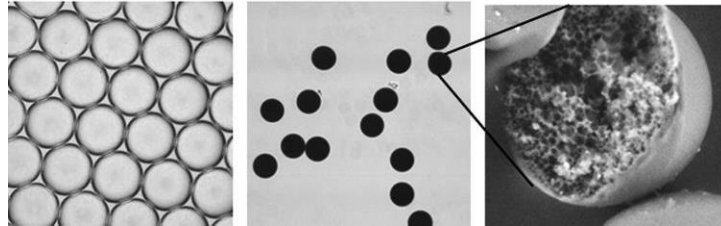
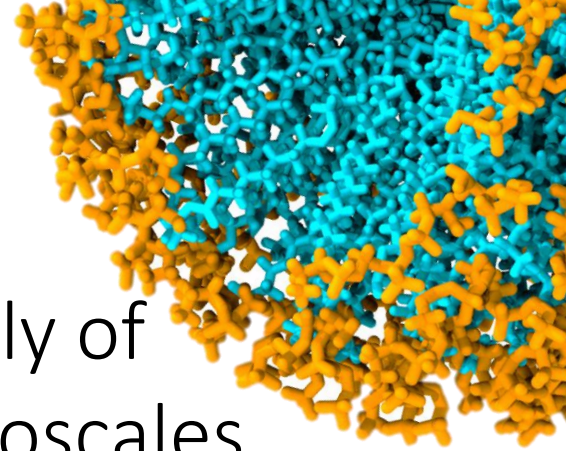


Spontaneous and directed-assembly of polymers from the molecular to macroscales



João T. Cabral

Department of Chemical Engineering, Imperial College London

j.cabral@ic.ac.uk

a personal introduction...



TÉCNICO
LISBOA

Dep Física
1992-1997



Department of
Physics 1996



1997



Imperial College of Science
Technology & Medicine
PhD 1998-2002



NIST

**National Institute of
Standards and Technology**

U.S. Department of Commerce

Postdoc
2002-2005



**Imperial College
London**

2005-



2008

復旦大學 高分子科学系
DEPARTMENT OF
MACROMOLECULAR SCIENCE
FUDAN UNIVERSITY



PRITZKER SCHOOL OF
MOLECULAR ENGINEERING

THE UNIVERSITY OF CHICAGO

2014-2015



Imperial College London



Imperial College London

- Imperial is a **research-led** university specialised in:

- **Engineering**

10 departments, incl Chemical Engineering

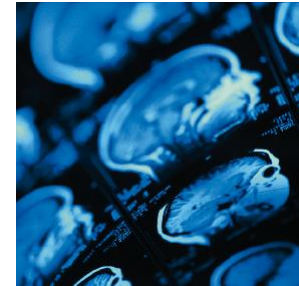
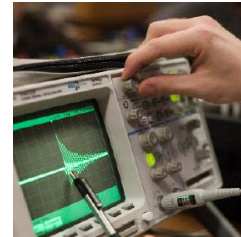
- **Natural Sciences**

Physics, Chemistry, Maths, Biosciences

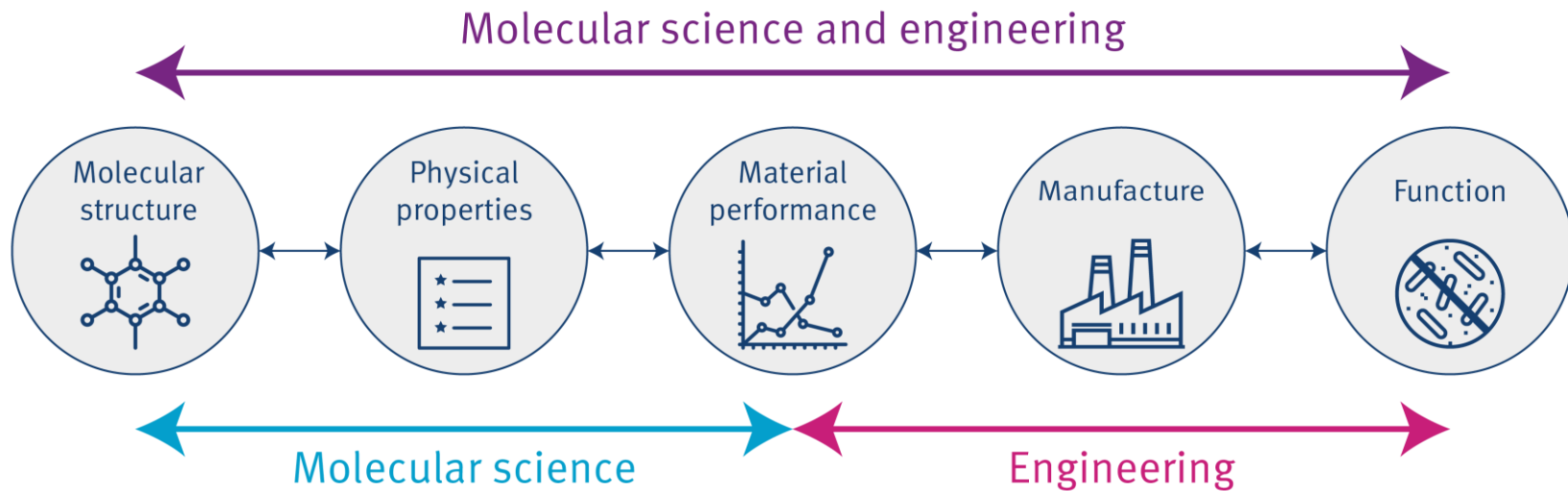
- **Medicine**

- **Business**

Humanities programme



Institute of Molecular science and engineering



Outline

1. Polymer thermodynamics & demixing

Cahn-Hilliard theory & spinodal decomposition.

Membranes and photovoltaics

2. Particle and capsule formation

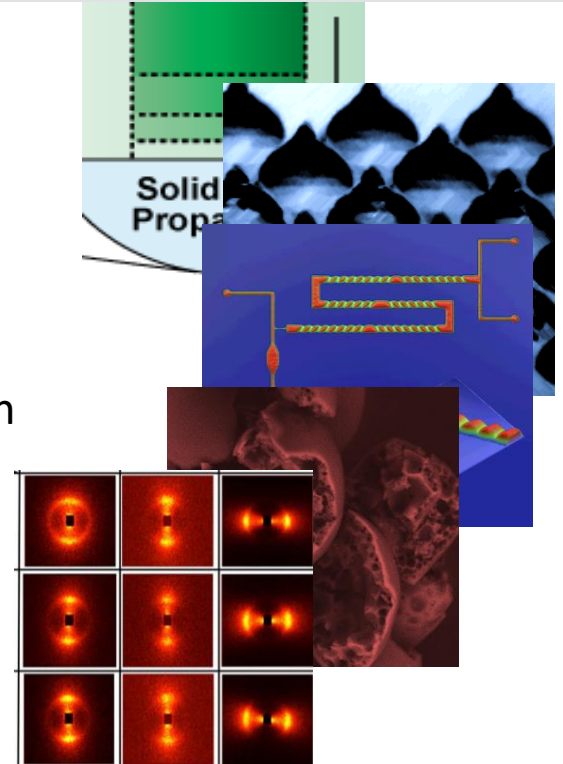
Levitation & microfluidics

Solution thermodynamics, phase inversion and solidification

Functional capsules with tunable, triggered release.

3. Waves of network formation

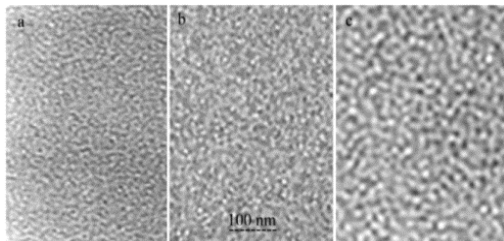
3D printing with light



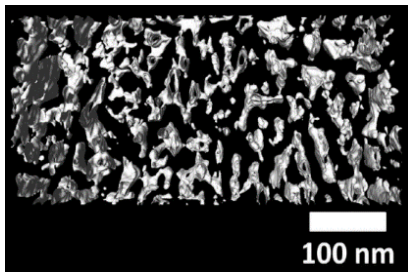
Functional polymeric spinodal nanostructures

(bicontinuous interpenetrating)

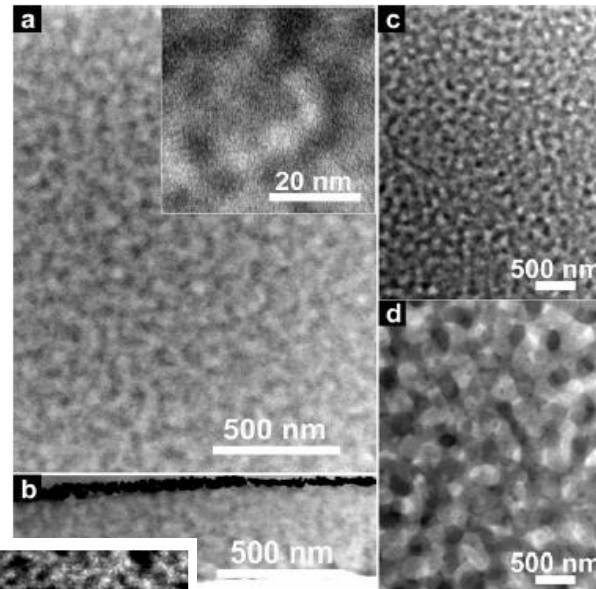
Organic photovoltaics (donor/acceptor)



Heeger Adv Func Mater 2005 2012

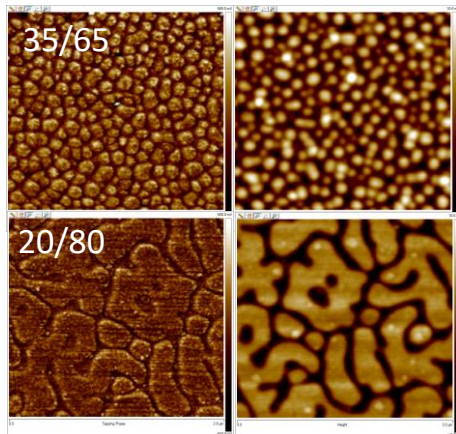


Reactive blending

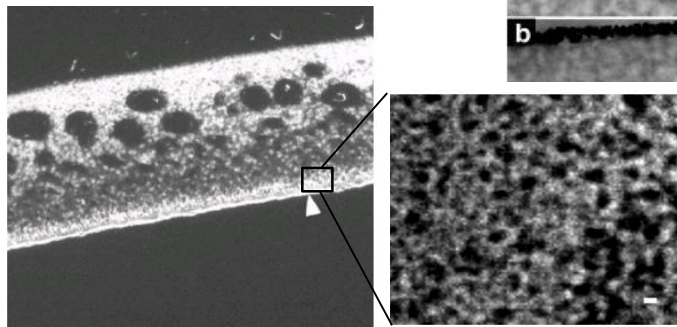


T Russell 2012

PS/PCBM (S Pont)



Membranes/separations

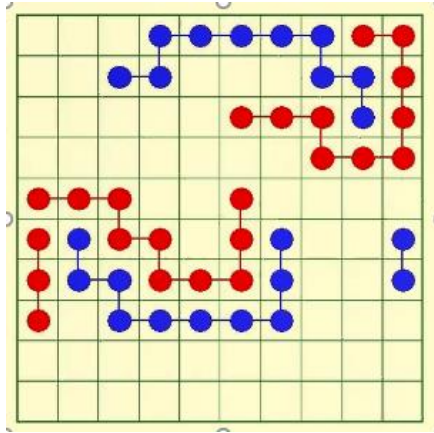


PVD, Liu JMS 2011

Phase inversion via ternary polymer/solvent/non-solvent quench

Polymer miscibility & demixing

Flory-Huggins lattice



Binary mixture

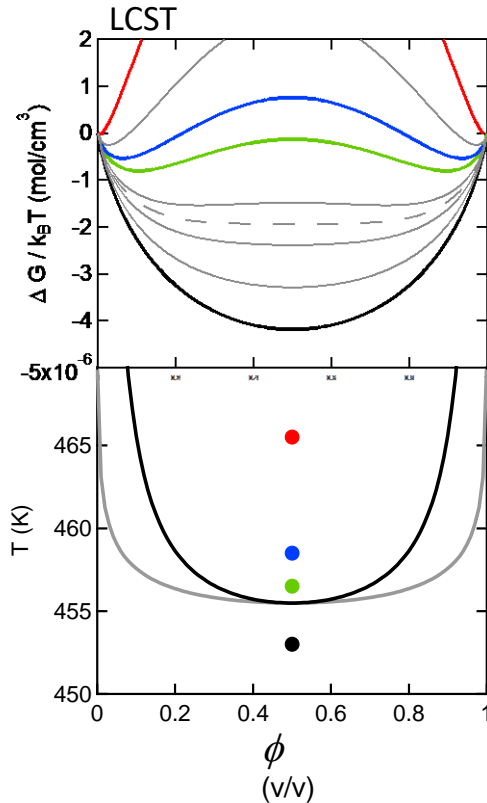
Thermodynamics $\Delta G_m = \Delta H_m - T\Delta S_m$

$$\frac{\Delta G_m}{k_B T} = \frac{\phi}{v_A N_A} \ln \phi + \frac{(1-\phi)}{v_B N_B} \ln(1-\phi) + \frac{\phi(1-\phi)}{v} \chi$$

Combinatorial
entropy

Enthalpy

Polymer miscibility & demixing



Thermodynamics $\Delta G_m = \Delta H_m - T\Delta S_m$

$$\frac{\Delta G_m}{k_B T} = \frac{\phi}{v_A N_A} \ln \phi + \frac{(1-\phi)}{v_B N_B} \ln(1-\phi) + \frac{\phi(1-\phi)}{v} \chi$$

Combinatorial
entropy

Enthalpy

Phase boundaries?

Binodal

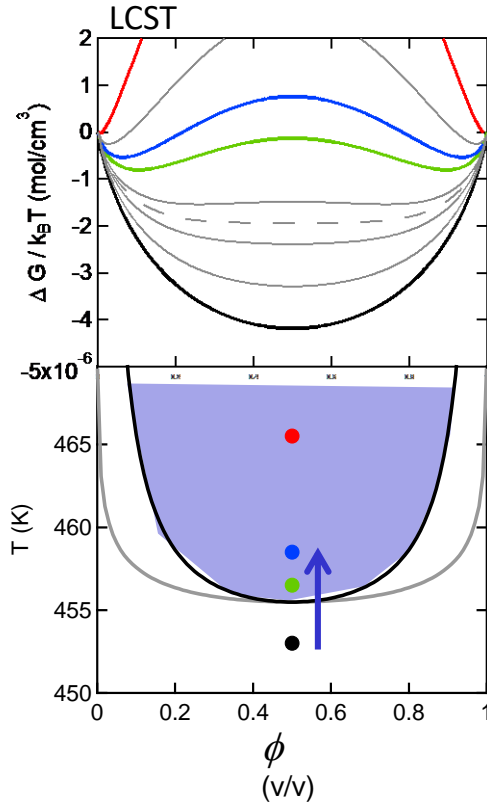
$$\left\{ \begin{array}{l} \frac{\partial \Delta G_m}{\partial \phi_{B1}} = \frac{\partial \Delta G_m}{\partial \phi_{B2}} \equiv \mu \quad \text{‘minima’} \\ \Delta G_m(\phi_{B1}) + \Delta G_m(\phi_{B2}) = \min \end{array} \right.$$

Spinodal

$$\frac{\partial^2 \Delta G_m}{\partial \phi^2} = 0$$

inflection points

Polymer miscibility & demixing

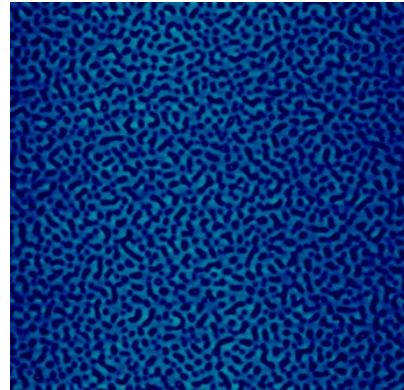


Thermodynamics $\Delta G_m = \Delta H_m - T\Delta S_m$

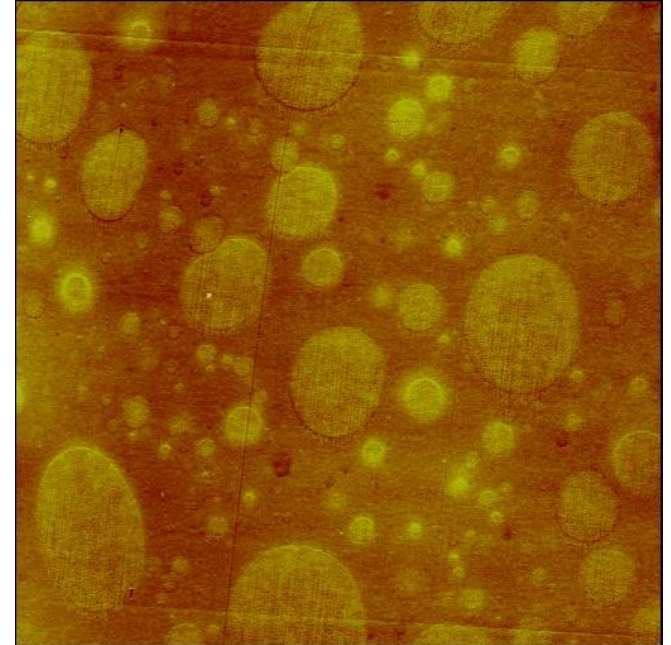
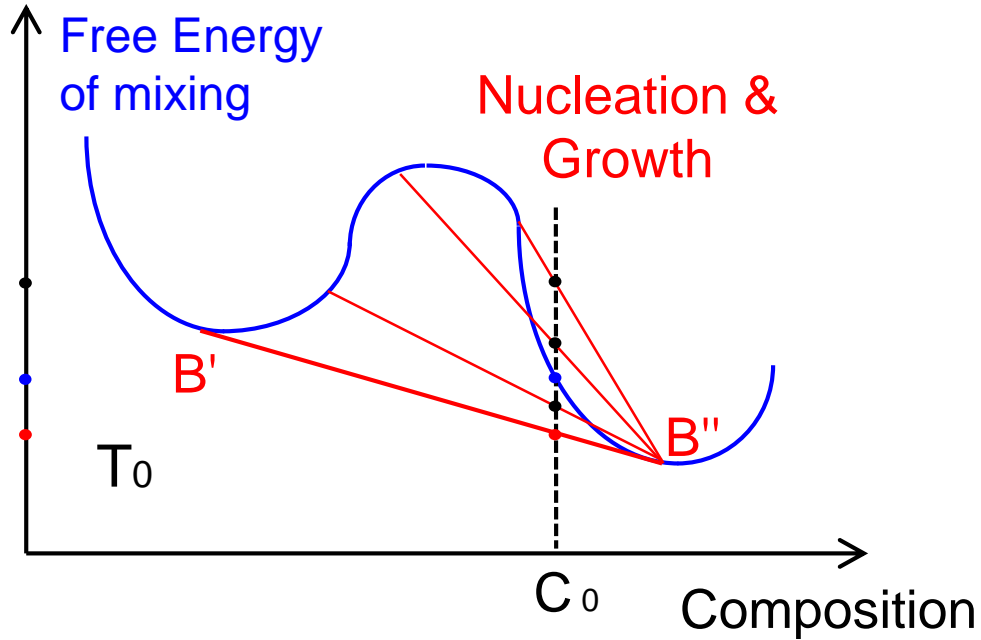
$$\frac{\Delta G_m}{k_B T} = \frac{\phi}{v_A N_A} \ln \phi + \frac{(1-\phi)}{v_B N_B} \ln(1-\phi) + \frac{\phi(1-\phi)}{v} \chi$$

