Coordination Strategies for Supply Chain Management of Perishable Goods using Model Predictive Control

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Abstract

Food loss and waste has become an issue of great concern and, according to the Food and Agriculture Organization (FAO), in 2016, around 14 percent of the food was lost in the Supply Chain from the post-harvest stage up to the retail stage, not included. Hence, reducing food loss and waste is crucial, since it has several positive impacts, such as increasing environmental sustainability and contributing to end hunger, which lead to an improvement in the quality of life of communities and populations. One way to achieve this is by improving the efficiency of Perishable Food Supply Chains.
In this work, the Perishable Food Supply Chains are modeled using a state-space representation, which considers multiple sub-inventories for each Supply Chain player, based on the products' lifetime.
Furthermore, two Model Predictive Control strategies – Decentralized and Distributed – are developed with the objective of being applied to the Supply Chain Management of Perishable Food Supply Chains, focusing on the minimization of the quantity of overdue products across the Supply Chain.
The developed strategies are explained for a Supply Chain configuration addressed in the literature for regular Supply Chains. Then, the results of applying these strategies to a more complex configuration, also addressed in the literature, are presented, in order to evaluate their performance. The obtained results validate the proposed strategies for this case study and encourage further work development.
Keywords: Supply Chain, Perishable Goods, Decentralized Model Predictive Control, Distributed Model Predictive Control, Food loss reduction

1. Introduction

According to the Food and Agriculture Organization (FAO) [3], in 2016, around 14 percent of the food produced was lost in the Supply Chain from the post-harvest stage up to the retail stage, not included. Although some food loss and waste is inevitable, reducing it is crucial to improve the quality of life of communities and populations. It decreases production costs, contributes to end hunger, to achieve food security and improved nutrition, and to environmental sustainability. In detail, decreasing food loss leads to a reduction of the greenhouse gas emissions related to waste management.
One way to reduce food loss is by improving the efficiency of Perishable Food Supply Chain agents' individual operation at production, inventory and transportation levels as well as the coordination between them. This way, the possibility of product deterioration, due to the perishable nature of the products, before arriving to the retailers and later to the consumers is reduced.
According to the literature, Supply Chain does not have a unique and consensual definition, as multiple authors present distinct definitions [15]. This work adopts the Supply Chain definition presented in [13], where "a supply chain is defined as a set of three or more entities (organizations or individuals) directly involved in the upstream and downstream flows of products, services, finances, and/or information from a source to a customer".
Perishable Food Supply Chains (PFSC) differ from other regular Supply Chains, mainly due to the perishable nature of the products and their decreasing quality over time, while being moved from production to the customer, which leads to additional challenges than the ones observed for other regular Supply Chains [8].
Supply Chain Management is the business area responsible for dealing with the coordination performance between the different agents. It focuses on coordinating material, information and financial flows, in order to fulfill customer demand requirements, while involving all Supply Chain stakeholders in the decision-making process and its major goal is to improve the overall performance of the

Supply Chain [5].
Modeling and optimization of Supply Chains have been studied for years. Operations Research techniques have been used to optimize Supply Chains over the years. Although these techniques present some advantages, namely, reaching optimal or acceptable solutions, being methodical, clear and accessible, they also present some limitations, mainly related to the problem's dynamics. Supply Chain models involve high dimension models, non-stationary and nonlinear operations with complex dynamics that are not well captured by Operations Research techniques. The problems are either simplified or heuristics are applied. Also, models of planning and execution control are not explicitly interconnected in terms of uncertainty [6].
Operations Research techniques have been applied to regular Supply Chains for many years. However, Perishable Food Supply Chains have not received much attention until recently. In [10], a literature review of the Operations Research methods applied to perishable Supply Chain Management modeling and optimization, focusing on loss minimization along the Supply Chain are presented. The authors highlight that, for these type of problems, the majority of papers analysed - 55% - were published during the two years prior to the publication of the article. This evidence means that the relevance of this topic is increasing significantly. Besides, they conclude that, despite the fact that researchers acknowledge the occurrence of high food losses in the Supply Chain, the main objective of Supply Chain Management research works continues to be maximizing the revenue, instead of reducing the food loss.
In more recent years, Control Theory techniques, such as Model Predictive Control algorithms, have gained some relevance dealing with the optimization of Supply Chain operations. One of its main advantages is the capacity to accurately deal with Supply Chain's dynamics, contrary to Operations Research. In addition, Optimal Control considers planning and scheduling as a continuous adaptive process, instead of discrete operations. Furthermore, it allows a goal-oriented design of Supply Chain structures, which are addressed as whole multi-agent integrated systems. However, Optimal Control techniques also have their limitations associated to modelling complexity, the discretization of continuous variables and mathematics limitations related, for example, to numerical instability and the nonexistence of gradients. In the literature it is possible to find several research works that apply Model Predictive Control strategies to Supply Chains[18]. And, in more recent years, some authors have already started to apply Model Predictive Control strategies to Perishable Food Supply Chains [4, 11]
The effect of information sharing and visibility in Supply Chains has been discussed in many research works over the years [7, 12] and is regarded as one of the most effective ways of improving the Supply Chain performance, leading to a significant reduction of uncertainties of the orders from downstream agents. As a consequence, the inventory levels and the bullwhip effect are reduced, which leads to a decrease in overproduction of products [9, 19]. When dealing with Supply Chains of perishable commodities, information sharing and visibility gain an additional importance, since the overproduction leads to losses.
The problem addressed in this work consists in developing management strategies associated to agents' coordination that improve the Supply Chain performance prioritizing sustainable goals, while guaranteeing high levels of customer satisfaction. Specifically, the challenge consists in defining the control structure to apply to Supply Chain agents in order to minimize the quantity of overdue perishable goods, while satisfying all the customer demand at the retailers. Hence, the objective is to minimize food loss by increasing Supply Chain visibility.
One of the ultimate goals is to build a generic multi-scenario simulator for Supply Chains of perishable goods, that would design the Supply Chain according to the parameters chosen by managers and stakeholders, such as storage and transport capacities and the number of entities involved. The Supply Chain performance would be evaluated in terms of quantity of overdue goods, storage usage, production, overproduction and quantity of commodity movements. Supply Chain managers and stakeholders would be able to test distinct tactical plans as well as monitor in real-time the storage levels and commodities flows of the multiple players involved. So, the contributions of this work are: develop two Model Predictive Control strategies - Decentralized and Distributed - in order to be applied to Perishable Food Supply Chains management and be integrated in the simulator alongside the Centralized strategy present in [5]; validate these strategies and show their relevance, considering one case study present in the literature; consider sustainability as the main driver to the Supply Chain Management; develop analytical models that promote the increase of Supply Chain visibility, namely, through the introduction of new parameters, such as product lifetime across the Supply Chain.
This work is divided into four chapters. The first chapter presents the introduction of this work. Chapter 2 presents the design principles and the formulation of the Perishable Food Supply Chain

