Coordination Strategies for Supply Chain Management of Perishable Goods using Model Predictive Control

Margarida Cardoso dos Reis

Thesis to obtain the Master of Science Degree in

Mechanical Engineering

Supervisor: Prof. Miguel Afonso Dias de Ayala Botto

Examination Committee

Chairperson: Prof. Duarte Pedro Mata de Oliveira Valério
Supervisor: Prof. Miguel Afonso Dias de Ayala Botto
Member of the Committee: Prof. Bruna Alexandra Elias Mota

July 2021

To my parents

Acknowledgments

I want to thank my supervisor, Prof. Miguel Ayala Botto, and my tutor, Eng. Tomás Pedro Hipólito, from Instituto Superior Técnico, for their close supervision and orientation, during the course of this work, and their availability and patience to answer all my questions.

I also want to take this opportunity to thank my friends and colleagues from Instituto Superior Técnico for all the hours we spent working for lab projects and for going through this experience with me.

Lastly, I want to express my profound gratitude to my parents, for all their support and encouragement during these 5 years and for reminding me, many times, that college is a marathon, not a sprint and that it is important to find a balance. And, for helping me find that balance, I want to give a special thanks to my friends and masters from STAT.

Resumo

O desperdício de comida tem-se vindo a tornar um assunto de crescente importância. Segundo a Food and Agriculture Organization (FAO), em 2016, cerca de 14% da comida deteriorou-se no intervalo de tempo compreendido entre a colheita e a sua chegada ao retalho. Reduzir o desperdício de comida é crucial, uma vez que tem muitos impactos positivos, como aumentar a sustentabilidade ambiental e contribuir para acabar com a fome, o que se traduz numa melhoria na qualidade de vida das populações. Uma forma de atingir este objetivo passa por aumentar a eficiência das cadeias de abastecimento de produtos perecíveis.

Nesta dissertação de mestrado, as cadeias de abastecimento de produtos perecíveis são modeladas recorrendo a uma representação em espaço de estados, considerando múltiplos sub-inventários, baseados no tempo de vida dos produtos, para cada agente da cadeia de abastecimento.

Além disso, são desenvolvidas duas estratégias de controlo preditivo - descentralizado e distribuído - para serem aplicadas à gestão destas cadeias de abastecimento de produtos perecíveis, sendo o principal foco a minimização da quantidade de produtos estragados ao longo da cadeia de abastecimento.

As estratégias desenvolvidas são aplicadas a duas configurações de cadeias de abastecimento, presentes na literatura para cadeias de abastecimento de produtos não perecíveis, de maneira a avaliar a sua performance. Os resultados obtidos validam as estratégias propostas para os casos apresentados e incentivam um desenvolvimento futuro do trabalho.

Palavras-chave: Cadeia de abastecimento, Produtos perecíveis, Controlo Descentralizado, Controlo Distribuído, Redução do desperdício de comida

Abstract

Food loss and waste has become an issue of great concern and, according to the Food and Agriculture Organization (FAO), in 2016, around 14 percent of the food was lost from the post-harvest stage up to the retail stage, not included. Reducing food loss and waste is crucial, since it has several positive impacts, such as increasing environmental sustainability and contributing to end hunger, which lead to an improvement in the quality of life of communities and populations. One way to achieve this is by improving the efficiency of Perishable Food Supply Chains.

In this thesis, Perishable Food Supply Chains are modeled using a state-space representation, which considers multiple sub-inventories for each Supply Chain player, based on the products' lifetime.

Furthermore, two Model Predictive Control strategies – Decentralized and Distributed – are developed with the objective of performing Supply Chain Management of Perishable Food Supply Chains, focusing on the minimization of the quantity of overdue products across the Supply Chain.

The developed strategies are applied to two Supply Chain network configurations, addressed in the literature for regular Supply Chains, in order to evaluate their performance. The obtained results validate the proposed strategies for these two case studies and encourage further work development.

Keywords: Supply Chain, Perishable Goods, Decentralized Model Predictive Control, Distributed Model Predictive Control, Food loss reduction

Contents

List of Tables

List of Figures

Nomenclature

$\tilde{\mathbf{u}}_k$ Vector composed of the control actions at each sampling time instance *k*, over the prediction horizon, N_p

$\tilde{\mathbf{x}}_k$ Vector composed of the state vectors at each sampling time instance *k*, over the prediction horizon, N_p

$\mathbf{A}$ State-space dynamic matrix

$\mathbf{bwr}_{jm}$ Measure of the bullwhip effect generated by node j for product m

$\mathbf{B}_\mathrm{u}$ State-space input matrix

$\mathbf{C}$ State-space output matrix

$\mathbf{D}$ Customer Demand

$\mathbf{F}_{\mathrm{man\_in}i}$ Inflow of perishable commodities to manufacturer i from production

$\mathbf{P_d}$ Projection matrix from the control action set $\mathcal{U}$ into the customer demand set $\mathcal{D}$

$\mathbf{P_{uu}}$ Projection matrix from the control action set $\mathcal{U}$ into the maximum transport capacity set $\mathbf{U_{max}}$

$\mathbf{P_{ux}}$ Projection matrix from the state-space set $\mathcal{X}$ into the control action set $\mathcal{U}$

$\mathbf{P_{xu}}$ Projection matrix from the control action set $\mathcal{U}$ into the state-space set $\mathcal{X}$

$\mathbf{P_{xx}}$ Projection matrix from the state-space set $\mathcal{X}$ into the maximum storage capacity set $\mathbf{X_{max}}$

$\mathbf{U_{max}}$ Maximum transport capacity per Supply Chain connection

$\mathbf{u}$ Input vector of the system

$\mathbf{u}_{\mathrm{dist\_in}ij}$ Inflow of perishable commodities to distributor i from manufacturer j

$\mathbf{u}_{\mathrm{dist\_out}ij}$ Outflow of perishable commodities from distributor i to retailer j

$\mathbf{u}_{\mathrm{man\_out}ij}$ Outflow of perishable commodities from manufacturer i to distributor j

$\mathbf{u}_{\mathrm{ret\_in}ij}$ Inflow of perishable commodities to retailer i from distributor j

$\mathbf{u}_{\mathrm{ret\_out}i}$ Outflow of perishable commodities from retailer i

$\mathbf{X_{max}}$ Maximum storage capacity per Supply Chain player

$\mathbf{x}$ State vector of the system

$\mathbf{y}$ Output vector of the system

a_p Time until expiration of perishable commodity *p* measured in days

$F_{dist\_out\mathit{j}}$ Outflow of perishable commodity from distributor with remaining time until expiration *j*

F_{man_in} Inflow of perishable commodity from production

$F_{man\_out\mathit{j}}$ Outflow of perishable commodity from manufacturer with remaining time until expiration *j*

$F_{ret\_out\mathit{j}}$ Outflow of perishable commodity from retailer with remaining time until expiration *j*

I_{dist0} Distributor inventory of overdue perishable commodity

I_{DIST_OD} Total distributor quantity of overdue perishable commodity

$I_{dist\mathit{i}}$ Distributor inventory of perishable commodity with remaining time until expiration *i*

I_{DIST} Total distributor inventory

I_{man0} Manufacturer inventory of overdue perishable commodity

I_{MAN_OD} Total manufacturer quantity of overdue perishable commodity

$I_{man\mathit{i}}$ Manufacturer inventory of perishable commodity with remaining time until expiration *i*

I_{MAN} Total manufacturer inventory

I_{ret0} Retailer inventory of overdue perishable commodity

I_{RET_OD} Total retailer quantity of overdue perishable commodity

$I_{ret\mathit{i}}$ Retailer inventory of perishable commodity with remaining time until expiration *i*

I_{RET} Total retailer inventory

L_p Lifetime of perishable commodity *p* measured in days

m Number of days since production of perishable commodity *p* measured in days

N_p Prediction horizon

n_E Number of echelons

n_P Number of perishable commodities handled by the Supply Chain

n_{FL} Total number of links in the Supply Chain

n_{pl_e} Number of Supply Chain players belonging to echelon *e*

n_{PL} Total number of players in the Supply Chain

$\mathrm{N}_{\mathrm{inv}_i}$ Number of sub-inventories of Supply Chain player *i*

N_{pl_e} Number of sub-inventories of Supply Chain player *pl* of echelon *e*

$\mathrm{q}_{\mathrm{FL}j}$ Weights associated to Supply Chain connection *j*

$\mathrm{q}_{\mathrm{PL}i}$ Weights associated to Supply Chain player *i*

$\mathrm{tt}_{e_{ij}}$ Transportation time, in days, of moving commodities to player *i* of echelon $e+1$ from player *j* of echelon *e*

$\mathrm{u}_{\mathrm{in}ijm}$ Commodity quantity moved to sub-inventory *j* of Supply Chain player *i* from Supply Chain player *m* of the previous echelon

$\mathrm{u}_{\mathrm{out}ijn}$ Commodity quantity to move from sub-inventory *j* of Supply Chain player *i* to Supply Chain player *n* of the next echelon

x_{ij} Commodity quantity stored in sub-inventory *j* of Supply Chain player *i*

$\mathrm{y}_{\mathrm{OD}i}$ Total quantity of overdue perishable commodity of Supply Chain player *i*

y_i Total commodity quantity stored by Supply Chain player *i*

$\sum_i \mathbf{O_{ijm}}$ Orders placed from node *j* to all upstream nodes *i* for product *m*

$\sum_l \mathbf{O_{jlm}}$ Orders placed from all downstream nodes *l* to node *j* for product *m*

Chapter 1

Introduction

1.1 Motivation

According to the Food and Agriculture Organization (FAO) [1], in 2016, around 14 percent of the food produced was lost from the post-harvest stage up to the retail stage, not included. This percentage was obtained using the Food Loss Index, a sub-indicator used to evaluate the progress towards the Sustainable Development Goal target 12.3 which ”calls for halving per capita global food waste at retail and consumer levels and reducing food loss along production and supply chains, including post-harvest loss, by 2030.”

In Figure 1.1, the percentage of food loss is presented geographically, globally and per region. The food loss by region ranges from 6 percent, in Australia and New Zealand, to 21 percent in Central and Southern Asia.

In terms of commodities, the total percentage of food loss is divided in five major groups: cereals and pulses, fruits and vegetables, meat and animal product, roots, tubers and oil-bearing crops and others. According to the percentages shown in Figure 1.2, roots, tubers and oil-bearing crops are the goods that register the highest loss, followed by the group of fruits and vegetables. The group of roots has the highest levels due to cassava and potato losses, since potatoes require a careful handling and proper storage, while cassava has a very perishable nature. Like cassava, also vegetables and fruits have a very perishable nature, which justifies their high loss percentage. Food loss and food waste have two different meanings and although there are no common definitions for these two concepts, in FAO’s 2019 report [1], food loss is defined as ”the decrease in the quantity or quality of food resulting from decisions and actions by food suppliers in the chain, excluding retail, food service providers and consumers”. Additionally, food waste is defined as ”the decrease in the quantity or quality of food resulting from decisions and actions by retailers, food services and consumers.”

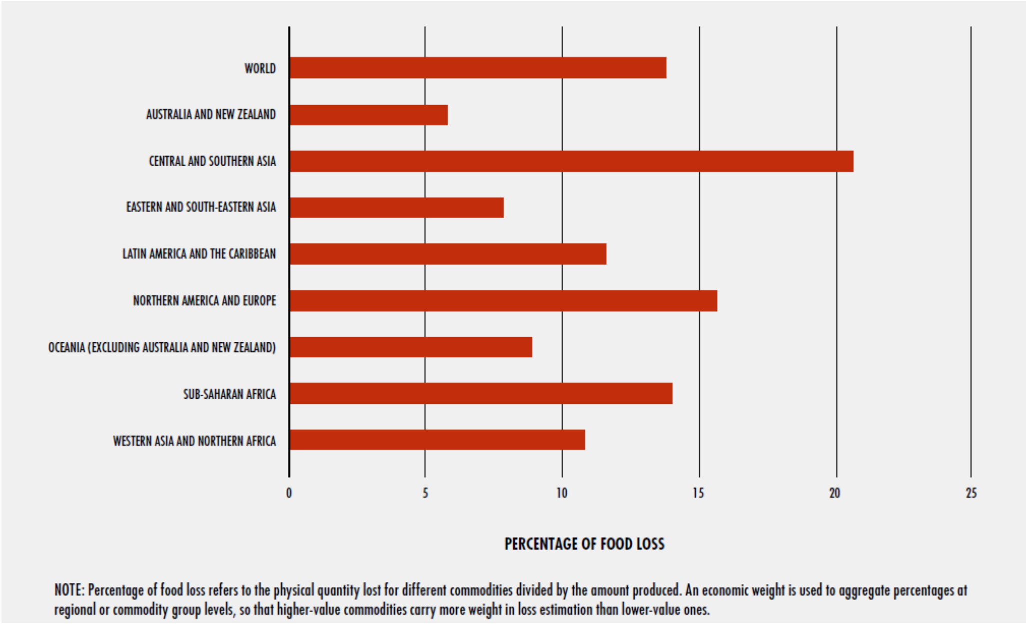


Figure 1.1: Geographic distribution of food loss, globally and per region, in percentage, from post-harvest to distribution in 2016 [1].

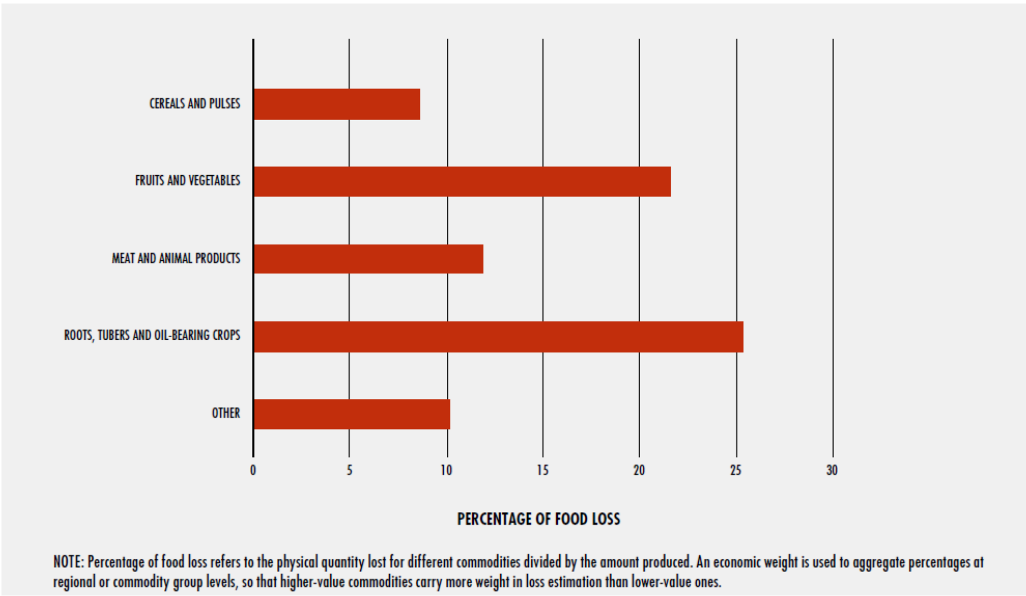


Figure 1.2: Food loss, in percentage, by commodity type, from post-harvest to distribution in 2016 [1].

Although some food loss and waste is inevitable, reducing it is crucial to improve the quality of life of communities and populations, because it:

- decreases production costs;
- contributes to environmental sustainability, since decreasing food loss leads to an ease in pressure of the natural resources used and to a reduction of the greenhouse gas emissions related to the management of waste;
- contributes to end hunger and achieve food security and improved nutrition.

One way to reduce food loss is by improving the efficiency of Supply Chain agents' individual operation at production, inventory and transportation levels as well as the coordination between them. This way, the possibility of product deterioration, due to the perishable nature of the products, before arriving to the retailers and later to the consumers is reduced. When there is lack of coordination, the agents do not have access to all information or have incentives that are not aligned with the interests of the entire Supply Chain, leading to a poorer performance and, consequently, higher food loss rates. This way, a good coordination is crucial for the Supply Chain to perform effectively.

1.2 Literature Review

According to the literature, Supply Chain does not have a unique and consensual definition, as multiple authors present distinct definitions [2]. This thesis adopts the Supply Chain definition presented in [3], where "a supply chain is defined as a set of three or more entities (organizations or individuals) directly involved in the upstream and downstream flows of products, services, finances, and/or information from a source to a customer".

Perishable Food Supply Chains (PFSC) differ from other regular Supply Chains, mainly due to the perishable nature of the products and their decreasing quality over time, while being moved from production to the customer, which leads to additional challenges than the ones observed for other regular Supply Chains [4].

Supply Chain Management is the business area responsible for dealing with the coordination performance between different agents. It focuses on coordinating material, information and financial flows, in order to fulfill customer demand requirements, while involving all Supply Chain stakeholders in the decision-making process. The major goal of Supply Chain Management is to improve the overall performance of the Supply Chain [5].

Modeling and optimization of Supply Chains have been studied for years. Operations Research techniques like Linear Programming, Mixed Integer Programming, Dynamic Programming and Evolutionary algorithms have been used to optimize Supply Chains over the years [6]. Although these techniques present some advantages, namely, reaching optimal or acceptable solutions, being methodical, clear and accessible, they also present some limitations, mainly related to avoid considering the problem's dynamics. Supply Chain models involve high dimension models, non-stationary and nonlinear operations with complex dynamics that are not well captured by Operations Research techniques. The problems are either simplified or heuristics are applied. Also, models of planning and execution control are not explicitly interconnected in terms of uncertainty [6].

Operations Research techniques have been applied to regular Supply Chains for many years. However, Perishable Food Supply Chains have not received much attention until recently. Lemma et al. [7] presents a literature review of the Operations Research methods applied to perishable Supply Chain Management modeling and optimization, focusing on loss minimization along the Supply Chain. The authors highlight that, for these type of problems, the majority of papers analysed - 55% - were published during the two years prior to the publication of their work. This evidence means that the relevance of this topic is increasing significantly. Besides, the authors conclude that, despite the fact that researchers acknowledge the occurrence of high food losses in the Supply Chain, the main objective of Supply Chain Management research works continues to be maximizing the revenue, instead of reducing the food loss. Designing the model with the main objective of reducing food loss is different from designing the model with the main objective of maximizing revenue, because the relationship between these goals is not obvious and reducing food loss may imply costs. However, it means greater sustainability and more organizations and entities are moving towards sustainability and responsible business models. Thus, aiming to reduce food loss may lead to Supply Chain dynamics and behaviors that differ to the dynamics of a Supply Chain that aims to maximize revenue.

In more recent years, Control Theory has gained some relevance dealing with the optimization of Supply Chain operations. One of its main advantages is the capacity to accurately deal with Supply Chain's dynamics, contrary to Operations Research [6]. In addition, Optimal Control considers planning and scheduling as a continuous adaptive process, instead of discrete operations. Furthermore, it allows a goal-oriented design of Supply Chain structures, which are addressed as whole multi-agent integrated systems. However, Optimal Control techniques also have their limitations associated to modelling complexity, the discretization of continuous variables and mathematics limitations related, for instance with numerical instability. Ivanov et al. [6] propose a model that integrates Operations Research and Optimal Control techniques. In detail, the static part of the problem is handled by Operations Research and the dynamic part is handled by Optimal Control, meaning it became possible to solve an integrated problem that considered production planning, transportation planning and execution control at once, instead of solving partial problems in isolated approaches. This upcoming research trend of combining Operations Research techniques and Optimal Control to solve Supply Chain Management problems considering real world settings seems a promising research path [5].

Model Predictive Control is an optimization-based control technique that has been used in process industries, such as chemical plants and oil refineries since the 1980s. Nowadays, it is used in many fields, like refining, solar plants and aerospace due to its ability to handle complex dynamical problems subject to operational constraints [8–11]. And, in more recent years, some authors explored the application of Model Predictive Control strategies to Perishable Food Supply Chains [12, 13]. To predict the system's behaviour and eventually obtain the optimal control actions, a mathematical model of the system being studied is designed, an objective function that represents the desirable system's behaviour is selected and constraints are explicitly defined.