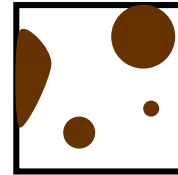
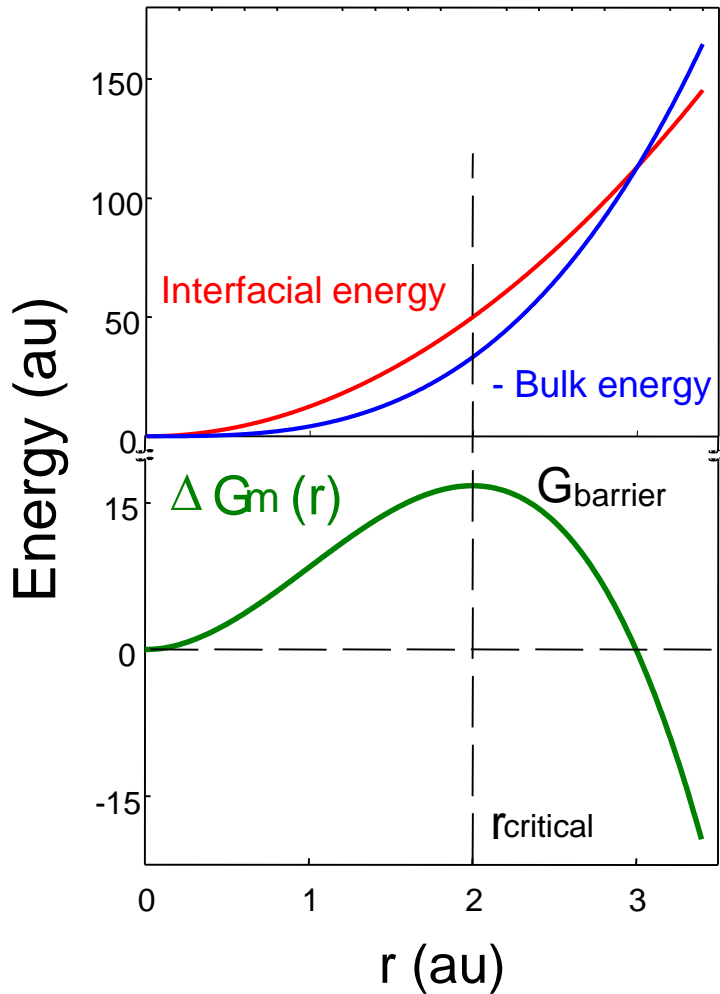
Combinatorial
entropy

Enthalpy



Nucleation and growth





Energy balance

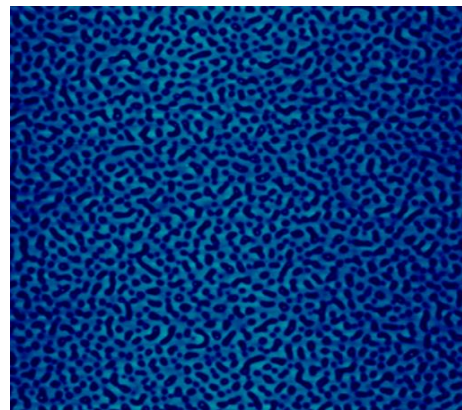
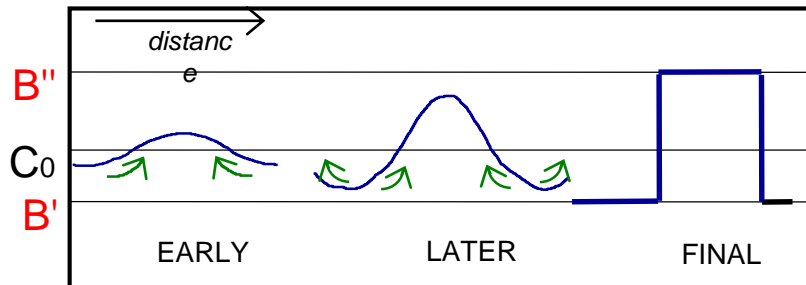
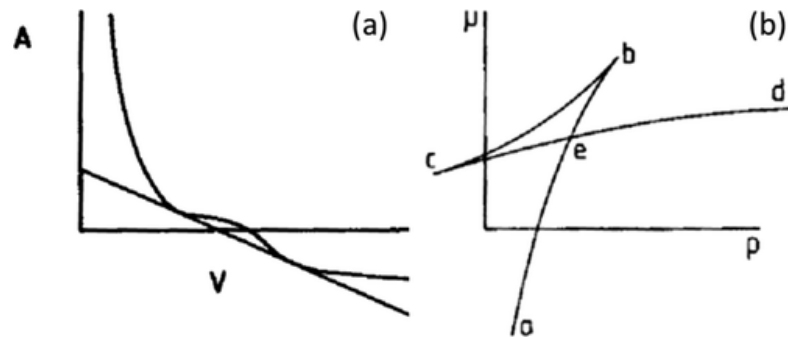
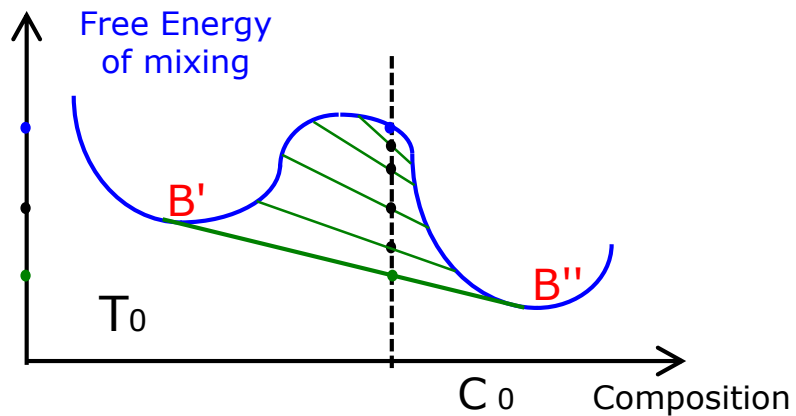
$$\Delta G(r) = -\frac{4\pi}{3} r^3 \Delta g + 4\pi r^2 \sigma$$

bulk interface

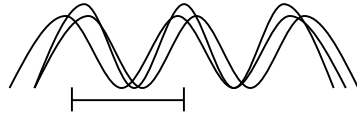
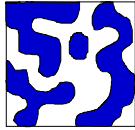
→ **Critical droplet**

$$\Delta G_{\text{barrier}} = \frac{16\pi\sigma^3}{3\Delta g^2} \quad r_c = 2\sigma/\Delta g$$

Spinodal decomposition



Cahn-Hilliard (Cook) linearised theory



$$d = \frac{2\pi}{q_m}$$

$$\phi = \phi_o + \tilde{\phi}$$

equation of motion
concentration fluctuations

d characteristic length of phase separation

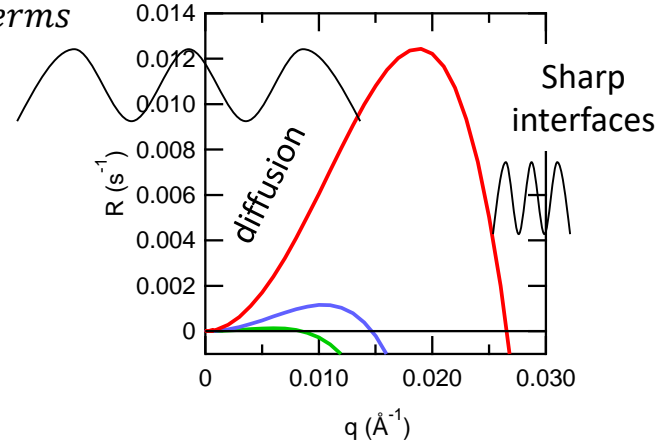
$$-\frac{\partial \phi}{\partial t} = \underbrace{MG''}_{-D_{app}} \nabla^2 \phi - 2Mk \nabla^4 \phi + \text{non linear terms}$$

Growth rate of Fourier components

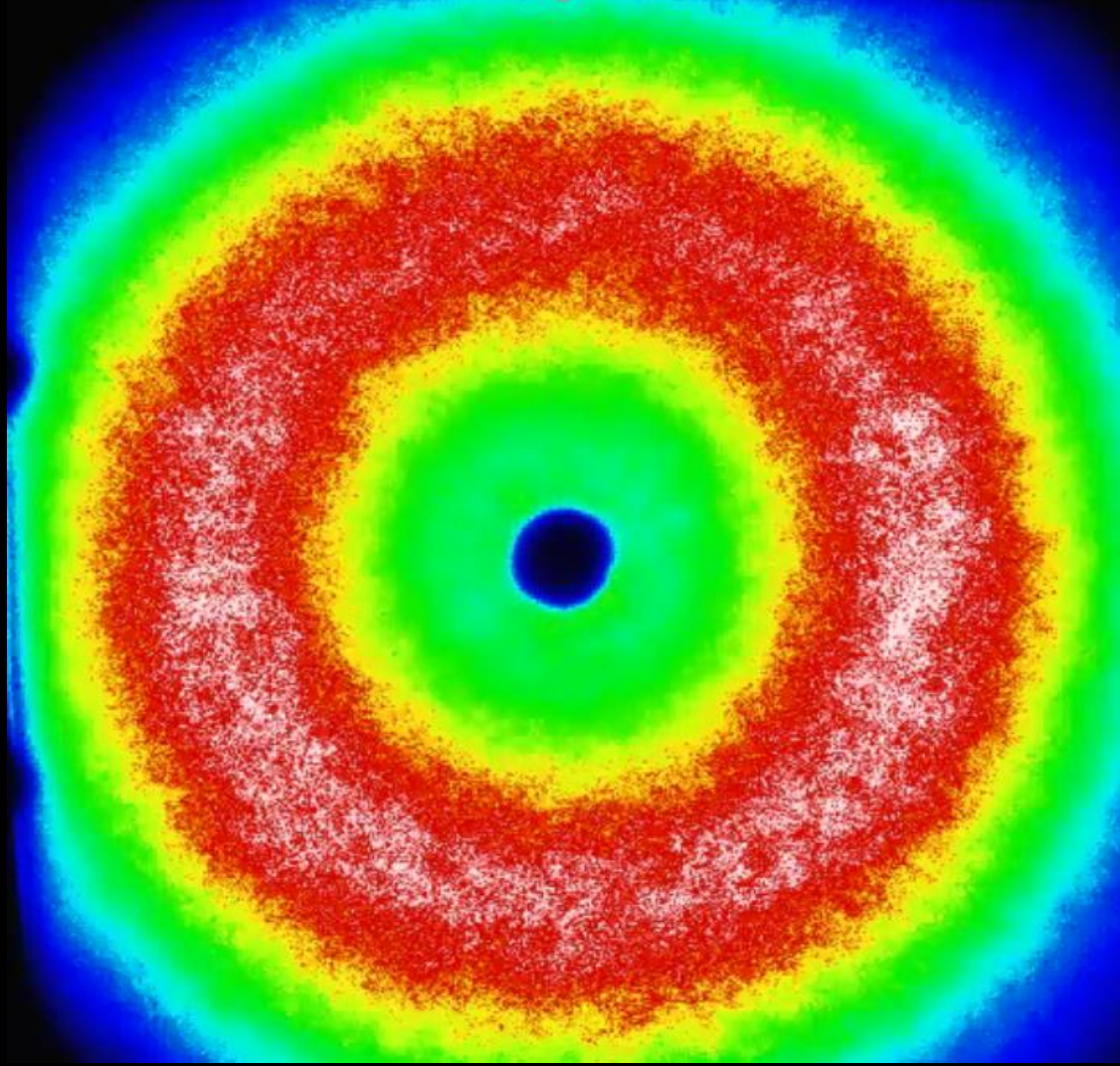
$$R(q) = \underbrace{-MG''}_{\text{diffusion}} q^2 - \underbrace{2Mk}_{\text{sharp interfaces}} q^4$$

$$q_m = \sqrt{-\frac{G''}{4k}} \quad \text{dominant wavelength}$$

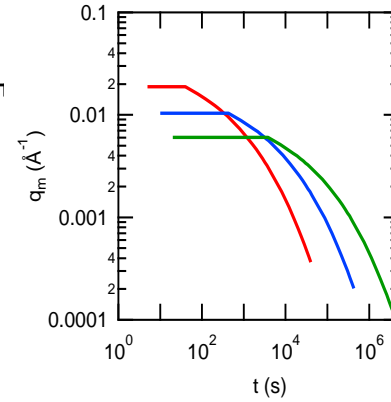
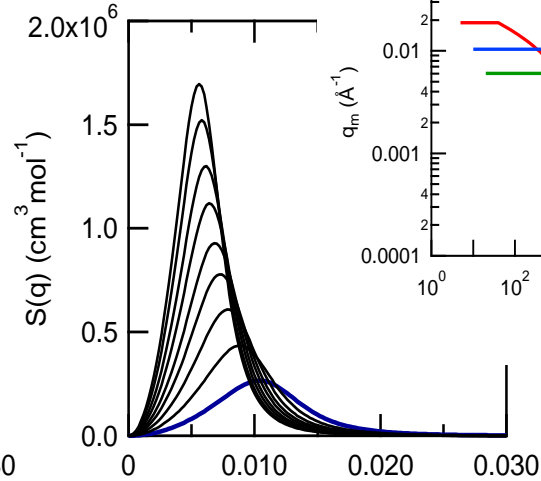
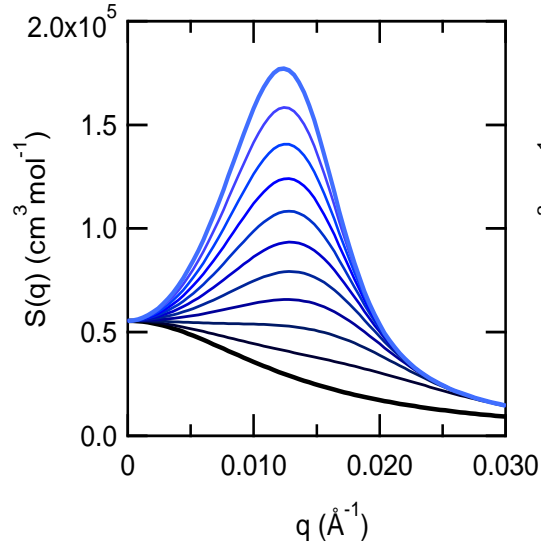
$$\tau_c \sim \left(\frac{MG''^2}{8k} \right)^{-1} \quad \text{duration of 'early stage'}$$



$$R(q_m) = \frac{MG''^2}{8k}$$

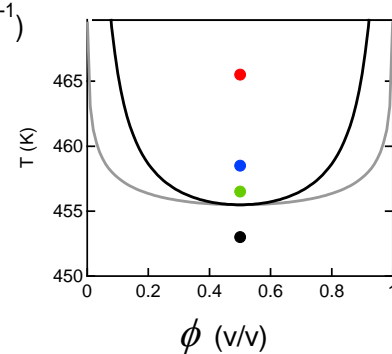


Early stages and coarsening

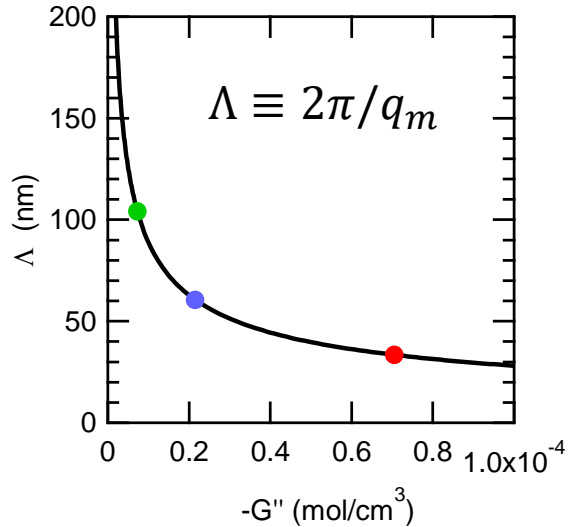


$$q_m = \sqrt{-\frac{G''}{4k}}$$

$\Delta T = 2.5^\circ\text{C}$, where $G'' = 1.8 \times 10^{-5} \text{ mol/cm}^3$, and $k = 0.05 \text{ \AA}^2 \text{ mol/cm}^3$, $M = 10^6 \text{ \AA}^2 \text{ s}^{-1} \text{ cm}^3 \text{ mol}^{-1}$



The dominant lengthscale



$$q_m = \sqrt{-\frac{G''}{4k}}$$

Second derivative of ΔG $G'' \equiv \frac{\partial^2 \left(\frac{\Delta G}{k_B T} \right)}{\partial \phi^2}$

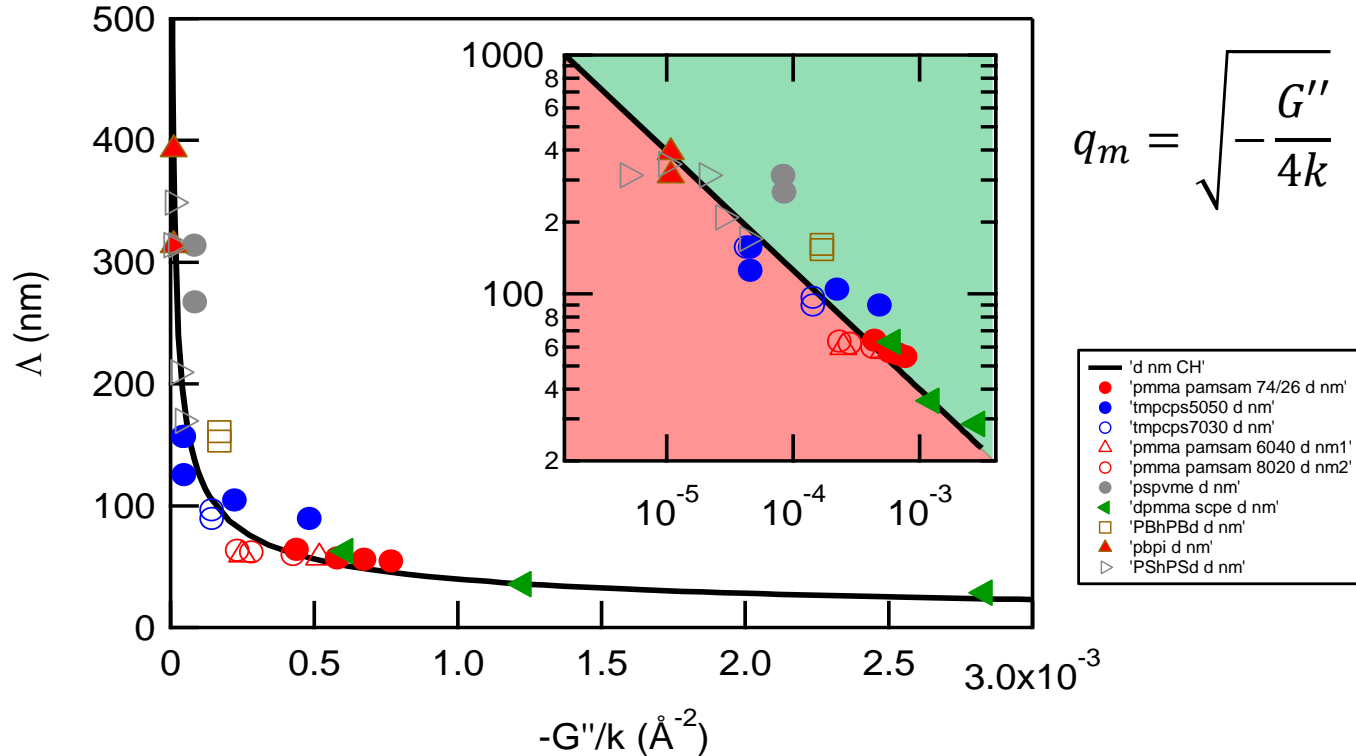
if we assume Flory-Huggins

$$G'' = \frac{1}{\phi_A v_A N_A} + \frac{1}{\phi_B v_B N_B} - 2 \frac{\chi_{AB}(T)}{v}$$