model. Besides, it explains the Model Predictive Control strategies developed to coordinate and optimize the Supply Chain. Then, Chapter 3 presents the results of the numerical experiments drawn to evaluate the performance of the developed Model Predictive Control strategies - Decentralized and Distributed. Lastly, in Chapter 4, conclusions of this work are presented and possible future work developments are described.

2. Background

2.1. Supply Chain Model Design

2.1.1 Network Configuration

Each agent involved in the Supply Chain operation can be interpreted as a Supply Chain player which performs specific tasks necessary to deliver the required quantity of products or services with the required quality to customers, at the required time and location. Supply Chain players performing similar tasks are grouped in echelons, such as procurement, manufacturing, distribution, marketing or retailing. Hence, the Supply Chain network configuration consists of $\mathrm{n_E}$ echelons, where each echelon is composed of $\mathrm{n_{pl_e}}$ players, $e = 1, ..., \mathrm{n_E}$, resulting in a total number of $\mathrm{n_{PL}} = \sum_{e=1}^{\mathrm{n_E}} \mathrm{n_{pl_e}}$ Supply Chain players. It is assumed that Supply Chain player pl of echelon e provides commodities to all Supply Chain players of the nearest downstream echelon $e{+}1$. The transportation time, in days, of moving commodities to player *i* of echelon $e+1$ from player *j* of echelon *e* is described by variable $\mathrm{tt}_{e_{ij}}, e = 1, ...\mathrm{n_E} - 1, i = 1, ..., \mathrm{n_{pl_{e+1}}}, j = 1, ..., \mathrm{n_{pl_e}}$.

2.1.2 Perishability

It is assumed that the Supply Chain is capable of handling $\mathrm{n_P}$ perishable commodities simultaneously.

Perishability is addressed assuming that perishable commodities have a lifetime of L_p days, $p = 1, ..., \mathrm{n_P}$, from the moment they are produced until expiring, after which they lose their commercial value [16]. This way, it is relevant to track the time until expiration of perishable commodities. The time until expiration is an intrinsic characteristic of each perishable commodity *p*, denoted by a_p, $p = 1, ..., \mathrm{n_P}$, measured in days and described by:

$$a_p = \mathrm{L}_p - m, \tag{1}$$

where *m* is the number of days since its production. Perishable commodity *p* expires when $\mathrm{a}_p = 0$.

2.1.3 Inventory Management

It is considered that the Supply Chain player pl from echelon *e* has its inventory divided into N_{pl_e} sub-inventories, $pl = 1, ..., \mathrm{n_{pl_e}}$, $e = 1, ..., \mathrm{n_E}$, corresponding to the inventory of each perishable commodity *p* with a_p days until expiration, stored by Supply Chain player pl of echelon *e*.