Figure 1.3 shows three Model Predictive Control strategies - Centralized, Distributed and Decentralized. In terms of visibility and information sharing, the classical Model Predictive Control strategy used is the Centralized strategy, where the system is modeled as a unit and there is one controller managing the entire system. However, the Centralized strategy is not always the most suitable choice. Even though the computational power has increased, the Centralized strategy takes too much time solving large and complex problems. The constraints on the information flows of multi-agent systems is another concern issue of the Centralized strategy. For instance, a system such as a Supply Chain is composed of multiple sub-systems controlled by different entities that may be unavailable or unable to share information. For

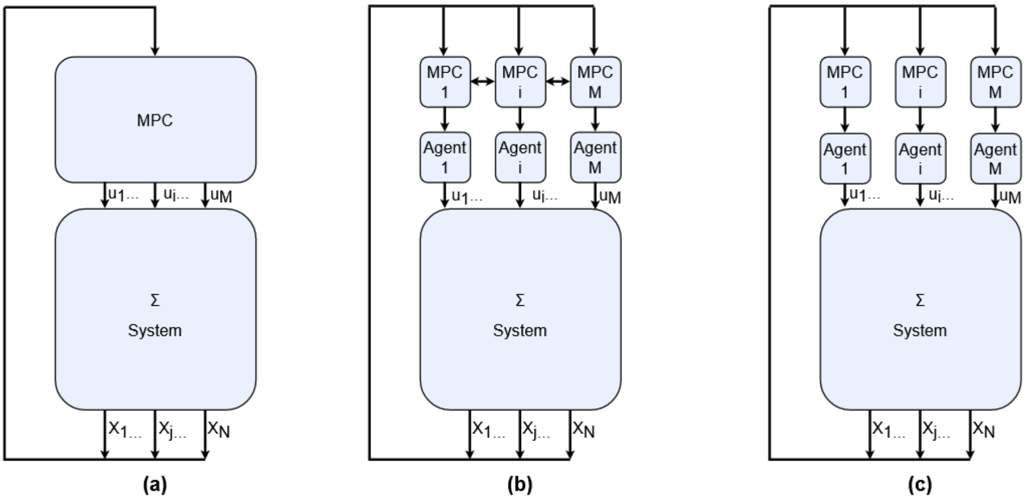


Figure 1.3: (a) Centralized, (b) Distributed and (c) Decentralized Model Predictive Control Strategies (adapted from [14]).

these situations, other strategies may be more suitable, namely, the Decentralized and the Distributed Model Predictive Control strategies [14].

Decentralized and Distributed strategies consider that each sub-system is controlled by a different controller. The main difference between these strategies is related to the communication level - local objectives, local constraints, local states - between sub-systems' controllers [15]. When using a Decentralized strategy, there is no direct communication between controllers. Thus, the influence of the different sub-systems is only considered when the controller needs to respond to the dynamics of the sub-system it is controlling. This may lead to conflicts between control actions of different sub-systems, which limits the performance of this strategy. In fact, due to these conflicts, when optimizing Supply Chains, some adjustments are required. In detail, Supply Chains can be considered chains of several independent nodes where only the nodes at the retail level have direct access to the customers' orders (Figure 1.4). This means the only way the next nodes have access to the customer demand is through the retail nodes. And so on, until reaching the most upstream node of the Supply Chain.

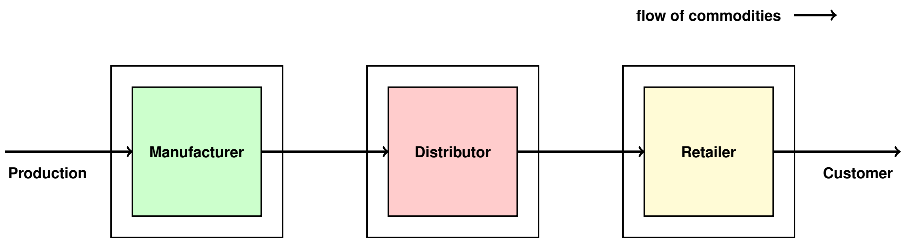


Figure 1.4: Supply Chain composed of three echelons: manufacturing, distribution and retailing (adapted from Braun et al. [16]).

Due to these specific problem constraints, some information needs to be shared between sub-systems, in the form of inputs and outputs, and the strategy may still be considered Decentralized, since each player still controls its own control actions - the inflows and outflows of products [17]. When using a Distributed strategy, the conflicts may be prevented since the controllers share information, leading to control actions that suit all sub-systems [14, 18]. Specifically for the case of Supply Chains, the level of information shared increases, when compared to the Decentralized strategy, and there is a negotiation and an agreement between the control actions of the entities. In the literature it is possible to find several research works that apply Model Predictive Control strategies to Supply Chains. Pinho et al. [19] review the state-of-the-art of Model Predictive Control applied in Supply Chains.

The effect of information sharing and visibility in Supply Chains has been discussed in many research works over the years [20, 21]. According to Wang and Wei [22], visibility in the Supply Chain is ”the degree to which the supply chain partners have on-hand information related to demand and supply for planning and control management”. It has been shown that an increase in visibility brings significant advantages, being regarded as one of the most effective ways of improving the Supply Chain performance. However, visibility does not mean that all information is shared with all entities at any time. It means that the shared information is meaningful and shared at a time when it can be useful and entities can respond to it. This is crucial because exchanging information, at the right time, leads to a significant reduction of uncertainties of the orders from downstream agents. As a consequence, the inventory levels and the bullwhip effect are reduced. In detail, the uncertainty causes distortion in the orders of downstream agents, misguiding the upstream agents in their inventory and production decisions. As one moves upstream, the quantity of products ordered tends to increase [22, 23], which leads to an overproduction of products. When dealing with Supply Chains of perishable commodities, the overproduction leads to losses, meaning information sharing and visibility gain an additional importance in these Supply Chain problems.

The problem addressed in this thesis consists in developing management strategies associated to agents' coordination that improve the Supply Chain performance prioritizing sustainable goals, while guaranteeing high levels of customer satisfaction. Specifically, the challenge consists in defining the control structure to apply to Supply Chain agents in order to minimize the quantity of overdue perishable goods while satisfying all the customer demand at the retailers. Hence, the objective is to minimize food loss by increasing Supply Chain visibility.

1.3 Contribution

One of the ultimate goals of this line of research is to build a generic multi-scenario simulator for Supply Chains of perishable goods, that would design the Supply Chain according to the parameters chosen by managers and stakeholders, such as storage and transport capacities and the number of entities involved. The Supply Chain performance would be evaluated in terms of quantity of overdue goods, storage usage, production, overproduction and quantity of commodity movements. Supply Chain managers and stakeholders would be able to test distinct tactical plans as well as monitor in real-time the storage levels and commodities flows of the multiple players involved.

So, the contributions of this work are:

- develop two Model Predictive Control strategies - Decentralized and Distributed - in order to be applied to Perishable Food Supply Chains management and be integrated in the simulator alongside the Centralized strategy present in [5], which was the starting point;

- validate these strategies and show their relevance, considering two case studies present in the literature;
- consider sustainability as the main driver to the Supply Chain Management. In detail, the cost function focuses on minimizing overdue products, instead of maximizing revenue;
- develop analytical models that promote the increase of Supply Chain visibility. Namely, through the introduction of new parameters, such as product lifetime across the Supply Chain and the transportation time between Supply Chain players.

1.4 Thesis Outline

This thesis is divided into four chapters. The first chapter presents the introduction of this work. Chapter 2 presents the design principles and the formulation of the Perishable Food Supply Chain model. Besides, it explains in detail the Model Predictive Control strategies developed to coordinate and optimize the Supply Chain. Then, Chapter 3 presents the results of the numerical experiments drawn to evaluate the performance of the developed Model Predictive Control strategies - Decentralized and Distributed. Lastly, in Chapter 4, conclusions of this thesis are presented and possible future work developments are described. Additionally, it is possible to consult two appendices - Appendix A and Appendix B - that present a further explanation of the developed strategies applied to the Supply Chain configurations presented.

Chapter 2

Model and Control Strategies for Supply Chains of perishable goods

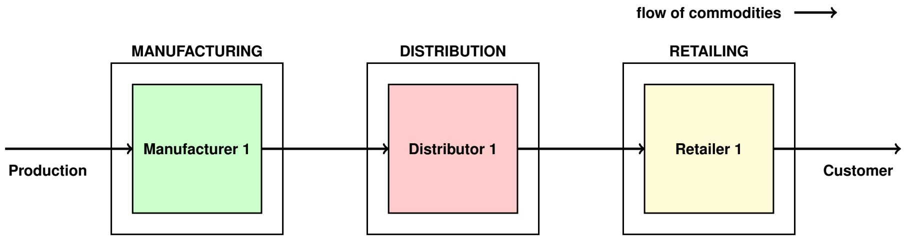
The methods chosen to solve the problem described in chapter 1, minimize food loss in Perishable Food Supply Chains, are explained next. The Supply Chain design is modular and defined by specific parameters. Additionally, the dynamics of the Supply Chain is modeled using a state-space representation. Then, a Decentralized Model Predictive Control strategy and a Distributed Model Predictive Control strategy manage the dynamics of the Supply Chain in order to optimize its performance according to sustainable goals.

2.1 Supply Chain Model Design

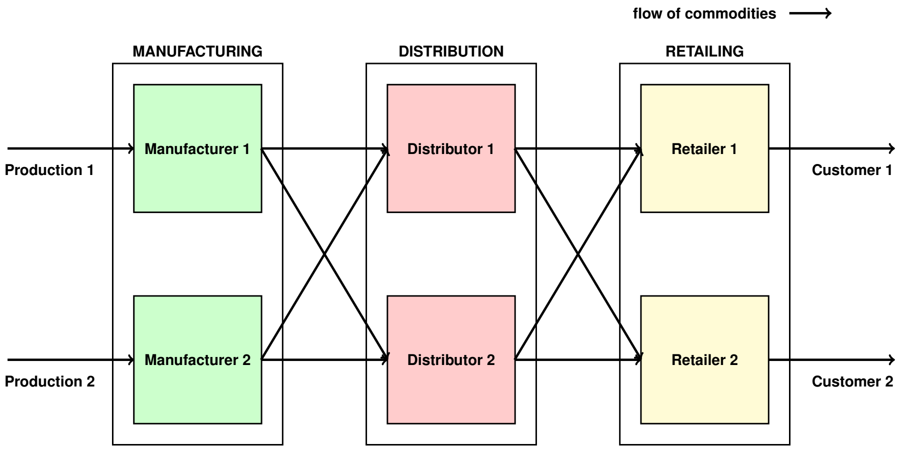
2.1.1 Network Configuration

Each agent involved in the Supply Chain operation can be interpreted as a Supply Chain player which performs specific tasks necessary to deliver the required quantity of products or services with the required quality to customers, at the required time and location. Supply Chain players performing similar tasks are grouped in echelons, such as procurement, manufacturing, distribution, marketing or retailing. Hence, the Supply Chain network configuration consists of $\mathrm{n_E}$ echelons, where each echelon is composed of $\mathrm{n_{pl_e}}$ players, $e = 1, ..., \mathrm{n_E}$, resulting in a total number of $\mathrm{n_{PL}} = \sum_{e=1}^{\mathrm{n_E}} \mathrm{n_{pl_e}}$ Supply Chain players. Furthermore, it is assumed that Supply Chain player pl of echelon e provides commodities to all Supply Chain players of the nearest downstream echelon $e{+}1$. The transportation time, in days, of moving commodities to player *i* of echelon $e+1$ from player *j* of echelon *e* is described by variable $\mathrm{tt}_{e_{ij}}, e = 1, ...\mathrm{n_E} - 1, i = 1, ..., \mathrm{n_{pl_{e+1}}}, j = 1, ..., \mathrm{n_{pl_e}}$.

A schematic representation of the network configuration and the player interaction of a Supply Chain composed of three echelons - manufacturing, distribution and retailing - is presented in Figure 2.1. All three echelons have the same number of Supply Chain players: one, in Figure 2.1(a), and two, in Figure 2.1(b).



(a) One Supply Chain player per echelon



(b) Two Supply Chain players per echelon.

Figure 2.1: Representation of the network configuration and the player interaction of a Supply Chain composed of three echelons: manufacturing, distribution and retailing (adapted from Braun et al. [16]).

The design of the Supply Chain model labels echelons and players in the network using the following policies:

- considering echelons and players simultaneously - i) echelons are numbered from upstream to downstream; and ii) players belonging to an echelon are numbered from top to bottom;

- considering players solely - players are numbered cumulatively from top to bottom of each echelon, following echelons from upstream to downstream.

Exemplifying, in Figure 2.1(b), ”Distributor 1” belonging to the ”Distribution” echelon corresponds to Supply Chain player 1 of echelon 2, when considering players and echelons simultaneously, and corresponds to Supply Chain player 3, when taking into account players solely.

2.1.2 Perishability

It is assumed that the Supply Chain is capable of handling $\mathrm{n_P}$ perishable commodities simultaneously. Perishability is addressed assuming that perishable commodities have a lifetime of L_p days, $p = 1, ..., \mathrm{n_P}$, from the moment they are produced until expiring, after which they lose their commercial value (Nahmias [24]). This way, it is relevant to track the time until expiration of perishable commodities. Thus, the time until expiration is an intrinsic characteristic of each perishable commodity *p*, denoted by a_p, $p = 1, ..., \mathrm{n_P}$, measured in days and described by:

$$a_p = \mathrm{L}_p - m, \tag{2.1}$$

where *m* is the number of days since its production. Perishable commodity *p* expires when $\mathrm{a}_p = 0$.

2.1.3 Inventory Management

The inventory of Supply Chain player pl from echelon *e* is divided into N_{pl_e} sub-inventories, $pl = 1, ..., \mathrm{n_{pl}}_e$, $e = 1, ..., \mathrm{n_E}$, where:

$$\begin{cases} \bullet\ \mathrm{N}_{pl_e} = \sum_{p=1}^{\mathrm{n_P}} \mathrm{L}_p, \quad e = 1, \\ \bullet\ \mathrm{N}_{pl_e} = \sum_{p=1}^{\mathrm{n_P}} \mathrm{L}_p - \min_{\forall i}(\mathrm{tt}_{1_{pli}}), \quad e = 2, \\ \bullet\ \mathrm{N}_{pl_e} = \sum_{p=1}^{\mathrm{n_P}} \mathrm{L}_p - \min_{\forall i,j}(\mathrm{tt}_{1_{ij}} + \mathrm{tt}_{2_{pli}}), \quad e = 3, \\ \bullet\ \mathrm{N}_{pl_e} = \sum_{p=1}^{\mathrm{n_P}} \mathrm{L}_p - \min_{\forall i,j,l}(\mathrm{tt}_{1_{jl}} + \mathrm{tt}_{2_{ij}} + \mathrm{tt}_{3_{pli}}), \quad e = 4, \\ \quad \vdots \\ \bullet\ \mathrm{N}_{pl_e} = \sum_{p=1}^{\mathrm{n_P}} \mathrm{L}_p - \min_{\forall i,j,l,m,n}(\mathrm{tt}_{1_{lm}} + \mathrm{tt}_{2_{nl}} + \quad \dots \quad + \mathrm{tt}_{k-2_{ij}} + \mathrm{tt}_{k-1_{pli}}), \quad e = k. \end{cases}$$

Each sub-inventory corresponds to the inventory of each perishable commodity *p* with a_p days until expiration, stored by Supply Chain player pl of echelon *e*. Considering Supply Chain players independently, it is possible to define $\mathrm{N}_{\mathrm{inv}_i}, i = 1, ..., \mathrm{n_{PL}}$ as the number of sub-inventories of Supply Chain player i. Considering Supply Chain configuration represented in Figure 2.1(a), $\mathrm{n_E} = 3$ because the Supply Chain is composed of three echelons and $\mathrm{n_{pl_1}} = \mathrm{n_{pl_2}} = \mathrm{n_{pl_3}} = 1$, since each echelon is composed of one player. Assuming that the transportation time from one echelon e to the next echelon $e+1$ is equal to 1 and that the Supply Chain handles only one product with a lifetime of 4 days, then $\mathrm{tt}_{1_{11}} = \mathrm{tt}_{2_{11}} = 1$ and $\mathrm{L}_1 = 4$. This way, the number of sub-inventories per player of echelon e is:

$$\begin{cases} \bullet\ \mathrm{N}_{1_1} = 4, \quad e = 1, \\ \bullet\ \mathrm{N}_{1_2} = 4 - \min(\mathrm{tt}_{1_{11}}) = 4 - 1 = 3, \quad e = 2, \\ \bullet\ \mathrm{N}_{1_3} = 4 - \min(\mathrm{tt}_{1_{11}} + \mathrm{tt}_{2_{11}}) = 4 - (1+1) = 2 \quad e = 3. \end{cases}$$

2.1.4 Model Formulation

The Supply Chain model proposed describes the movement of commodities along the multiple sub-inventories of Supply Chain players of the distinct echelons from the moment commodities are produced until being delivered to the customer. Furthermore, the model proposed is systematic and dimensionally flexible, being applicable to multiple network configurations handling multiple perishable commodities with distinct lifetimes. Hence, the model of the Supply Chain is fully defined by parameters $\mathrm{n_E}$, $\mathrm{n_{pl_e}}$,

$\mathrm{tt}_{e_{ij}}$, $\mathrm{n_P}$ and L_p.

Figure 2.2 illustrates schematically the dynamics of the model proposed, considering a Supply Chain characterized by the same parameters presented earlier in subsection 2.1.3: $\mathrm{n_E} = 3$, $\mathrm{n_{pl_1}} = \mathrm{n_{pl_2}} = \mathrm{n_{pl_3}} = 1$, $\mathrm{tt}_{1_{11}} = \mathrm{tt}_{2_{11}} = 1$, $\mathrm{n_P} = 1$ and $\mathrm{L_1} = 4$ (see Figure 2.1(a)).

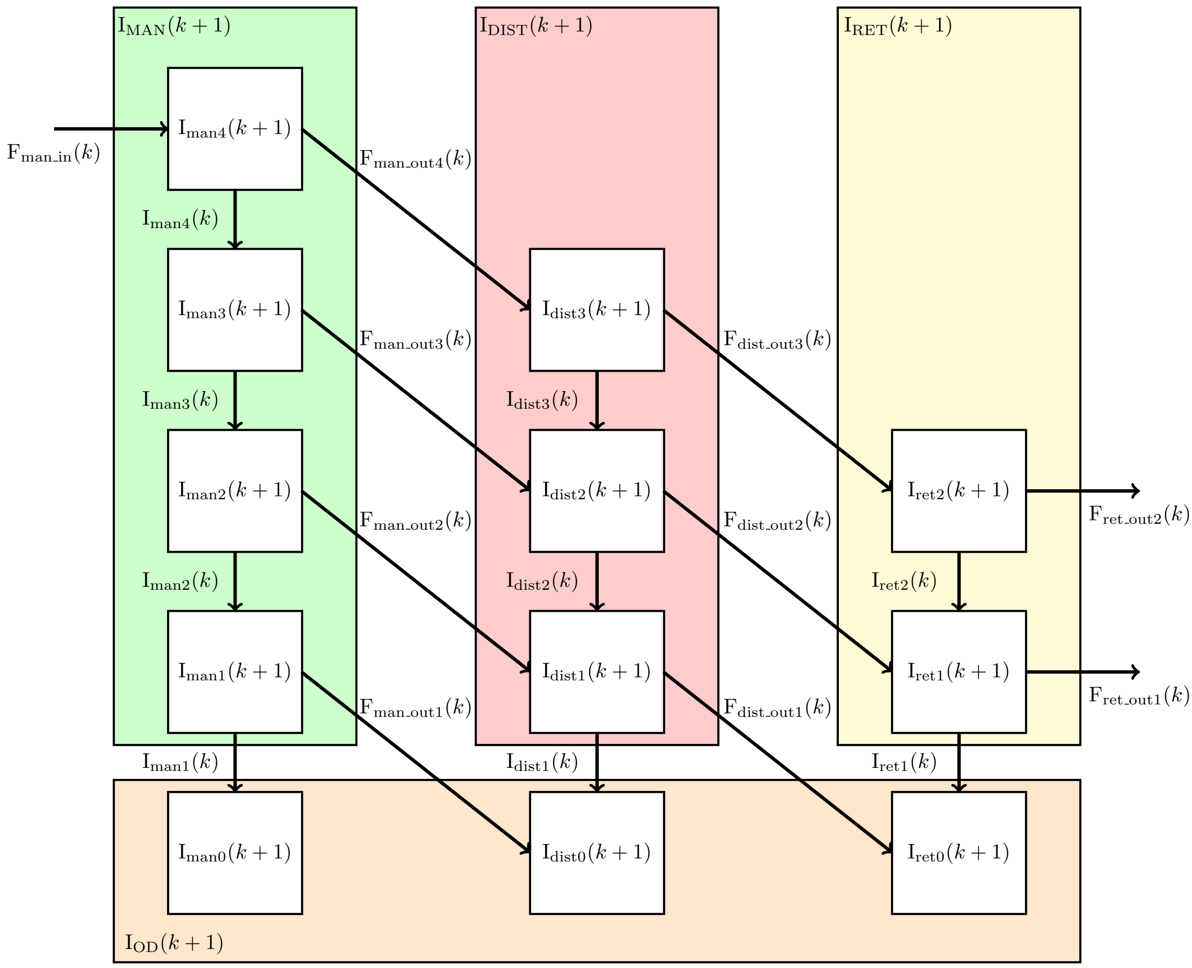


Figure 2.2: Dynamics scheme of a Supply Chain composed of three echelons with one player per echelon, handling a perishable commodity with a lifetime of four days.