Gradient coefficient ($k=0.05 \text{ \AA}^2 \text{ mol/cm}^3$)

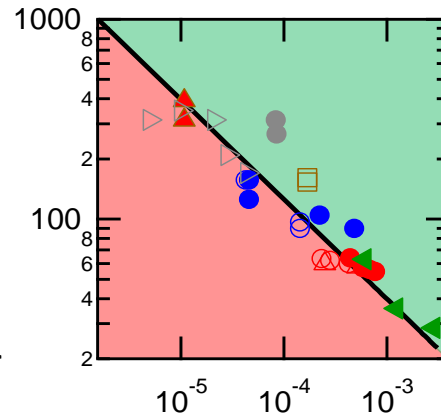
$$k = \frac{1}{6} \left(\frac{\langle R_{gA}^2 \rangle_z}{\phi_A v_A \langle N_A \rangle_w} + \frac{\langle R_{gB}^2 \rangle_z}{\phi_B v_B \langle N_B \rangle_w} \right) \approx k = \frac{1}{6} \left(\frac{b_A^2}{6\phi_A v_A} + \frac{b_B^2}{6\phi_B v_B} \right)$$

Re-examining Cahn-Hilliard scale against (all?) early stage SD experiments

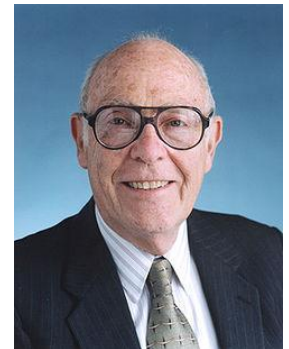


Perspective

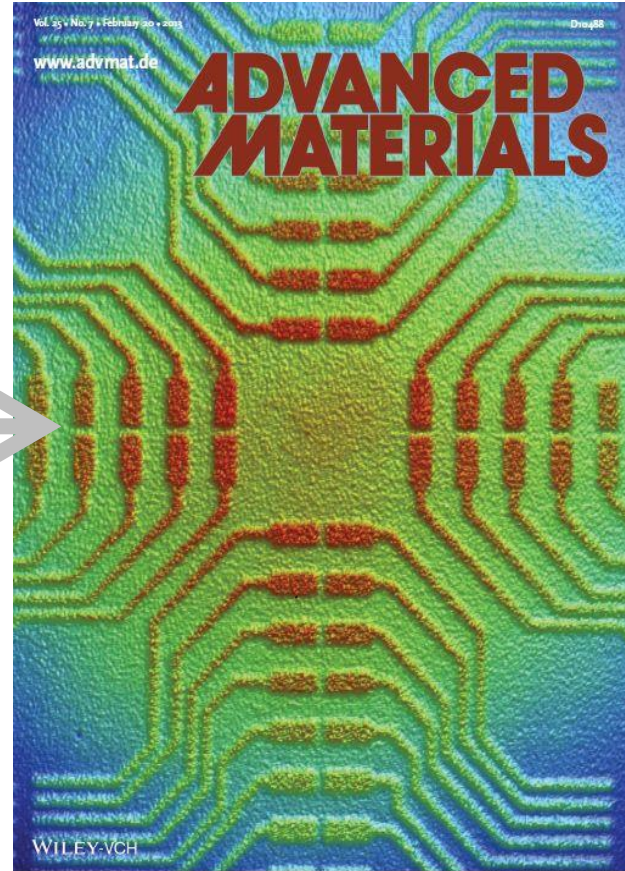
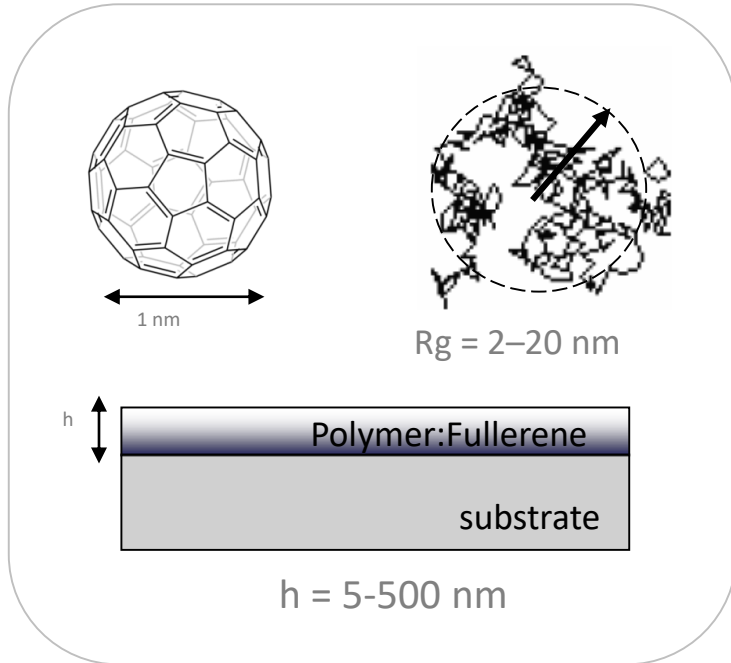
- C-H theory is an excellent predictor of initial S-D lengthscales
- Despite extensive $\chi(\phi,T)$, F-H theory is not helpful for prediction of G'' and thus behaviour in new blends.
- Remains great need for thermodynamic theory capable of correlating ΔG or G'' to molecular architecture to enable first principles material design by spinodal decomposition (LCT, Prism, LCL, ...)



“Having been the father of this equation, it really is very similar to being a father, because it has a life of its own. I long ago lost the ability to keep guiding it. It’s just going, and I’m very proud of it, *but it’s on its own.*”

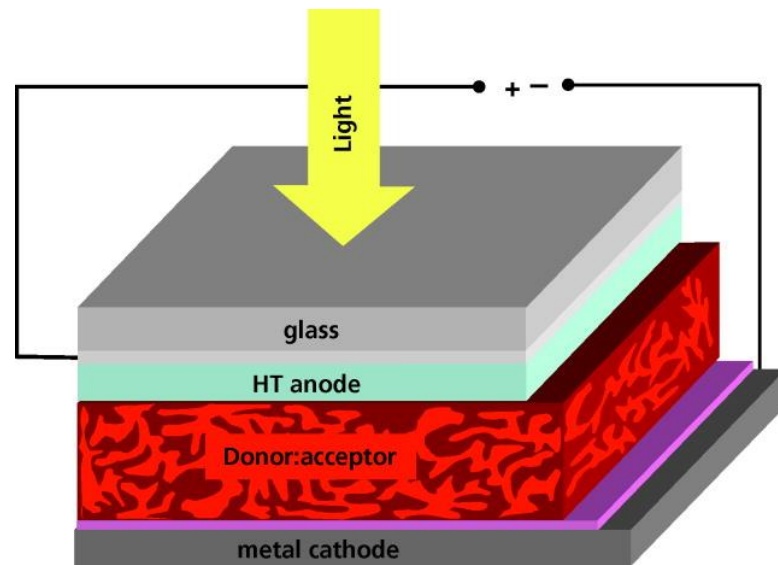
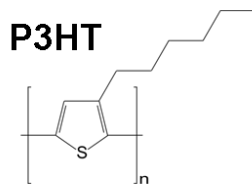
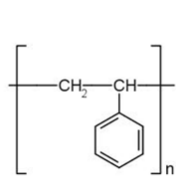
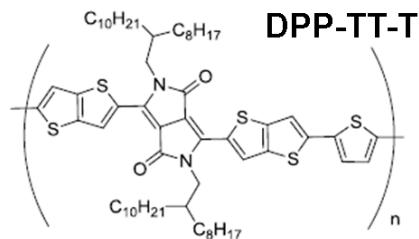
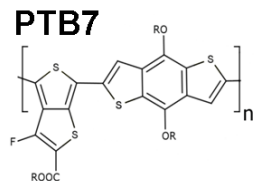
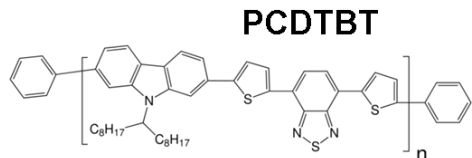


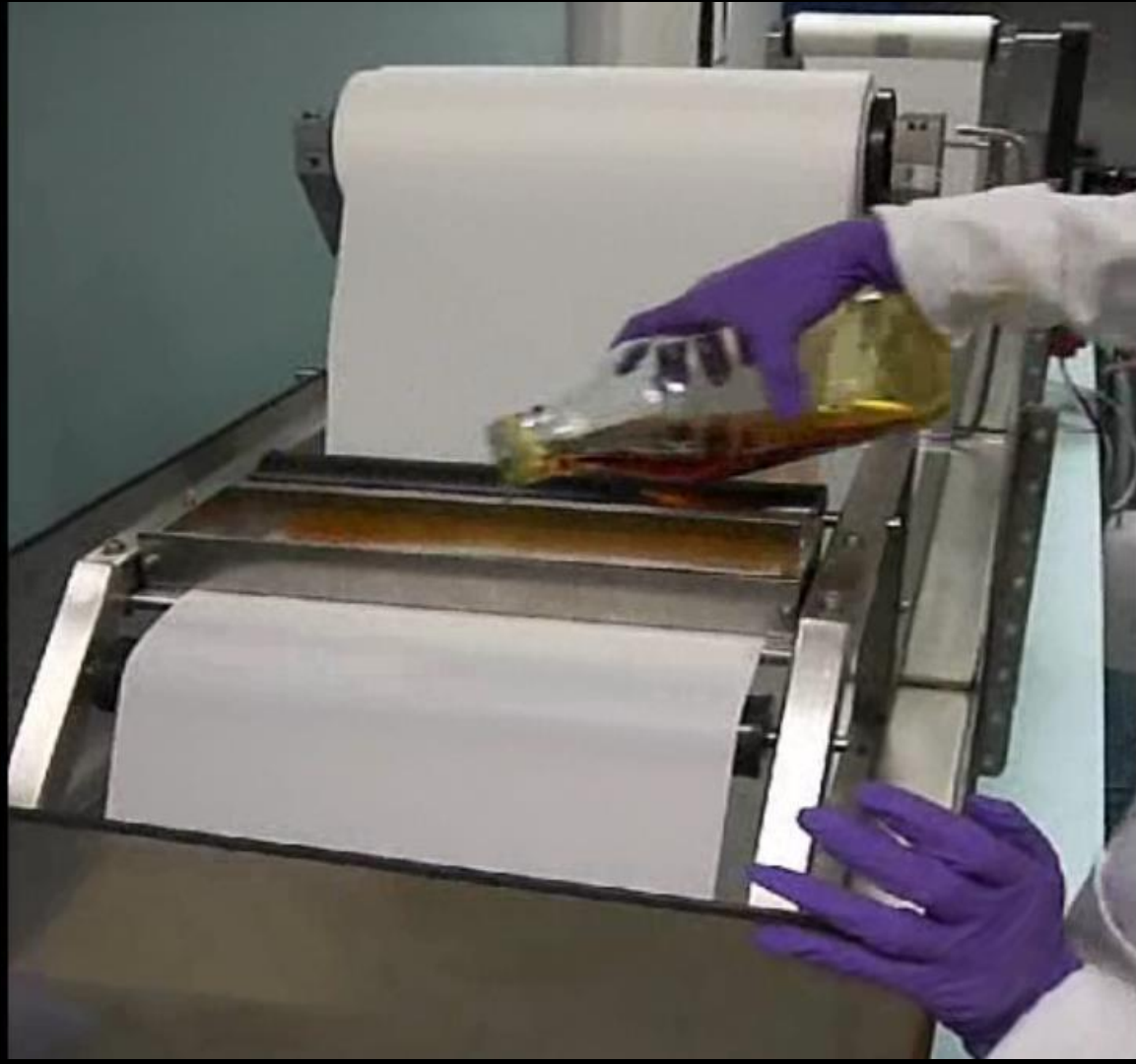
Demixing in organic photovoltaics



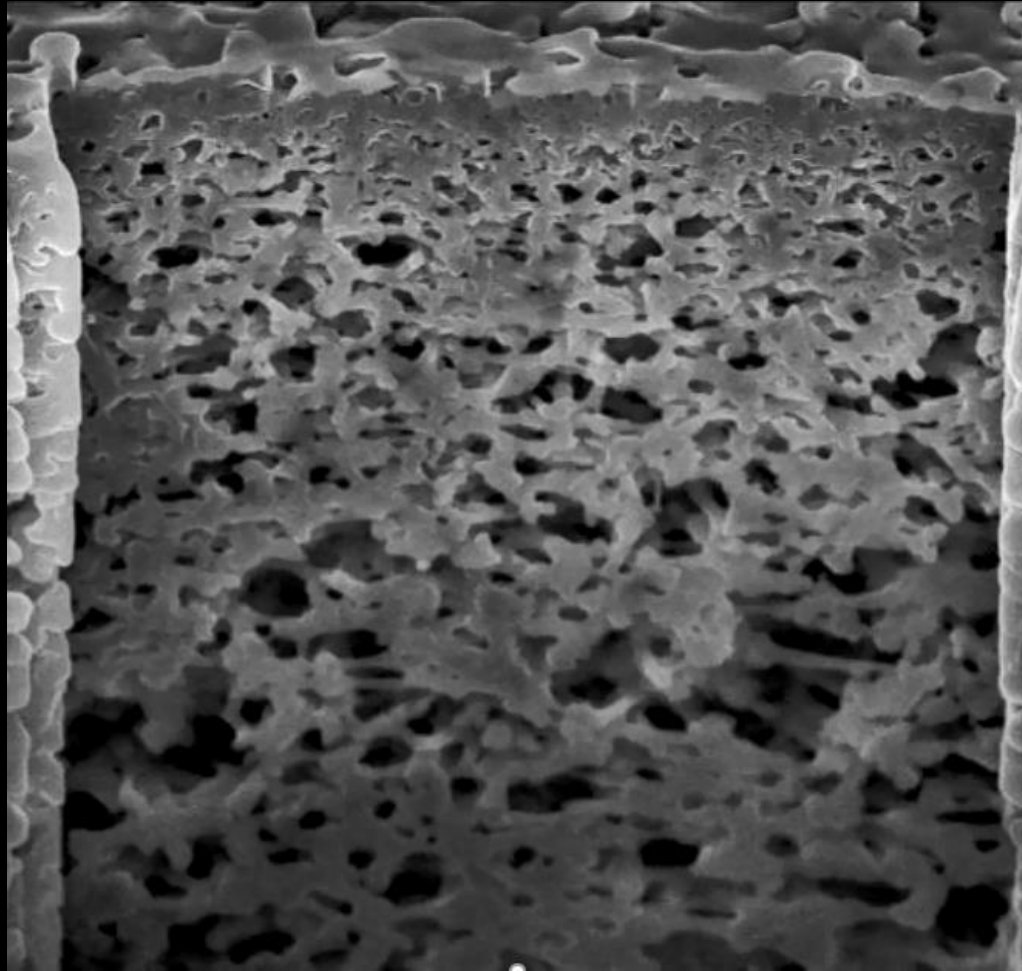
Organic photovoltaics

Morphology of efficient photovoltaics: *phase separation* on a length scale of the exciton diffusion length (~ 10 nm) in the active layer with interconnected phases and a thickness (~ 100 nm) optimised for light absorption...





Polymeric membranes for separations



Outline

1. Polymer thermodynamics & demixing

Cahn-Hilliard theory & spinodal decomposition.

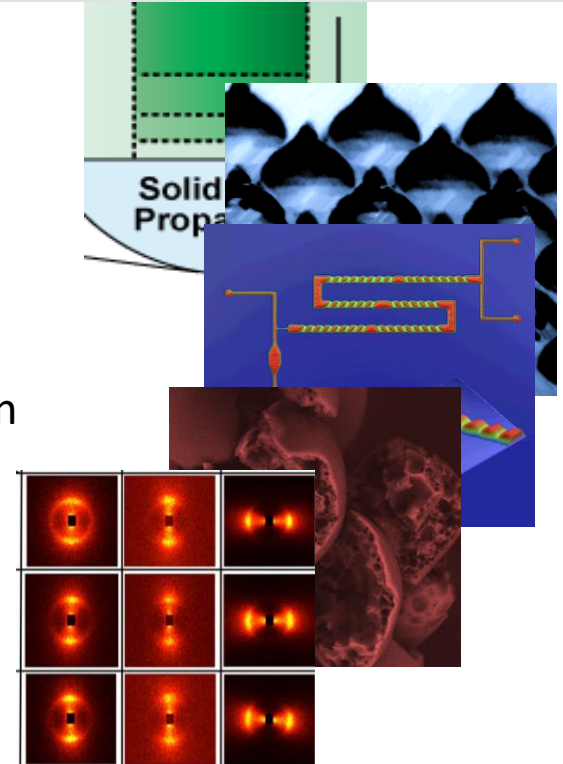
Membranes and photovoltaics

2. Particle and capsule formation

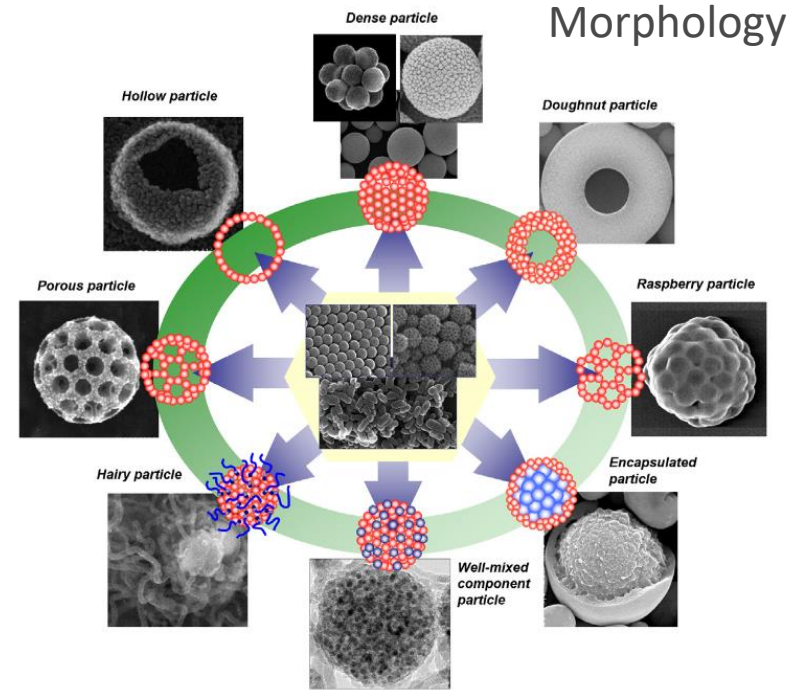
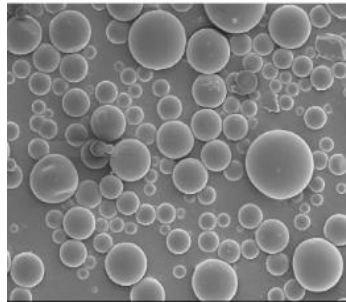
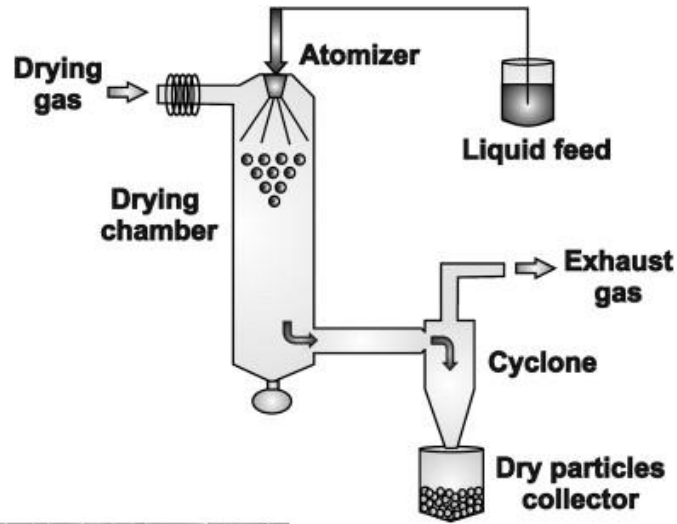
Levitation & microfluidics

Solution thermodynamics, phase inversion and solidification

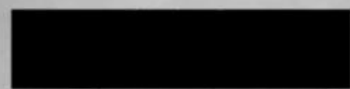
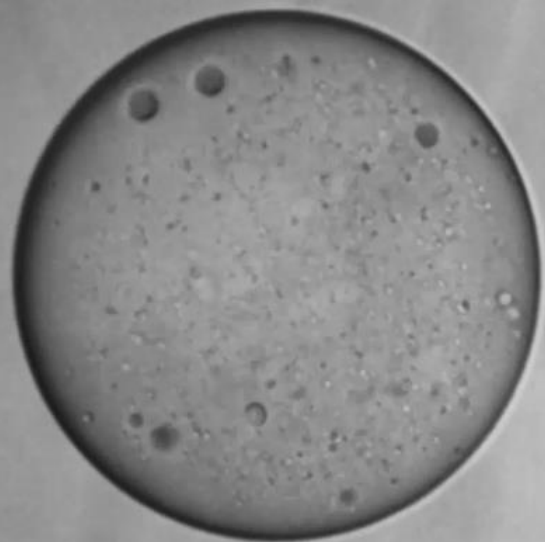
Functional capsules with tunable, triggered release.