2.1.4 Model Formulation

The Supply Chain model proposed describes the movement of commodities along the multiple sub-inventories of Supply Chain players of the distinct echelons from the moment commodities are produced until being delivered to the customer. Furthermore, the model proposed is systematic and dimensionally flexible, being applicable to multiple network configurations handling multiple perishable commodities with distinct lifetimes. The Supply Chain model is fully defined by the parameters $\mathrm{n_E}$, $\mathrm{n_{pl_e}}$, $\mathrm{tt}_{e_{ij}}$, $\mathrm{n_P}$ and L_p.

2.1.5 State-Space Representation

Each Supply Chain player is interpreted as an autonomous subsystem. The inputs of the model are the commodity quantity inflows and outflows from the sub-inventories of the Supply Chain players. In its turn, the total inventories per perishable commodity of Supply Chain players are the measured outputs. Therefore, the generic model of the Supply Chain can be synthesized as follows:

$$\mathrm{x}_{ij}(k+1) = \mathrm{x}_{ij}(k) + \sum_m \mathrm{u}_{\mathrm{in}ijm}(k) - \sum_n \mathrm{u}_{\mathrm{out}ijn}(k) \tag{2}$$

$$\mathrm{y}_i(k) = \sum_j \mathrm{x}_{ij}(k) \tag{3}$$

$$\mathrm{y}_{\mathrm{OD}i}(k) = \mathrm{x}_{i0}(k), \tag{4}$$

$$i = 1, ..., \mathrm{n_{PL}},\ \ j = 1, ..., \mathrm{N}_{\mathrm{inv}_i}, \tag{5}$$

$$m = 1, ..., \mathrm{n_{pl_{e-1}}},\ \ n = 1, ..., \mathrm{n_{pl_{e+1}}}. \tag{6}$$

where $\mathrm{x}_{ij}(k)$ represents the commodity quantity stored in sub-inventory *j* of Supply Chain player *i*, at time instant *k*. Furthermore, $\mathrm{u}_{\mathrm{in}ijm}(k)$ stands for the commodity quantity moved to sub-inventory *j* of Supply Chain player *i* from Supply Chain player *m* of the previous echelon, at time instant *k* and $\mathrm{u}_{\mathrm{out}ijn}(k)$ is the commodity quantity to move from sub-inventory *j* of Supply Chain player *i* to Supply Chain player *n* of the next echelon, at time instant *k*. Lastly, $\mathrm{y}_i(k)$ represents the total commodity quantity stored by Supply Chain player *i*, at time instant *k* and $\mathrm{y}_{\mathrm{OD}i}(k)$ represents the total quantity of overdue perishable commodity of Supply Chain player *i*.

From a control perspective, $\mathbf{x}(k) = \mathrm{x}_{ij}(k)$ is the state vector of the system, $\mathbf{u}(k) = \sum_m \mathrm{u}_{\mathrm{in}ijm}(k) - \sum_n \mathrm{u}_{\mathrm{out}ijn}(k)$ is the input vector of the system and

$\mathbf{y}(k) = \mathrm{y}_i(k)$ is the output vector of the system. Thus, the Supply Chain model can be presented using the following state-space representation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}_{\mathrm{u}}\mathbf{u}(k) \tag{7}$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k), \tag{8}$$

where $\mathbf{A}$, $\mathbf{B}_{\mathrm{u}}$ and $\mathbf{C}$ are the state-space matrices associated to the state-space vectors that describe the dynamic model of the system.

3. Implementation

3.1. Model Predictive Control

Model Predictive Control predicts the optimal future behaviour (the future outputs $\hat{y}$) of a dynamical model based on current and past measurements of its operation. At each sampling instance, the Model Predictive Control algorithm receives as inputs the measures of the current state of the model, the current model parameters and the intensity of disturbances, as well as predicted values of these variables over a prediction horizon, N_p. Next, making use of the known dynamical model and the collected inputs, it formulates and solves an optimization problem in order to optimize the performance of the dynamical model. In detail, the optimization problem consists of finding the optimal sequence of control actions, ***u*** (over the prediction horizon, N_p), that optimizes the objective function, while satisfying the dynamics and constraints of the model. Then, the state of the system is updated by applying only the first predicted control action of the optimal sequence. At the next sampling instance, the measurements of the state of the model, the model parameters and the intensity of disturbances are collected and the Model Predictive Control algorithm is applied again following the same steps [18].

In the context of Supply Chains, the output *y* corresponds to the commodity quantity stored by each player and the control action ***u*** to the commodity quantity to move between the sub-inventories, calculated over the prediction horizon N_p, which corresponds to the demand forecasting interval. Model Predictive Control techniques are suitable to manage and optimize Supply Chains performance, often subjected to uncertain operation conditions and demand variability [14]. Any desired objective function can be used and process input and output constraints are included directly in the problem formulation so that future constraint violations are anticipated and prevented.