The dynamics of the Supply Chain model illustrated in Figure 2.2 can be described as follows:

- **Manufacturer dynamics**

$$\mathrm{I_{man4}}(k+1) = \mathrm{F_{man\_in}}(k) - \mathrm{F_{man\_out4}}(k) \tag{2.2}$$

$$\mathrm{I_{man3}}(k+1) = \mathrm{I_{man4}}(k) - \mathrm{F_{man\_out3}}(k) \tag{2.3}$$

$$\mathrm{I_{man2}}(k+1) = \mathrm{I_{man3}}(k) - \mathrm{F_{man\_out2}}(k) \tag{2.4}$$

$$\mathrm{I_{man1}}(k+1) = \mathrm{I_{man2}}(k) - \mathrm{F_{man\_out1}}(k) \tag{2.5}$$

$$\mathrm{I_{man0}}(k+1) = \mathrm{I_{man1}}(k) + \mathrm{I_{man0}}(k) \tag{2.6}$$

$$\mathrm{I_{MAN}}(k+1) = \mathrm{I_{man4}}(k+1) + \mathrm{I_{man3}}(k+1) + \mathrm{I_{man2}}(k+1) + \mathrm{I_{man1}}(k+1) \tag{2.7}$$

$$\mathrm{I_{MAN}}(k+1) = \sum_{i=2}^{\mathrm{L_1}} \mathrm{I}_{\mathrm{man}i}(k) + \mathrm{F_{man\_in}}(k) - \sum_{i=1}^{\mathrm{L_1}} \mathrm{F}_{\mathrm{man\_out}i}(k) \tag{2.8}$$

$$\mathrm{I_{MAN\_OD}}(k+1) = \mathrm{I_{man0}}(k+1) = \mathrm{I_{man1}}(k) + \mathrm{I_{man0}}(k), \tag{2.9}$$

- **Distributor dynamics**

$$\mathrm{I_{dist3}}(k+1) = \mathrm{F_{man\_out4}}(k) - \mathrm{F_{dist\_out3}}(k) \tag{2.10}$$

$$\mathrm{I_{dist2}}(k+1) = \mathrm{I_{dist3}}(k) + \mathrm{F_{man\_out3}}(k) - \mathrm{F_{dist\_out2}}(k) \tag{2.11}$$

$$\mathrm{I_{dist1}}(k+1) = \mathrm{I_{dist2}}(k) + \mathrm{F_{man\_out2}}(k) - \mathrm{F_{dist\_out1}}(k) \tag{2.12}$$

$$\mathrm{I_{dist0}}(k+1) = \mathrm{I_{dist1}}(k) + \mathrm{I_{dist0}}(k) + \mathrm{F_{man\_out1}}(k) \tag{2.13}$$

$$\mathrm{I_{DIST}}(k+1) = \mathrm{I_{dist3}}(k+1) + \mathrm{I_{dist2}}(k+1) + \mathrm{I_{dist1}}(k+1) \tag{2.14}$$

$$\mathrm{I_{DIST}}(k+1) = \sum_{i=2}^{\mathrm{L_1}-1} \mathrm{I}_{\mathrm{dist}i}(k) + \sum_{i=1}^{\mathrm{L_1}} \mathrm{F}_{\mathrm{man\_out}i}(k) - \sum_{i=1}^{\mathrm{L_1}-1} \mathrm{F}_{\mathrm{dist\_out}i}(k) \tag{2.15}$$

$$\mathrm{I_{DIST\_OD}}(k+1) = \mathrm{I_{dist0}}(k+1) = \mathrm{I_{dist1}}(k) + \mathrm{I_{dist0}}(k) + \mathrm{F_{man\_out1}}(k), \tag{2.16}$$

- **Retailer dynamics**

$$\mathrm{I_{ret2}}(k+1) = \mathrm{F_{dist\_out3}}(k) - \mathrm{F_{ret\_out2}}(k) \tag{2.17}$$

$$\mathrm{I_{ret1}}(k+1) = \mathrm{I_{ret2}}(k) + \mathrm{F_{dist\_out2}}(k) - \mathrm{F_{ret\_out1}}(k) \tag{2.18}$$

$$\mathrm{I_{ret0}}(k+1) = \mathrm{I_{ret1}}(k) + \mathrm{I_{ret0}}(k) + \mathrm{F_{dist\_out1}}(k) \tag{2.19}$$

$$\mathrm{I_{RET}}(k+1) = \mathrm{I_{ret2}}(k+1) + \mathrm{I_{ret1}}(k+1) \tag{2.20}$$

$$\mathrm{I_{RET}}(k+1) = \sum_{i=2}^{\mathrm{L_1}-2} \mathrm{I}_{\mathrm{ret}i}(k) + \sum_{i=1}^{\mathrm{L_1}-1} \mathrm{F}_{\mathrm{dist\_out}i}(k) - \sum_{i=1}^{\mathrm{L_1}-2} \mathrm{F}_{\mathrm{ret\_out}i}(k) \tag{2.21}$$

$$\mathrm{I_{RET\_OD}}(k+1) = \mathrm{I_{ret0}}(k+1) = \mathrm{I_{ret1}}(k) + \mathrm{I_{ret0}}(k) + \mathrm{F_{dist\_out1}}(k), \tag{2.22}$$

where:

- $\mathrm{I_{mani}}$, $\mathrm{I_{disti}}$ and $\mathrm{I_{reti}}$ are the manufacturer, distributor and retailer inventories of perishable commodity with remaining time until expiration *i*, respectively. For the manufacturer, *i* $= 1, 2, 3, 4$, for the distributor, *i* $= 1, 2, 3$ and for the retailer, *i* $= 1, 2$, as shown previously in subsection 2.1.3.
- $\mathrm{I_{man0}}$, $\mathrm{I_{dist0}}$ and $\mathrm{I_{ret0}}$ are the manufacturer, distributor and retailer inventories of overdue perishable commodity ($\mathrm{a_1} = 0$).
- $\mathrm{I_{MAN}}$, $\mathrm{I_{DIST}}$ and $\mathrm{I_{RET}}$ are the manufacturer, distributor and retailer total inventories.
- $\mathrm{I_{MAN\_OD}}$,$\mathrm{I_{DIST\_OD}}$ and $\mathrm{I_{RET\_OD}}$ are the manufacturer, distributor and retailer total quantity of overdue perishable commodity ($\mathrm{a_1} = 0$).
- $\mathrm{F_{man\_in}}$ is the inflow of perishable commodity from production to the manufacturer ($\mathrm{a_1} = 4$)

- $\mathrm{F}_{\mathrm{man\_out}j}$, $\mathrm{F}_{\mathrm{dist\_out}j}$ and $\mathrm{F}_{\mathrm{ret\_out}j}$ are the outflows of perishable commodity from the manufacturer, distributor and retailer with remaining time *j*, in days, until expiration. Once more, for the manufacturer *j* $= 1, 2, 3, 4$, for the distributor *j* $= 1, 2, 3$ and for the retailer *j* $= 1, 2$

2.1.5 State-Space Representation

As previously described, each Supply Chain player is interpreted as an autonomous subsystem. The inputs of the model are the commodity quantity inflows and outflows from the sub-inventories of the Supply Chain players. In its turn, the total inventories per perishable commodity of Supply Chain players are the measured outputs. Therefore, the generic model of the Supply Chain can be synthesized as a mass balance dynamical system as follows:

$$\mathrm{x}_{ij}(k+1) = \mathrm{x}_{ij}(k) + \sum_{m} \mathrm{u}_{\mathrm{in}_{ijm}}(k) - \sum_{n} \mathrm{u}_{\mathrm{out}_{ijn}}(k) \tag{2.23}$$

$$\mathrm{y}_i(k) = \sum_{j} \mathrm{x}_{ij}(k) \tag{2.24}$$

$$\mathrm{y}_{\mathrm{OD}i}(k) = \mathrm{x}_{i0}(k), \tag{2.25}$$

$$i = 1, ..., \mathsf{n}_{\mathsf{PL}},\ \ j = 1, ..., \mathsf{N}_{\mathsf{inv}_i},\ \ m = 1, ..., \mathrm{n}_{\mathrm{pl}_{e-1}},\ \ n = 1, ..., \mathrm{n}_{\mathrm{pl}_{e+1}}. \tag{2.26}$$

where $\mathrm{x}_{ij}(k)$ represents the commodity quantity stored in sub-inventory *j* of Supply Chain player *i*, at time instant *k*. Furthermore, $\mathrm{u}_{\mathrm{in}_{ijm}}(k)$ stands for the commodity quantity moved to sub-inventory *j* of Supply Chain player *i* from Supply Chain player *m* of the previous echelon, at time instant *k* and $\mathrm{u}_{\mathrm{out}_{ijn}}(k)$ is the commodity quantity to move from sub-inventory *j* of Supply Chain player *i* to Supply Chain player *n* of the next echelon, at time instant *k*. Lastly, $\mathrm{y}_i(k)$ represents the total commodity quantity stored by Supply Chain player *i*, at time instant *k* and $\mathrm{y}_{\mathrm{OD}i}(k)$ represents the total quantity of overdue perishable commodity of Supply Chain player *i*.

From a control perspective, $\mathbf{x}(k) = \mathrm{x}_{ij}(k)$ is the state vector of the system, $\mathbf{u}(k) = \sum_m \mathrm{u}_{\mathrm{in}_{ijm}}(k) - \sum_n \mathrm{u}_{\mathrm{out}_{ijn}}(k)$ is the input vector of the system and $\mathbf{y}(k) = \mathrm{y}_i(k)$ is the output vector of the system. Thus, the Supply Chain model can be presented using the following generic state-space representation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}_{\mathrm{u}}\mathbf{u}(k) \tag{2.27}$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k), \tag{2.28}$$

where $\mathbf{A}$, $\mathbf{B}_{\mathrm{u}}$ and $\mathbf{C}$ are the state-space matrices associated to the state-space vectors that describe the dynamic model of the system.

Exemplifying, using the set of parameters presented before (Figure 2.1(a)), $\mathrm{n}_{\mathrm{E}} = 3$, $\mathrm{n}_{\mathrm{pl}_1} = \mathrm{n}_{\mathrm{pl}_2} = \mathrm{n}_{\mathrm{pl}_3} = 1$, $\mathrm{tt}_{1_{11}} = \mathrm{tt}_{2_{11}} = 1$, $\mathrm{n}_{\mathrm{P}} = 1$ and $\mathrm{L}_1 = 4$, the state-space representation of the Supply Chain model, divided by Supply Chain player, is the following:

- **Manufacturer**

$$\begin{bmatrix} x_{man1}(k+1) \\ x_{man2}(k+1) \\ x_{man3}(k+1) \\ x_{man4}(k+1) \\ x_{man0}(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{man1}(k) \\ x_{man2}(k) \\ x_{man3}(k) \\ x_{man4}(k) \\ x_{man0}(k) \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F_{man\_in}(k) \\ F_{man\_out1}(k) \\ F_{man\_out2}(k) \\ F_{man\_out3}(k) \\ F_{man\_out4}(k) \end{bmatrix} \quad (2.29)$$

$$y_{MAN}(k) = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{man1}(k) \\ x_{man2}(k) \\ x_{man3}(k) \\ x_{man4}(k) \\ x_{man0}(k) \end{bmatrix}, \quad (2.30)$$

- **Distributor**

$$\begin{bmatrix} x_{dist1}(k+1) \\ x_{dist2}(k+1) \\ x_{dist3}(k+1) \\ x_{dist0}(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{dist1}(k) \\ x_{dist2}(k) \\ x_{dist3}(k) \\ x_{dist0}(k) \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} F_{man\_out1}(k) \\ F_{man\_out2}(k) \\ F_{man\_out3}(k) \\ F_{man\_out4}(k) \\ F_{dist\_out1}(k) \\ F_{dist\_out2}(k) \\ F_{dist\_out3}(k) \end{bmatrix} \quad (2.31)$$

$$y_{DIST}(k) = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{dist1}(k) \\ x_{dist2}(k) \\ x_{dist3}(k) \\ x_{dist0}(k) \end{bmatrix}, \quad (2.32)$$

• **Retailer**

$$\begin{bmatrix} \mathrm{x}_{\mathrm{ret1}}(k+1) \\ \mathrm{x}_{\mathrm{ret2}}(k+1) \\ \mathrm{x}_{\mathrm{ret0}}(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{x}_{\mathrm{ret1}}(k) \\ \mathrm{x}_{\mathrm{ret2}}(k) \\ \mathrm{x}_{\mathrm{ret0}}(k) \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{F}_{\mathrm{dist\_out1}}(k) \\ \mathrm{F}_{\mathrm{dist\_out2}}(k) \\ \mathrm{F}_{\mathrm{dist\_out3}}(k) \\ \mathrm{F}_{\mathrm{ret\_out1}}(k) \\ \mathrm{F}_{\mathrm{ret\_out2}}(k) \end{bmatrix} \tag{2.33}$$

$$\mathrm{y}_{\mathrm{RET}}(k) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{x}_{\mathrm{ret1}}(k) \\ \mathrm{x}_{\mathrm{ret2}}(k) \\ \mathrm{x}_{\mathrm{ret0}}(k) \end{bmatrix}. \tag{2.34}$$

2.2 Model Predictive Control

Model Predictive Control predicts the optimal future behaviour (the future outputs $\hat{y}$) of a dynamical model based on current and past measurements of its operation. At each sampling instant, the Model Predictive Control algorithm receives as inputs the measurements of the current state of the model, the current model parameters and the intensity of disturbances, as well as predicted values of these variables over a prediction horizon, N_p. Next, making use of the known dynamical model and the collected inputs, the Model Predictive Control algorithm formulates and solves an optimization problem in order to optimize the performance of the dynamical model. In detail, the optimization problem consists in finding the optimal sequence of control actions, $\boldsymbol{u}$, (over the prediction horizon, N_p) that optimizes the objective function, while satisfying the dynamics and constraints of the model (see Figure 2.3). Then, the state of the system is updated by applying only the first predicted control action of the optimal sequence. At the next sampling instant, the measurements of the state of the model, the model parameters and the intensity of disturbances are collected and the Model Predictive Control algorithm is applied again following the same steps (receding horizon principle) [19, 25].

In the context of Supply Chains, the Model Predictive Control algorithm would be applied considering the output y as the commodity quantity stored by each Supply Chain player and the control action $\boldsymbol{u}$ as the commodity quantity to move between the sub-inventories of the Supply Chain players, that would be calculated over the prediction horizon N_p, which corresponds to the demand forecasting interval.

Model Predictive Control techniques are suitable to manage and optimize Supply Chains performance, often subjected to uncertain operational conditions and demand variability (Mestan et al. [15]). Specifically, this means the model parameters, such as storage and transport capacities, might vary over time and customer demand is volatile. Model Predictive Control is suitable to manage Supply Chain operation due to its ability to:

- formulate and solve an optimization problem which evaluates the overall Supply Chain performance;
- integrate constraints regarding Supply Chain operation, e.g., production quantity, storage and transport capacities, and desired inventory levels;
- be stable and robust even in the presence of disturbances;
- provide expanded Supply Chain visibility, by tracking the location and status of goods over the entire Supply Chain.

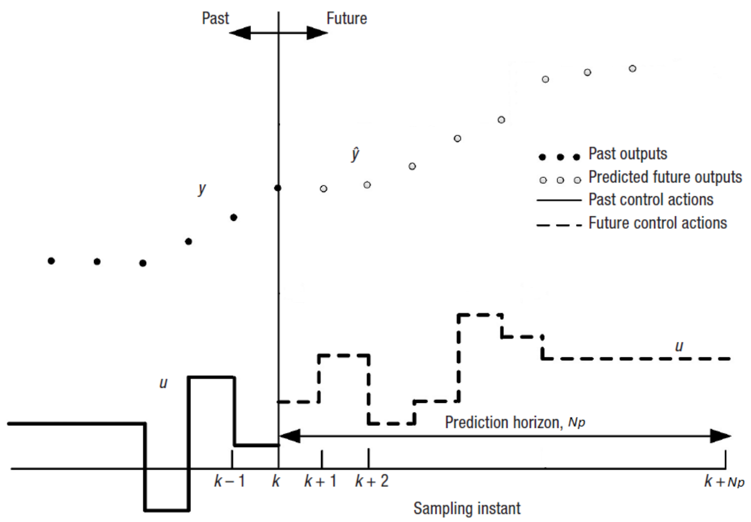


Figure 2.3: Basic concept of Model Predictive Control operation (adapted from [19]).

The success of a Model Predictive Control algorithm depends on the description accuracy of the model of the system. In Supply Chains, any desired objective function can be used. The Supply Chain operation dynamics are described by an explicit process model which can take, in principle, any required mathematical form. Process input and output constraints are included directly in the problem formulation so that future constraint violations are anticipated and prevented.

For this problem, the cost function used is linear, solved at each instant k using a mixed integer linear programming formulation, and is defined as (adapted from Hipólito et al. [5]):

$$J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) = \sum_{l=0}^{N_\mathrm{p}-1} \mathbf{q_{PL}}\left(k+l\right)\mathbf{x}\left(k+1+l\right) + \sum_{l=0}^{N_\mathrm{p}-1} \mathbf{q_{FL}}\left(k+l\right)\mathbf{u}\left(k+l\right), \tag{2.35}$$

where $\tilde{\mathbf{x}}_k$ and $\tilde{\mathbf{u}}_k$ are vectors composed of the state vectors and control actions, respectively, of each sampling instance k, over the prediction horizon N_p,

$$\tilde{\mathbf{x}}_k = \left[\begin{array}{ccc} \mathbf{x}^\mathrm{T}(k+1) & ,\dots, & \mathbf{x}^\mathrm{T}(k+N_\mathrm{p}) \end{array}\right]^\mathrm{T} \tag{2.36}$$

$$\tilde{\mathbf{u}}_k = \left[\begin{array}{ccc} \mathbf{u}^\mathrm{T}(k+1) & ,\dots, & \mathbf{u}^\mathrm{T}(k+N_\mathrm{p}-1) \end{array}\right]^\mathrm{T}. \tag{2.37}$$

The objective function has two sets of weights:

- $q_{PL_i}(k)$, $i = 1, ..., n_{PL}$, associated to the players - their sub-inventories and overdue goods - over the prediction horizon, N_p
- $q_{FL_j}(k)$, $j = 1, ..., n_{FL}$ - where n_{FL} is the number of links in the model - associated to the links, over the prediction horizon, N_p.

The weights of the cost function may vary according to the management policies chosen by Supply Chain managers as different Supply Chain goals require distinct operational behaviour. This cost function is used in a minimization problem. If a certain component has positive weights associated, that means its contribution to the overall value of the cost function will be positive. Since the objective is to minimize the cost function, then one of the objectives will be to have the lowest possible value for that component, that still respects the problem constraints. On the other hand, if a component has a negative weight associated, its contribution to the overall value of the cost function will be negative and one of the objectives will be to have the highest possible value for that component, that still respects the problem constraints.

In the next sections, three Model Predictive Control strategies - Centralized, Decentralized and Distributed - will be explained based on the configuration presented in Figure 2.1(a), composed by three echelons, each with one player.

The explanation for the configuration represented in Figure 2.1(b) is presented in Appendix B.

2.2.1 Centralized Model Predictive Control

The Centralized strategy applied to this Supply Chain model is adapted from Hipólito et al. [5] and it is the starting point to the development and study of Decentralized and Distributed control strategies applied to Supply Chains of perishable goods.

This strategy, represented in Figure 2.4, considers an additional player - the Global Control Center [26] - that receives all of Supply Chain players' information, namely, storage and transport capacities, sub-inventory levels, the demand for each time instant and their predictions in future time instants. Making use of that information, it runs a Centralized Model Predictive Control Algorithm that finds the control actions that optimize the entire Supply Chain as a whole, meaning the quantity of goods to move between players. Then, the control actions are presented to the players and are implemented.

This problem can be formulated as follows:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{2.38}$$

$$\text{s.t.} \quad \mathbf{x}(k+1+l) = \mathbf{A}\mathbf{x}(k+l) + \mathbf{B}_{\mathrm{u}}\mathbf{u}(k+l), \tag{2.39}$$

$$\mathbf{x}(k+1+l) \geq \mathbf{0}, \tag{2.40}$$

$$\mathbf{u}(k+l) \geq \mathbf{0}, \tag{2.41}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}(k+1+l) \leq \mathbf{X}_{\max}, \tag{2.42}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}(k+l) \leq \mathbf{U}_{\max}, \tag{2.43}$$

$$\mathbf{P}_{\mathrm{xu}}\mathbf{u}(k+l) \leq \mathbf{P}_{\mathrm{ux}}\mathbf{x}(k+l), \tag{2.44}$$

$$\mathbf{P}_{\mathrm{d}}\mathbf{u}(k+l) = \mathbf{D}(k+l), \tag{2.45}$$

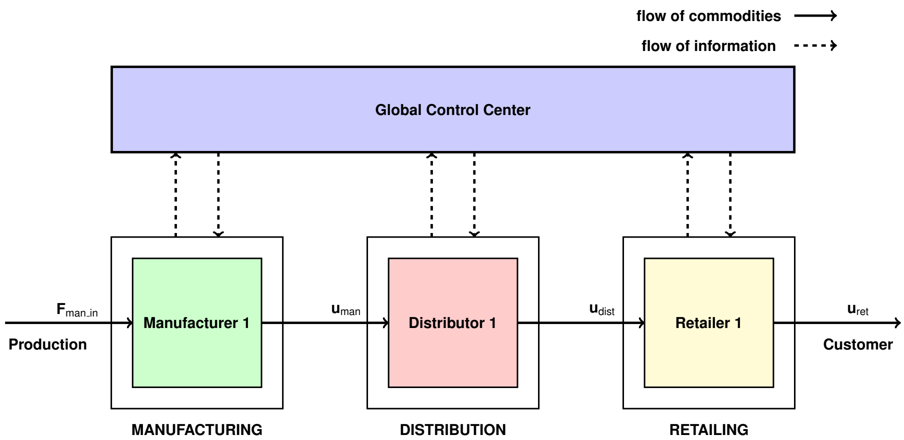


Figure 2.4: Centralized Control Strategy for a Supply Chain configuration with three echelons and one player per echelon, considering the additional Supply Chain player - the Global Control Center.

for $l = 0, \ldots, N_{\mathrm{p}} - 1$, where $\mathbf{X_{max}}$ is the maximum storage capacity per Supply Chain player, $\mathbf{U_{max}}$ corresponds to the maximum transport capacity per Supply Chain connection, $\mathbf{P_{xx}}$ is the projection matrix from the state-space set $\mathcal{X}$ into the maximum storage capacity set $\mathbf{X_{max}}$, $\mathbf{P_{uu}}$ is the projection matrix from the control action set $\mathcal{U}$ into the maximum transport capacity set $\mathbf{U_{max}}$, $\mathbf{P_{xu}}$ is the projection matrix from the control action set $\mathcal{U}$ into the state-space set $\mathcal{X}$, $\mathbf{P_{ux}}$ is the projection matrix from the state-space set $\mathcal{X}$ into the control action set $\mathcal{U}$, $\mathbf{P_d}$ is the projection matrix from the control action set $\mathcal{U}$ into the customer demand set $\mathcal{D}$ and $\mathbf{D}$ is the customer demand.

