

$$\int_{-\infty}^{\infty} X(f)Y^*(f)\,df = 0$$

This definition is at the basis of the signal frequency multiplexing (FDMA=frequency division multiple access).

There is a third form to render two signals $f(t)$ and $g(t)$ orthogonal. It is sufficient that

$$x(t) = c_1(t)f(t)$$

$$y(t) = c_2(t)g(t)$$

where $c_1(t)$ and $c_2(t)$ are two spread spectrum codes (orthogonal) or spreading signals. These signals have a very large bandwidth compared with the bandwidths of $f(t)$ and

$g(t)$. This concept is at the basis of the signal code multiplexing (CDMA=code division multiple access). All users in a CDMA system use the same carrier frequency and may transmit simultaneously.

The three multiple access schemes are sketched in Fig. 6 [11].

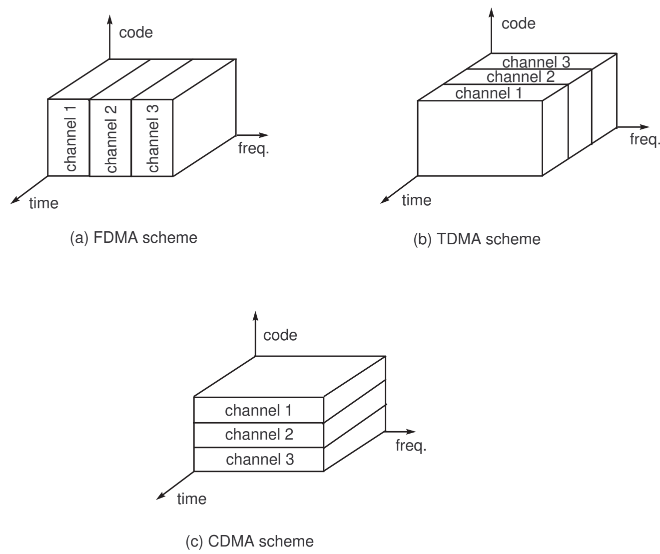

Figure 6: FDMA, TDMA and CDMA multiple access schemes

1. Frequency-division multiple access (FDMA).

In this technique, disjoint sub-bands of frequencies are allocated to the different users on a continuous-time basis. In order to reduce interference between users, allocated adjacent channel bands, *guard bands*, are used to act as buffer zones. These guard bands are necessary because of the impossibility of achieving ideal filtering for separating the different users.

2. Time-division multiple access (TDMA).

In this second technique, each user is allocated the full spectral occupancy of the channel, but only for a short duration of time called *slot*. Buffer zones in the form of *guard times* are inserted between the assigned time slots. This is done to reduce interference between users by allowing for time uncertainty that arises due to system imperfections, especially in synchronization schemes.

3. Code-division multiple access (CDMA).

In the CDMA scheme all users operate with a nominal frequency and use *simultaneously* all the available bandwidth. An important feature is that, contrarily to the FDMA and TDMA schemes, practically no coordination (in time or frequency) is required among multiple users. The CDMA scheme uses a class of signaling techniques known collectively as *spread-spectrum modulation*.

Sometimes it is also considered a fourth type of multiplexing, the *space-division multiple access* (SDMA) where multibeam antennas are used to separate radio signals by pointing them along different directions. Thus, different users are enabled to access the channel simultaneously on the same frequency or in the same time slot [11].

2.2 spread-spectrum modulations

Spread spectrum systems were originally developed for military applications, to provide anti-jam and low probability of detection communications by spreading the signal over a large bandwidth and transmitting it with a low power spectral density. However, there are important commercial applications as well. Currently, it is used in local area network standards such as IEEE801.11 (commonly known as WiFi) and wireless personal area networks such as Bluetooth [3]. The definition of a spread-spectrum modulation may be stated in two parts [13], [10]:

1. Spread spectrum is a means of transmission in which the transmitted signal occupies a bandwidth in excess of the minimum bandwidth necessary to send it.