For this problem, the cost function used is linear, solved at each instance $\mathbf{k}$ using a mixed integer linear programming formulation, and is defined as (adapted from [5]):

$$J(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k) = \sum_{l=0}^{N_{\mathrm{p}}-1} \mathbf{q_{PL}}(k+l)\,\mathbf{x}(k+1+l) + \sum_{l=0}^{N_{\mathrm{p}}-1} \mathbf{q_{FL}}(k+l)\,\mathbf{u}(k+l), \tag{9}$$

where $\tilde{\mathbf{x}}_k$ and $\tilde{\mathbf{u}}_k$ are vectors composed of the state vectors and control actions, respectively, of each sampling instance k, over the prediction horizon N_p:

$$\tilde{\mathbf{x}}_k = \left[\ \mathbf{x}^{\mathrm{T}}(k+1) \quad ,\ldots, \quad \mathbf{x}^{\mathrm{T}}(k+N_{\mathrm{p}})\ \right]^{\mathrm{T}} \tag{10}$$

$$\tilde{\mathbf{u}}_k = \left[\ \mathbf{u}^{\mathrm{T}}(k+1) \quad ,\ldots, \quad \mathbf{u}^{\mathrm{T}}(k+N_{\mathrm{p}}-1)\ \right]^{\mathrm{T}}. \tag{11}$$

The objective function has two sets of weights:

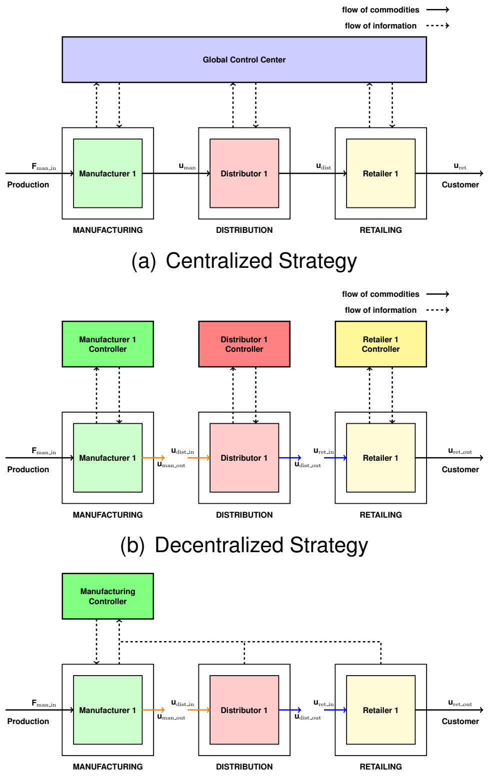
- $\mathrm{q_{PL}}_i(k)$, $i = 1, ..., \mathrm{n_{PL}}$, associated to the players - their sub-inventories and overdue goods - over the prediction horizon, N_p
- $\mathrm{q_{FL}}_j(k)$, $j = 1, ..., \mathrm{n_{FL}}$ - where $\mathrm{n_{FL}}$ is the number of links in the model - associated to the links, over the prediction horizon, N_p.

The weights of the cost function may vary according to the management policies chosen by Supply Chain managers as different Supply Chain goals require distinct operational behaviour. This cost function is used in a minimization problem. If a certain component has positive weights associated, that means its contribution to the overall value of the cost function will be positive. Since the objective is to minimize the cost function, then one of the objectives will be to have the lowest possible value for that component, that still respects the problem constraints. On the other hand, if a component has a negative weight associated, its contribution to the overall value of the cost function will be negative and one of the objectives will be to have the highest possible value for that component, that still respects the problem constraints.

Three Model Predictive Control strategies - Centralized, Decentralized and Distributed - represented in Figure 1, are explained next, based on a Supply Chain configuration composed by three echelons, each with one player.

The Centralized strategy is the starting point to the development and study of Decentralized and Distributed control strategies applied to Supply Chains of perishable goods and is adapted from [5].

This strategy, represented in Figure 1(a), considers an additional player - the Global Control Center [2] - that receives all of Supply Chain players' information, namely, storage and transport capacities, sub-inventory levels, the demand for each time instant and their predictions in future time instants. Making use of that information, it runs a Centralized Model Predictive Control Algorithm that finds the control actions that optimize the entire Supply



(a) Centralized Strategy

(b) Decentralized Strategy

(c) Distributed Strategy

Figure 1: Representation of three Model Predictive Control Strategies - Centralized, Decentralized and Distributed, considering a Supply Chain configuration with three echelons and one player per echelon.

Chain as a whole, meaning the quantity of goods to move between players. Then, the control actions are presented to the players and they implement them.
The formulation for this problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{12}$$

$$\text{s.t.} \quad \mathbf{x}(k+1+l) = \mathbf{A}\mathbf{x}(k+l) + \\ + \mathbf{B}_{\mathrm{u}}\mathbf{u}(k+l), \tag{13}$$

$$\mathbf{x}(k+1+l) \geq \mathbf{0}, \tag{14}$$

$$\mathbf{u}(k+l) \geq \mathbf{0}, \tag{15}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}(k+1+l) \leq \mathbf{X}_{\mathrm{max}}, \tag{16}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}(k+l) \leq \mathbf{U}_{\mathrm{max}}, \tag{17}$$

