

## 4.5 Asymptotic behaviour of reducible Markov chains

### Quiz 4.87

```
In[7]:= (*P={{1/2,1/2,0,0,0,0},{1/2,1/2,0,0,0,0},{0,0,1/3,2/3,0,0},
{0,0,2/3,1/3,0,0},{0,0,0,0,1,0},{1/6,1/6,1/6,1/6,1/6,1/6}};*)
P = {{0.5, 0.5, 0, 0, 0, 0}, {0.5, 0.5, 0, 0, 0, 0}, {0, 0, N[1/3, 6], 1 - N[1/3, 6],
0, 0}, {0, 0, N[2/3, 6], 1 - N[2/3, 6], 0, 0}, {0, 0, 0, 0, 1, 0},
{N[1/6, 5], N[1/6, 5], N[1/6, 5], N[1/6, 5], N[1/6, 5], N[1/6, 5]}};
MatrixForm[Table[{n, MatrixForm[MatrixPower[P, n]]}, {n, 1, 100, 10}]]
```

Out[8]//MatrixForm=

$$\begin{aligned}
1 & \left( \begin{array}{cccccc} 0.5 & 0.5 & 0. & 0. & 0. & 0. \\ 0.5 & 0.5 & 0. & 0. & 0. & 0. \\ 0. & 0. & 0.333333 & 0.666667 & 0. & 0. \\ 0. & 0. & 0.666667 & 0.333333 & 0. & 0. \\ 0. & 0. & 0. & 0. & 1. & 0. \\ 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 & 0.166667 \end{array} \right) \\
11 & \left( \begin{array}{ccccc} 0.5 & 0.5 & 0. & 0. & 0. \\ 0.5 & 0.5 & 0. & 0. & 0. \\ 0. & 0. & 0.499997 & 0.500003 & 0. \\ 0. & 0. & 0.500003 & 0.499997 & 0. \\ 0. & 0. & 0. & 0. & 1. \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 2.75636 \times 10^{-9} \end{array} \right) \\
21 & \left( \begin{array}{ccccc} 0.5 & 0.5 & 0. & 0. & 0. \\ 0.5 & 0.5 & 0. & 0. & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0. & 0. & 1. \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 4.55852 \times 10^{-17} \end{array} \right) \\
31 & \left( \begin{array}{ccccc} 0.5 & 0.5 & 0. & 0. & 0. \\ 0.5 & 0.5 & 0. & 0. & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0. & 0. & 1. \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 7.53896 \times 10^{-25} \end{array} \right) \\
41 & \left( \begin{array}{ccccc} 0.5 & 0.5 & 0. & 0. & 0. \\ 0.5 & 0.5 & 0. & 0. & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0. & 0. & 1. \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 1.24681 \times 10^{-32} \end{array} \right) \\
51 & \left( \begin{array}{ccccc} 0.5 & 0.5 & 0. & 0. & 0. \\ 0.5 & 0.5 & 0. & 0. & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0. & 0. & 1. \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 2.06199 \times 10^{-40} \end{array} \right) \\
61 & \left( \begin{array}{ccccc} 0.5 & 0.5 & 0. & 0. & 0. \\ 0.5 & 0.5 & 0. & 0. & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0. & 0. & 1. \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 3.41015 \times 10^{-48} \end{array} \right) \\
71 & \left( \begin{array}{ccccc} 0.5 & 0.5 & 0. & 0. & 0. \\ 0.5 & 0.5 & 0. & 0. & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0. & 0. & 1. \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 5.63977 \times 10^{-56} \end{array} \right) \\
81 & \left( \begin{array}{ccccc} 0.5 & 0.5 & 0. & 0. & 0. \\ 0.5 & 0.5 & 0. & 0. & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0. & 0. & 1. \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 9.32714 \times 10^{-64} \end{array} \right) \\
91 & \left( \begin{array}{ccccc} 0.5 & 0.5 & 0. & 0. & 0. \\ 0.5 & 0.5 & 0. & 0. & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0.5 & 0.5 & 0. \\ 0. & 0. & 0. & 0. & 1. \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 1.54254 \times 10^{-71} \end{array} \right)
\end{aligned}$$