The first constraint (2.39) corresponds to the model dynamics. Besides that, some additional constraints are added to guarantee that feasible and meaningful results are obtained. Constraints (2.40) and (2.41) impose that there are no negative states nor negative flows of goods. Constraints (2.42) and (2.43) impose that the storage and transport capacities cannot be exceeded. Constraint (2.44) imposes that the quantity of goods that are taken from a player do not exceed the quantity of goods stored in that player. And constraint (2.45) guarantees that the customer demand is satisfied.

As said previously, the Centralized strategy was the starting point. This control strategy is ideal and utopic. In reality, there is no external player that has total access to the information from all players and total control over the entire Supply Chain. For that reason, a Decentralized strategy was developed.

2.2.2 Decentralized Model Predictive Control

In this strategy, each player has control over its own inflows of goods, which means that instead of having one controller for the entire Supply Chain, each player has its own controller and has access to limited information, specifically, the current quantity of goods stored in their sub-inventories, the storage capacity they have available and the transport capacity of the links directly connected to them. Furthermore, only retailers have access to the demand.

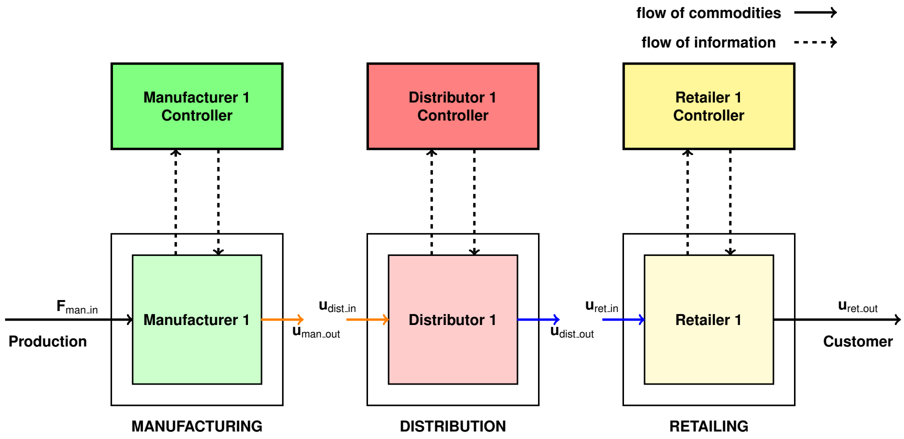


Figure 2.5: Decentralized Control Strategy for a Supply Chain configuration with three echelons and one player per echelon.

As it can be seen in Figure 2.5, the Supply Chain players are not totally independent from each other and their control actions are found sequentially and backward, starting on the retailing echelon, where the customer demand needs to be guaranteed, and ending on the manufacturing echelon, where the quantity of goods to produce on each day is decided.

When solving the retailer's problem, the outflow $\mathbf{u}_{\text{ret\_out}}$ is equal to the demand, so the only control action that needs to be determined is the inflow $\mathbf{u}_{\text{ret\_in}}$.

Then, the distributor's problem is solved. The outflow is already determined because it needs to be equal to the retailer's inflow, so the only control action to determine is the inflow $\mathbf{u}_{\text{dist\_in}}$.

Finally, the manufacturer's problem is solved, where the only variable yet to determine is the quantity of goods that will enter in the Supply Chain, $\mathbf{F}_{\text{man\_in}}$.

Then, after solving all these problems, for a specific time instant, the flows are implemented backwards, from the manufacturing echelon to the retailing echelon and the process repeats itself until arriving to the last time instant of the simulation.

The problems for each Supply Chain player are similar, having some differences related with the constraints. So, the formulation of the problem of the manufacturer is presented next and the formulation of the other players' problems is explained in Appendix A.

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{2.46}$$

$$\text{s.t.} \quad \mathbf{x}_{\text{man}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{man}}(k+l) + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{man}}(k+l), \tag{2.47}$$

$$\mathbf{x}_{\text{man}}(k+1+l) \geq \mathbf{0}, \tag{2.48}$$

$$\mathbf{u}_{\text{man}}(k+l) \geq \mathbf{0}, \tag{2.49}$$

$$\mathbf{P}_{\text{xx}}\mathbf{x}_{\text{man}}(k+1+l) \leq \mathbf{X}_{\text{max}_{\text{man}}}, \tag{2.50}$$

$$\mathbf{P}_{\text{uu}}\mathbf{u}_{\text{man}}(k+l) \leq \mathbf{U}_{\text{max}_{\text{man}}}, \tag{2.51}$$

$$\mathbf{P}_{\text{xu}}\mathbf{u}_{\text{man}}(k+l) \leq \mathbf{P}_{\text{ux}}\mathbf{x}_{\text{man}}(k+l), \tag{2.52}$$

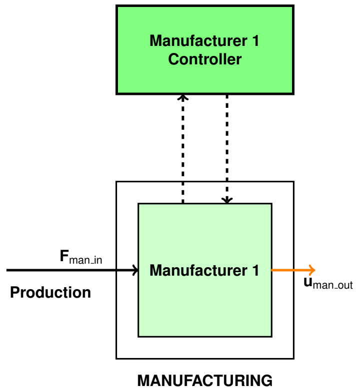


Figure 2.6: Decentralized Control Strategy for a Supply Chain configuration with three echelons and one player per echelon - Manufacturer Controller.

$$\mathbf{u}_{\text{man\_out}}(k+l) = \mathbf{u}_{\text{dist\_in}}(k+l), \tag{2.53}$$

for $l = 0, \ldots, N_{\text{p}} - 1$.

Similarly to the Centralized strategy, the first constraint, (2.47) refers to the model dynamics. Constraints (2.48) and (2.49) impose the states and the control actions to be non-negative. In this case, the state vector $\mathbf{x}_{\text{man}}(k+l)$ refers to the goods' sub-inventories levels of the manufacturer at each time instant and the control actions are the inflows entering from production - $\mathbf{F}_{\text{man\_in}}$ - and the outflows leaving to distributor - $\mathbf{u}_{\text{man\_out}}$. Constraints (2.50) and (2.51) impose that the storage and transport capacities cannot be exceeded, in this case only for the manufacturer player. Constraint (2.52) imposes that what leaves from the manufacturer to the distributor cannot exceed the quantity stored at the manufacturer. And constraint (2.53) is a compatibility constraint that imposes that what leaves from the manufacturer to the distributor is equal to what enters in the distributor from the manufacturer, which means there are no losses between players and that the only flow that needs to be determined in this problem is the inflow $\mathbf{F}_{\text{man\_in}}$.

It is expectable that this strategy generates worse results than the Centralized one, since the access to information decreases significantly. For that reason, a Distributed model predictive algorithm was developed, with the purpose to have a representation in which players have more access to information, which is closer to reality.

2.2.3 Distributed Model Predictive Control

This strategy differs from the Decentralized one mainly in terms of access to information. Now, each echelon is managed by one controller, meaning each Supply Chain player has access to the information of the other players of the same echelon and the best control actions for Supply Chain players of the same echelon are found simultaneously. Additionally, the communication between echelons increases.

The explanation of the Distributed strategy for the configuration of Figure 2.1(a) is presented next. The problems are still solved sequentially, starting on the retailing echelon and ending on the manufacturing echelon.

The retailer has access to the same amount of information as before - its storage and transport capacities, the sub-inventory levels and demand for each time instant and their predictions for future time instants. The distributor has now access to more information, knowing its own information plus the information related to the retailer. The manufacturer has access to its own information and also the information regarding both the distributor and the retailer.

The flows of information and goods for the manufacturer are represented in Figure 2.7.

Next, the formulation of the manufacturing echelon problem is presented. The formulation for the other echelons' problems is explained in Appendix A.

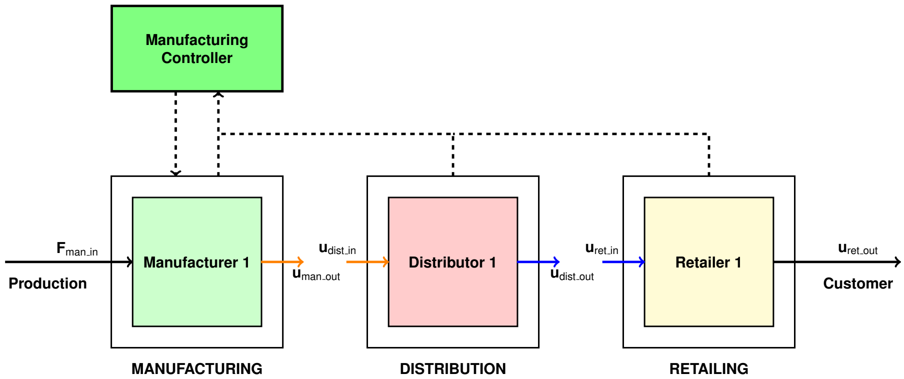


Figure 2.7: Distributed Control Strategy for a Supply Chain configuration with three echelons and one player per echelon - Manufacturer Controller.

The formulation for the manufacturers' echelon problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{2.54}$$

$$\text{s.t.} \quad \mathbf{x}_{\text{man}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{man}}(k+l) + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{man}}(k+l), \tag{2.55}$$

$$\mathbf{x}_{\text{man}}(k+1+l) \geq \mathbf{0}, \tag{2.56}$$

$$\mathbf{u}_{\text{man}}(k+l) \geq \mathbf{0}, \tag{2.57}$$

$$\mathbf{P}_{\text{xx}}\mathbf{x}_{\text{man}}(k+1+l) \leq \mathbf{X}_{\text{max}_{\text{man}}}, \tag{2.58}$$

$$\mathbf{P}_{\text{uu}}\mathbf{u}_{\text{man}}(k+l) \leq \mathbf{U}_{\text{max}_{\text{man}}}, \tag{2.59}$$

$$\mathbf{P}_{\text{xu}}\mathbf{u}_{\text{man}}(k+l) \leq \mathbf{P}_{\text{ux}}\mathbf{x}_{\text{man}}(k+l), \tag{2.60}$$

$$\mathbf{u}_{\text{man\_out}}(k+l) = \mathbf{u}_{\text{dist\_in}}(k+l), \tag{2.61}$$

$$\mathbf{u}_{\text{dist\_out}}(k+l) = \mathbf{u}_{\text{ret\_in}}(k+l), \tag{2.62}$$

$$\mathbf{P}_{\text{d}}\mathbf{u}_{\text{man}}(k+l) = \mathbf{D}(k+l), \tag{2.63}$$

for $l = 0, \ldots, N_{\text{p}} - 1$.

The first seven constraints, (2.55) - (2.61), have the same meaning as seen before for the Decentralized strategy. However, the state vector $\mathbf{x}_{\text{man}}(k+l)$ now includes the sub-inventory levels of the entire Supply Chain, meaning the quantity of goods stored by the manufacturer, the distributor and the retailer. Likewise, the control action vector $\mathbf{u}_{\text{man}}(k+l)$ now includes the control actions of the three Supply Chain players. Besides the constraint (2.61), it is now necessary to have an additional compatibility constraint, (2.62), which guarantees that what leaves from the distribution echelon is equal to what enters the retailing echelon, since the manufacturer has visibility of the entire Supply Chain. Constraint (2.63), that is not included in the manufacturer's problem in the Decentralized formulation because, in that strategy, its visibility is reduced, guarantees the customer demand is fulfilled.

Chapter 3

Numerical Experiments

As stated previously in section 1.3, one of the ultimate goals of this research work is to design a generic multi-scenario simulator for Supply Chains of perishable goods that designs the Supply Chain according to the parameters presented in section 2. Besides, the simulator accounts for three distinct Model Predictive Control strategies, regarding the information coordination - Centralized, Decentralized and Distributed - and multiple management policies, which are defined by the desired choice of cost function weights. The simulator evaluates the performance of the Supply Chain in terms of quantity of overdue goods, storage usage, production, overproduction and quantity of commodity movements.
In order to validate the Model Predictive Control strategies developed - Decentralized and Distributed - numerical simulations were performed, considering both network configurations presented in Figure 2.1(a) and Figure 2.1(b). This thesis focuses on these two configurations because many authors addressed them in literature for regular Supply Chains [16, 27].
The intention of this research work is to test the developed strategies using literature case studies, but it was difficult to find case studies considering the network parameters used in the presented models, namely, due time of good until expiration and the quantity of overdue goods. Hence, the framework and simulator developed in this work are used as benchmark to perform tests and simulations to plan Supply Chains of perishable goods. Additionally, the Supply Chain performance using the present models is compared with the literature in terms of the bullwhip effect in section 3.2.5.
In this chapter, the results of the numerical simulations considering configuration 1 (Figure 2.1(a)) and then configuration 2 (Figure 2.1(b)) are presented.

3.1 Configuration 1

3.1.1 Case Study

Simulation Specifications

All the simulations performed have a duration of 100 days and a sampling time of 1 day. The default prediction horizon, N_p, is set to 12 days [5].

Network Configuration

The Supply Chain is composed by three echelons - manufacturing, distribution and retailing - each one composed of one player. The goods enter in the Supply Chain from production to the manufacturer echelon. Then, they are moved across the Supply Chain, being handled by the distinct players, until reaching the customer or becoming overdue goods. It is considered that the transportation time of moving commodities from a player of a given echelon to a player of the next echelon is equal to 1 day.
The storage and transport capacities of the Supply Chain are assumed constant over the entire simulation and can be consulted in Table 3.1 and Table 3.2 for each player and each link. The presented values are generic and could be changed within certain operational limits, that have not been calculated and are outside the scope of this work, and taking into account the demand characteristics.

Table 3.1: Storage capacity of all Supply Chain players (units) for a Supply Chain composed by 3 echelons with 1 player per echelon.

Player	Storage Capacity
Manufacturer	40
Distributor	30
Retailer	25

Table 3.2: Transport capacity of all Supply Chain connections (units) for a Supply Chain composed by 3 echelons with 1 player per echelon.

Connection	Transport Capacity
Production - Manufacturer	40
Manufacturer - Distributor	30
Distributor - Retailer	30
Retailer - Customer	25

The manufacturer has the highest storage capacity and can store up to 40 products simultaneously, while the distributor and the retailer have the capacity to store 30 and 25 products at once, respectively. Regarding the transport, the manufacturer can receive 40 products simultaneously, from production. The transport of commodities from the manufacturer to the distributor and from the distributor to the retailer has a limit of 30 products. And the retailer can only move 25 commodities to the customer at once.

Demand

For this case study, it is considered that only one type of perishable commodity, with a lifetime of 5 days, is handled by this Supply Chain and its demand profile is represented in Figure 3.1.
The customer demand only starts 10 days after the beginning of the simulation. It has a maximum peak of 22 units on the 86^{th} day of the simulation, a minimum equal to 1 on several days and a mean of 4 units per day.
The demand profile is obtained stochastically, using a gamma distribution and considering the parameter values k = 2 and θ = 1 [5, 28].

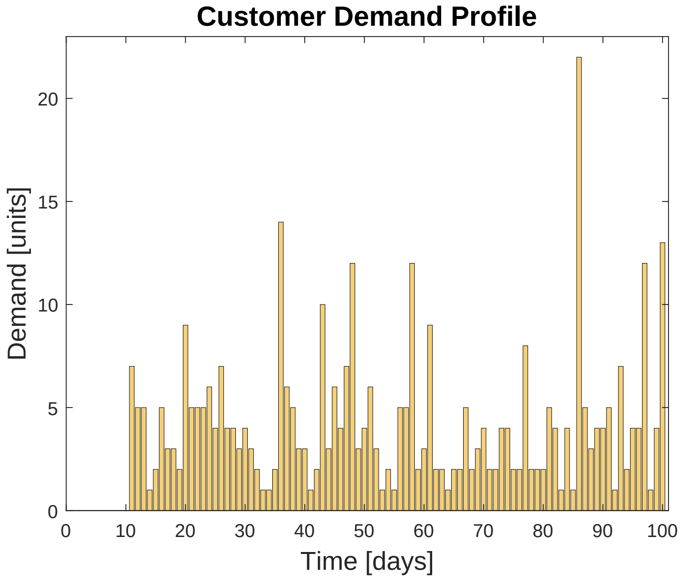


Figure 3.1: Demand Profile for a perishable commodity with a lifetime of 5 days.

Optimal Control techniques usually consider the worst case scenarios, close to the operational limits of the system. However, it would be necessary to test the system's limits, which is outside the scope of this work. Hence, it is considered that the presented profile is demanding enough in terms of variability and intensity to test the performance and validate the developed strategies. And, if the strategies are validated for demanding cases, then it is expected to work for less demanding cases.

Management Policy

The management policy used is described by the cost function's weights of the optimization problem. The cost function has three components, related to:

- inventory levels at each player
- minimization of overdue goods
- connections

The cost function weights for configuration 1 are presented in Table 3.3.
The weight value associated to the overdue goods is the highest positive value, meaning waste minimization is prioritized relatively to the other two components of the cost function.
The weights related to the inventory levels - Manufacturer, Distributor and Retailer - are positive to penalize the storage of perishable products.
The weights related to connections are positive or null. The weight of the connection from production to the manufacturing echelon is highly positive to penalize the overproduction of goods. The weights of connections between Supply Chain players are equal, meaning none of these connections is neither

Table 3.3: Cost function weights for a Supply Chain composed by 3 echelons with 1 player per echelon.

Cost Function Weights	
Overdue	100
Manufacturer	1
Distributor	1
Retailer	1
Production - Manufacturer	50
Manufacturer - Distributor	0
Distributor - Retailer	0
Retailer - Customer	0

penalized or prioritized over the others and null, meaning the transport of commodities between Supply Chain players has a neutral contribution to the cost function.

The weight values are assigned according to the managers' decisions.

To maintain the management policy considered, the values of the weights may vary. However, the relative values must stay the same. But the optimal allocation of weights to follow the desired management policies is outside the scope of this work.

Performance measures

The performance measures considered to evaluate numerically the performance of the Model Predictive Control strategies are:

- total production - the total quantity of goods that are produced, measured in units, to fulfill the customer demand at the retailers;
- total storage - the total quantity of goods stored, measured in units, considering all Supply Chain and the entire length of the simulation;
- overproduction - the percentage of goods produced that exceed the customer demand at the retailers;
- quantity of commodities movements - the total quantity of goods being moved through the links of the Supply Chain, measured in units, considering all links excluding the production links and considering the entire length of the simulation;
- total quantity of overdue products - the quantity of goods that expire and become overdue before being sold to the customers at the retailers, measured in units;
- computation time - the time that it takes to run the entire simulation, measured in seconds.

3.1.2 Results

In this section, the results of the numerical simulation for configuration 1 (Figure 2.1(a)) are presented, first considering the Decentralized strategy and then the Distributed strategy. Finally, the results of the numerical simulations are compared for these two strategies and the Centralized one.

Decentralized Strategy

Figure 3.2 presents the storage intensity evolution of the entire Supply Chain and the storage intensity discriminated by player.

The total storage intensity is zero for the first 8 time instants, since the customer demand only starts 10 days after the beginning of the simulation. The Supply Chain player that has non-zero storage intensity levels on the 9th day is the manufacturer, that receives the commodities from the production to be available at the retailer on the 11th day.

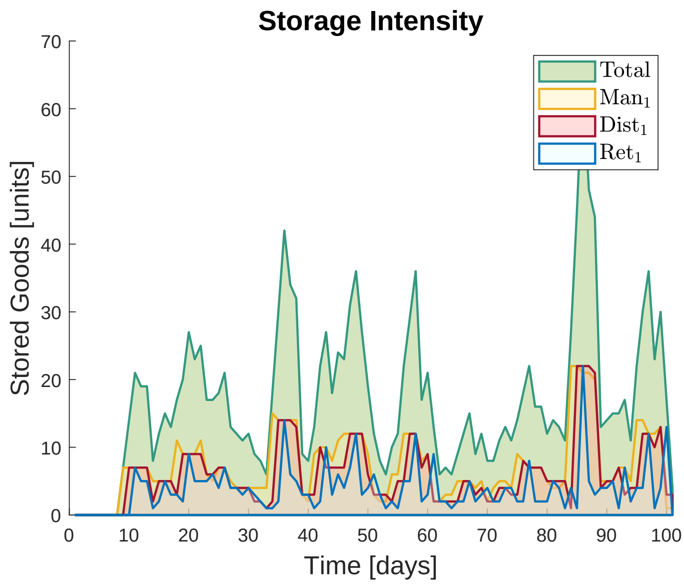


Figure 3.2: Decentralized Strategy for a configuration with 3 echelons and 1 player per echelon - Storage Intensity.