$$\mathbf{P}_{\mathrm{xu}}\mathbf{u}(k+l) \leq \mathbf{P}_{\mathrm{ux}}\mathbf{x}(k+l), \tag{18}$$

$$\mathbf{P}_{\mathrm{d}}\mathbf{u}(k+l) = \mathbf{D}(k+l), \tag{19}$$

for $l = 0, \ldots, N_{\mathrm{p}} - 1$, where $\mathbf{X_{max}}$ is the maximum storage capacity per Supply Chain player, $\mathbf{U_{max}}$ corresponds to the maximum transport capacity per Supply Chain connection, $\mathbf{P_{xx}}$ is the projection matrix from the state-space set $\mathcal{X}$ into the maximum storage capacity set $\mathbf{X_{max}}$, $\mathbf{P_{uu}}$ is the projection matrix from the control action set $\mathcal{U}$ into the maximum transport capacity set $\mathbf{U_{max}}$, $\mathbf{P_{xu}}$ is the projection matrix from the control action set $\mathcal{U}$ into the state-space set $\mathcal{X}$, $\mathbf{P_{ux}}$ is the projection matrix from the state-space set $\mathcal{X}$ into the control action set $\mathcal{U}$, $\mathbf{P_{d}}$ is the projection matrix from the control action set $\mathcal{U}$ into the customer demand set $\mathcal{D}$ and $\mathbf{D}$ is the customer demand.
Even though the Centralized strategy is the classical Model Predictive Control strategy used in terms of visibility and information sharing, it has some limitations, namely, the time it takes to solve large and complex problems and the constraints on the information flows of multi-agent systems, such as Supply Chains, where the agents may be unavailable or unable to share information [17].
For the Decentralized and Distributed strategies, the entire Supply Chain problem is divided into sub-problems and the information that each Supply Chain player has access to is reduced. For both cases, the problems are solved sequentially, starting from the retailing echelon and ending on the manufacturing echelon. However, the visibility and information sharing differs between strategies.
When applying the Decentralized strategy, represented in Figure 1(b), each Supply Chain player has its own controller that manages its flows of commodities and has access to the current quantity of goods stored in its sub-inventories, the storage capacity it has available and the transport capacity of the links directly connected to it. Furthermore, only the retailing echelon has access to the demand. The problems for each Supply Chain player are similar, having some differences related to the constraints. Exemplifying, the formulation of the manufacturer's problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{20}$$

$$\text{s.t.} \quad \mathbf{x}_{\mathrm{man}}(k+1+l) = \mathbf{A}\mathbf{x}_{\mathrm{man}}(k+l) + \\ + \mathbf{B}_{\mathrm{u}}\mathbf{u}_{\mathrm{man}}(k+l), \tag{21}$$

$$\mathbf{x}_{\mathrm{man}}(k+1+l) \geq \mathbf{0}, \tag{22}$$

$$\mathbf{u}_{\mathrm{man}}(k+l) \geq \mathbf{0}, \tag{23}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}_{\mathrm{man}}(k+1+l) \leq \mathbf{X}_{\mathrm{max_{man}}}, \tag{24}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}_{\mathrm{man}}(k+l) \leq \mathbf{U}_{\mathrm{max_{man}}}, \tag{25}$$

$$\mathbf{P}_{\mathrm{xu}}\mathbf{u}_{\mathrm{man}}(k+l) \leq \mathbf{P}_{\mathrm{ux}}\mathbf{x}_{\mathrm{man}}(k+l), \tag{26}$$

$$\mathbf{u}_{\mathrm{man\_out}}(k+l) = \mathbf{u}_{\mathrm{dist\_in}}(k+l), \tag{27}$$

for $l = 0, \ldots, N_{\mathrm{p}} - 1$.
When considering the Distributed Strategy, represented in Figure 1(c), each echelon is managed by one controller, meaning each Supply Chain player has access to the information of the other players of the same echelon and the best control actions for Supply Chain players of the same echelon are found simultaneously. Additionally, the communication between echelons increases and

there is a negotiation between entities.

For this configuration, the retailer has access to the same amount of information as for the Decentralized approach. The distributor has access to its own information plus the information related to the retailer. And the manufacturer has access to its own information plus the information regarding both the distributor and the retailer. Once more, the formulation for the manufacturer problem is presented and given by:

$$
\begin{aligned}
\min_{\tilde{\mathbf{u}}_k} \quad & J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) && (28)\\
\text{s.t.} \quad & \mathbf{x}_{\text{man}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{man}}(k+l) + \\
& \qquad + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{man}}(k+l), && (29)\\
& \mathbf{x}_{\text{man}}(k+1+l) \geq \mathbf{0}, && (30)\\
& \mathbf{u}_{\text{man}}(k+l) \geq \mathbf{0}, && (31)\\
& \mathbf{P}_{\text{xx}}\mathbf{x}_{\text{man}}(k+1+l) \leq \mathbf{X}_{\text{max}_{\text{man}}}, && (32)\\
& \mathbf{P}_{\text{uu}}\mathbf{u}_{\text{man}}(k+l) \leq \mathbf{U}_{\text{max}_{\text{man}}}, && (33)\\
& \mathbf{P}_{\text{xu}}\mathbf{u}_{\text{man}}(k+l) \leq \mathbf{P}_{\text{ux}}\mathbf{x}_{\text{man}}(k+l), && (34)\\
& \mathbf{u}_{\text{man\_out}}(k+l) = \mathbf{u}_{\text{dist\_in}}(k+l), && (35)\\
& \mathbf{u}_{\text{dist\_out}}(k+l) = \mathbf{u}_{\text{ret\_in}}(k+l), && (36)\\
& \mathbf{P}_{\text{d}}\mathbf{u}_{\text{man}}(k+l) = \mathbf{D}(k+l), && (37)
\end{aligned}
$$

for $l = 0, \ldots, N_{\text{p}} - 1$.

For the three strategies, the objective function considered is the same and the first six constraints have the same meaning. Considering the Centralized strategy, constraint (13) corresponds to the model dynamics. Constraints (14) and (15) impose that there are no negative states nor negative flows of goods. Constraints (16) and (17) impose that the storage and transport capacities cannot be exceeded. Finally, constraint (18) imposes that the quantity of goods that is taken from a player does not exceed the quantity of goods stored in that player. The difference between these constraints is related to the state and the control action vectors, and consequently, the matrices dimension. The state and control action vectors for the formulations presented for the Centralized and Distributed strategies consist on the states and control actions of all Supply Chain players and connections between them, respectively. For the Decentralized strategy, the state and the control action vector presented consist on the manufacturer's states and on the connections directly connected to it.

Specifically for the Centralized strategy, constraint (19) guarantees the customer demand is fulfilled.

For the Decentralized strategy, the manufacturer does not have direct access to the demand. The information regarding the demand reaches the manufacturer through the distributor's orders or inflows of commodities, $\mathbf{u}_{\text{dist\_in}}$, that the manufacturer needs to fulfill. This way, constraint (27) is a compatibility constraint that imposes that what leaves from the manufacturer to the distributor is equal to what enters in the distributor from the manufacturer, guaranteeing there are no losses between players.

For the Distributed strategy, constraint (37) guarantees the customer demand is fulfilled. Regarding the compatibility constraints, constraint (35) guarantees that what leaves the manufacturing echelon to each player of the distribution echelon is equal to what enters that player. Considering only one player per echelon, this constraint is the same as for the Decentralized strategy. However, when there is more than one player per echelon, the manufacturing controller has more freedom to find the best control actions to apply, reflecting the negotiation dynamics for the Distributed strategy. Additionally, it is necessary to consider constraint (36), which guarantees that what leaves from the distribution echelon is equal to what enters the retailing echelon, since the manufacturer has visibility of the entire Supply Chain.

4. Results & discussion

In order to validate the developed Model Predictive Control strategies, numerical simulations were performed, considering the Supply Chain configuration presented in Figure 2.

4.1. Simulation Specifications

The performed simulations have a duration of 100 days and a sampling time of 1 day and the default prediction horizon, N_p, is set to 12 days [5].

4.2. Network Configuration

The Supply Chain considered is composed by 3 echelons - manufacturing, distribution and retailing - each one composed of 2 players. The goods enter in the Supply Chain from production to the manufacturing echelon. Then, they are moved across the Supply Chain, being handled by the distinct players, until reaching the customer or becoming overdue goods. Any player of a given echelon, that has stored goods, can deliver products to any player of the next echelon, if the transport capacity between players and the storage capacity of the player receiving the goods are not exceeded. It is considered that the transportation time of moving commodities from a player of a given echelon to a player of the next echelon is equal to 1 day.

These storage and transport limits are assumed constant over the entire simulation.

Regarding the storage capacity, each manufacturer can store up to 30 products simultaneously, while each distributor and each retailer have the capacity to store 25 and 20 products at once, respectively. Regarding the transport, each manufacturer can receive 20 products simultaneously, from production. The transport capacity of commodities between players 1 and players 2 of neighbouring echelons is equal to 20 and the transport capacity from player 1 (player 2) of a given echelon to player 2 (player 1) of the next echelon is equal to 15, since it is considered that these players are further away. As for the retailing echelon, each retailer can deliver 15 products to the customer at once.

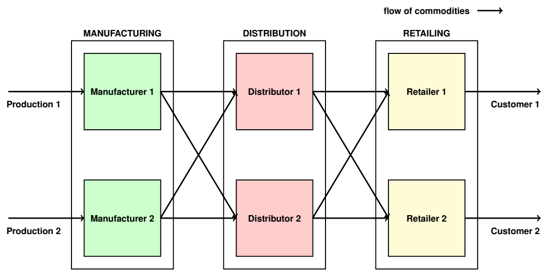


Figure 2: Representation of the network configuration and the player interaction of a Supply Chain composed of three echelons - manufacturing, distribution and retailing - with two players per echelon (adapted from [1]).