Precision 'spray drying'



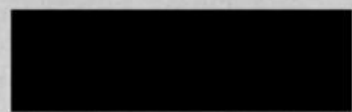
114.0 s



200 μm

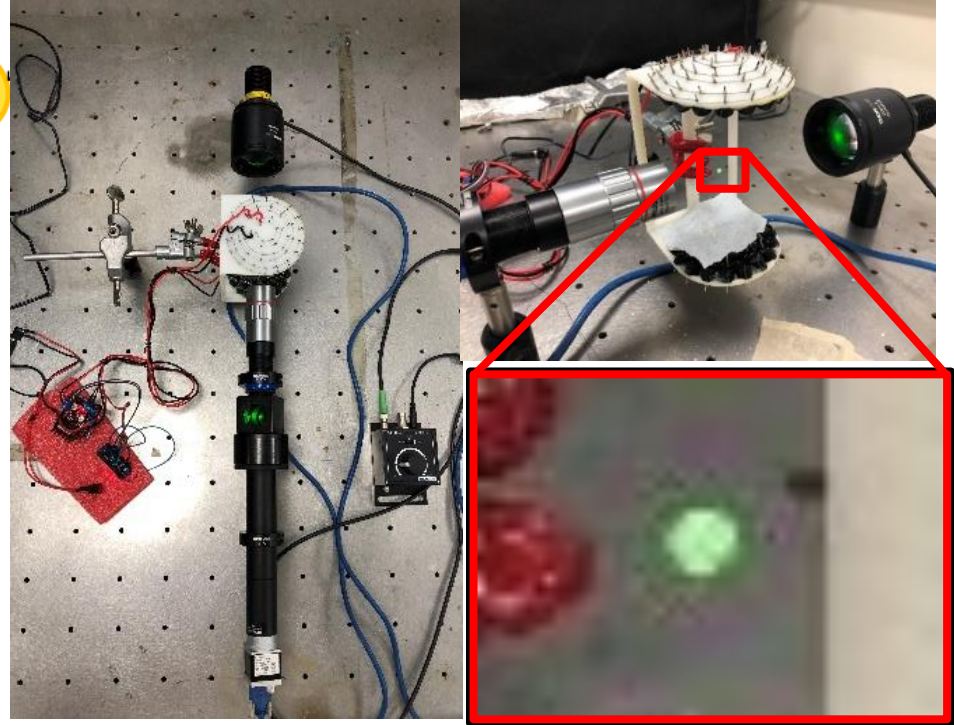
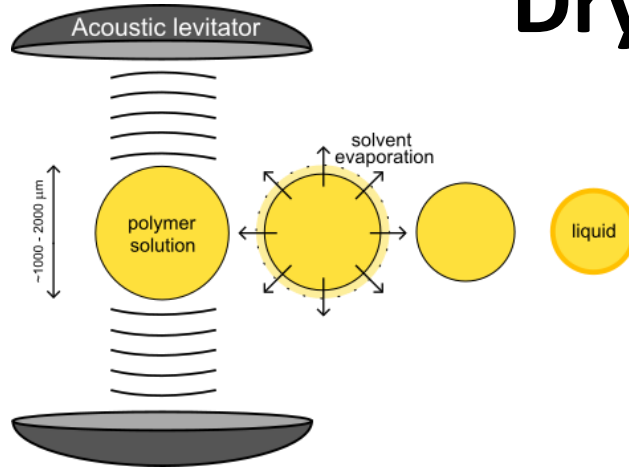
NaPSS

460 s

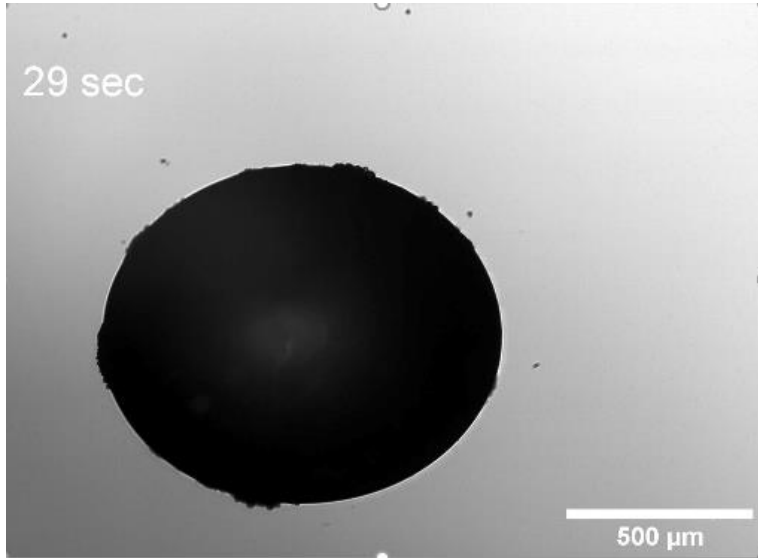
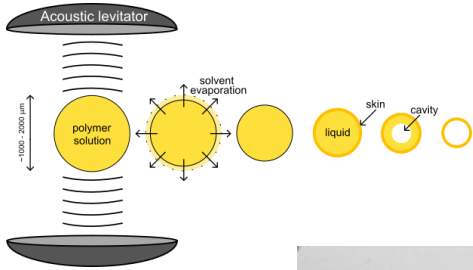


200 μm

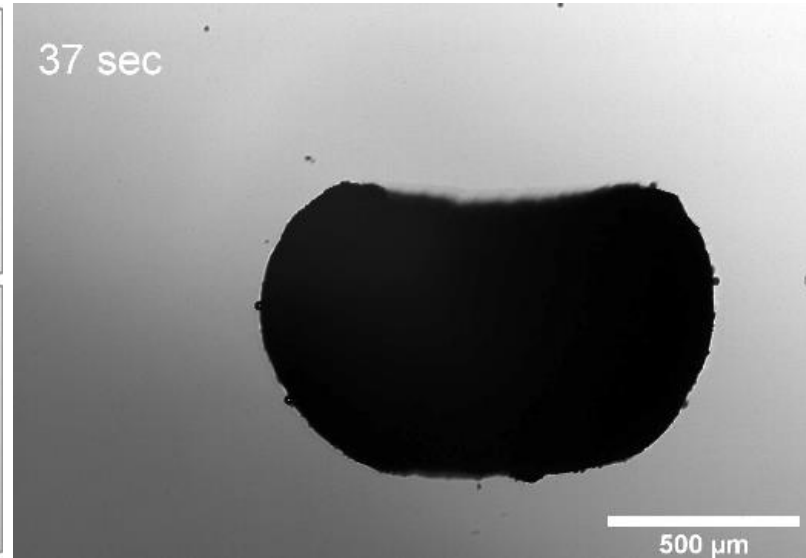
Drying in acoustic levitation



Drying in acoustic levitation

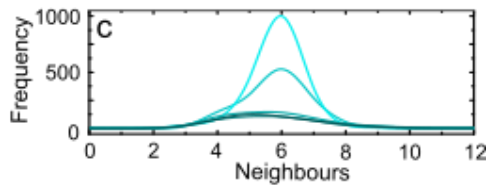
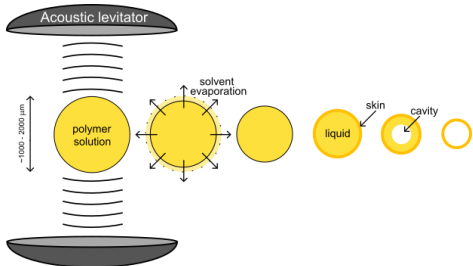


0.5 w/w%



2 w/w%

Drying in acoustic levitation



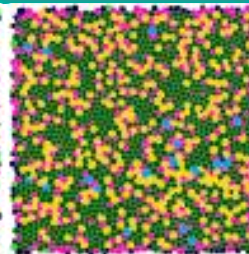
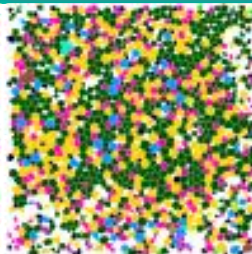
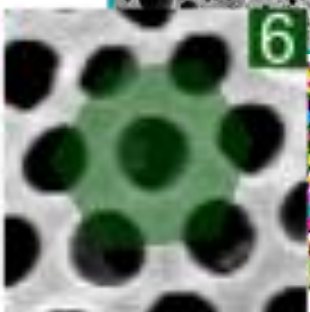
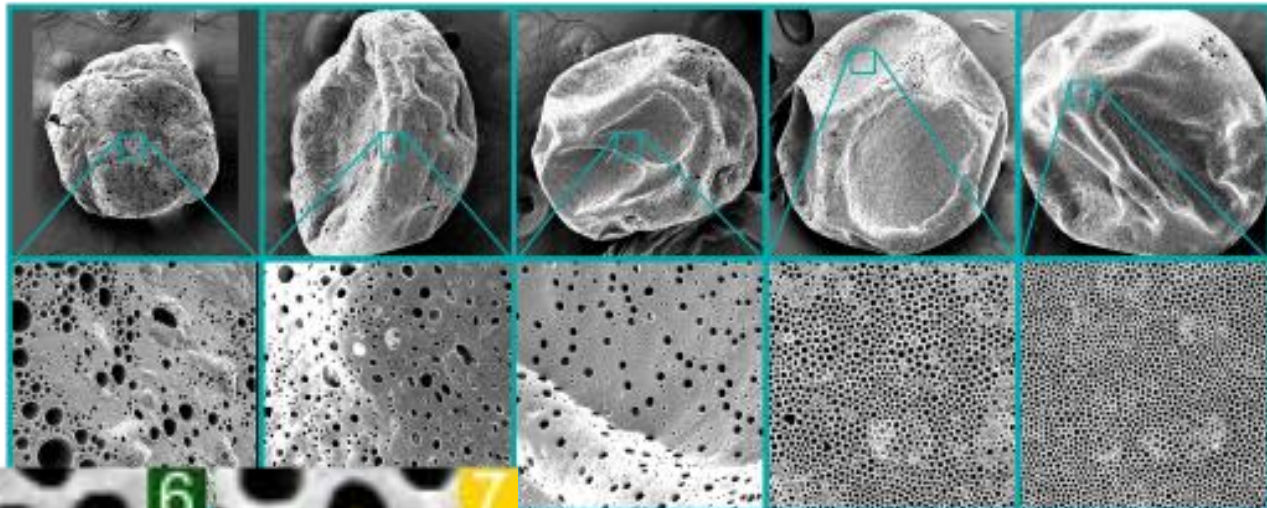
0.5%

2%

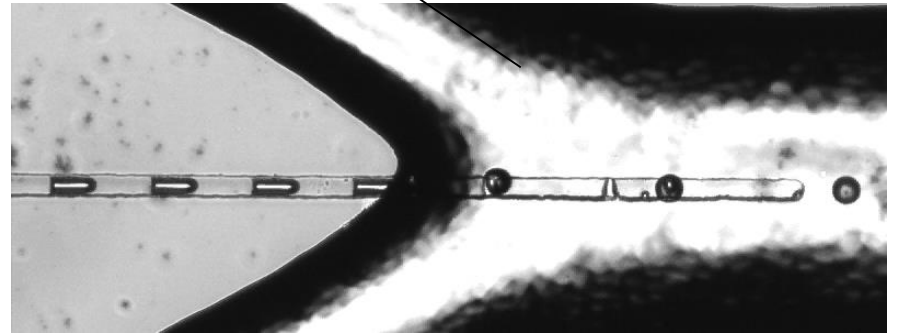
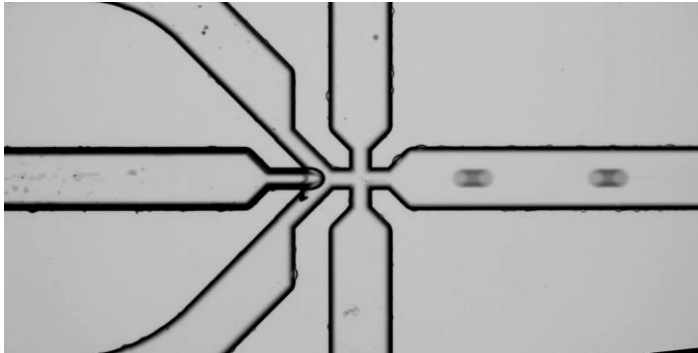
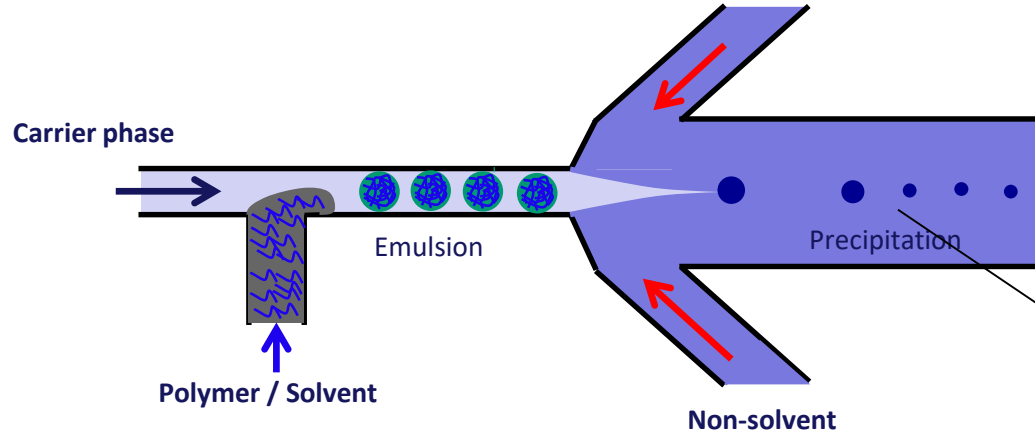
4%

6%

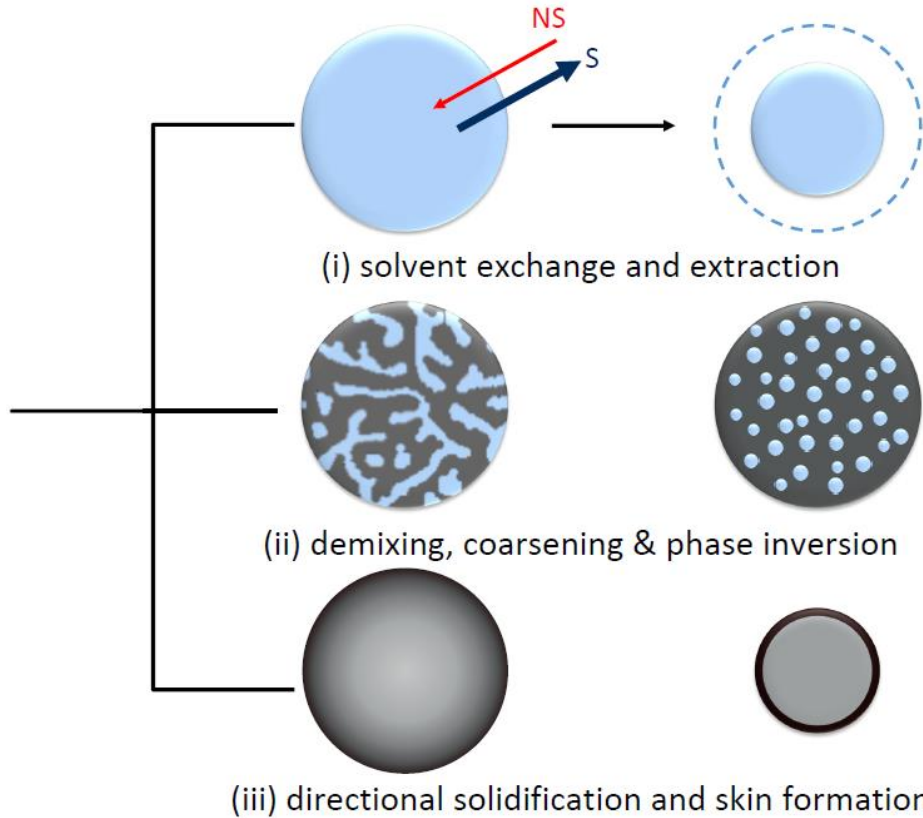
8%



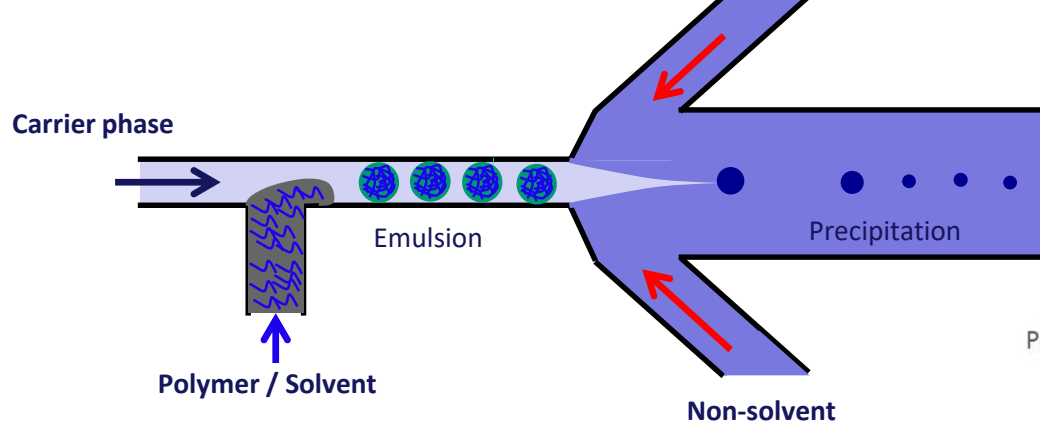
Microfluidic extraction



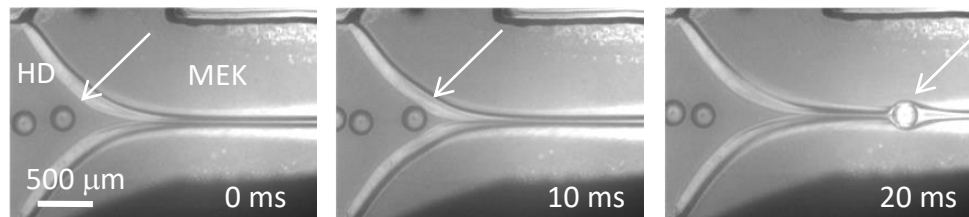
Coupling of mechanisms...