2. The spectrum spreading is accomplished before transmission through the use of a code (spreading) signal that is independent of the data sequence. The same code is used in the receiver (operating in synchronism with the transmitter) to despread the received signal so that the original data sequence may be recovered.

The two main ways used to produce spread-spectrum signals are the *direct sequence* (DS) and the *frequency-hopping* (FH) techniques. In a DS technique, two stages of modulation are used. First, the incoming data sequence is used to modulate a wide-band code signal. The code transforms the narrowband data sequence into a noiselike wideband signal. The resulting wideband signal undergoes a second modulation using a PSK technique. In a FH technique, on the other hand, the spectrum of the data-modulated carrier is widened by changing the carrier frequency in a pseudo-random manner. For their operation, both of these techniques rely on the availability of a noiselike spreading code called a *pseudo-random* or *pseudo-noise sequence*.

2.3 pseudo-noise sequences

A *pseudo-noise (PN) sequence* is a periodic binary sequence with a noiselike waveform that is usually generated by means of a feedback shift register, as shown in Fig. 7. It consists of an ordinary *shift register* made up of m flip-flops and a *logic circuit* that are interconnected to form a multiloop feedback circuit. The flip-flops in the shift register are regulated by a single timing clock. At each pulse (tick) of the clock, the state of each flip-flop is shifted to the next one down the line. With a total number of m flip-flops, the number of possible states of the shift-register is at most $2^m - 1$ (the only state not used in the all-zeros state). Thus, the PN sequence generated by a feedback shift register is periodic with a period of at most $2^m - 1$.

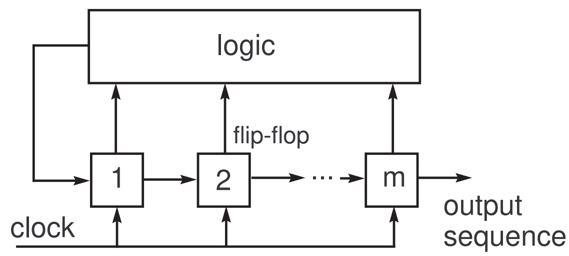

Figure 7: Feedback shift register

Note that, if the PN sequence was truly random, the generated signal would be a polar NRZ line code with power spectral density given by

$$G_c(f) = T_c \operatorname{sinc}^2(fT_c) \tag{20}$$

which is a continuous power spectrum in contrast with the power spectrum of the PN sequence which is a line spectrum. However, for PN sequences with long periods (T), (20) is still a good approximation. The (time-average) autocorrelation function over the chip interval, corresponding to (20) is [8]

$$\begin{aligned} R_c(\tau) &= \frac{1}{T_c}\int_0^{T_c} E\{c(t)c(t-\tau)\}\,dt \\ &= \Lambda\left(\frac{\tau}{T_c}\right) = \begin{cases} 1 - \frac{|\tau|}{T_c}, & 0 \le |\tau| < T_c \\ 0, & |\tau| \ge T_c \end{cases} \end{aligned} \tag{21}$$

Unlike the autocorrelation of the PN sequences, (21) is not periodic (see Fig. 8).

2.4 Direct-sequence spread spectrum (DS)

2.4.1 DS transmitter and receiver

Figure 9a) represents the main blocks of a DS spread spectrum transmitter for 2-PSK signals. The transmitter first converts the incoming binary data sequence $\{b_k\}$ into a polar NRZ waveform $b(t)$, which is followed by two stages of modulation:

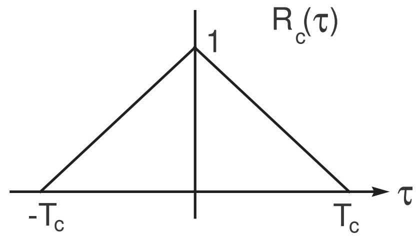

Figure 8: Autocorrelation function of the code signal (with infinite period)

• The first stage consists of a product modulator with the data (message) signal $b(t)$ and the PN signal $c(t)$ (PN sequence) as inputs. The result is the baseband signal $v(t)$, as illustrated in Fig. 10.