4.3. Demand

To evaluate the performance of the developed strategies, three demand profiles for one type of commodity with a lifetime of 5 days - D1, D2 and D3 - were considered and their characteristics are presented in Table 1.

Table 1: Demand Profiles Characteristics.

Characteristics	Demand Profile		
	D1	D2	D3
Mean	4.3	4.3	4.3
Variance	11.8	0.2	29.2
Total	389	389	389

The three demand profiles have a total amount of commodities equal to 389 units, distributed over the 100 days of the simulation. The main difference between the demand profiles is related to the variance. Profiles D2 and D3 present the lowest and highest variance, respectively.

Optimal Control techniques usually consider the worst case scenarios, close to the operational limits of the system. However, it would be necessary to test the system's limits, which is outside the scope of this work. Hence, it is considered that the presented profiles are demanding enough in terms of variability and intensity to test the performance and validate the developed strategies.

4.4. Management Policy

The management policy is described by the cost function weights of the optimization problem. The cost function has three components, related to the minimization of overdue goods, inventory levels at each player and connections:

- The weight value associated to the overdue goods is the highest positive value - 100 - meaning waste minimization is prioritized relatively to the other two cost function components;
- The weights related to inventory levels are positive and equal for all players - 1 - to penalize equally the storage of perishable products in every Supply Chain player;
- The weights related to connections are positive or null. The connections with positive weights associated are the ones from production to the manufacturing echelon - 50 - to penalize the overproduction of goods and the ones from player 1 of a given echelon to player 2 of the next echelon - 10 - because the distance between them is assumed to be bigger than the distance between player 1 of a given echelon and player 1 of the next echelon. The same happens between players 2. This way, the connections between players that are closer are chosen as a first option, leading to lower costs.

4.5. Performance measures

The performance measures considered to evaluate numerically the performance of the Model Predictive Control strategies are:

- total production (TP) - the total quantity of goods that are produced, measured in units, to fulfill the customer demand at the retailers;
- total storage (TS) - the total quantity of goods stored, measured in units, considering all Supply Chain and the entire length of the simulation;
- overproduction (OP) - the percentage of goods produced that exceed the customer demand at the retailers;
- quantity of commodities movements (QCM)- the total quantity of goods being moved through the links of the Supply Chain, measured in units, considering all links excluding the production links and considering the entire length of the simulation;
- total quantity of overdue products (TQOP) - the quantity of goods that expire and become overdue before being sold to the customers at the retailers, measured in units;
- computation time (CT) - the time that it takes to run the entire simulation, measured in seconds.

4.6. Results

4.6.1 Strategies Comparison

The three strategies were tested considering the configuration, simulation specifications and management policy presented previously. Additionally, the considered demand has the characteristics presented for demand D1 of Table 1.
The performance measures obtained can be consulted in Table 2.

Table 2: Performance measures for the three Model Predictive Control Strategies for a configuration with 3 echelons and 2 players per echelon

	Strategy		
	Decentralized	Distributed	Centralized
TP	690	677	523
TS	1736	1672	667
OP	44	43	26
QCM	1319	1635	1167
TQOP	297	284	131
CT	16	43.23	32.22

The Centralized strategy presents the best values for almost every performance measure, when the main objective is to minimize the quantity of overdue products. All three strategies have proven to satisfy the customer demand, but the Centralized one does it with a lower overproduction, lower values of storage intensity and with less commodity movements. However, in terms of computation time, the Decentralized strategy is the fastest, despite of solving more problems – one per player – than the other two strategies – one problem for the entire Supply Chain and one problem per echelon. But, these problems are simpler, leading to a lower computation time. The Distributed strategy presents the highest computation time, which is related to the size of the problems. In detail, the manufacturing problem is the same size of the whole Centralized problem, even though some variables are already fixed. In addition, this strategy solves the retailing and distribution problems.
Even though the Centralized strategy presents, in general, the best results, this strategy is ideal, since, in reality, there is no external player that has total access to the information from all players and total control over the entire Supply Chain. Between the developed strategies, when the Distributed strategy is applied to the Supply Chain, it shows a lower overproduction and it has lower storage intensity values, leading to a lower quantity of overdue products. Additionally, this strategy is closer to the real-world settings than the Centralized or the Decentralized strategies.

4.7. Literature Benchmark

After the proposed strategies were validated, the bullwhip effect was measured and compared to the results obtained in [14], where three Model Predictive Control strategies - Centralized, Decentralized and semi-Decentralized - are developed to manage the flows of multi-product Supply Chains. The Supply Chains models used present some differences to the ones developed in this thesis, namely, the authors consider:

- non-perishable products;
- different objectives;
- each product is produced by a specific manufacturer, distributed by a specific distributor and sold by a specific retailer. This means there is no interaction or cooperation between Supply Chain agents.