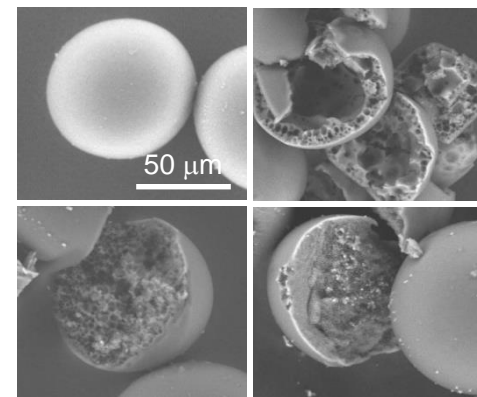
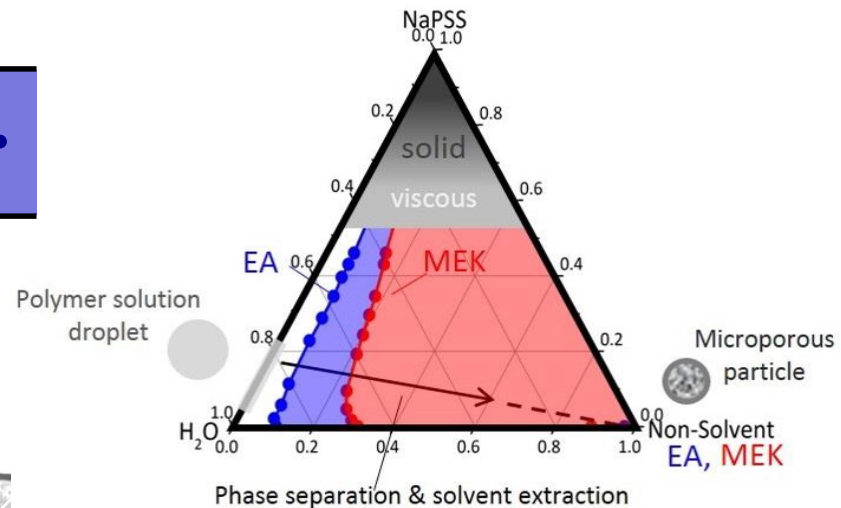
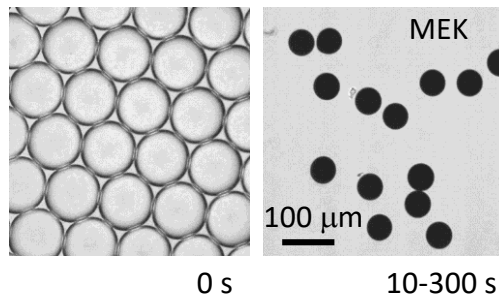
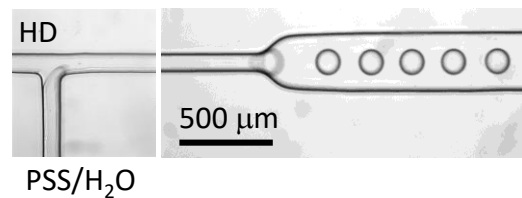
$$Pe = \frac{Ru}{D}$$



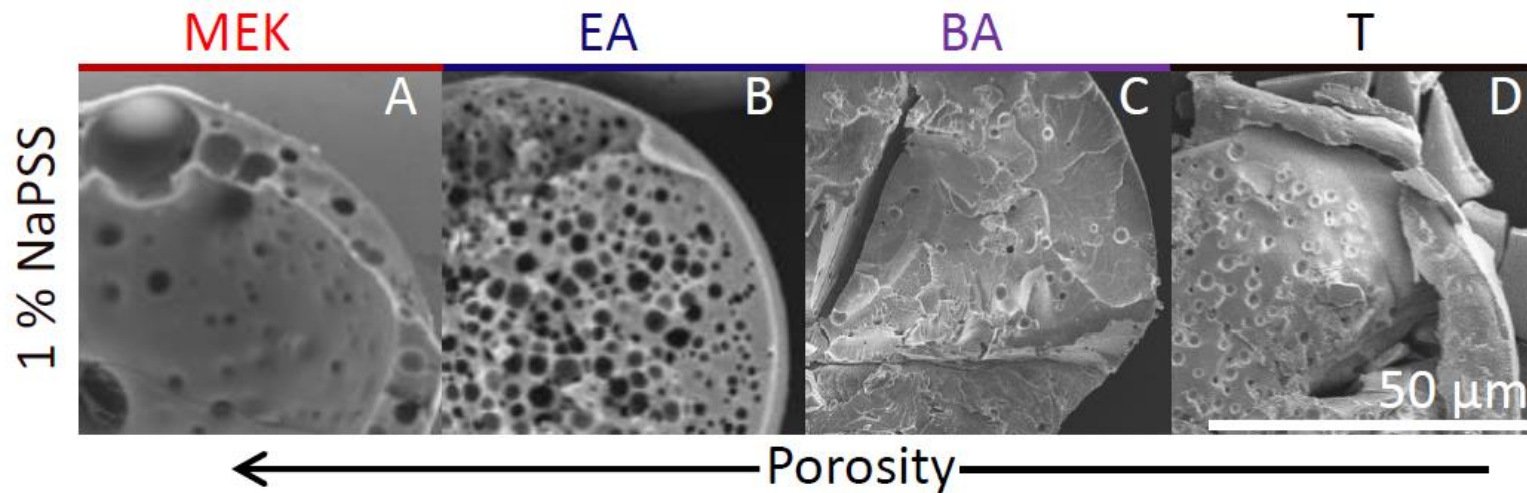
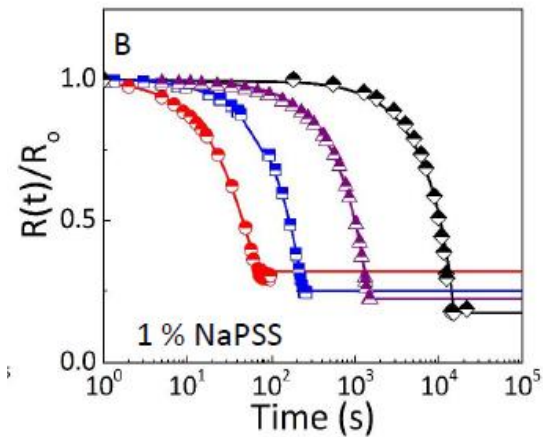
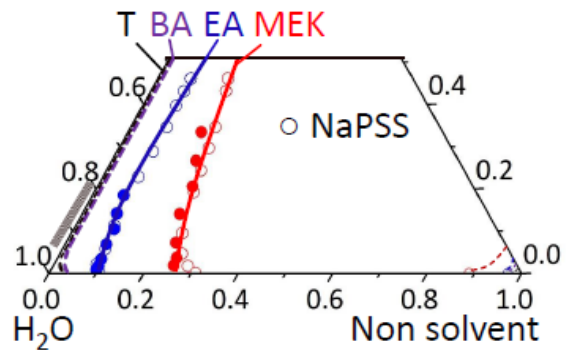
(a) *in-situ* precipitation



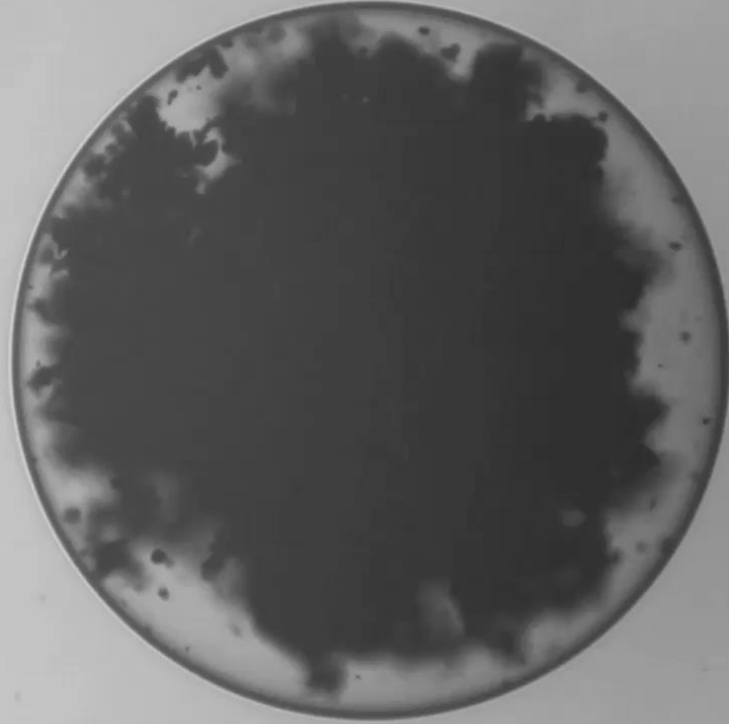
(b) *ex-situ* precipitation




non-solvent quality

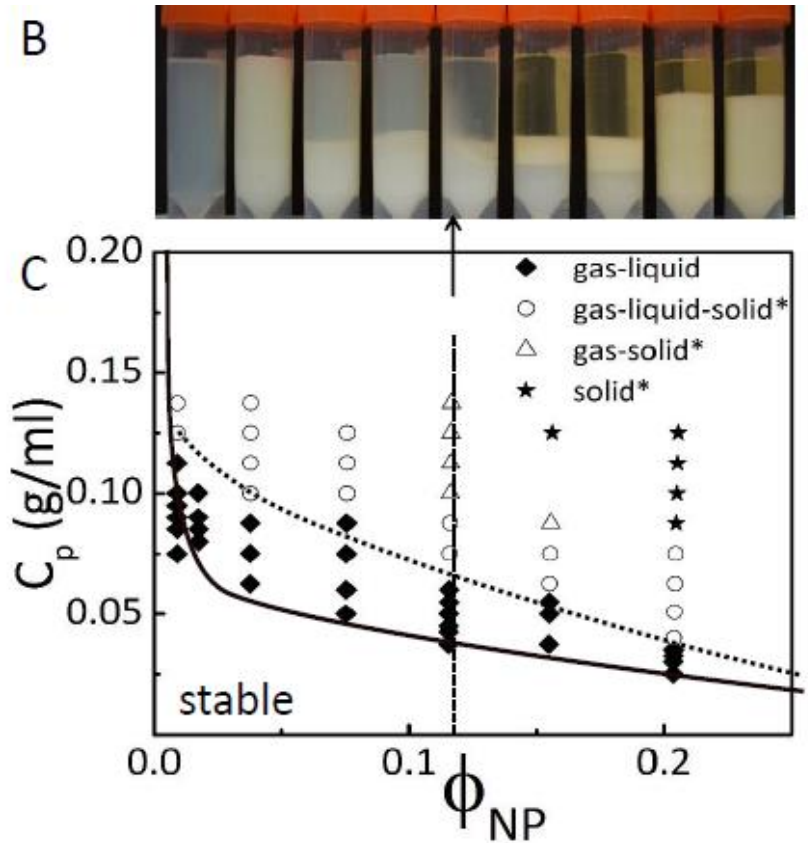
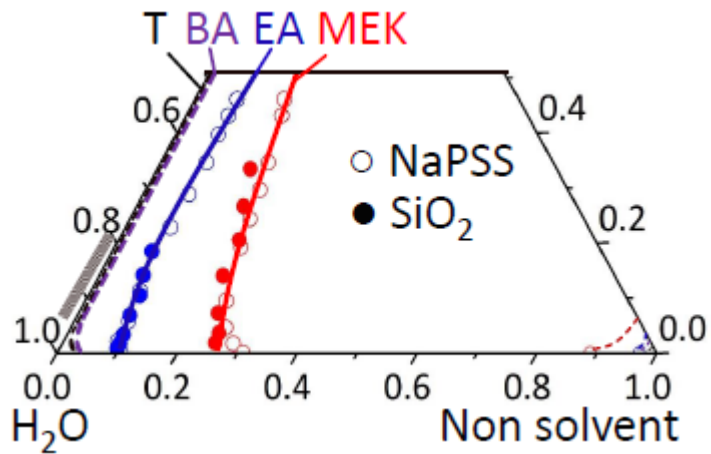


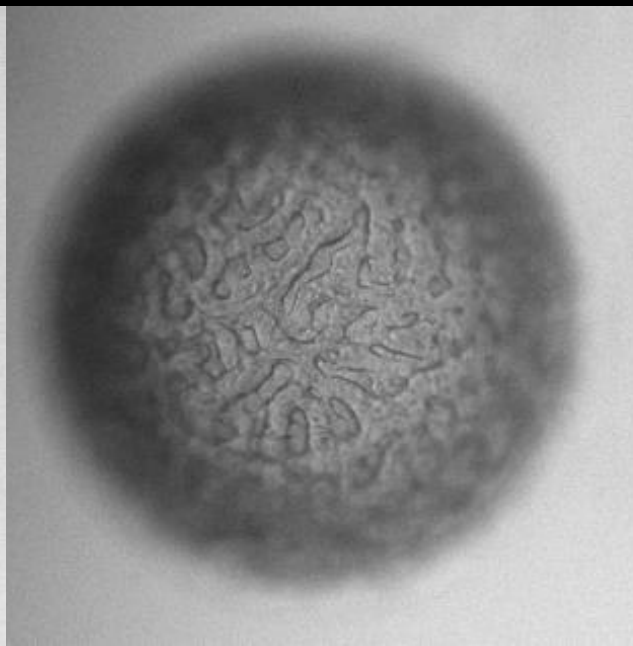
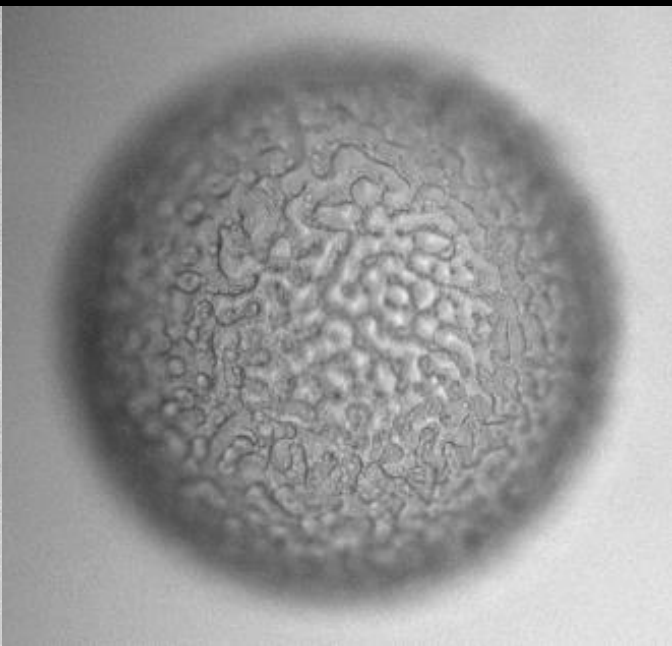
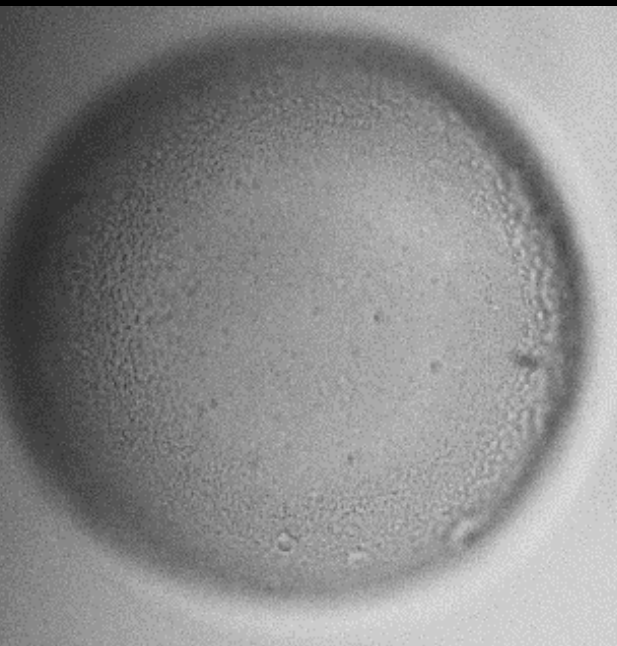
0.0 s




200 μm

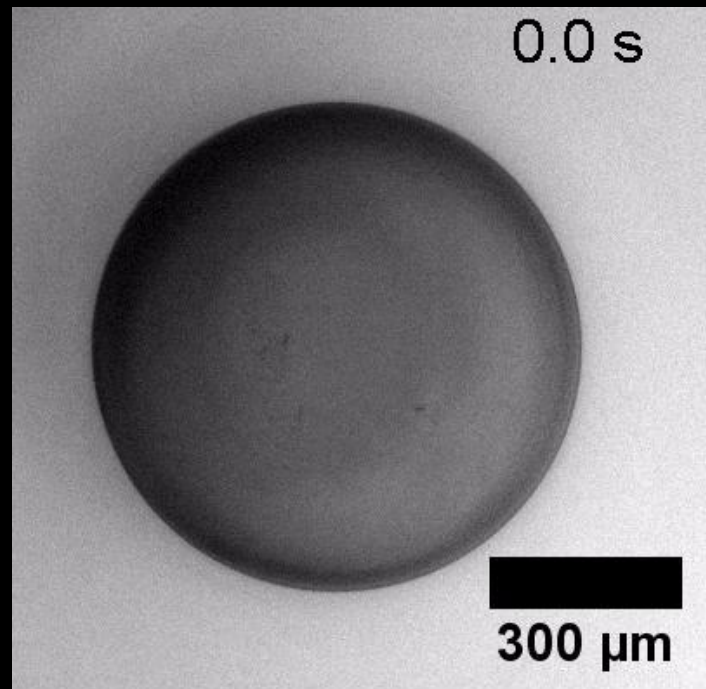
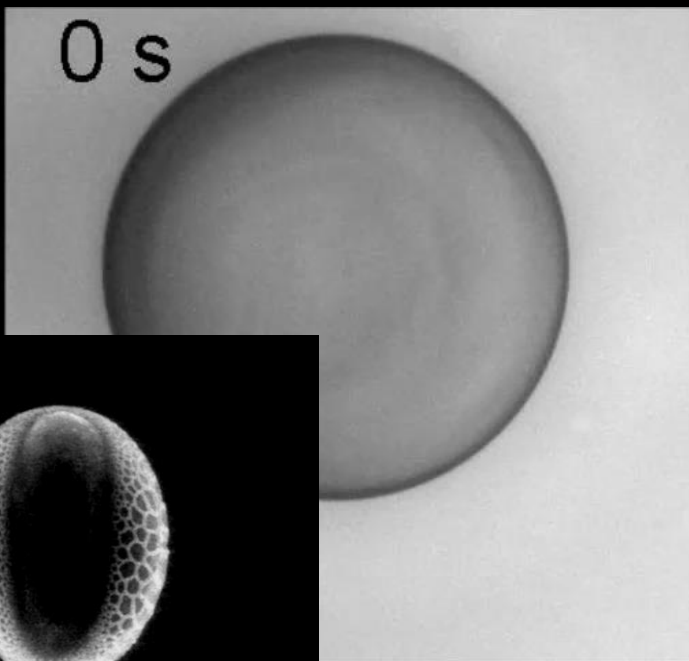
Polymer-colloid thermodynamics