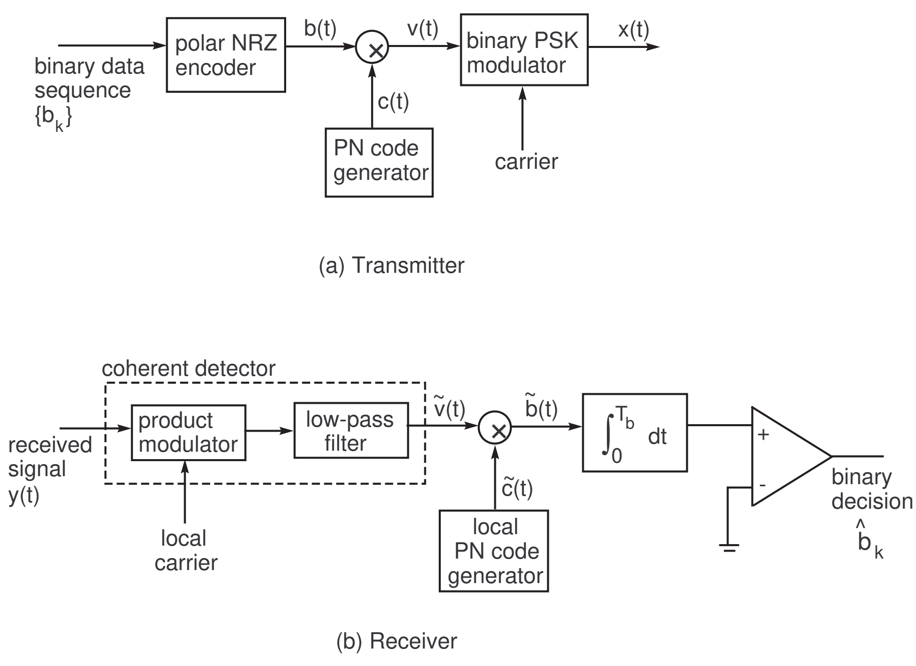

Figure 9: Direct-sequence spread spectrum coherent PSK: a) transmitter, b) receiver

• The second stage consists of a binary PSK modulator. The transmitted signal $x(t)$ is thus a *direct-sequence spread binary phase-shift-keyed* (DS/BPSK) *signal*.

Note that the chip duration is much smaller than the bit duration, that is $T_c \ll T_b$; thus, the bandwidth of $v(t) = b(t)c(t)$ will be much larger than the bandwidth of $b(t)$. In fact, the (time-average) autocorrelation of $v(t)$ over the bit interval is

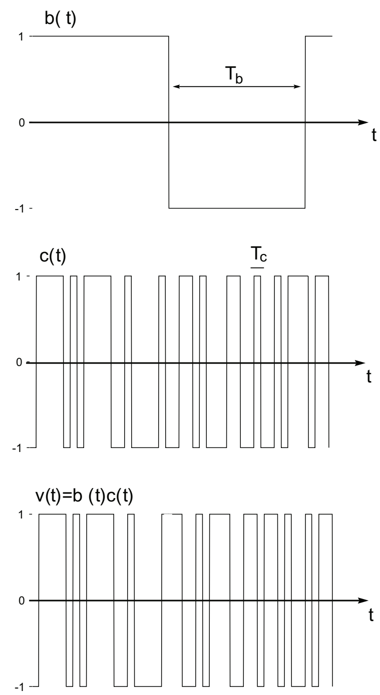

Figure 10: Baseband signals: a) message $b(t)$, b) code $c(t)$, c) spread spectrum signal $v(t)$

$$R_v(\tau) = \frac{1}{T_b}\int_0^{T_b} E\{b(t)b(t-\tau)\}E\{c(t)c(t-\tau)\}\,dt$$

$$= \frac{1}{N}\sum_{n=1}^{N}\int_{(n-1)T_c}^{nT_c} E\{b(t)b(t-\tau)\}E\{c(t)c(t-\tau)\}\,dt$$

$$\approx \frac{1}{N}\sum_{n=1}^{N} R_c(\tau) = R_c(\tau)$$

where N stands for the number of chips per code period, leading to the power spectrum

$$G_v(f) \approx G_c(f)$$

The signal bandwidth of signal $v(t)$ is approximately equal to the one of the code signal, that is, $B \approx 1/T_c$ (measured between nulls). Thus, the radio-frequency bandwidth of the spread spectrum signal is $2B$. The effect of bandwidth growth is illustrated in Fig. 11.