During the entire simulation, the total storage intensity shows peaks that are present mainly due to the storage intensity pattern of the retailer. This is a result of the short time storage of products at this Supply Chain stage. For the manufacturer and distributor, the storage intensity has more plateaus, meaning the products are stored for longer time periods at those stages. Additionally, there is almost never a time instant, for any player, where the storage intensity is zero.

Figure 3.3 shows the production and the customer demand evolution over time.

The area under the production curve is bigger than the area under the customer demand curve, meaning commodities are being produced in excess, which leads to an overproduction that contributes to the quantity of overdue products. Additionally, it is possible to observe a delay between the production and the customer demand peaks. This is an expected result, since the delivery of products from a given echelon to the next one takes one day, meaning a product takes at least two days to be moved from production to retailing, without being stored in the process.

Figure 3.4 presents the evolution over time of the total quantity of products that have deteriorated from

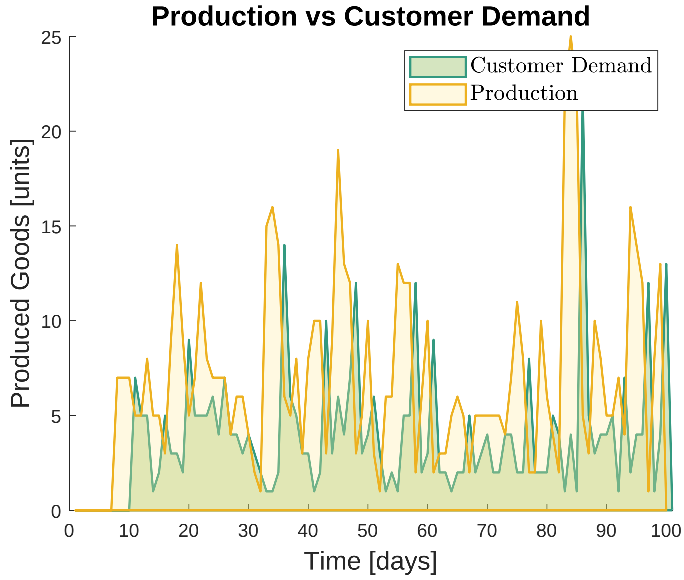


Figure 3.3: Decentralized Strategy for a configuration with 3 echelons and 1 player per echelon - Production VS Demand.

production to the retailing stage, before arriving to the customer. It also presents the quantity of products lost by each Supply Chain player.

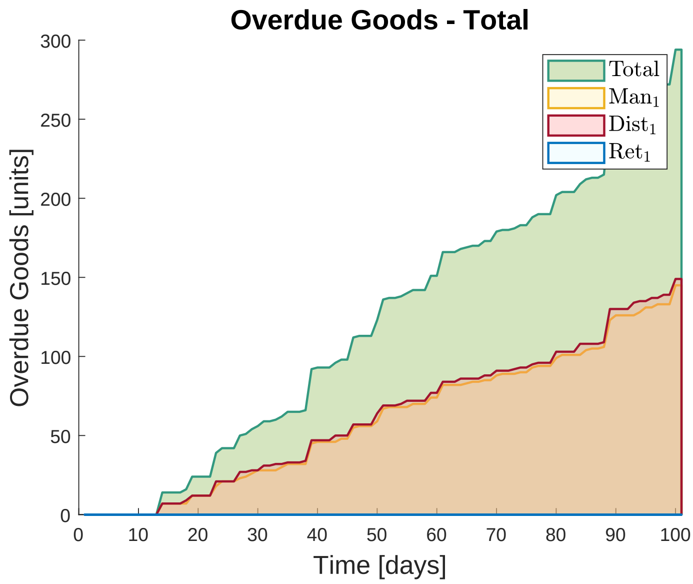


Figure 3.4: Decentralized Strategy for a configuration with 3 echelons and 1 player per echelon - Overdue Goods.

At the end of the simulation, the total quantity of overdue goods is close to 300 units and the major contributors are the manufacturer and the distributor, which is in accordance with the results obtained for the storage intensity.

Distributed Strategy

The same set of results shown for the Decentralized strategy is now presented and analyzed for the Distributed strategy.

Figure 3.5 presents the storage intensity for the entire Supply Chain and discriminated by player.

Once more, the retailing echelon is the one that presents more peaks and the distribution and manufacturing echelons present more plateaus, meaning products are stored longer at the first stages of the Supply Chain.

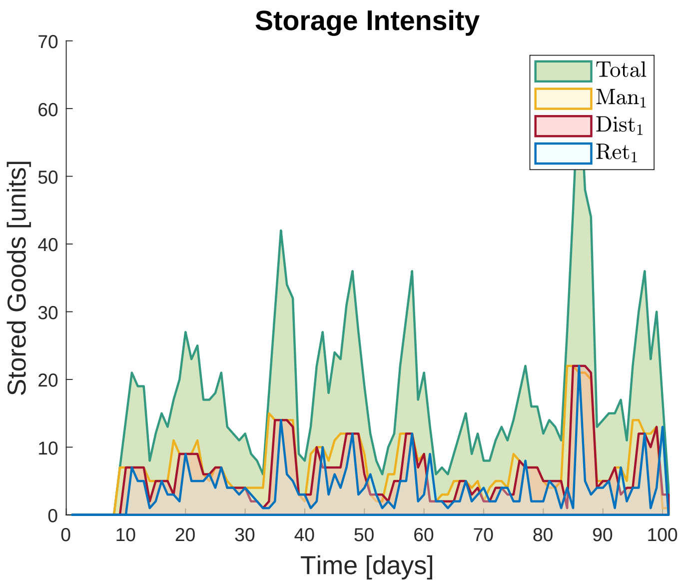


Figure 3.5: Distributed Strategy for a configuration with 3 echelons and 1 player per echelon - Storage Intensity.

Figure 3.6 presents the customer demand and the production intensity for the Distributed strategy.

As seen previously for the Decentralized strategy, the production exceeds the customer demand, contributing to the quantity of overdue products generated across the Supply Chain. And the same delay is present between the higher production peaks and the higher customer demand peaks.

Figure 3.7 presents the evolution of the quantity of overdue products over time for the entire Supply Chain and discriminated by players. It is possible to observe that the retailer does not contribute to the total quantity of overdue products across the Supply Chain and that the manufacturer and distributor contribute approximately with the same amount of overdue products. Once more, this result is consistent with the values presented in Figure 3.5 for the storage intensity discriminated by Supply Chain player.

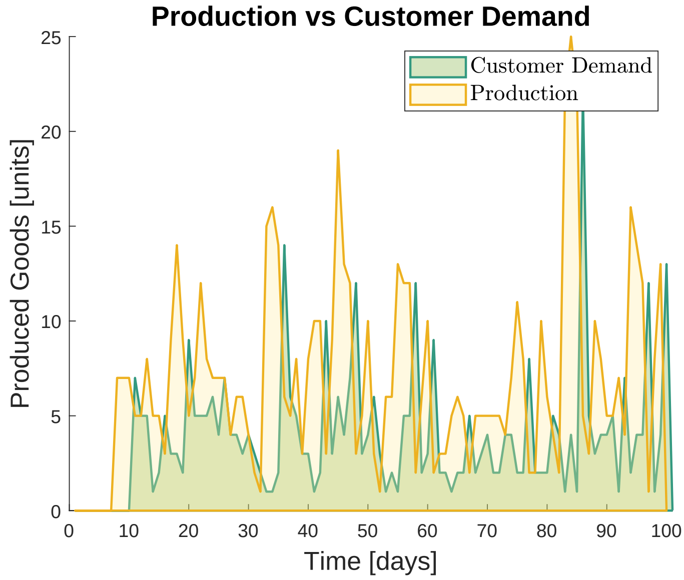


Figure 3.6: Distributed Strategy for a configuration with 3 echelons and 1 player per echelon - Production VS Demand.

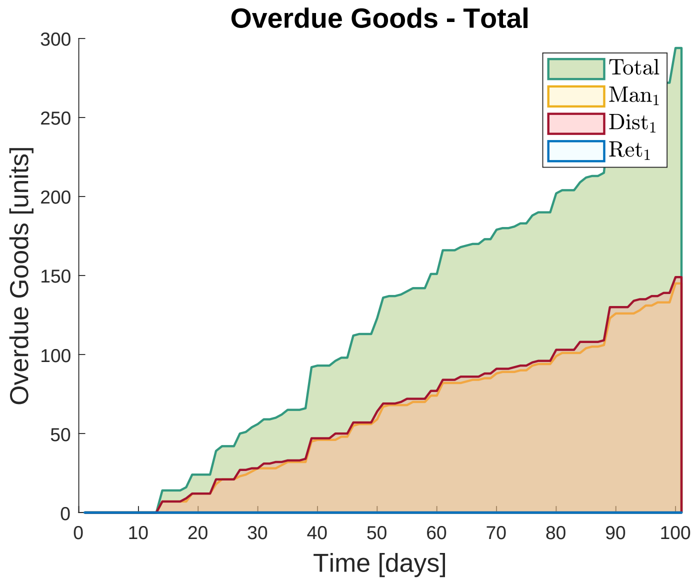


Figure 3.7: Distributed Strategy for a configuration with 3 echelons and 1 player per echelon - Overdue Goods.

Strategies Comparison

Figure 3.8 displays the storage intensity evolution over the entire simulation for the whole Supply Chain, considering the three Model Predictive Control strategies - Decentralized, Distributed and Centralized.

The Centralized strategy is the one that presents the lowest values of storage intensity levels over the entire simulation. The Decentralized and Distributed strategies present exactly the same storage intensity evolution over the entire simulation, which can be related with the low number of control actions

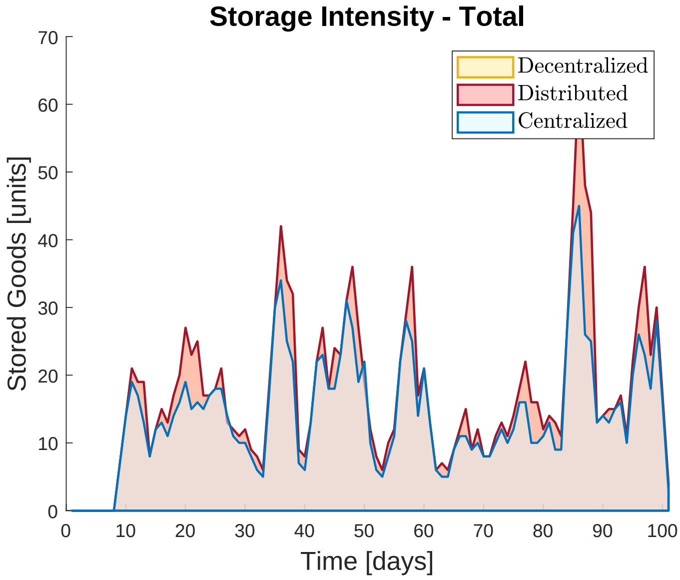


Figure 3.8: Model Predictive Control Strategies Comparison - Decentralized, Distributed and Centralized - for a configuration with 3 echelons and 1 player per echelon - Storage Intensity.

that need to be optimized. In this case, the visibility increase is not significant, leading to an equal performance of the Supply Chain when using either the Decentralized or the Distributed strategy.

Figure 3.9 presents the production of the three strategies over the entire simulation.

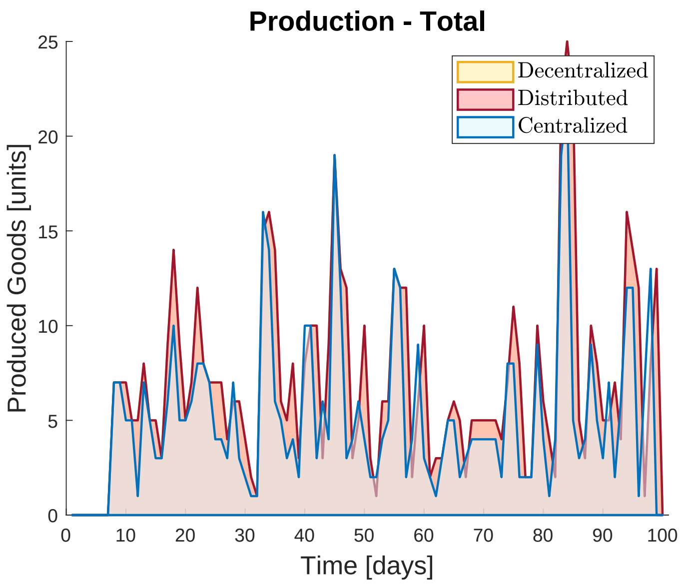


Figure 3.9: Model Predictive Control Strategies Comparison - Decentralized, Distributed and Centralized - for a configuration with 3 echelons and 1 player per echelon - Production.

The evolution of the production for the Decentralized and Distributed strategies is exactly the same and similar to the Centralized strategy. However, the Centralized strategy shows the lowest values of production for almost every time instant of the entire simulation.

Figure 3.10 shows the amount of overdue goods obtained for the Supply Chain, over the entire simulation for the three Model Predictive Control strategies.

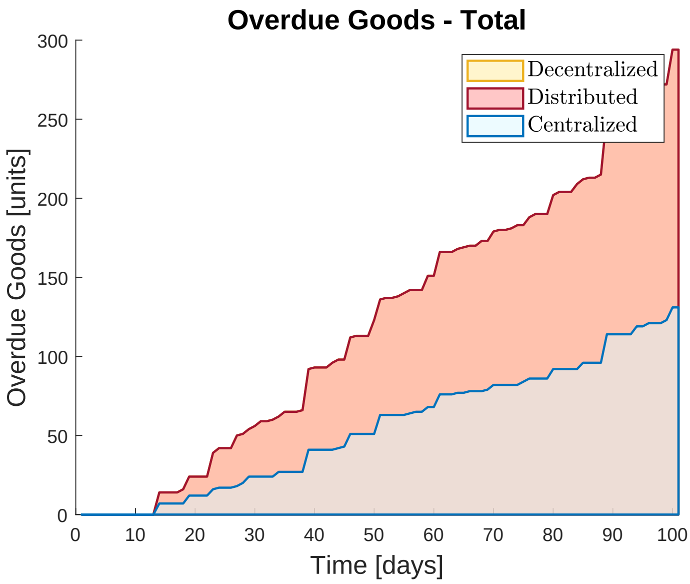


Figure 3.10: Model Predictive Control Strategies Comparison - Decentralized, Distributed and Centralized - for a configuration with 3 echelons and 1 player per echelon - Overdue Goods.

The Decentralized and the Distributed strategies present, once more, the same evolution for the quantity of overdue goods produced across the Supply Chain. The Centralized strategy is the one that presents the lowest amount of overdue products. This is in accordance with the previous results, since excess of production will lead to more stored goods that will eventually deteriorate before arriving the retailing echelon.

Table 3.4 presents the performance measures obtained for the three strategies.

Table 3.4: Performance measures for the three Model Predictive Control Strategies for a configuration with 3 echelons and 1 player per echelon.

Performance Measures	Strategy		
	Decentralized	Distributed	Centralized
Total Production (units)	687	687	523
Total Storage (units)	1722	1722	667
Overproduction (%)	43	43	26
Quantity of commodities movement (units)	1319	1319	1167
Total quantity of overdue products (units)	294	294	131
Computation time (sec)	7.75	10.52	5.22

The results show that the Centralized strategy is the best when the main objective is to minimize the quantity of overdue products. All three strategies have proven to satisfy the customer demand, but the Centralized strategy does it with the lowest values for every performance measure. However, this strategy is, as stated before, ideal. Between the developed strategies, they present exactly the same results for almost every performance measure, since the increase in visibility is not significant, resulting in the same control actions. In terms of computation time, the values are close, since the optimization problem is not very complex.

3.2 Configuration 2

3.2.1 Case Study

Simulation Specifications

All the simulations performed have a duration of 100 days and a sampling time of 1 day. The default prediction horizon, N_p, is set to 12 days [5].

Network Configuration

The Supply Chain is composed by 3 echelons - manufacturing, distribution and retailing - each one composed of 2 players. The goods enter in the Supply Chain from production to the manufacturing echelon. Then, they are moved across the Supply Chain, being handled by the distinct players, until reaching the customer or becoming overdue goods. Any player of a given echelon, that has stored goods, can deliver products to any player of the next echelon, if the transport capacity between players and the storage capacity of the player receiving the goods are not exceeded. It is considered that the transportation time of moving commodities from a player of a given echelon to a player of the next echelon is equal to 1 day.
These storage and transport limits are assumed constant over the entire simulation and can be consulted in Table 3.5 and Table 3.6 for each player and each link. As stated previously for configuration 1, the presented values are generic and could be changed within certain operational limits, that have not been calculated and are outside the scope of this work, and taking into account the demand characteristics.

Table 3.5: Storage capacity of all Supply Chain players (units) for a Supply Chain composed by 3 echelons and 2 players per echelon.

Player	Storage Capacity
Manufacturer 1	30
Manufacturer 2	30
Distributor 1	25
Distributor 2	25
Retailer 1	20
Retailer 2	20

Each manufacturer can store 30 products simultaneously, while each distributor and each retailer have the capacity to store 25 and 20 products at once, respectively. Regarding the transport, each manufacturer can receive 20 products simultaneously, from production. The transport capacity of commodities

Table 3.6: Transport capacity of all Supply Chain connections (units) for a Supply Chain composed by 3 echelons and 2 players per echelon.

Connection	Transport Capacity
Production - Manufacturer 1	20
Production - Manufacturer 2	20
Manufacturer 1 - Distributor 1	20
Manufacturer 1 - Distributor 2	15
Manufacturer 2 - Distributor 1	15
Manufacturer 2 - Distributor 2	20
Distributor 1 - Retailer 1	20
Distributor 1 - Retailer 2	15
Distributor 2 - Retailer 1	15
Distributor 2 - Retailer 2	20
Retailer 1 - Customer	15
Retailer 2 - Customer	15

between players 1 and players 2 of neighbouring echelons is equal to 20 and the transport capacity from player 1 (player 2) of a given echelon to player 2 (player 1) of the next echelon is equal to 15, since it is considered that these players are further away. As for the retailing echelon, each retailer can deliver 15 products to the customer at once.

Demand

The demand considered is the same presented previously in section 3.1.1.

Management Policy

The management policy used is described by the cost function weights of the optimization problem. The cost function has the same three components presented previously in section 3.1.1, which are related to:

- inventory levels at each player
- minimization of overdue goods
- connections

The cost function weights considered are presented in Table 3.7.

The weight value associated to the overdue goods is the highest positive value, meaning waste minimization is prioritized relatively to the other two components of the cost function.

The weights related to inventory levels are positive and equal for all players to penalize equally the storage of perishable products in every Supply Chain player.

The weights related to connections are positive or null. The weights of the connections from production to the manufacturing echelon are highly positive to penalize the overproduction of goods. The weights

Table 3.7: Cost function weights for a Supply Chain composed by 3 echelons and 2 players per echelon.

Cost Function Weights	
Overdue	100
Manufacturer 1	1
Manufacturer 2	1
Distributor 1	1
Distributor 2	1
Retailer 1	1
Retailer 2	1
Production - Manufacturer 1	50
Production - Manufacturer 2	50
Manufacturer 1 - Distributor 1	0
Manufacturer 1 - Distributor 2	10
Manufacturer 2 - Distributor 1	10
Manufacturer 2 - Distributor 2	0
Distributor 1 - Retailer 1	0
Distributor 1 - Retailer 2	10
Distributor 2 - Retailer 1	10
Distributor 2 - Retailer 2	0
Retailer 1 - Customer	0
Retailer 2 - Customer	0

of connections starting from player 1 of a given echelon to player 2 of the next echelon are positive because the distance between them is assumed to be bigger than the distance between player 1 of a given echelon and player 1 of the next echelon. The same happens between players 2. This way, the connections between players that are closer are chosen as a first option, leading to lower costs.

Once more, the weight values are assigned according to the managers' decisions.
To maintain the management policy considered, the values of the weights may vary. However, the relative values must stay the same. But the optimal allocation of weights to follow the desired management policies is outside the scope of this work.

Performance measures

The performance measures used are the same presented in section 3.1.1.

3.2.2 Results

In this section, the results of the numerical simulation for configuration 2 (Figure 2.1(b)) are presented, first considering the Decentralized strategy and then the Distributed strategy. Finally, the results of the numerical simulations are compared for these 2 strategies and the Centralized one.

Decentralized Strategy

Figure 3.11 presents the storage intensity evolution of the entire Supply Chain and the storage intensity discriminated by player.

The total storage intensity is zero for the first 8 time instants, since the customer demand only starts 10 days after the beginning of the simulation. The Supply Chain players that have non-zero storage intensity levels on the 9^{th} day are the manufacturers, that receive the commodities from the production to be available at the retailers on the 11^{th} day.