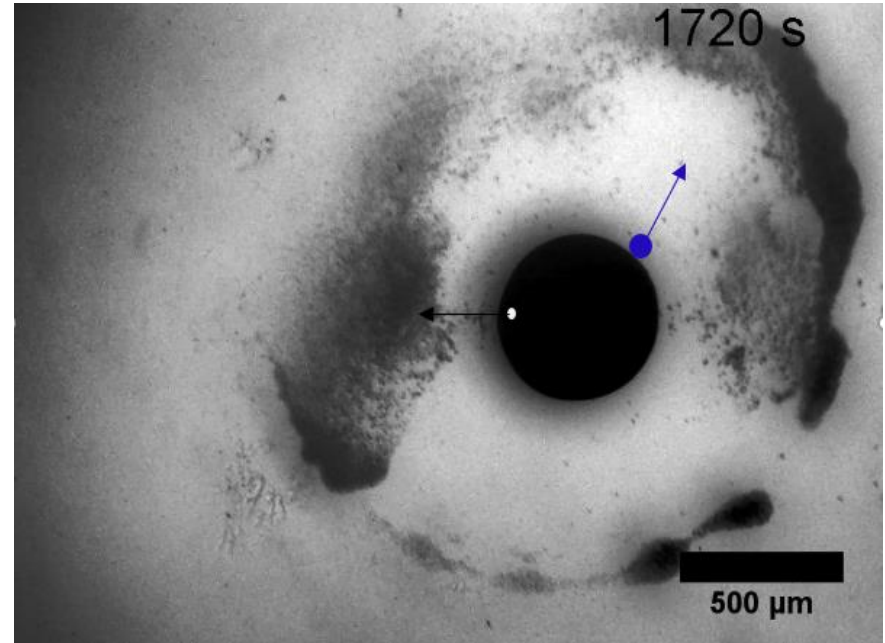
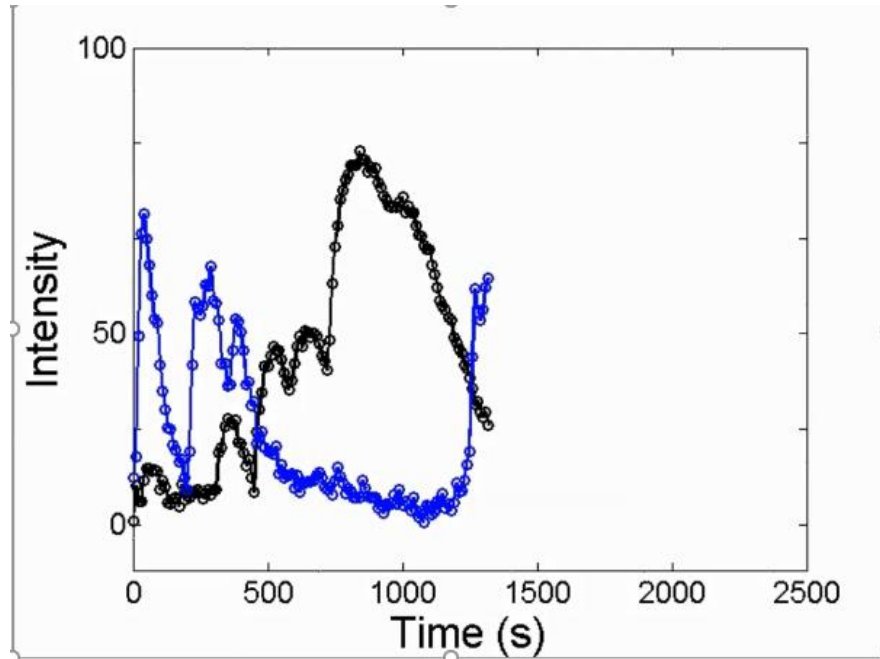
1% NaPSS, 2wt% SiO₂ 22 nm/H₂O/ MEK

Polymer-nanoparticle capsules



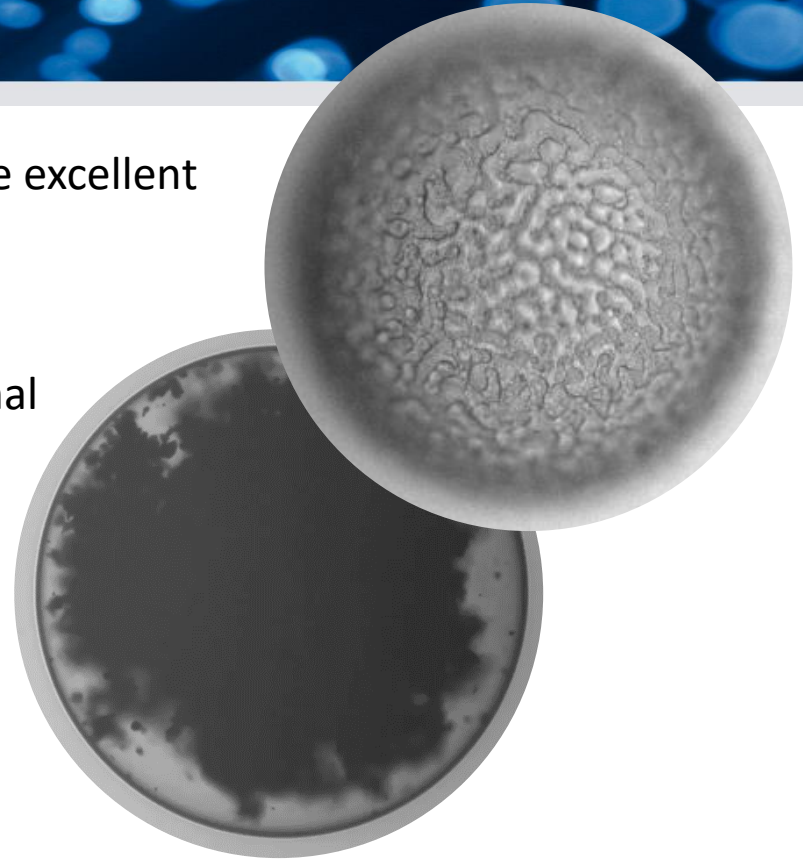
Pulsed, directional, triggered release

1 wt% P 10 wt% NP (DI H₂O)



Perspective

- Levitation and microfluidic (osmotic) drying are excellent platforms to examine “spray drying”
- Solution thermodynamics, demixing and coarsening can be engineered to yield functional particles with unexpected performance
- Transformative predictive design requires molecular insight (eg scattering) and theory/simulation



Outline

1. Polymer thermodynamics & demixing

Cahn-Hilliard theory & spinodal decomposition.

Membranes and photovoltaics

2. Particle and capsule formation

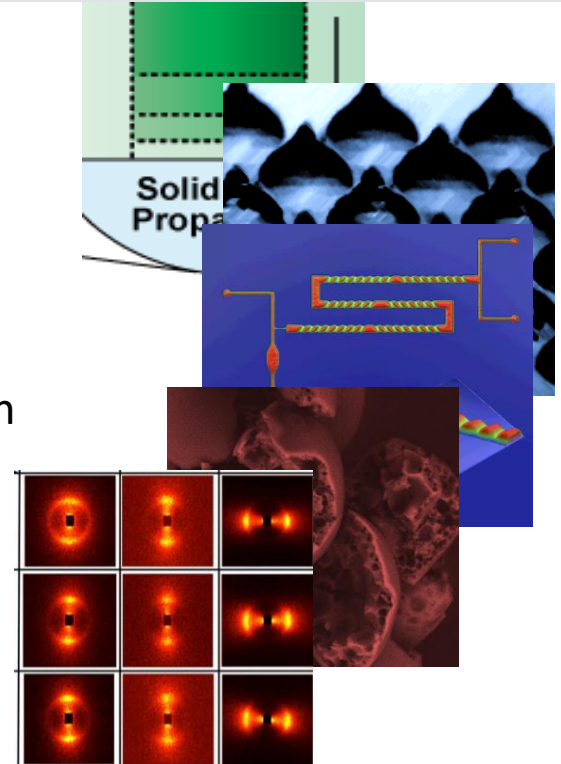
Levitation & microfluidics

Solution thermodynamics, phase inversion and solidification

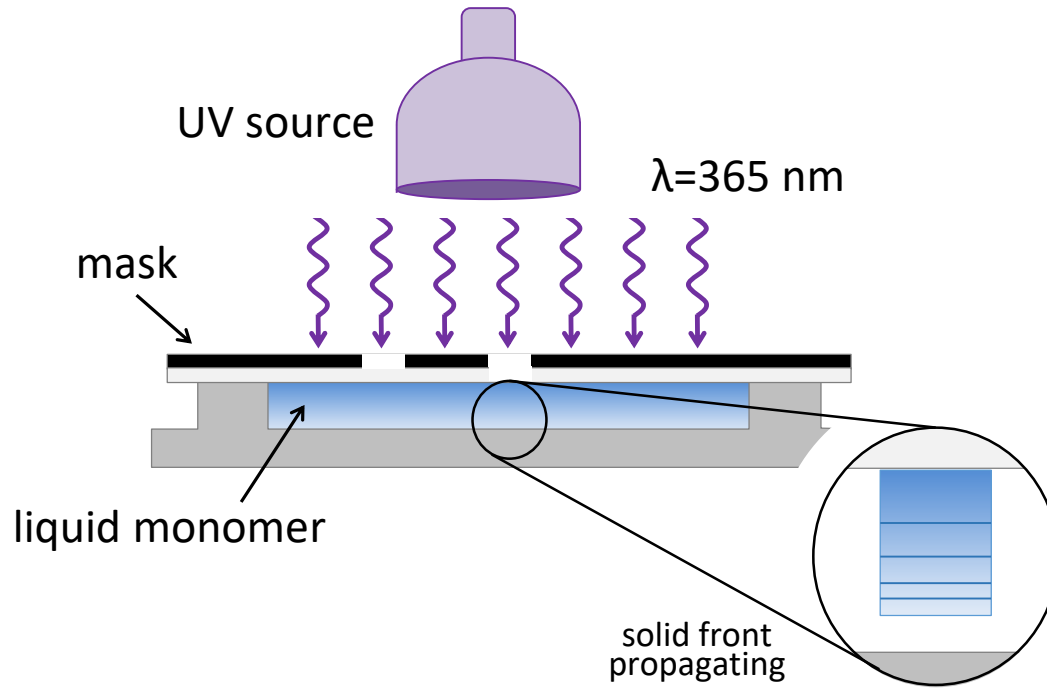
Functional capsules with tunable, triggered release.

3. Waves of network formation

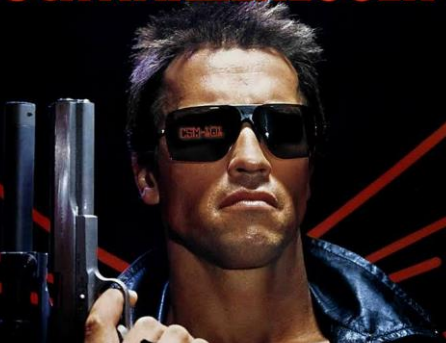
3D printing with light



Photopolymerisation & 3D printing



SCHWARZENEGGER



In the Year of Darkness, 2029,
the rulers of this planet
devised the ultimate plan.
They would reshape the
Future by changing the Past.
The plan required something
that felt no pity. No pain.
No fear.
Something unstoppable. They created

**THE
TERMINATOR**

Hamdale Presents a Pacific Western Production of a James Cameron Film
Arnold Schwarzenegger "The Terminator" Michael Biehn, Linda Hamilton and Paul Winfield
Make-Up Effects By Stan Winston - Executive Producers John Daly and Derek Gibson
Written by James Cameron with Gale Anne Hurd - Produced by Gale Anne Hurd
Directed by James Cameron - Prints by DeLuxe

ORION

October 26

1984

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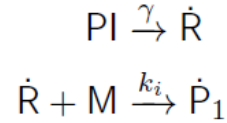
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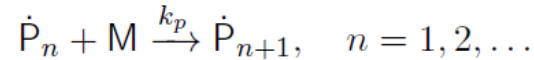


Explicit photopolymerization models

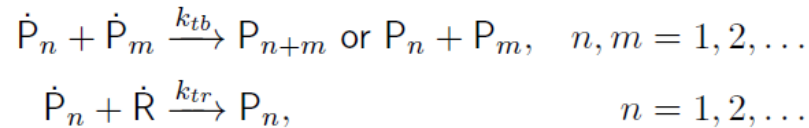
Initiation



Propagation



Termination



Inhibition



Explicit photopolymerization models

$$\frac{\partial[\text{PI}]}{\partial t} = -R_i,$$

$$\frac{\partial[\dot{\text{R}}]}{\partial t} = R_i - k_i[\dot{\text{R}}][\text{M}] - k_{tr}[\dot{\text{P}}][\dot{\text{R}}],$$

$$\frac{\partial[\text{M}]}{\partial t} = -k_i[\dot{\text{R}}][\text{M}] - k_p[\dot{\text{P}}][\text{M}],$$

$$\frac{\partial[\dot{\text{P}}]}{\partial t} = k_i[\dot{\text{R}}][\text{M}] - k_{tb}[\dot{\text{P}}]^2 - k_{tr}[\dot{\text{P}}][\dot{\text{R}}] - k_z[\dot{\text{P}}][\text{Z}],$$

$$\frac{\partial[\text{Z}]}{\partial t} = -k_z[\dot{\text{P}}][\text{Z}],$$

$$\frac{\partial[\text{P}]}{\partial t} = k_{tb}[\dot{\text{P}}]^2 + k_{tr}[\dot{\text{P}}][\dot{\text{R}}]$$

$$\frac{\partial[\text{PZ}]}{\partial t} = k_z[\dot{\text{P}}][\text{Z}].$$

Explicit photopolymerisation models

Parameter	Value	Units	Ref.	Remarks
$[M]_0$	8.2	mol/l		
MW_w	130.14	g/mol		
MW_z	92.09	g/mol		
R	4	l/mol	Goodner et al.	
α_M	0.0005	1/°C	Goodner et al.	
α_T	0.00075	1/°C	Goodner et al.	
α_S	0.0005	1/°C		Same as α_M
ρ_M	1.073	g/ml	Aldrich	
ρ_P	1.15	g/ml	Goodner et al.	
ρ_S	1.261	g/ml	Aldrich	
T_{ref}	213	K	Goodner et al.	
$T_{0,P}$	328	K	Goodner et al.	
$T_{0,S}$	173	K		Estimated
ϕ	0.6		Goodner et al.	
c	150	l/mol e	Goodner et al.	
k_{p0}	1.6×10^6	l/mol s	Goodner et al.	
E_p	18.23×10^3	J/mol	Gao and Penlidis	For MMA
A_p	0.66		Goodner et al.	
f_{sp}	0.042		Goodner et al.	
k_{t0}	3.6×10^6	l/mol s	Goodner et al.	
E_t	2.94×10^3	J/mol	Gao and Penlidis	For MMA
A_t	1.2		Goodner et al.	
f_{st}	0.060		Goodner et al.	
k_{t0}	1.6×10^6	l/mol s		Same as k_{p0}
k_{20}	52.8×10^3	l/mol s	Odian	For O ₂ /MMA
k_{120}, A'_{t0}	0.0	l/mol s		Unless otherwise noted
E_1, E_{2s}, E'_1	18.23×10^3	J/mol		Same as E_p
E_{1p}	2.94×10^3	J/mol		Same as E_t
A_{1s}, A_{1p}, A'_1	0.66			Same as A_p
A_2	0.287			Calculated from diffusivity
f_{cs}, f_{cp}, f'_{ct}	0.042		Goodner et al.	Same as f_{sp}
f_{ct}	0.010		Goodner et al.	Estimated
ΔH	-5.48×10^4	J/mol	Cook	From -13.1 kcal/mol
k	0.292	W/cm	Lide	For glycerine
ρ_{cp}	1900	J/K	Gao and Penlidis	Monomer/polymer average
D_{00}	23.6×10^{-3}	cm ² /s	Lide	Small species in ethyl acetate
D_{050}	23.6×10^{-3}	cm ² /s	Lide	Small species in ethyl acetate
D_{20}	49.1×10^{-3}	cm ² /s	Brandrup and Immergut	O ₂ in ethyl methacrylate

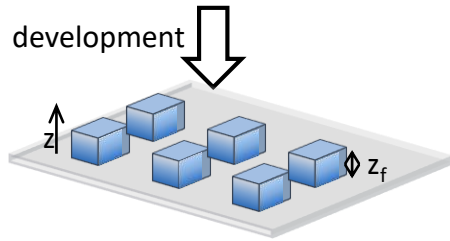
Figure: Taken from Goodner & Bowman (2002)

A 'minimal' FPP model

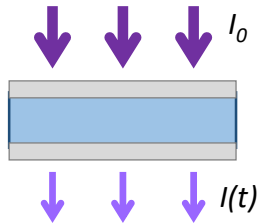
(Self-consistent & neglecting all chemical "details"!)

Physical observables :

- **pattern height**
(position of the solid/liquid front)



- **light transmission**



$$\left\{ \begin{array}{l} \frac{\partial \phi(z,t)}{\partial t} = K[1 - \phi(z,t)]I(z,t) \\ \frac{\partial I(z,t)}{\partial z} = -[\mu_0[1 - \phi(z,t)] + \mu_\infty \phi(z,t)]I(z,t) \end{array} \right.$$
$$\phi(z,0) = 0, \quad I(0,t) = 1$$

ϕ conversion fraction, I light intensity, μ_0 and μ_∞ initial and final attenuation coefficients, K conversion rate, ϕ_C solid threshold, z depth, t time.

A 'minimal' FPP model

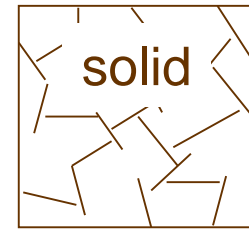
(Self-consistent & neglecting all chemical "details"!)

$$\left\{ \begin{array}{l} \frac{\partial \phi(z,t)}{\partial t} = K[1 - \phi(z,t)]I(z,t) \\ \frac{\partial I(z,t)}{\partial z} = -[\mu_0[1 - \phi(z,t)] + \mu_\infty \phi(z,t)]I(z,t) \\ \phi(z,0) = 0, \quad I(0,t) = 1 \end{array} \right.$$

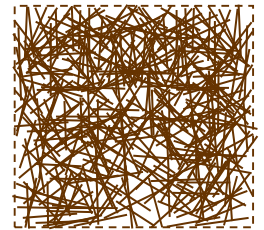
ϕ conversion fraction, I light intensity,
 μ_0 and μ_∞ initial and final attenuation coefficients, K
 conversion rate, ϕ_C solid threshold, z depth, t time.