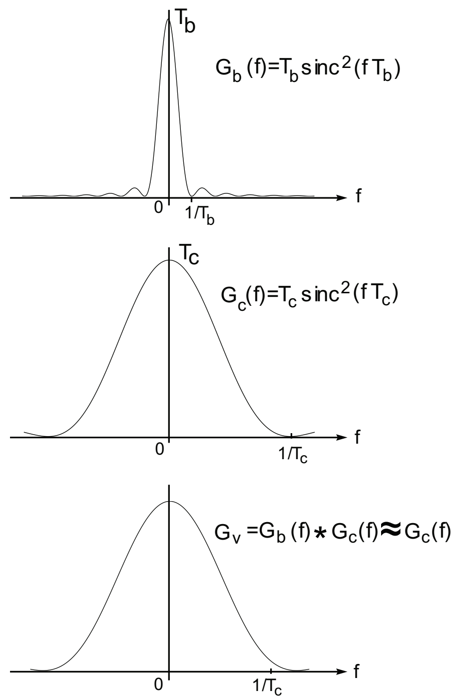

Figure 11: Baseband power spectra of a) message $b(t)$, b) code $c(t)$, c) spread spectrum signal $v(t)$

The receiver, shown in Fig. 9b), consists of two stages of demodulation. In the first stage, the received signal $y(t)$ and a locally generated carrier are applied to a product modulator followed by a low-pass filter whose bandwidth is equal to the original message signal $m(t)$. This stage reverses the phase-shift keying applied to the transmitted signal. The second stage of demodulation performs spectrum despreading by multiplying the low-pass filter output by a locally generated replica of the PN signal $c(t)$, followed by the integration over a bit interval $0 \leq t \leq T_b$ and finally by the decision making.

Consider that the modulated signal in Fig. 9a) is

$$x(t) = Av(t)\cos(\omega_c t) = Ab(t)c(t)\cos(\omega_c t)$$

and the received signal is

$$y(t) = x(t) + w(t)$$

where $w(t)$ is AWGN. The receiver's local carrier is $\cos(\omega_c t + \phi)$ with ϕ representing a possible phase error. Multiplying $y(t)$ by the local carrier and performing lowpass filtering yields

$$\widetilde{v}(t) = \frac{A}{2}b(t)c(t)\cos\phi + n(t)$$

where $n(t)$ is a lowpass Gaussian noise. Next, we multiply $\widetilde{v}(t)$ by a locally generated code signal as

$$\widetilde{b}(t) = \widetilde{v}(t)c(t-\tau) = \frac{A}{2}b(t)c(t)c(t-\tau)\cos\tau + \widetilde{n}(t)$$

where τ denotes the delay error of the locally generated code signal and $\widetilde{n}(t)$ is lowpass noise. This operation aims at removing the code signal from $\widetilde{v}(t)$ (despreading operation) and results in a narrowband signal. In fact, if $\tau = 0$ we obtain $c(t)c(t-\tau) = c^2(t) = 1$. Finally, integrating in the bit interval leads to

$$z(T_b+\epsilon) = \int_{\epsilon}^{T_b+\epsilon} \widetilde{b}(t)\,dt = \pm\frac{A}{2}\cos\phi \int_{\epsilon}^{T_b+\epsilon} c(t)c(t-\tau)\,dt + n \qquad (22)$$

where signs $\pm$ indicate the bits 0 or 1 and n is a zero-mean Gaussian random variable. The quantity ϵ denotes the bit synchronization error.