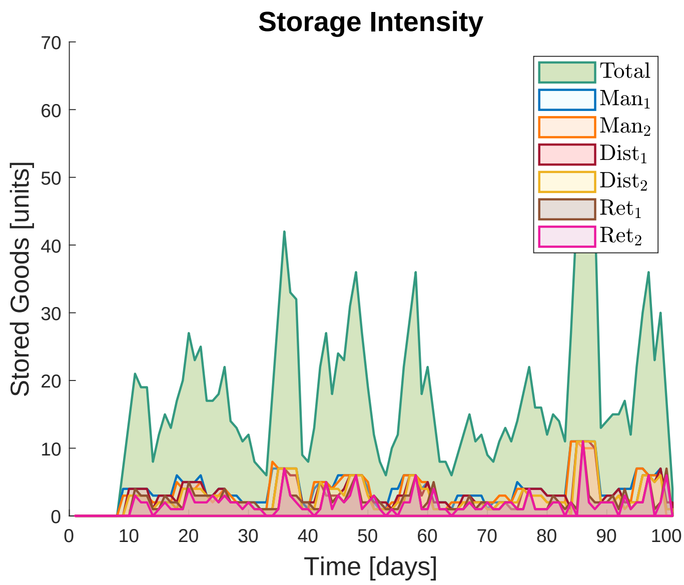


Figure 3.11: Decentralized Strategy for a configuration with 3 echelons and 2 players per echelon - Storage Intensity.

During the entire simulation, the total storage intensity shows peaks that are present mainly due to the storage intensity pattern of both retailers. This is a result of the short time storage of products at this Supply Chain stage. For the manufacturing and distribution echelons, the storage intensity has more plateaus, meaning the products are stored for longer time periods. Additionally, for these Supply Chain players, there is almost never a time instant where the storage intensity is zero.

Figure 3.12 presents the storage intensity level only for the manufacturing echelon. For this strategy, the lack of visibility results in a safety stock to deal with the uncertain demand.

Figure 3.13 shows the production and the customer demand evolution over time. The area under the production curve is bigger than the area under the customer demand curve, meaning commodities are being produced in excess. This leads to an overproduction that contributes to the total amount of overdue products. Additionally, it is possible to observe a delay between the production and the customer demand peaks. This is an expected result, since the delivery of products from a given echelon to the next echelon takes one day, which means that a product takes at least two days to be moved from production to retailing, if it is not stored at any stage.

Figure 3.14 presents the evolution over time of the total quantity of products that have deteriorated from

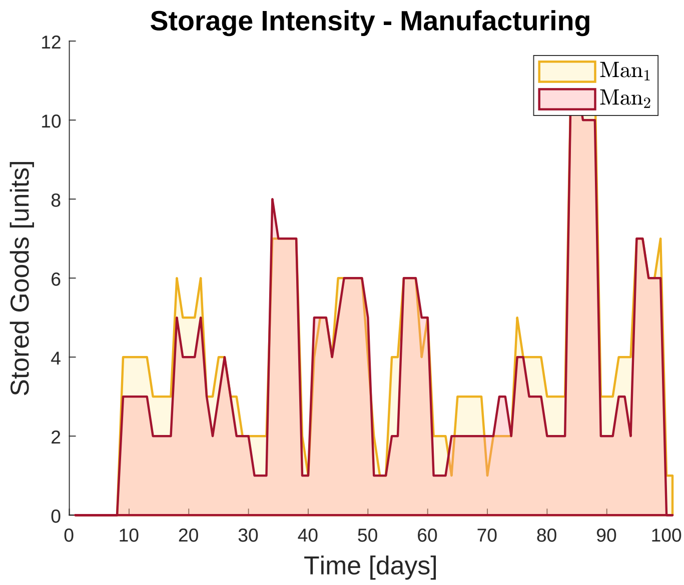


Figure 3.12: Decentralized Strategy for a configuration with 3 echelons and 2 players per echelon - Manufacturing Storage Intensity.

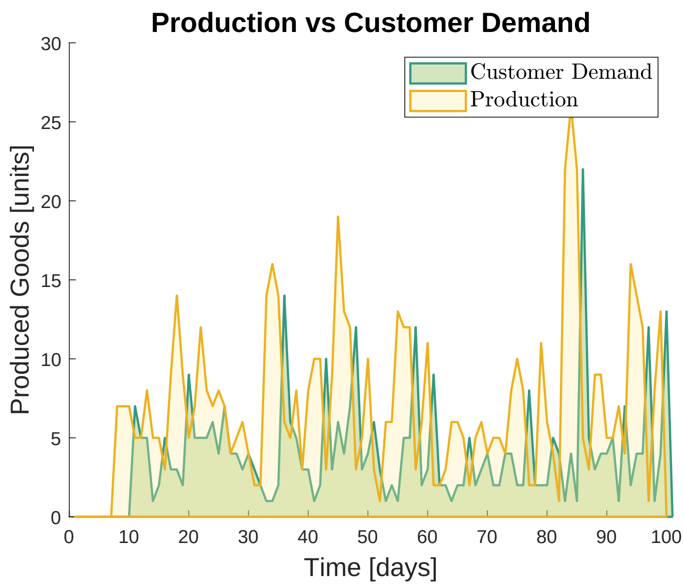


Figure 3.13: Decentralized Strategy for a configuration with 3 echelons and 2 players per echelon - Production VS Demand.

production to the retailing stage, before arriving to the customer. It also presents the quantity of products lost per Supply Chain player. At the end of the simulation, the total quantity of overdue goods is close to 300 units and the major contributors are the manufacturers and the distributors, which is in accordance with the results obtained for the storage intensity.

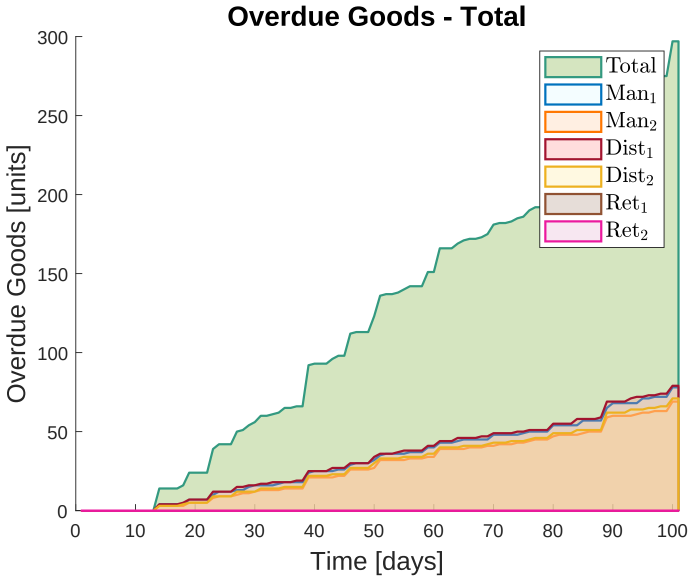


Figure 3.14: Decentralized Strategy for a configuration with 3 echelons and 2 players per echelon - Overdue Goods.

Distributed Strategy

The same set of results presented for the Decentralized strategy is now presented and analyzed for the Distributed strategy.

Figure 3.15 presents the storage intensity for the entire Supply Chain and discriminated by player.

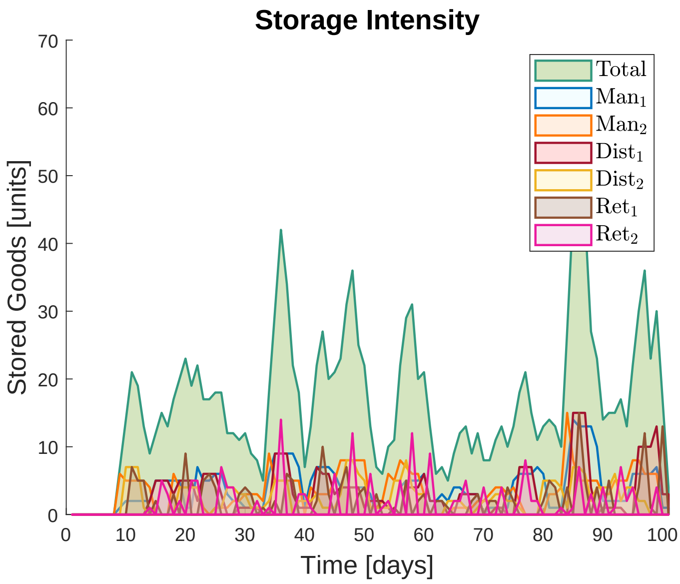


Figure 3.15: Distributed Strategy for a configuration with 3 echelons and 2 players per echelon - Storage Intensity.

Once more, the retailing echelon presents more peaks and the distribution and manufacturing echelons present more plateaus. However, all Supply Chain players have higher oscillations for the levels of storage intensity, compared to the Decentralized strategy.

Figure 3.16 presents the storage intensity for the manufacturing echelon. For this strategy, the manufacturing echelon has time instants where the storage intensity is equal to zero, meaning there is no safety stock. This can be related with the increase in visibility among Supply Chain players, which reduces the uncertainty and leads to a lower necessity of having safety stocks. The same happens for the distribution and retailing echelons.

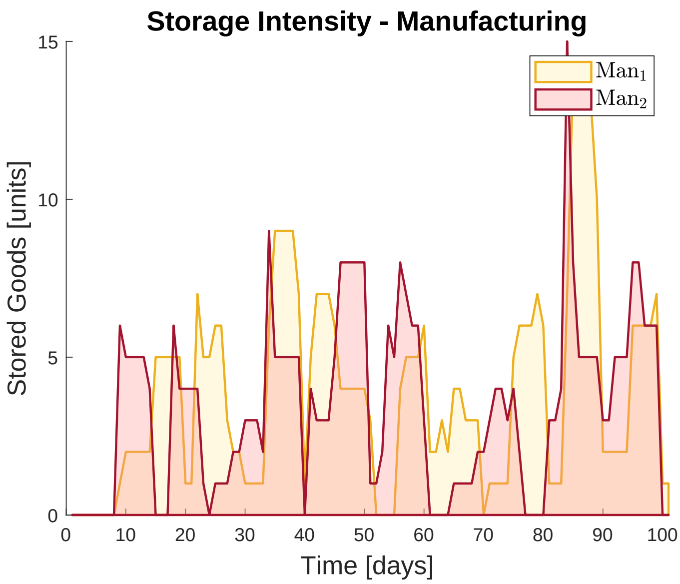


Figure 3.16: Distributed Strategy for a configuration with 3 echelons and 2 players per echelon - Manufacturing Storage Intensity.

Figure 3.17 presents the customer demand and the production intensity for the Distributed strategy. Once more, the production exceeds the customer demand, contributing to the quantity of overdue products generated across the Supply Chain. Additionally, the higher production peaks precede the higher customer demand peaks, mostly, in 2 to 3 time instants, which is consistent with the necessary time to move a product from the manufacturing to the retailing echelon.

Figure 3.18 displays the evolution of the quantity of overdue products over time for the entire Supply Chain and discriminated by players. The retailing echelon does not contribute to the total quantity of overdue products across the Supply Chain. Manufacturers and distributors contribute approximately with the same amount of overdue products. Once more, this result is consistent with the values presented in Figure 3.15 for the storage intensity discriminated by Supply Chain player.

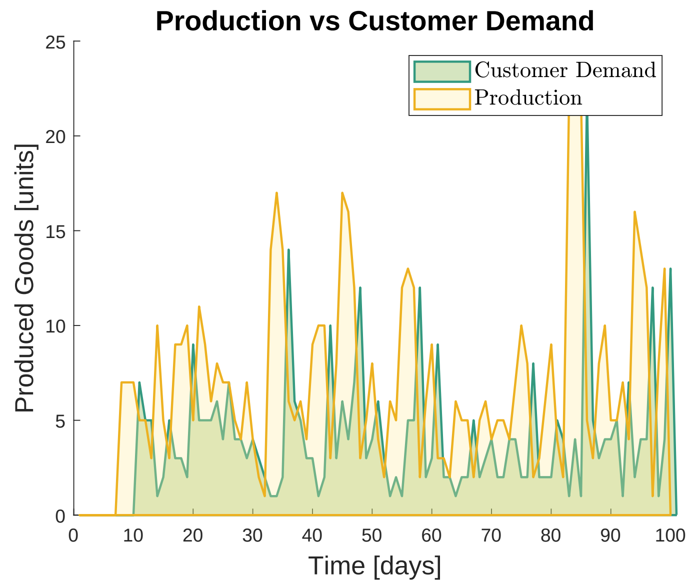


Figure 3.17: Distributed Strategy for a configuration with 3 echelons and 2 players per echelon - Production VS Demand.

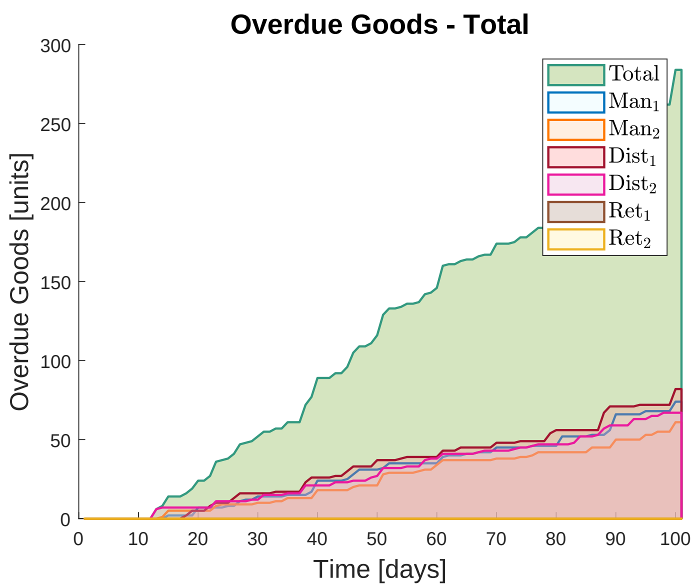


Figure 3.18: Distributed Strategy for a configuration with 3 echelons and 2 players per echelon - Overdue Goods

Strategies Comparison

Figure 3.19 presents the storage intensity evolution over the entire simulation for the entire Supply Chain, considering the three Model Predictive Control strategies - Decentralized, Distributed and Centralized. All three strategies present a similar evolution of the total storage intensity. However, the Centralized strategy is the one that has the lowest values over the entire simulation. Although the Decentralized and Distributed strategies present closer values, the Decentralized strategy presents, for many instants, higher peaks than the Distributed one.

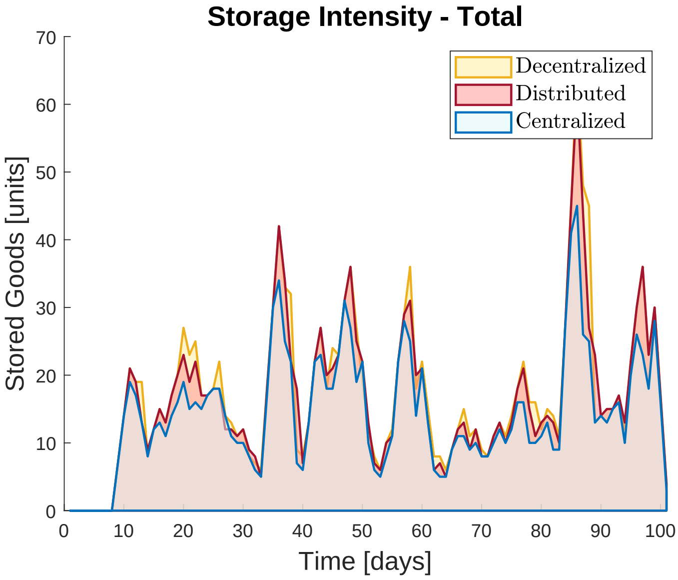


Figure 3.19: Model Predictive Control Strategies Comparison - Decentralized, Distributed and Centralized - for a configuration with 3 echelons and 2 players per echelon - Storage Intensity.

Figure 3.20 presents the production of the three strategies over the entire simulation.

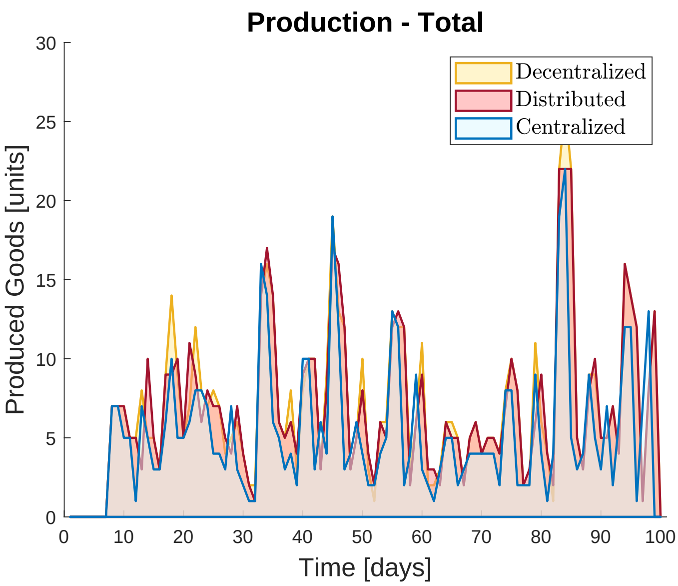


Figure 3.20: Model Predictive Control Strategies Comparison - Decentralized, Distributed and Centralized - for a configuration with 3 echelons and 2 players per echelon - Production.

Once more, the evolution among strategies is similar. However, the Centralized strategy shows the lowest values of production for almost every instant of the entire simulation, followed by the Distributed strategy. The Decentralized strategy continues to be the one presenting the highest values, which results in higher values of overproduction, that can be consulted on Table 3.8.

Figure 3.21 shows the amount of overdue goods obtained across the Supply Chain, over the entire simulation for the three Model Predictive Control strategies.

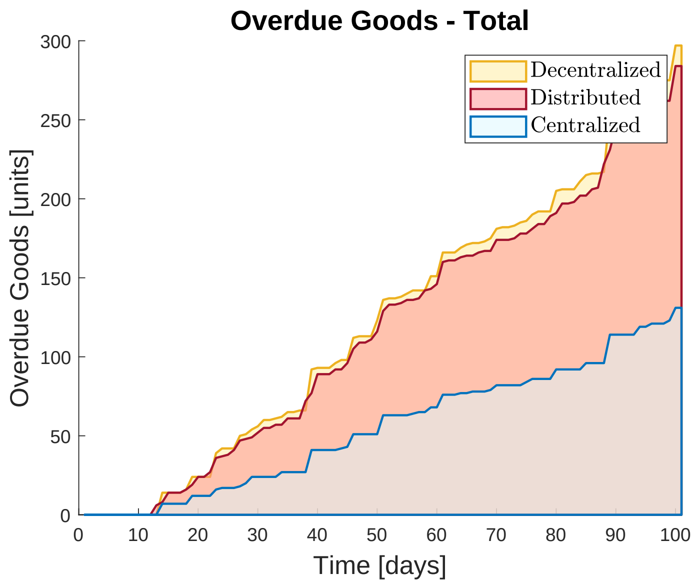


Figure 3.21: Model Predictive Control Strategies Comparison - Decentralized, Distributed and Centralized - for a configuration with 3 echelons and 2 players per echelon - Overdue Goods.

The strategy that presents the highest values of overdue goods is the Decentralized one, followed by the Distributed strategy. The Centralized strategy is the one that presents the lowest amount of overdue products. This is in accordance with the previous results, since excess of production will lead to more stored goods that will eventually deteriorate before arriving the retailing echelon.

Table 3.8 presents the performance measures obtained for the three strategies.

Table 3.8: Performance measures for the three Model Predictive Control Strategies for a configuration with 3 echelons and 2 players per echelon.

Performance Measures	Strategy		
	Decentralized	Distributed	Centralized
Total Production (units)	690	677	523
Total Storage (units)	1736	1672	667
Overproduction (%)	44	43	26
Quantity of commodities movement (units)	1319	1635	1167
Total quantity of overdue products (units)	297	284	131
Computation time (sec)	16	43.23	32.22

The Centralized strategy presents the best values for almost every performance measure, when the main objective is to minimize the quantity of overdue products. All three strategies have proven to satisfy the customer demand, but the Centralized one does it with a lower overproduction, lower values of storage intensity and with less commodity movements. However, in terms of computation time, the Decentralized strategy is the fastest, despite of solving more problems – one per player – than the

other two strategies – one problem for the entire Supply Chain and one problem per echelon. But, these problems are simpler, leading to a lower computation time. The Distributed strategy presents the highest computation time, which is related to the size of the problems. In detail, the manufacturing problem is the same size of the whole Centralized problem, even though some variables are already fixed. In addition, this strategy needs to solve the retailing and distribution problems. Although the Centralized strategy presents, in general, the best results, this strategy is, as stated before, not realistic because, in reality, there is no external player that has total access to the information from all players and total control over the entire Supply Chain. Between the developed strategies, when the Distributed strategy is applied to the Supply Chain, it shows a lower overproduction and it has lower storage intensity values, leading to a lower quantity of overdue products. And this strategy is closer to the real-world settings than the Centralized or the Decentralized strategies. For this reason, new scenarios were developed to continue to evaluate the performance of the Distributed strategy.
To summarize the study of the three coordination strategies, Table 3.9 presents a qualitative comparison between the strategies, highlighting their performance level , advantages, limitations and applicability.