Nevertheless, Mestan et al. [14] present a bullwhip effect measure to quantify the performance of the strategies, which can also be applied to the control strategies developed in this work, given by (adapted from [14]):

$$\mathbf{bwr}_{jm} = \left| \frac{\sum_i \mathbf{O_{ijm}}(k)}{\sum_l \mathbf{O_{jlm}}(k)} \right|, \qquad (38)$$

where $\mathbf{bwr}_{jm}$ is the measure of the bullwhip effect generated by node j for product m, $\sum_i \mathbf{O_{ijm}}(k)$ represents the orders that are placed from node j to all upstream nodes i for product m and $\sum_l \mathbf{O_{jlm}}(k)$ are the orders from all downstream nodes l to node j for product m. Ideally, this ratio would be 1, meaning the orders placed by node j would be equal to the orders placed by the downstream nodes to j and no bullwhip effect would be registered.
Table 3 presents the average measure of the bullwhip effect for the three strategies developed in [14] discriminated by product and Figure 3 shows the demand profiles of the three products.

Table 3: Comparison of the Bullwhip Effect under Different Configurations - [14] results.

	Average Measure of the Bullwhip Effect		
	Decentralized	semi-Decentralized	Centralized
A	2.10	1.43	1.17
B	3.06	1.54	1.22
C	2.55	2.62	2.13

In terms of quantity, product B presents the highest total amount of units, followed by product A. In terms of variability, product A has the most constant demand profile from the three and products B and C present a similar variability.
Analyzing Table 3 results, product A is the one that

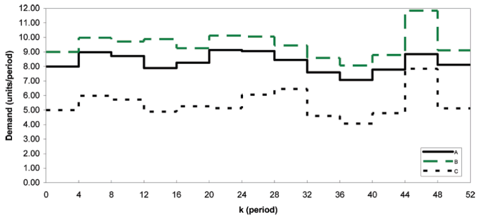


Figure 3: Demand profiles for products A, B and C [14].

presents the lowest values for the average measure of the bullwhip effect for the three strategies. For the Decentralized strategy, product B is the one that presents the highest value and for the remaining two strategies, product C presents the highest values. From the results shown, it may be concluded that a more constant demand leads to a lower bullwhip effect for all strategies. Since product C has the lowest amount of units, it may also be concluded that the total quantity of demand is not a determinant factor for the bullwhip effect.

Although the storage and transport capacities are not specified and the total amount of units for the three products is different from the total amount of the demand profiles presented in Table 1, the values obtained for the bullwhip effect measure are compared with the ones obtained for the strategies developed, since a ratio is considered and the applied strategies are similar. The values for the semi-Decentralized strategy are compared with the values obtained for the Distributed strategy.

The values of the average bullwhip effect measure for the developed strategies are shown in Table 4, for the three demand profiles presented in Table 1.

Table 4: Comparison of the Bullwhip Effect under Different Configurations and for different demand profiles.

	Average Measure of the Bullwhip Effect		
	Decentralized	Distributed	Centralized
D1	1.39	1.21	1.11
D2	1.32	1.15	1.07
D3	1.45	1.28	1.16

The values of the performance measure obtained for these strategies are lower than the ones obtained for the corresponding strategies developed in [14], for the product with the lowest performance values. Additionally, the results show that, when the variability increases, the average bullwhip effect measure also increases. The demand of scenario 2 presents the highest variability among all demand profiles, higher than the one presented for any of the three products, A, B and C, and even for this scenario, the values obtained are lower. This way, considering less demanding profiles, for the same Supply Chain configuration would lead to similar or better results. In addition, considering similar case studies, the strategies may still be validated as long as there is a compromise between the demand intensity and variability and the Supply Chain constraints. However, finding the generic class of problems for which these conclusions hold and the compromise that needs to exist involves a more mathematical analysis that is outside the scope of this work.

5. Conclusions

5.1. Overview

The proposed strategies - Decentralized and Distributed - are evaluated and compared to the Centralized strategy. The Decentralized strategy is the one that presents the poorest performance, which is expected due to its lack of visibility. Contrary, the Centralized strategy has the best performance. However, as stated previously, it is ideal and further away from real-world settings than the Distributed strategy. Nevertheless, it presents the target values for the Distributed strategy, being the objective to minimize the gap between these two strategies.

The results validated the proposed strategies when applied to a case study presented in the literature adapted to Perishable Food Supply Chains and presented better results in terms of the bullwhip effect than the strategies developed in [14] for a similar case study.

The worst case scenario was not considered. However, the demand profiles were demanding. Finding the worst case scenario and the compromise between the demand and the model constraints would imply finding mathematical relations, which was outside the scope of this work.

Nevertheless, the obtained results encourage the further development of the work and the simulator, that intends to be generic, in order to be applied to other configurations and different scenarios.

5.2. Future Work

Even though the proposed strategies were validated, there are some aspects that could be further studied.

One improvement would be to expand the computational implementation of the Supply Chain in order to simulate any possible configuration.

Additionally, the amount of overdue products obtained across the Supply Chain could be reduced by considering different cost functions or considering different cost function weights, across the prediction horizon, assigning higher weights to the time instants closer to the current time instant.

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