Consider now that $T_b = PT$, with $P = 1, 2, \ldots$, and T is the period of the code signal. Taking into account that the code signal is characterized by the autocorrelation given in (21), we may express the correlator output in terms of the autocorrelation as

$$z(T_b+\epsilon) = \pm\gamma\frac{AT_b}{2}R_c(\tau)\cos\phi + n$$

where γ is a loss factor due to the bit synchronization error ϵ. Since $R_c(\tau) \neq 0$ only in the interval $-T_c \leq \tau \leq T_c$, it means that the receiver cannot detect the bit if the code misalignment is $|\tau| \geq T_c$.

In short. The receiver for spread spectrum signals using the DS technique requires 3 types of synchronization:

- carrier frequency and phase synchronization ($\phi \approx 0$)
- spreading code synchronization ($\tau \approx 0$)
- bit synchronization ($\epsilon \approx 0$).

In general, the code and bit synchronization are not independent.

In order to achieve the 3 types of synchronization the spread spectrum receivers have 2 different modes of operation:

- acquisition mode
- tracking mode.

In acquisition mode the receiver obtains coarse estimates of the Doppler frequency and the code delay τ. In tracking mode the frequency/phase closed loops (FLL or PLL) and the code delay closed loop (DLL) permit maintaining the different types of synchronization.

2.5 Multiple access interference in DS-CDMA

Consider a CDMA system in which L users occupy the same frequency band and transmit at the same time, according to the scenario described in Fig. 12. This scenario occurs, for instance, in mobile radio when the users are sending their messages to a central base station (uplink) in an asynchronous way (sometimes named A-CDMA [12]). If the base station is processing the signal from user m, all the remaining $L-1$ transmissions will be considered interference. Let the received signal from a generic user k be given by [11]

$$s_k(t-\tau_k) = \sqrt{2P_k}c_k(t-\tau_k)b_k(t-\tau_k)\cos(\omega_0 t + \phi_k), \quad k = 0, \ldots, L-1 \tag{23}$$

where P_k is the received power, $c_k(t)$ is the spreading (chip) sequence, τ_k is the delay of user k relative to a reference user 0, $b_k(t)$ is the data (bit) sequence and ϕ_k is the phase offset of user k relative to a reference user 0. Since τ_k and ϕ_k are relative terms, we can define $\phi_0 = 0$ and $\tau_0 = 0$.

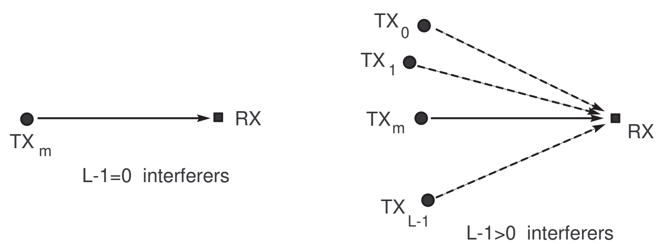

Figure 12: Scenarios of interference in CDMA communications

Let us assume that both $c_k(t)$ and $b_k(t)$ are binary sequences having values ± 1. The chip sequence $c_k(t)$ is of the form

$$c_k(t) = \sum_{j=-\infty}^{\infty} \sum_{i=0}^{N-1} c_{k,i} \Pi\left(\frac{t-(i+jN)T_c}{T_c}\right), \quad c_{k,i} = \pm 1$$

where N is the number of chips sent in the PN sequence period $T = NT_c$.