Table 3.9: MPC Strategy comparison - Qualitative analysis.

	MPC Strategies		
	Decentralized	Distributed	Centralized
Performance	Poor performance.	Good performance.	Optimal performance.
Advantages	Fast computation time. Independent controllers (players' privacy policies).	Cooperation between players. Integration of individual and global goals.	Optimal Supply Chain operation.
Limitations	Low Supply Chain visibility. Non-cooperative strategy.	Complex trade-offs between players. Slow computation time.	Utopic approach. Computationally demanding. Players' privacy policies.
When to use	Supply Chain players are not willing or unable to cooperate and share operational information.	Supply Chain players agree to cooperate to optimize the global operation.	Set the Supply Chain operational reference (to apply to Distributed strategy).

3.2.3 Alternative Customer Demand

The first set of alternative scenarios consists in changing the customer demand.

One Product Demand

Two new demand profiles, considering only one type of commodity with the same previously defined lifetime, equal to 5 days, were created and are presented in Figure 3.22 and in Figure 3.23. Both demand profiles start 10 days after the beginning of the simulation and both present a total demand for the entire simulation equal to 389 units, the same value obtained for the original demand profile, presented in section 3.1.1.

The first demand profile is almost constant. It starts with a demand of 4 units and remains with this value until the 41st day. Then, the demand increases slightly, to 5 units, for 29 consecutive days, and finally, returns to 4 units per day until the end of the simulation.
The second demand presents more variability than the two previously demand profiles, ranging from 0 to 22 units per day.

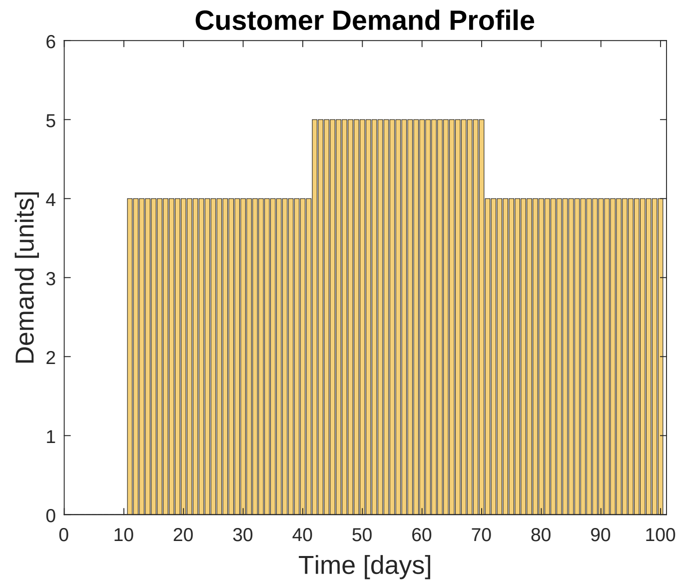


Figure 3.22: Alternative demand profile for a perishable commodity with a lifetime of 5 days - Scenario 1.

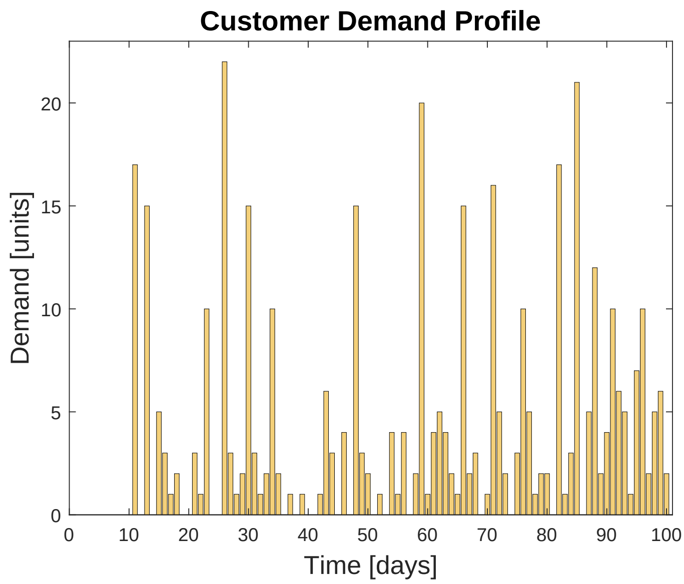


Figure 3.23: Alternative demand profile for a perishable commodity with a lifetime of 5 days - Scenario 2.

Table 3.10 presents the performance measures obtained for the three demand profiles applied to the Supply Chain with the configuration presented in section 3.2.1.

Table 3.10: Performance measures for the Distributed Strategy for a configuration with 3 echelons and 2 players per echelon - Alternative Demand Profiles.

Performance Measures	Demand Profile		
	Original	Scenario 1	Scenario 2
Total Production (units)	677	587	779
Total Storage (units)	1672	1277	2132
Overproduction (%)	43	34	50
Quantity of commodities movement (units)	1635	1733	1791
Total quantity of overdue products (units)	284	195	388
Computation time (sec)	43.23	42.78	40.25

The model was able to respond to the three imposed demands. However, its performance for the three cases presented significant differences. For the demand of scenario 1, the Distributed strategy obtained the lowest values for almost all performance measures. Contrary, when considering the demand of scenario 2, the performance measures took the highest values. This way, it can be concluded that, with a more constant demand the Supply Chain performance increases. The production and storage levels are more constant, leading to lower overproduction values and, consequently, to less overdue products across the Supply Chain.

Multi-Product Demand

Once established that, when applying the Distributed strategy, the Supply Chain can handle different demand profiles for one type of commodity, new scenarios were developed, considering multiple products at once.

Next, Figures 3.24 and 3.25 present scenarios 3 and 4 with a demand profile for two and three products, respectively, that are handled by the Supply Chain simultaneously.

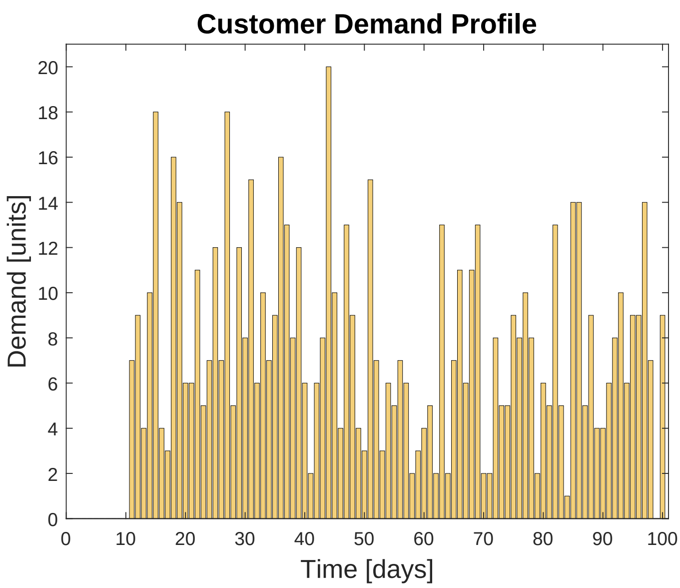


Figure 3.24: Demand profile for two perishable commodities with lifetimes of 5 and 7 days - Scenario 3.

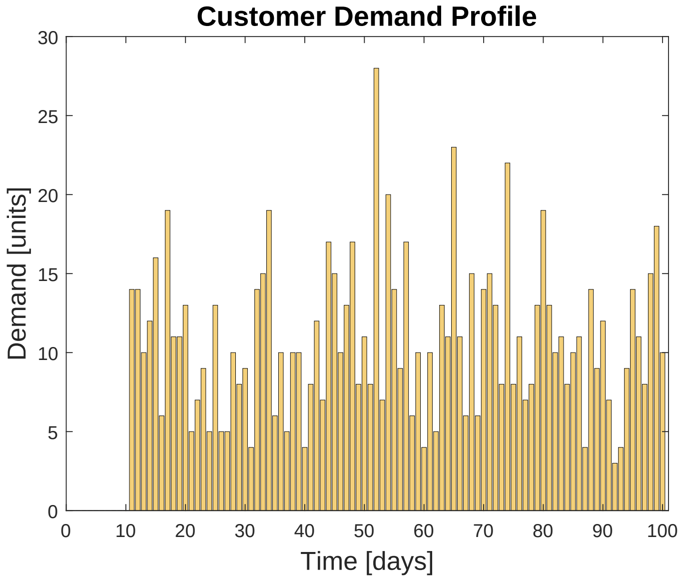


Figure 3.25: Demand profile for three perishable commodities with lifetimes of 5, 6 and 7 days - Scenario 4.

Table 3.11 presents the performance measures obtained for the Supply Chain with the characteristics presented in section 3.2.1, applying the Distributed strategy.

Table 3.11: Performance measures for the Distributed Strategy for a configuration with 3 echelons and 2 players per echelon - 1, 2 and 3 products.

Performance Measures	Demand Profile		
	Original	Scenario 3	Scenario 4
Total Production (units)	677	1175	1593
Total Storage (units)	1672	3152	4276
Overproduction (%)	43	40	39
Quantity of commodities movement (units)	1635	2944	3951
Total quantity of overdue products (units)	284	455	594
Computation time (sec)	43.23	216.09	479.5

The Supply Chain was able to fulfill the customer demand for the new demand profiles. Increasing the number of types of commodities in the Supply Chain, led to an increase in the total demand values, which justifies the higher total production and total storage values. Additionally, more types of commodities result in more states and more control actions, leading to higher-dimensional and more complex problems. As a consequence, the computation time also increases. However, the overproduction for scenarios 3 and 4 decreases.

Table 3.12 presents some statistic measures, namely mean, variance, their ratio and also the total quantity of goods for the five demand profiles presented.

Table 3.12: Statistical measures for the different demand profiles.

Statistic Measures	Demand Profile				
	Original	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Mean	4.3	4.3	4.3	7.9	10.9
Variance	11.8	0.2	29.2	18.5	22.7
Variance/Mean	2.7	0.1	6.8	2.3	2.1
Total	389	389	389	708	979

The ratio variance/mean is a normalized measure of distribution that allows to compare the variance of the demands with different mean values. Analyzing these ratios and the overproduction values, it can be concluded that the overproduction is strongly influenced by the variability, since a lower value of Variance/Mean leads to a lower value of overproduction. Scenario 2 presents the highest ratio and the highest overproduction and scenario 1 the lowest ratio and the lowest overproduction.
In terms of quantity of overdue goods, increasing the demand leads to an increase of overdue products. However, since scenarios 3 and 4 have lower variability than scenario 2 and the original demand, compared with these scenarios, the ratio overdue products/total production decreases.

3.2.4 Alternative Management policies

To evaluate the performance of the Distributed strategy when using different management policies, two new sets of weights were considered. Table 3.13 presents the original set of weights plus the weights assumed for the new management policies, scenarios 5 and 6.
For scenario 5, the changes from the original set of weights consist in modifying the weights of the state and outflow of retailer 2 from 1 to -50 and from 0 to -50, respectively. The negative weight related to the state means it is intended to have stored goods at the retailer 2, rather than at any other player, since the other players have a positive weight associated. Regarding the outflow, the negative weight prioritizes the flow of commodities to the customer through retailer 2. This way, retailer 1 only is only used when retailer 2 is no longer available, since its outflow has a null weight associated.
In scenario 6, distributor 2 and retailer 1 have a negative weight of -45 associated to their states. Additionally, retailer 1 has now a negative weight associated to its outflow, instead of retailer 2. This means the commodities should be stored at distributor 2 and retailer 1 and the flow of commodities should preferentially reach the customer through retailer 1.
Figures 3.26 and 3.27 present the storage intensity for scenarios 5 and 6, for the same Supply Chain configuration presented previously (section 3.2.1) and for the demand profile presented in Figure 3.1. Table 3.14 presents the performance measures obtained for the three management policies applied.

Figure 3.26 shows that the products are mainly stored in players 2 of the three echelons, which happens due to the weights associated to the links. In detail, the movement of goods from player 1 (player 2)

Table 3.13: Cost function weights for three Management Policies applied to the Distributed Strategy for a configuration with 3 echelons and 2 players per echelon

Cost Function Weights	Management Policy		
	Original	Scenario 5	Scenario 6
Overdue	100	100	100
Manufacturer 1	1	1	1
Manufacturer 2	1	1	1
Distributor 1	1	1	1
Distributor 2	1	1	-45
Retailer 1	1	1	-45
Retailer 2	1	-50	1
Production - Manufacturer 1	50	50	50
Production - Manufacturer 2	50	50	50
Manufacturer 1 - Distributor 1	0	0	0
Manufacturer 1 - Distributor 2	10	10	10
Manufacturer 2 - Distributor 1	10	10	10
Manufacturer 2 - Distributor 2	0	0	0
Distributor 1 - Retailer 1	0	0	0
Distributor 1 - Retailer 2	10	10	10
Distributor 2 - Retailer 1	10	10	10
Distributor 2 - Retailer 2	0	0	0
Retailer 1 - Customer	0	0	-45
Retailer 2 - Customer	0	-50	0

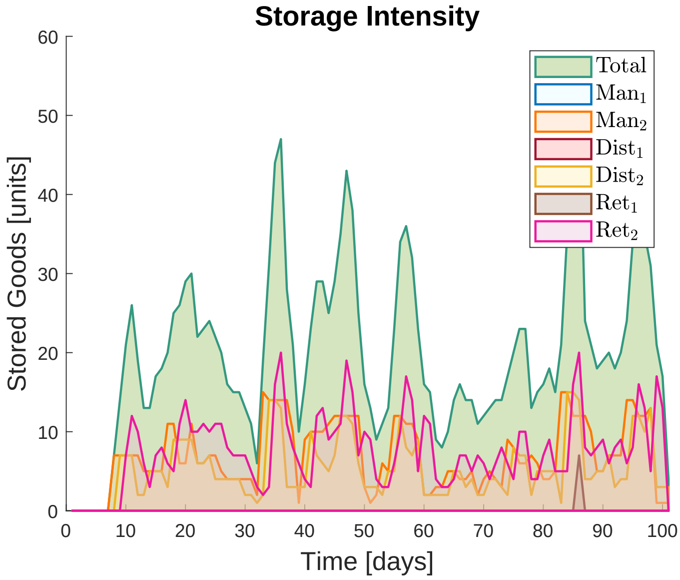


Figure 3.26: Storage Intensity for a Supply Chain with three echelons and two players per echelon, applying the Distributed Strategy - Management Policy - Scenario 5.

of a given echelon to player 1 (player 2) of the next echelon are prioritized in relation to the movement of goods from player 1 (player 2) of a given echelon and player 2 (player 1) of the next echelon. This way, given that the demand is not too challenging, the products move across the Supply Chain through

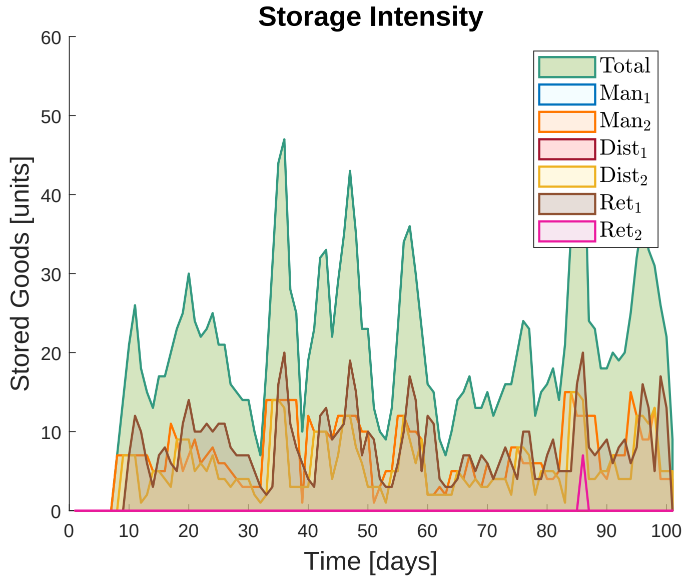


Figure 3.27: Storage Intensity for a Supply Chain with three echelons and two players per echelon, applying the Distributed Strategy - Management Policy - Scenario 6.

players 2.

Figure 3.27 shows that the goods are stored mainly in manufacturer 2, distributor 2 and retailer 1. Since the state of distributor 2 has a negative weight associated that has a higher absolute value ($|-45|$) than the penalty associated to the flow of commodities between distributor 2 and retailer 1 (10), the Supply Chain is forced to move products from distributor 2 to retailer 1, instead of prioritizing the movement from distributor 1 to retailer 1. In this case, manufacturer 1, distributor 1 and retailer 2 are only used when the other players from the same echelon are unavailable.

Table 3.14: Performance measures for the Distributed Strategy for a configuration with 3 echelons and 2 players per echelon, considering three different management policies.

Performance Measures	Management Policy		
	Original	Scenario 5	Scenario 6
Total Production (units)	677	665	672
Total Storage (units)	1672	2005	2022
Overproduction (%)	43	42	42
Quantity of commodities movement (units)	1635	1977	1999
Total quantity of overdue products (units)	284	272	274
Computation time (sec)	43.23	48.19	45.27

The results show that the new management policies lead to slightly lower values of total production and to a lower quantity of overdue products than the original management policy applied. However, the number of units stored increases, which is consistent with the new weights applied, and the quantity of commodity movement also increases. Despite the differences, with the three management policies considered, the Supply Chain managed to satisfy the customer demand.

3.2.5 Literature Benchmark

As stated previously, there is a lack of systematic and generic strategies in the literature to evaluate the performance of innovative features and models in Supply Chain strategies. Each Supply Chain has its own specificity and research techniques tend to model the specificity of the Supply Chains they focus on. Hence, it is difficult to benchmark the methods presented in this thesis. However, Mestan et al. [15] developed Model Predictive Control Centralized, Decentralized and semi-Decentralized strategies to manage the flows of multi-product Supply Chains. The Supply Chains models used present some differences to the ones developed in this thesis, namely, the authors consider:

- non-perishable products;
- different objectives;
- each product is produced by a specific manufacturer, distributed by a specific distributor and sold by a specific retailer. This means there is no interaction or cooperation between Supply Chain agents.

Nevertheless, Mestan et al. present a bullwhip effect measure to quantify the performance of the strategies, which can also be applied to the control strategies developed in this thesis.
The bullwhip effect consists in the amplification of the demand variability when moving from a downstream echelon to an upstream echelon and tends to increase as one moves upstream across the Supply Chain. Exemplifying, Figure 3.28 presents the bullwhip effect for a given Supply Chain.

Figure 3.28: Bullwhip effect (from [29])

This happens, for instance, when each Supply Chain player makes the demand forecasting independently, considering only the orders from immediate customers or when the orders are made in batches, in order to reduce processing and transportation costs. This way, the orders from the downstream members do not coincide with the actual demand, misguiding the upstream members in their inventory and production decisions [23, 30].

The bullwhip effect measure is given by (adapted from [15]):

$$\mathbf{bwr}_{jm} = \left| \frac{\sum_i \mathbf{O_{ijm}}(k)}{\sum_l \mathbf{O_{jlm}}(k)} \right|, \tag{3.1}$$

where $\mathbf{bwr}_{jm}$ is the measure of the bullwhip effect generated by node j for product m, $\sum_i \mathbf{O_{ijm}}(k)$ represents the orders that are placed from node j to all upstream nodes i for product m and $\sum_l \mathbf{O_{jlm}}(k)$ are the orders from all downstream nodes l to node j for product m. Ideally, this ratio would be 1, meaning the orders placed by node j would be equal to the orders placed by the downstream nodes to j and no bullwhip effect would be registered.

Exemplifying, for the Supply Chain presented in section 3.2.1, for manufacturer 1, considering only one type of commodity, the bullwhip effect measure is given by:

$$\mathbf{bwr}_{11} = \left| \frac{\mathbf{F_{man\_in1}}(k)}{\mathbf{u_{man11}(k)} + \mathbf{u_{man12}(k)}} \right|. \tag{3.2}$$

Table 3.15 presents the average measure of the bullwhip effect for the three strategies developed in [15] discriminated by product and Figure 3.29 shows the demand profiles of the three products.

Table 3.15: Comparison of the Bullwhip Effect under Different Configurations - [15] results.

Product	Average Measure of the Bullwhip Effect		
	Decentralized	semi-Decentralized	Centralized
A	2.10	1.43	1.17
B	3.06	1.54	1.22
C	2.55	2.62	2.13

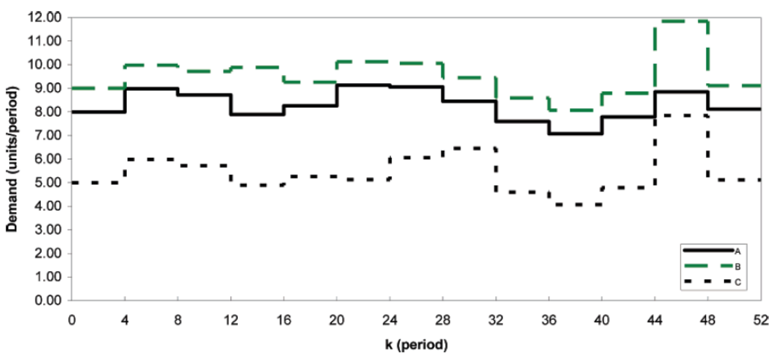


Figure 3.29: Demand profiles for products A, B and C [15].