$\phi = 0$

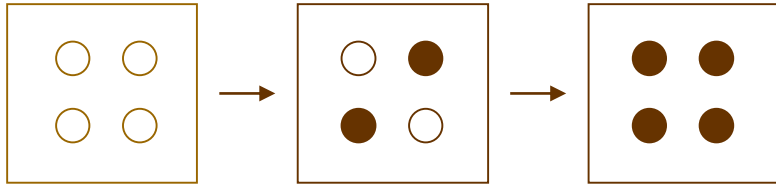


ϕ_C



$\phi = 1$

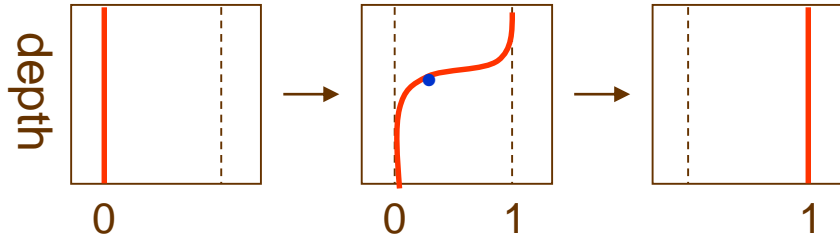
∇ depth



zero
conversion

total
conversion

conversion fraction ϕ

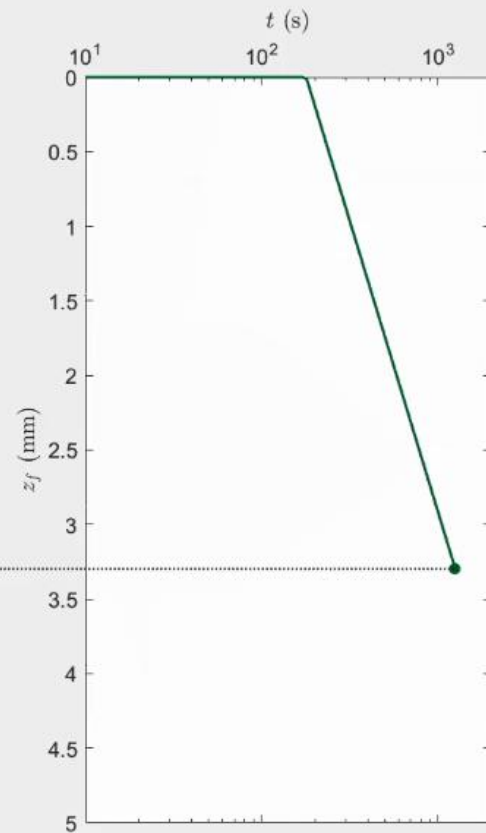
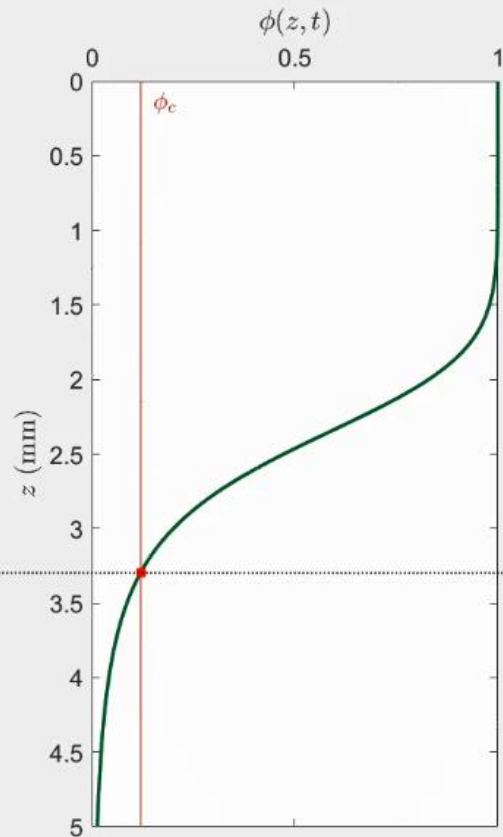
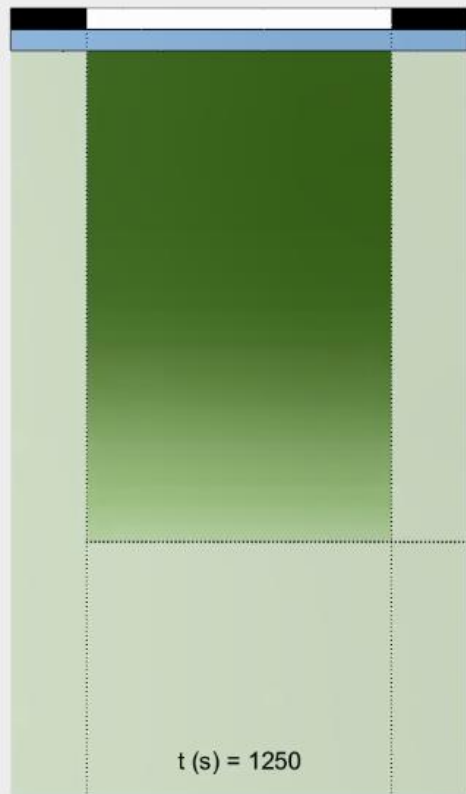


time \rightarrow

Solid
threshold

$$\begin{cases} \phi(x,t) > \phi_C, & \text{solid} \\ \phi(x,t) < \phi_C, & \text{liquid} \end{cases}$$

A 'minimal' FPP model



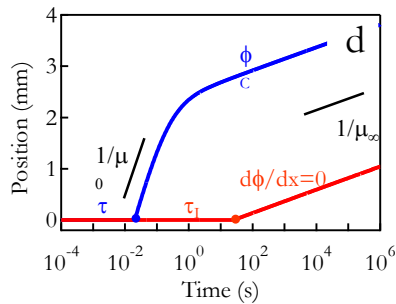
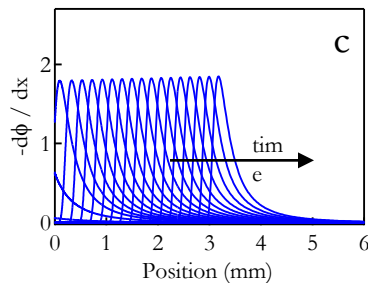
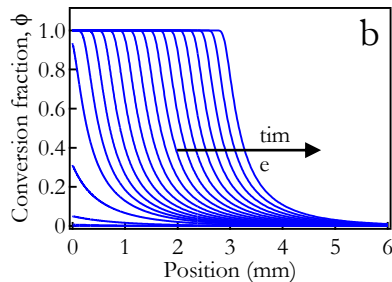
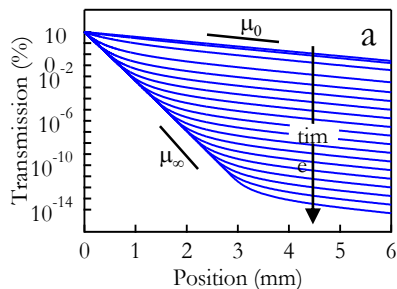
Traveling fronts

$\left\{ \begin{array}{l} \mu_0 > \mu_\infty (=0) \\ \mu_0 = \mu_\infty \\ \mu_0 < \mu_\infty \end{array} \right.$ Photobleaching (total)

\updownarrow

“Photodarkening”

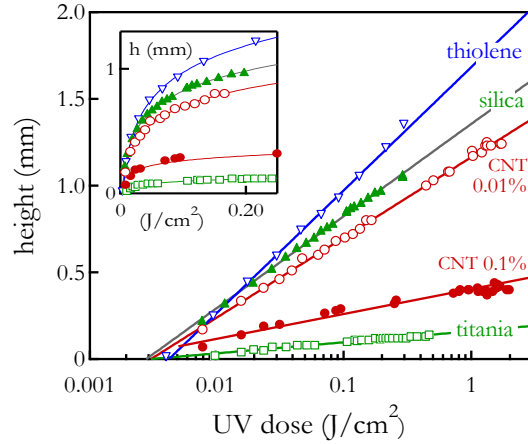
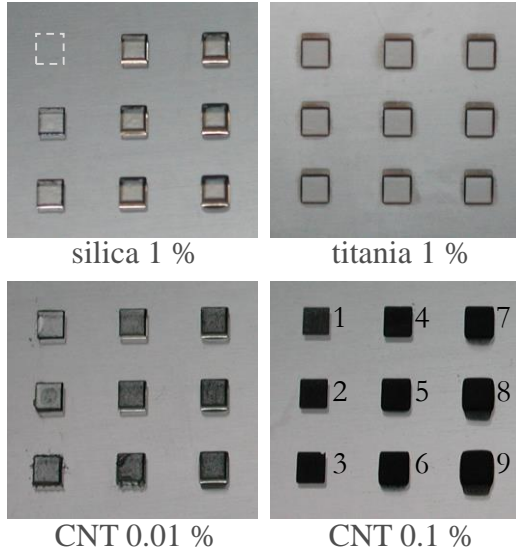
*Considerable attenuation,
negligible mass transfer*



$$height = \frac{\ln(t/\tau)}{\bar{\mu}}$$

$$induction\ time \equiv \frac{\ln[1/(1-\phi_C)]}{K}$$

FPP + nanofillers

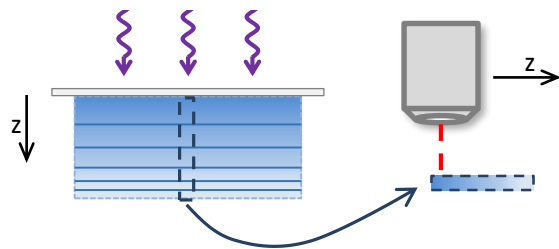


$$height = \frac{\ln(t/\tau)}{\bar{\mu}}$$

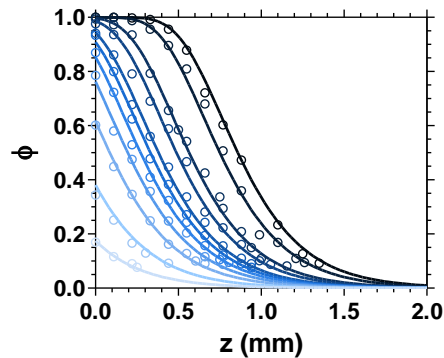
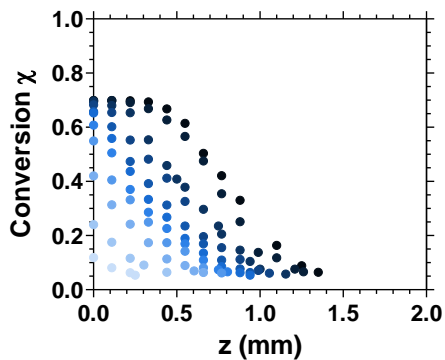
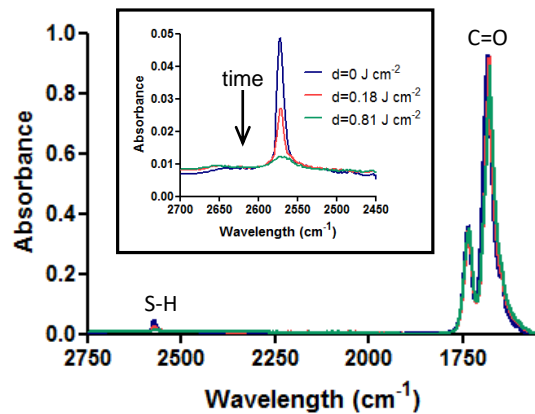
$$induction \text{ time} \equiv \frac{\ln[1/(1-\phi_c)]}{K}$$

Sets the **slope**

Spatio temporal conversion ϕ

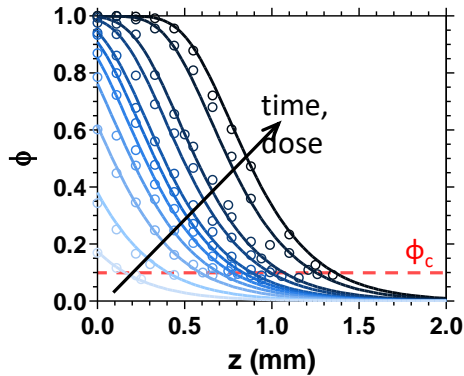


$$\text{Conversion } \chi \equiv 1 - \frac{A_{S-H}(t)/A_{C=O}(t)}{A_{S-H}(0)/A_{C=O}(0)}$$

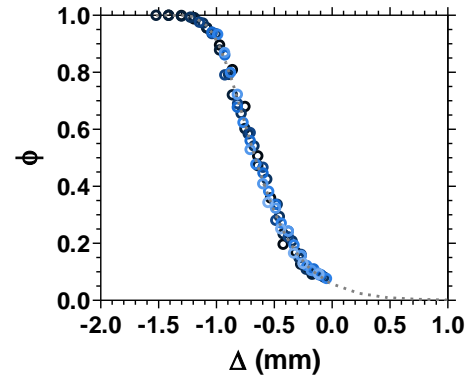


Shape-invariant travelling waves

$$\phi(z, t) = 1 - \exp[-KI_0 \exp(-\mu z) t]$$



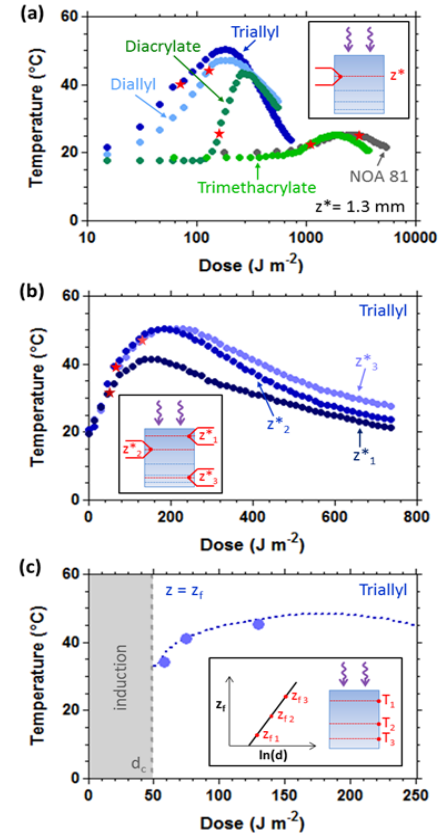
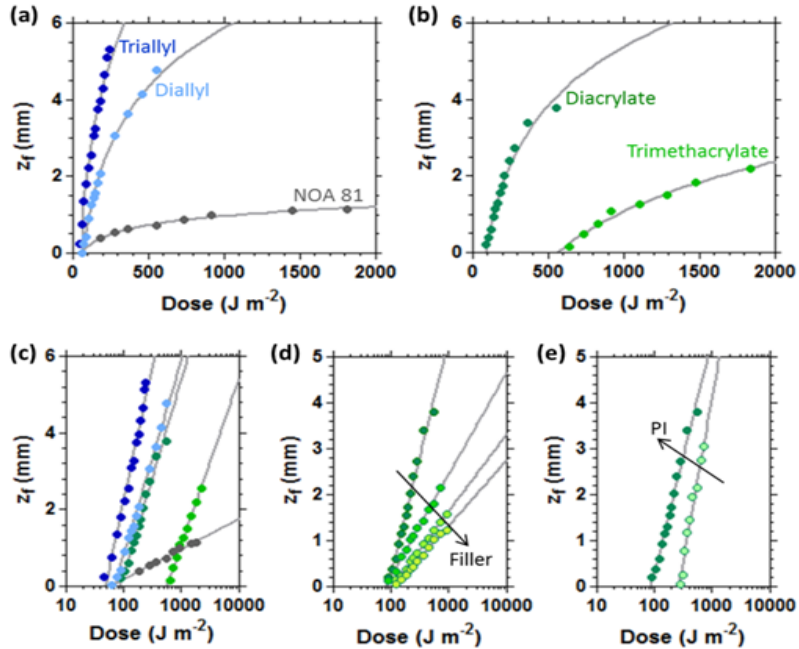
Thiol-ene 81
 $\mu = 3.74 \pm 0.21 \text{ mm}^{-1}$
 $KI_0 = 0.001598 \text{ s}^{-1}$
 $\phi_c = 0.052 \pm 0.0021$



travelling frame

$$w = \left| \frac{\partial \phi(z_1(t), t)}{\partial z} \right|^{-1}$$

Thermal and mass diffusion



Thermal and mass diffusion

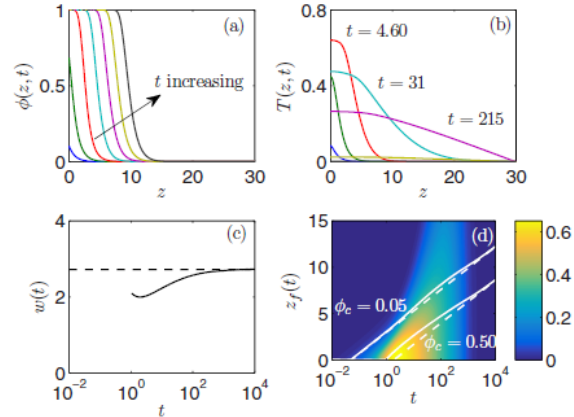
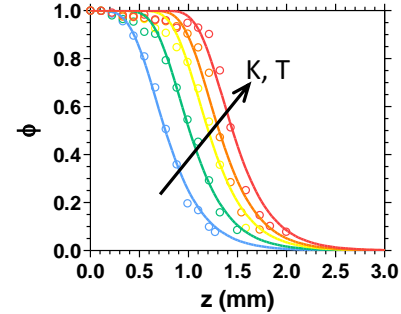
$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z} \left(D(\phi) \frac{\partial \phi}{\partial z} \right) + K(T)(1 - \phi)I$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(k(T) \frac{\partial T}{\partial z} \right) + (\Delta H) \frac{\partial \phi}{\partial t}$$

$$\frac{\partial I}{\partial z} = -[\mu_\infty \phi + \mu_0(1 - \phi)]I$$

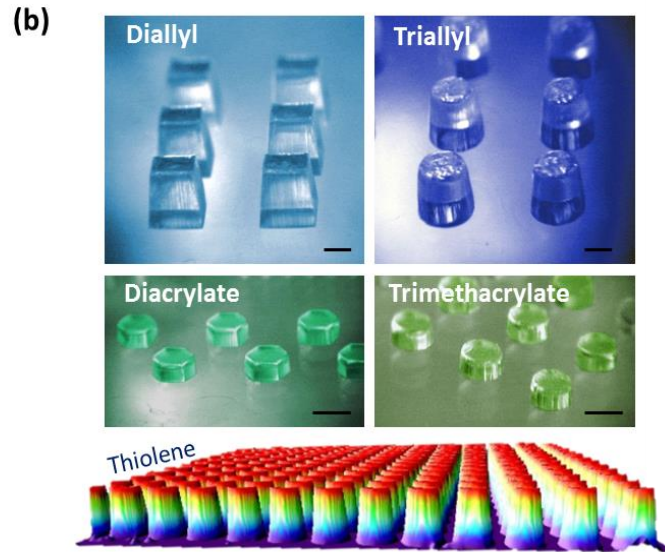
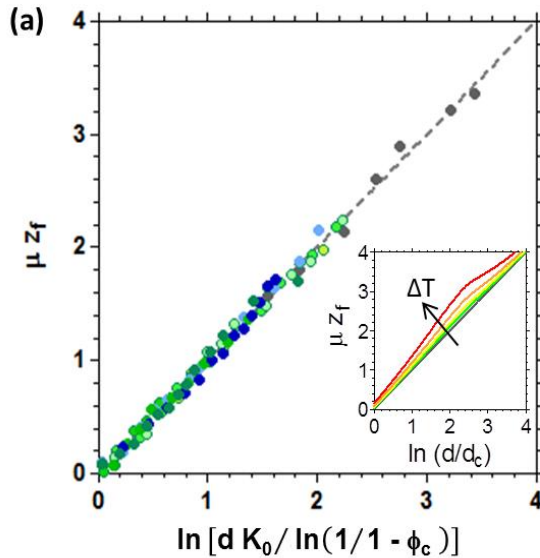
$$K(T) = K_0 \exp\left(-\frac{E_a}{RT}\right)$$

ϕ conversion fraction, D diffusivity, K conversion rate, I light intensity, ρc_p volumetric heat capacity, k thermal conductivity, ΔH enthalpy of polymerization, μ_0 and μ_∞ initial and final absorption coefficients, E_a activation energy, R ideal gas constant, T temperature, z depth, t time

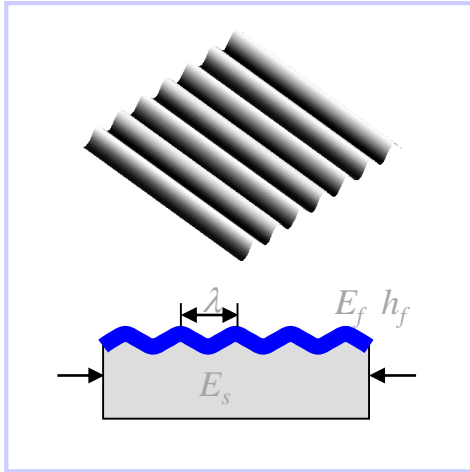


A unified FPP approach

- photopolymerization conditions (irradiation time, incident light intensity)
- monomer chemistry (reactivity and diffusivity)
- temperature (both externally applied heat and heat generated by exothermic reactions)
- material absorbing properties



Mechanical instability of a bilayer



thick substrate ($h_s \gg h_f$)

soft substrate ($E_s \ll E_f$)

interface must be well-bonded
(perfect adhesion, no slippage).