For the data sequence, $b_k(t)$, T_b is the bit period. We assume that the bit period is an integer multiple of the chip period: $T_b = MT_c$. Note that M and N do not need to be the same. The binary data sequence $b_k(t)$ is given by

$$b_k(t) = \sum_{j=-\infty}^{\infty} b_{k,j} \Pi\left(\frac{t-jT_b}{T_b}\right), \quad b_{k,j} = \pm 1$$

If we define the *spreading factor* or *processing gain* as

$$G = \frac{T_b}{T_c}$$

it can be shown that, when $K \gg 1$, the average bit error probability in the detection of signal m, with $m = 0, \ldots, L-1$, is approximately given by [11]

$$P_b = Q\left(\sqrt{\frac{P_m}{\frac{1}{3G}\sum_{\substack{k=0\\k\neq m}}^{L-1} P_k + \frac{\mathcal{N}_0}{2T_b}}}\right) \tag{24}$$

When all the incoming signals have the same power, (24) may be simplified to

$$P_b = Q\left(\sqrt{\frac{2E_b}{\mathcal{N}_0}}\frac{1}{\sqrt{1+\frac{2E_b}{\mathcal{N}_0}\frac{L-1}{3G}}}\right) \tag{25}$$

Fig. 13 plots the bit error probabilities obtained with formula (25) for $L = 100$ users and several values of the processing gain G. The figure shows that, for every value of G there is an irreducible error floor which is given by

$$\lim_{E_b/\mathcal{N}_0\to\infty} P_b = Q\left(\sqrt{\frac{3G}{L-1}}\right) \tag{26}$$

Fig. 14 plots the bit error probabilities achieved with a fixed processing gain $G = 1000$ for different numbers of users. This figure evidences a nice feature of the CDMA systems, known as *graceful degradation* property: the degradation of the quality of service can be traded off with a capacity increase. This means that the number of concurrently active users can be increased at the cost of a larger bit error error (BER) [12]. For example, consider $E_b/\mathcal{N}_0 = 10$ dB and $G = 1000$. In the absence of interferers ($L = 1$) the BER is equal to the one of BPSK, $P_b = Q(\sqrt{20}) = 3.9 \times 10^{-6}$. If the number of users increases to $L = 10$, the BER increases slightly to 7×10^{-6}, but if the active number of users grows to $L = 100$, the BER will degrade more significantly reaching 2.6×10^{-4}.

2.6 frequency-hopping spread spectrum (FH)

In the direct sequence (DS) type of spread spectrum, the use of a PN sequence to modulate a binary PSK signal achieves instantaneous spreading of the transmission bandwidth. By increasing the processing gain $G = T_b/T_c$ larger bandwidths are obtained. An alternative to this spread spectrum technique consists of randomly hopping the data-modulated carrier from one frequency to the next. In effect, the spectrum of the transmitted signal is spread *sequentially* rather than instantaneously.

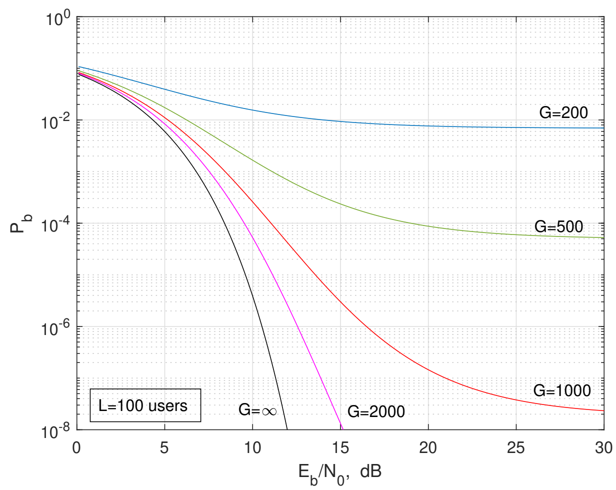

Figure 13: Bit error probabilities for a CDMA system using DS-PSK

The type of spread spectrum in which the carrier hops randomly from one frequency to another is called *frequency-hopping spread spectrum* (FH). A common modulation format for FH systems is that of *M-ary frequency-shift keying.* The resulting modulation is referred to simply as FH/MFSK. Since frequency hopping does not cover the entire spread spectrum instantaneously, we are led to two basic characterizations:
1. *Slow-frequency hopping*, in which the hop interval T_c is an integer multiple of the symbol duration T_s of the M-FSK signal. That is, several symbols are transmitted on each frequency hop.