Deformation of the substrate disfavors wrinkling of long wavelengths and competes with bending to select an intermediate wavelength.

$$\lambda = 2\pi h_f \left(\frac{\bar{E}_f}{3\bar{E}_s} \right)^{1/3}$$

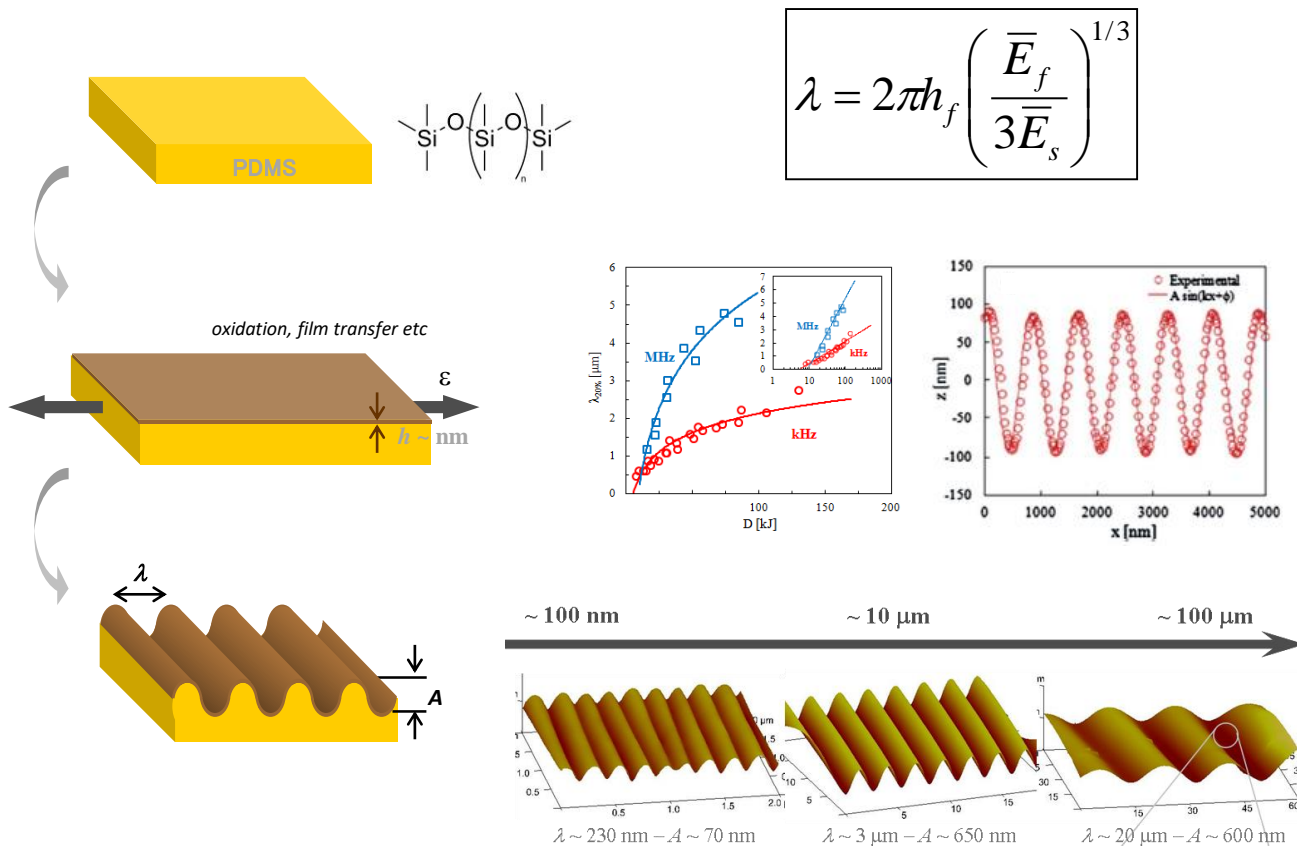
where
$$\bar{E}_x = \frac{E_x}{1 - \nu_x^2}$$

$$A = h_f \left(\frac{\varepsilon}{\varepsilon_c} - 1 \right)^{1/2}$$

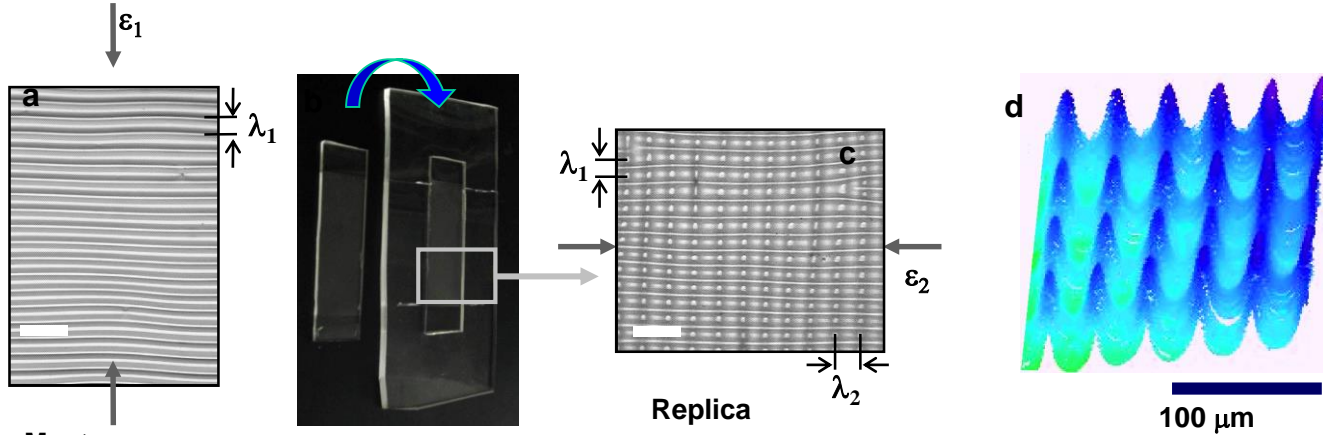
: amplitude

where
$$\varepsilon_c = -\frac{1}{4} \left(\frac{3\bar{E}_s}{\bar{E}_f} \right)^{2/3}$$
 : critical strain

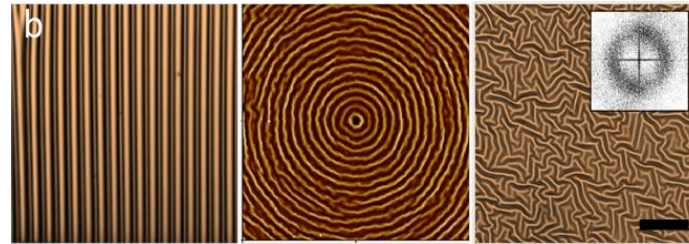
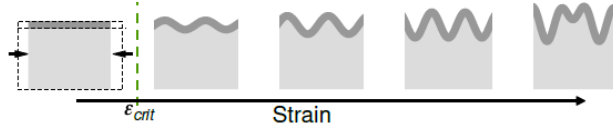
Mechanical instability of a bilayer



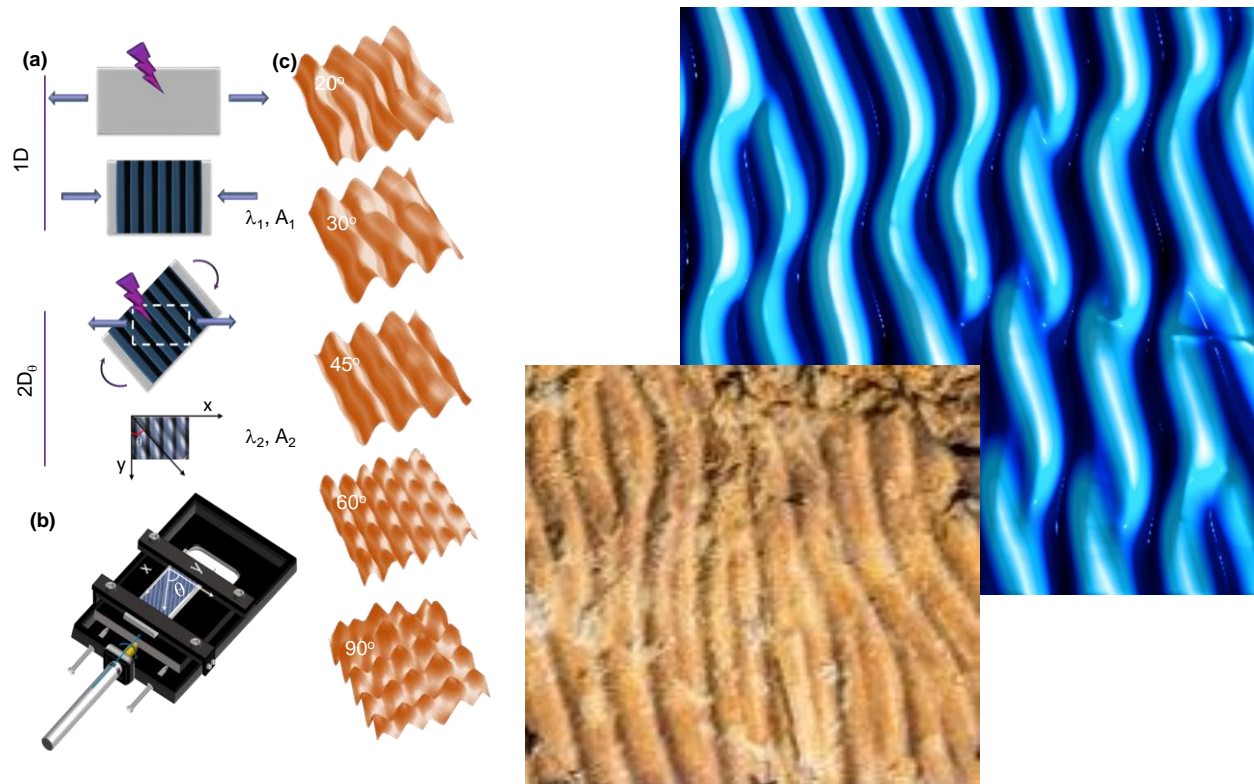
Multiaxial mechanical instabilities



Σ sin waves with *independently* tuned λ and A



Fourier superposition... nearly

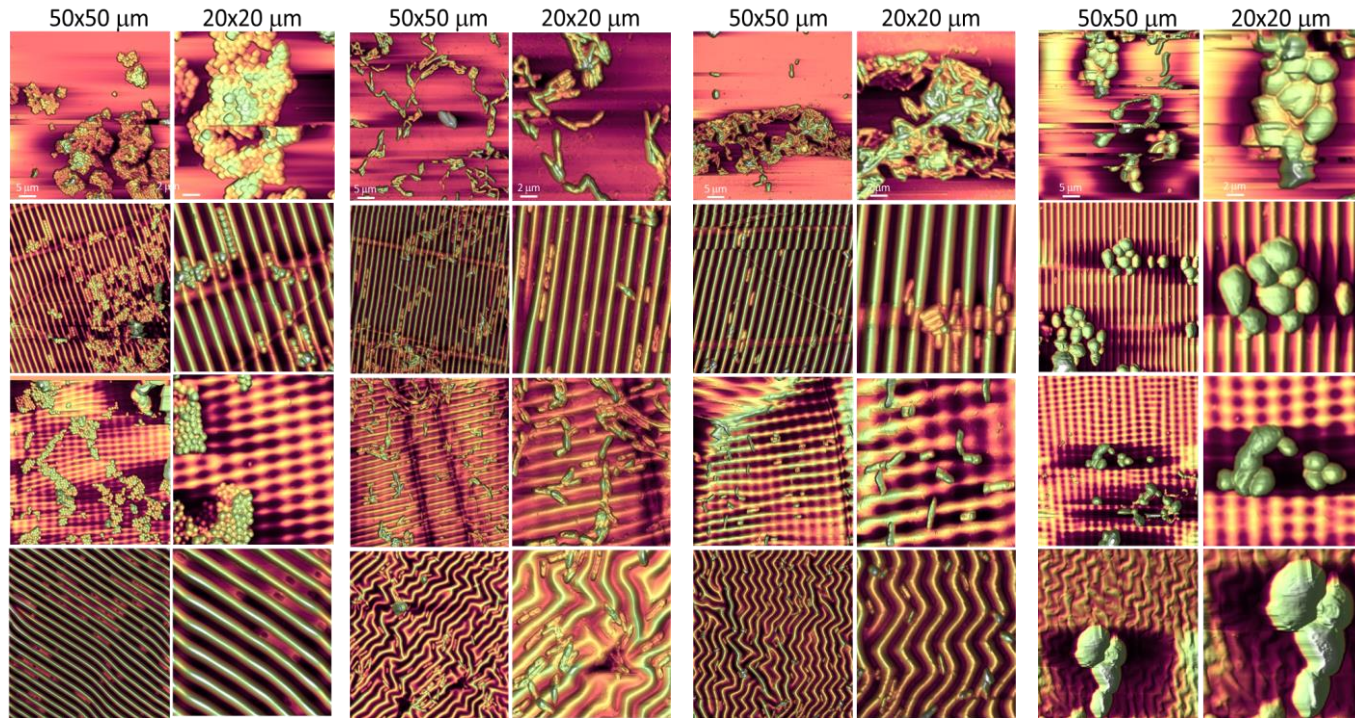


S. Aureus

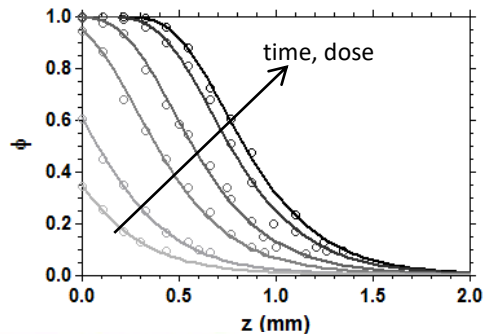
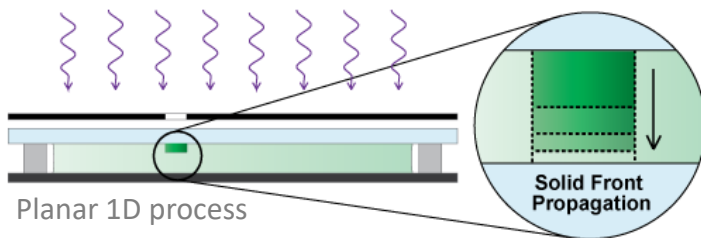
P. Aeruginosa

E. Coli

C. Albicans



Summary: FPP1D and 3D



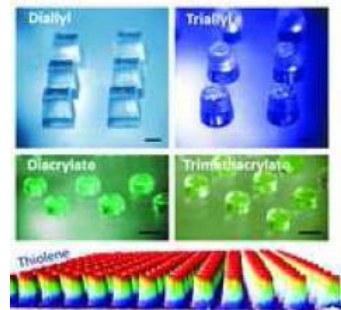
Spatio-temporal model

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z} \left(D(\phi) \frac{\partial \phi}{\partial z} \right) + K(T)(1 - \phi)I,$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(k(T) \frac{\partial T}{\partial z} \right) + (\Delta H) \frac{\partial \phi}{\partial t},$$

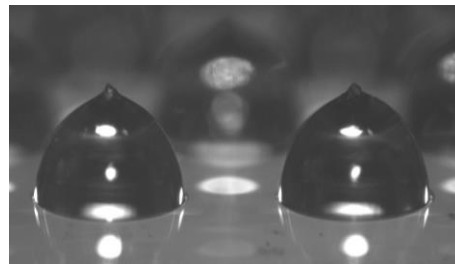
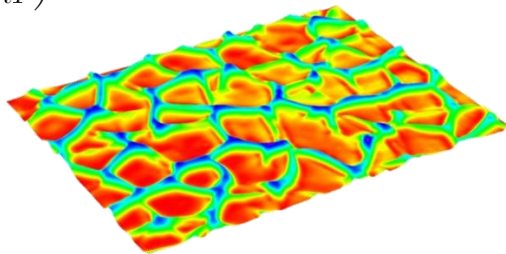
$$\frac{\partial I}{\partial z} = -[\mu_\infty \phi + \mu_0(1 - \phi)]I,$$

$$K(T) = K_0 \exp \left(-\frac{E_a}{RT} \right)$$

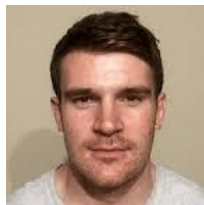


FPP1D

FPP3D



Thank you



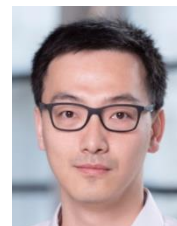
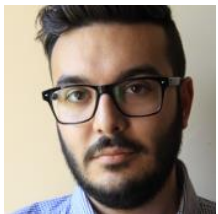
Roisin O'Connell

Will Sharratt

Yutaka Aoki

Sepiden
Khodaparast

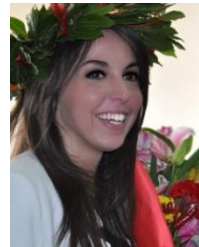
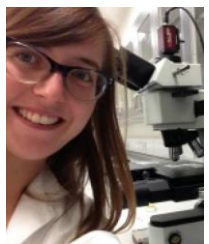
Gunjan Tyagi



Marco Adamo Christiana Udoh Carlos Gonzalez

Andreas
Poulos

Haoyu Wang



Ale Vitale Matt Hennessy Manuela Nania



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