2. *Fast-frequency hopping*, in which the symbol duration T_s is an integer multiple of the hop interval T_c. That is, the carrier frequency will change (or hop) several times during the transmission of one symbol.

Fig. 15 illustrates the frequency hopping modulation.

Fig. 16 describes a typical frequency-hopping M-FSK modulation system (transmitter+receiver).

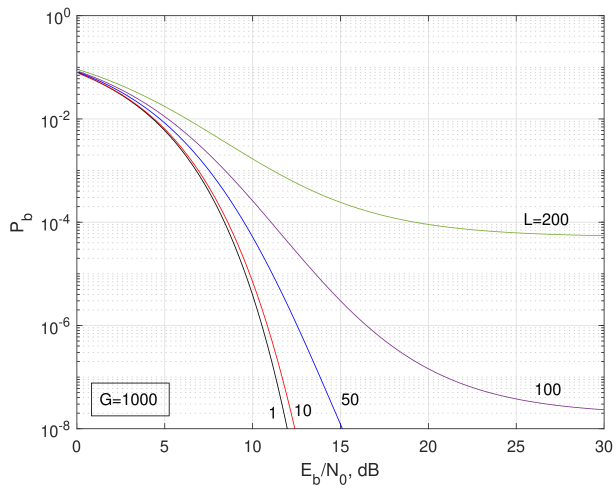

Figure 14: Bit error probabilities for a CDMA system with L users and processing gain $G = 1000$

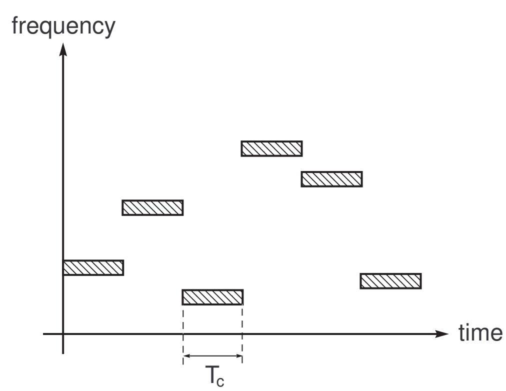

Figure 15: Illustrating frequency hopping

Frequency-hopping is used, for instance, in Bluetooth radio interface in the 2.4 GHz Industrial, Scientific and Medical (ISM) band. The standard uses a hopping rate of 1600 hops per second between the available 79 channels. The channels are spaced 1 MHz and are modulated with Gaussian frequency shift keying (GFSK) for 1 Mbps rate, $\pi/4$ DPSK for 2 Mbps rate or 8DPSK for 3 Mbps rate. Bluetooth devices are classified into three power classes based on their output power capabilities: class 1 (100 mW), class 2 (2.5 mW) and class 3 (1 mW).

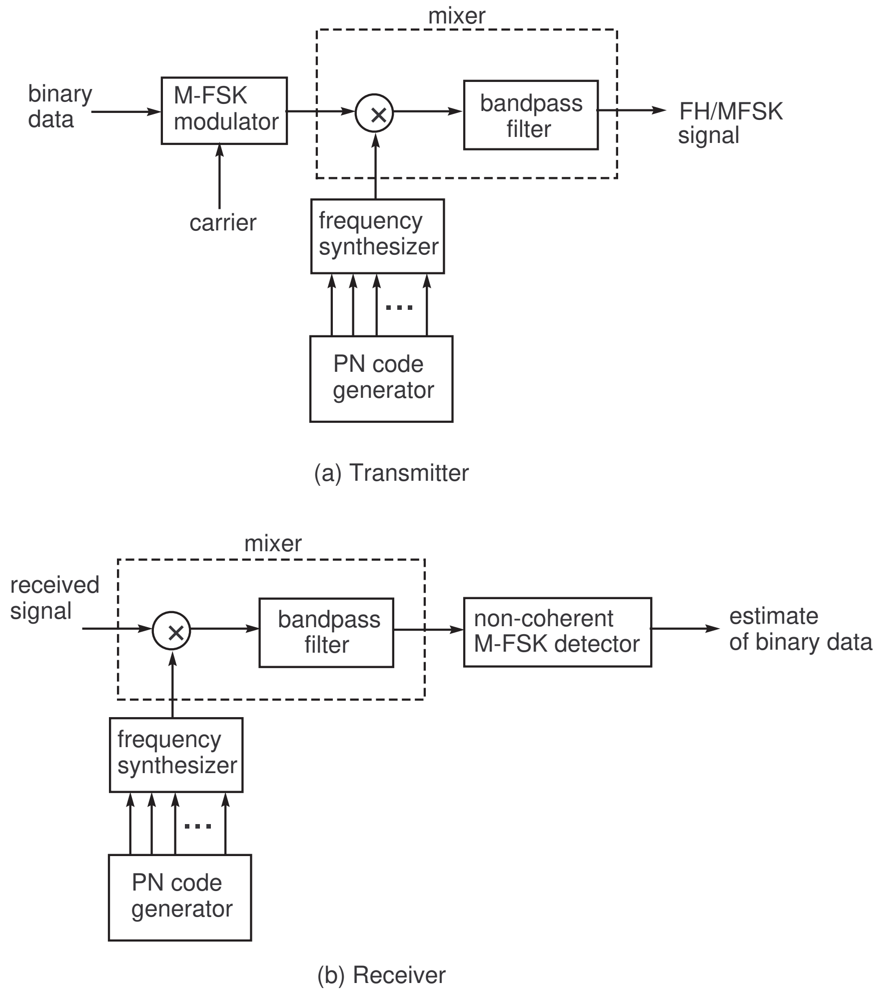

Figure 16: Frequency hopping system

References

[1] Andrea Goldsmith, "Wireless Communications", Cambridge University Press, N. York, 2005.

[2] Mounir Ghogho and Anathram Swami, "Carrier Frequency Synchronization for OFDM Systems", in Signal Processing for Mobile Communications Handbook, M. Ibnkahla (edt.), CRC Press, Boca Raton, Fl., 2005.

[3] Gordon Stüber, "Principles of Mobile Communication", fourth edition, Springer, Cham, Switzerland, 2017.

[4] Shinsuke Hara and Ramjee Prasad, "Multicarrer Techniques for 4G Mobile Communications", Artech House, Boston, 2003.

[5] Oran Brigham, “The Fast Fourier Transform and Its Applications”, Prentice Hall, Englewood Cliffs, NJ, 1988.

[6] T. Pollet, M. van Bladel and M. Moeneclaey, “BER Sensitivity of OFDM Systems to Carrier Frequency Offset and Wiener Phase Noise”, IEEE Trans. Commun., vol 43, February/March/April 1995.

[7] K. Sathananthan and C. Tellambura, “Probability of Error Calculation of OFDM Systems with Frequency Offset”, IEEE Trans. Commun., vol 49, November 2001.

[8] John Proakis and Masoud Salehi, “Digital Communications”, fifth edition, McGraw-Hill, Boston, 2008.

[9] ETSI EN 300 744 v1.6.2, European standard, “Digital Video Broadcasting (DVB): Framing structure, channel coding and modulation for digital terrestrial television”, October 2015.

[10] Bernard Sklar, “Digital Communications. Fundamentals and Aplications”, second edition, Prentice-Hall, Englewood Cliffs, N.J., 2001.

[11] Theodore S. Rappaport, “Wireless Communications. Principles and practice”, Prentice-Hall, Upper Saddle River, NJ, 2002.

[12] Fillipo Giannetti and Marco Luise, “Spread-Spectrum Techniques for Mobile Communications”, in Signal Processing for Mobile Communications Handbook, M. Ibnkahla (edt.), CRC Press, Boca Raton, Fl., 2005.

[13] Andrew Viterbi, “CDMA: Principles of Spread Spectrum Communication”, Addison-Wesley, Reading, MA, 1995.