The demand profile for the three products is different from the one presented in section 3.2.1. In terms of quantity, product B presents the highest total amount of units, followed by product A. In terms of variability, product A has the most constant demand profile from the three. Products B and C present a similar variability.

Analyzing Table 3.15 results, product A is the one that presents the lowest values of the average measure

of the bullwhip effect. For the Decentralized strategy, product B is the one that presents the highest value and for the remaining two strategies, product C presents the highest values. From the results shown, it may be concluded that a more constant demand leads to a lower bullwhip effect for all strategies.
Although the storage and transport capacities are not specified and the total amount of units for the three products is different from the total amount of the demand presented in section 3.2.1, the values obtained for the bullwhip effect measure are compared with the ones obtained for the strategies developed, since a ratio is considered and the applied strategies are similar. The values of the semi-Decentralized strategy are compared with the values obtained for the Distributed strategy.
The values of the average bullwhip effect measure for the three strategies presented in this thesis are shown in Table 3.16 for the demand profile of section 3.2.1 and also for the alternative scenarios 1 and 2, presented in section 3.2.3 for one type of commodity.

Table 3.16: Comparison of the Bullwhip Effect under Different Configurations and for different demand profiles.

Demand Profile	Average Measure of the Bullwhip Effect		
	Decentralized	Distributed	Centralized
Original	1.39	1.21	1.11
Scenario 1	1.32	1.15	1.07
Scenario 2	1.45	1.28	1.16

The results of the average measure of the bullwhip effect obtained for these strategies applied to the case study presented in this thesis are lower than the results obtained for the corresponding strategies developed in [15], for the product with the lowest values.
The three demand profile scenarios for one type of commodity were included in Table 3.16 to evaluate the impact of the demand variability on the bullwhip effect. The results show that, when the variability increases, the average bullwhip effect measure also increases. The demand of scenario 2 presents the highest variability among all demand profiles, higher than the one presented for any of the three products, A, B and C. And even for this scenario, the values obtained are lower.
As stated previously, worst case scenarios are usually considered to evaluate the performance and validate Optimal Control techniques. However, using worst case scenarios implies testing the limits of the system being studied, which is outside the scope of this work. Instead, the demand profiles considered were obtained with the objective of being demanding for the Supply Chain configuration presented and, for this case study, the strategies were validated and presented better values for the average measure of the bullwhip effect than the strategies developed in [15]. This way, considering less demanding profiles, for the same Supply Chain configuration would lead to similar or better results. In addition, considering similar case studies, the strategies may still be validated as long as there is a compromise between the demand intensity and variability and the Supply Chain constraints. However, finding the generic class of problems for which conclusions can be drawn and the compromise that needs to exist involves a more mathematical scientific body behind the work that is outside the scope of this thesis.

Chapter 4

Conclusions and future Work

Firstly, this chapter presents an overview of the most important aspects of this thesis and some considerations related with the proposed methods and the results obtained. Secondly, it presents possible future work to deepen the study of the proposed methods.

4.1 Overview

This thesis presents a model for Perishable Food Supply Chains that considers Supply Chain players as nodes, connected by links, with limited storage and transport capacities, respectively. Each Supply Chain player inventory is assumed to be partitioned into sub-inventories, based on the time the stored commodities have left until they expire. In a control perspective, the model is represented using a state-space representation, where the sub-inventory levels of goods are the states and the flow between players, across the links, are the control actions.
Two Model Predictive Control strategies - Decentralized and Distributed - are developed and studied, taking a Centralized Strategy as the starting point. The main differences between the developed strategies are the visibility of the Supply Chain players and the cooperation between them. In the Decentralized strategy, each player has access to limited information, namely its available storage capacity and the available transport capacity of the links directly connected to it. Additionally, there is no cooperation between players, meaning each Supply Chain player manages its own control actions by itself. In the Distributed strategy, it is considered that there is a controller for each Supply Chain echelon and the visibility across the Supply Chain, from the most upstream to the most downstream echelon, increases. In other words, each echelon has access to the information from the upstream echelons. This way, there is an influence and cooperation between players when deciding the best control actions to implement.
A computational model was implemented in order to analyze the performance of Perishable Food Supply Chains, considering the developed strategies and the Centralized one.
The proposed strategies are evaluated and compared to the Centralized strategy, using two case studies present in the literature. The Decentralized strategy is the one that presents the poorest performance, which is expected due to its lack of visibility. Contrary, the Centralized strategy has the best performance.

However, as stated previously, it is ideal and further away from real-world settings than the Distributed strategy. Nevertheless, it presents the target values for the Distributed strategy, being the objective to minimize the gap between these two strategies.
The results validated the proposed strategies when applied to two case studies presented in the literature for regular Supply Chains, adapted to Perishable Food Supply Chains. For configuration 2, the results showed that the Distributed strategy is robust to different demand profiles and that it can handle multiple products, with different lifetimes, at once. Additionally, for this configuration, the Distributed strategy proved to be flexible to different management policies and presented better results in terms of the bullwhip effect than a case study presented in the literature [15].
The worst case scenario was not considered. However, the demand profiles were demanding, which means considering less demanding profiles, that still respect the model constraints, would lead to a valid model response for this Supply Chain configuration. Finding the compromise between the demand and the model constraints would imply finding mathematical relations, which was outside the scope of this work. Likewise, finding the generic class of problems to which the obtained conclusions hold would require further mathematical tests.
Nevertheless, the obtained results encourage the further development of the work and the simulator, that intends to be generic, in order to be applied to other configurations and different scenarios.

4.2 Future Work

Even though the proposed strategies were validated, there are some aspects that could be further studied, in order to reach the goal of having a simulator flexible enough to evaluate the performance of different Perishable Food Supply Chain configurations.
The first improvement would be to improve the computational implementation of the Supply Chain in order to simulate any possible Supply Chain configuration. The computational implementation of the Supply Chain model was adapted to fit the two Supply Chain configurations presented in this thesis, both with three echelon. However, future work consists in developing the computational implementation of the Supply Chain model so that managerial insights regarding Supply Chain behaviour could be valid to any configuration.
The main objective of the developed strategies is to minimize the amount of overdue products across the Supply Chain. The results obtained can be improved by considering different cost functions, namely a quadratic cost function, or considering different cost function weights for the different time instants, across the prediction horizon, assigning higher weights to the time instants closer to the current time instant.
Additionally, the Distributed strategy is the one closer to the real-world settings. For this strategy it is considered that each echelon is managed by one controller. Another improvement would be to have one controller per player, which could result in the development of new relations and dynamics between players and lead to new cooperation possibilities.

Bibliography

[1] FAO. *The State of Food and Agriculture 2019. Moving forward on food loss and waste reduction*. Rome, 2019.

[2] A. Muysinaliyev and S. Aktamov. Supply chain management concepts: literature review. *IOSR Journal of Business and Management*, 15(6):60–66, Jan. 2017.

[3] J. Mentzer, W. Dewitt, J. Keebler, S. Min, N. Nix, C. Smith, and Z. Zacharia. Defining supply chain management. *Journal of Business Logistics*, 22(2), 2001. doi: 10.1002/j.2158-1592.2001.tb00001.x.

[4] A. Kumar, S. K. Mangla, P. Kumar, and S. Karamperidis. Challenges in perishable food supply chains for sustainability management: A developing economy perspective. *Business Strategy and the Environment*, 29(5):1809 – 1831, 2020. doi: 10.1002/bse.2470.

[5] T. Hipólito, J. L. Nabais, R. Carmona-Benítez, M. A. Botto, and R. R. Negenborn. A centralised model predictive control framework for logistics management of coordinated supply chains of perishable goods. *International Journal of Systems Science: Operations & Logistics*, July 2020. doi:10.1080/23302674.2020.1781953.

[6] D. Ivanov, B. Sokolov, and J. Kaeschel. Integrated supply chain planning based on a combined application of operations research and optimal control. *Central European Journal of Operations Research*, 19:299–317, 2011. doi: 10.1007/s10100-010-0185-0.

[7] Y. Lemma, D. Kitaw, and G. Gatew. Loss in perishable food supply chain: An optimization approach literature review. *International Journal of Scientific & Engineering Research*, 5(5), May 2014.

[8] J. S. Qin and T. A. Badgwell. A survey of industrial model predictive control technology. *Control Engineering Practice*, 11(7):733–764, 2003. doi: 10.1016/S0967-0661(02)00186-7.

[9] W. Wang and D. E. Rivera. Model predictive control for tactical decision-making in semiconductor manufacturing supply chain management. *IEEE Transactions on Control Systems Technology*, 16 (5):841–855, 2008. doi :10.1109/TCST.2007.916327.

[10] A. Alessandri, M. Gaggero, and F. Tonelli. Min-max and predictive control for the management of distribution in supply chains. *IEEE Transactions on Control Systems Technology*, 19(5):1075–1089, 2011. doi: 10.1109/TCST.2010.2076283.

[11] K. Subramanian, J. B. Rawlings, C. T. Maravelias, J. Flores-Cerrillo, and L. Megan. Integration of control theory and scheduling methods for supply chain management. *Computers and Chemical Engineering*, 51:4–20, 2013. doi: 10.1016/j.compchemeng.2012.06.012.

[12] M. Gaggero and F. Tonelli. Optimal control of distribution chains for perishable goods. *IFAC-PapersOnLine*, 48:1049–1054, 2015. doi: 10.1016/j.ifacol.2015.06.222.

[13] X. Lin, R. Negenborn, M. B. Duinkerken, and G. Lodewijks. Quality-aware modeling and optimal scheduling for perishable good distribution networks: The case of banana logistics. *International Conference on Computational Logistics*, page 483–497, 2017. doi: 10.1007/978-3-319-68496-3_32.

[14] R. R. Negenborn and J. M. Maestre. Distributed model predictive control: An overview and roadmap of future research opportunities. *IEEE Control Systems Magazine*, 34(4):87–97, Aug. 2014. doi: 10.1109/MCS.2014.2320397.

[15] E. Mestan, M. Türkay, and Y. Arkun. Optimization of operations in supply chain systems using hybrid systems approach and model predictive control. *Industrial & Engineering Chemistry Research*, 45: 6493–6503, 2006. doi: 10.1021/ie0511938.

[16] M. Braun, D. Rivera, M. Flores, M. Carlyle, and K. Kempf. A model predictive control framework for robust management of multi-product multi-echelon demand networks. *Annual Reviews in Control*, 27:229–245, 2003. doi: 10.1016/j.arcontrol.2003.09.006.

[17] E. Perea-López, I. Grossmann, B. Ydstie, and T. Tahmassebi. Dynamic modeling and decentralized control of supply chains. *Industrial & Engineering Chemistry Research*, 40:3369–3383, 2001.

[18] J. M. Maestre, D. M. de la Peña, and E. F. Camacho. Distributed model predictive control based on a cooperative game. *Optimal Control Applications and Methods*, 32:153–176, 2011. doi: 10.1002/oca.940.

[19] T. M. Pinho, A. P. Moreira, G. Veiga, and J. Boaventura-Cunha. Overview of mpc applications in supply chains: Potential use and benefits in the management of forest-based supply chains. *Forest Systems*, 24(3), 2015. doi: 10.5424/fs/2015243-08148.

[20] R. Kaipia and H. Hartiala. Information-sharing in supply chains: Five proposals on how to proceed. *The International Journal of Logistics Management*, 17(3):377–393, 2006. doi: 10.1108/09574090610717536.

[21] Z. Lotfi, M. Mukhtar, S. Sahran, and A. T. Zadeh. Information sharing in supply chain management. *Procedia Technology*, 11:298–304, 2013. doi: 10.1016/j.protcy.2013.12.194.

[22] E. T. G. Wang and H.-L. Wei. Interorganizational governance value creation: Coordinating for information visibility and flexibility in supply chains. *Decision Sciences*, 38(4):647–674, Nov. 2007. doi: 10.1111/j.1540-5915.2007.00173.x.

[23] H. L. Lee, V. Padmanabhan, and S. Whang. Information distortion in a supply chain: The bullwhip effect. *Management Science*, 43(4):546–558, 1997.

[24] S. Nahmias. Perishable inventory theory: A review. *Operations Research*, 30:680–708, 1982.

[25] J. Maciejowski. *Predictive control with constraints*. Prentice Hall,London, 2002.

[26] H. C. Dreyer, E. Alfnes, J. O. Strandhagen, and M. K. Thomassen. Global supply chain control systems: A conceptual framework for the global control centre. *Production Planning & Control*, 20: 147 – 157, 2009. doi: 10.1080/09537280802705013.

[27] D. Fu, C. M. Ionescu, E.-H. Aghezzaf, and R. D. Keyser. A constrained epsac approach to inventory control for a benchmark supply chain system. *International Journal of Production Research*, 54: 232 – 250, 2016. doi: 10.1080/00207543.2015.1070214.

[28] A. T. Burgin. The gamma distribution and inventory control. *Journal of the Operational Research Society*, 26:507–525, 1975.

[29] H. L. Lee, V. Padmanabhan, and S. Whang. The bullwhip effect in supply chains. *MIT Sloan Management Review*, 38:93–102, 1997.

[30] H. Sarimveis, P. Patrinos, C. D. Tarantilis, and C. T. Kiranoudis. Dynamic modeling and control of supply chain systems: A review. *Computers & Operations Research*, 35:3530 – 3561, 2008. doi: 10.1016/j.cor.2007.01.017.

Appendix A

Model Predictive Control: Configuration 1

In this appendix, the formulation for the distributor's and retailer's problems are presented and explained for the configuration represented in Figure 2.1(a) for the Decentralized and Distributed strategies.

A.1 Decentralized Model Predictive Control

The formulation for the distributor's and retailer's problems for the Decentralized strategy is very similar to the one presented in subsection 2.2.2 for the manufacturer. Figure A.1 presents the distributor and retailer controllers and the information they have access to.

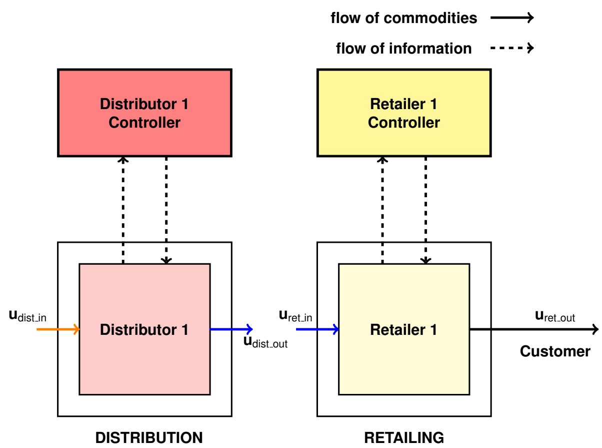


Figure A.1: Decentralized Control Strategy for a Supply Chain configuration with three echelons and one player per echelon - Distributor and Retailer Controllers.

The formulation for the distributor's problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{A.1}$$

$$
\begin{aligned}
\text{s.t.} \quad & \mathbf{x}_{\text{dist}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{dist}}(k+l) + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{dist}}(k+l), && \text{(A.2)}\\
& \mathbf{x}_{\text{dist}}(k+1+l) \geq \mathbf{0}, && \text{(A.3)}\\
& \mathbf{u}_{\text{dist}}(k+l) \geq \mathbf{0}, && \text{(A.4)}\\
& \mathbf{P}_{\text{xx}}\mathbf{x}_{\text{dist}}(k+1+l) \leq \mathbf{X}_{\text{max}_{\text{dist}}}, && \text{(A.5)}\\
& \mathbf{P}_{\text{uu}}\mathbf{u}_{\text{dist}}(k+l) \leq \mathbf{U}_{\text{max}_{\text{dist}}}, && \text{(A.6)}\\
& \mathbf{P}_{\text{xu}}\mathbf{u}_{\text{dist}}(k+l) \leq \mathbf{P}_{\text{ux}}\mathbf{x}_{\text{dist}}(k+l), && \text{(A.7)}\\
& \mathbf{u}_{\text{dist\_out}}(k+l) = \mathbf{u}_{\text{ret\_in}}(k+l), && \text{(A.8)}
\end{aligned}
$$

for $l = 0, \ldots, N_{\text{p}} - 1$.

And the formulation for the retailer's problem is given by:

$$
\begin{aligned}
\min_{\tilde{\mathbf{u}}_k} \quad & J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) && \text{(A.9)}\\
\text{s.t.} \quad & \mathbf{x}_{\text{ret}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{ret}}(k+l) + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{ret}}(k+l), && \text{(A.10)}\\
& \mathbf{x}_{\text{ret}}(k+1+l) \geq \mathbf{0}, && \text{(A.11)}\\
& \mathbf{u}_{\text{ret}}(k+l) \geq \mathbf{0}, && \text{(A.12)}\\
& \mathbf{P}_{\text{xx}}\mathbf{x}_{\text{ret}}(k+1+l) \leq \mathbf{X}_{\text{max}_{\text{ret}}}, && \text{(A.13)}\\
& \mathbf{P}_{\text{uu}}\mathbf{u}_{\text{ret}}(k+l) \leq \mathbf{U}_{\text{max}_{\text{ret}}}, && \text{(A.14)}\\
& \mathbf{P}_{\text{xu}}\mathbf{u}_{\text{ret}}(k+l) \leq \mathbf{P}_{\text{ux}}\mathbf{x}_{\text{ret}}(k+l), && \text{(A.15)}\\
& \mathbf{P}_{\text{d}}\mathbf{u}_{\text{ret}}(k+l) = \mathbf{D}(k+l), && \text{(A.16)}
\end{aligned}
$$

for $l = 0, \ldots, N_{\text{p}} - 1$.

The first six constraints for both problems have the same meaning as for the manufacturer's problem.

For the distributor's problem, the state vector $\mathbf{x}_{\text{dist}}$ consists on the sub-inventories levels and the quantity of overdue goods of the distributor and the control action vector $\mathbf{u}_{\text{dist}}$ consists on its control actions, $\mathbf{u}_{\text{dist\_in}}$ and $\mathbf{u}_{\text{dist\_out}}$. In this case, the constraint (A.8) states that what leaves from the distributor is equal to what reaches to the retailer.

For the retailer's problem, the state vector $\mathbf{x}_{\text{ret}}$ consists on the sub-inventories and the quantity of overdue goods of the retailer and the control action vector $\mathbf{u}_{\text{ret}}$ consists on its control actions, $\mathbf{u}_{\text{ret\_in}}$ and $\mathbf{u}_{\text{ret\_out}}$. The last constraint, (A.16) guarantees that the demand is fulfilled.

A.2 Distributed Model Predictive Control

The formulation for the distribution and retailing echelons' problems for the Distributed strategy is also very similar to the one presented in subsection 2.2.3 for the manufacturing echelon.

Figure A.2 and Figure A.3 present the distribution and retailing echelon controllers, respectively, and the information they have access to.

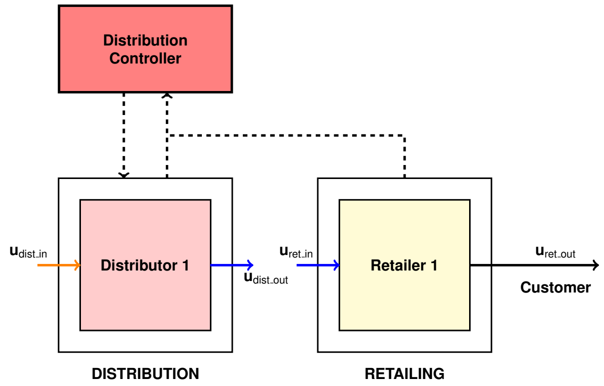


Figure A.2: Distributed Control Strategy for a Supply Chain configuration with three echelons and one player per echelon - Distribution Controller.

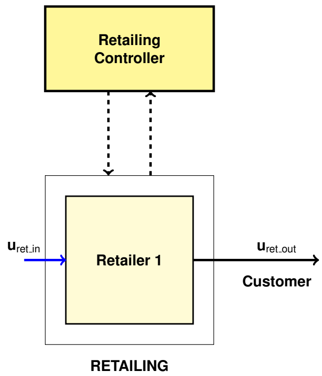


Figure A.3: Distributed Control Strategy for a Supply Chain configuration with three echelons and one player per echelon - Retailing Controller.

The formulation for the distributor's echelon problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{A.17}$$

$$\text{s.t.} \quad \mathbf{x}_{\mathrm{dist}}(k+1+l) = \mathbf{A}\mathbf{x}_{\mathrm{dist}}(k+l) + \mathbf{B}_{\mathrm{u}}\mathbf{u}_{\mathrm{dist}}(k+l), \tag{A.18}$$

$$\mathbf{x}_{\mathrm{dist}}(k+1+l) \geq \mathbf{0}, \tag{A.19}$$

$$\mathbf{u}_{\mathrm{dist}}(k+l) \geq \mathbf{0}, \tag{A.20}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}_{\mathrm{dist}}(k+1+l) \leq \mathbf{X}_{\mathrm{max}_{\mathrm{dist}}}, \tag{A.21}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}_{\mathrm{dist}}(k+l) \leq \mathbf{U}_{\mathrm{max}_{\mathrm{dist}}}, \tag{A.22}$$

$$\mathbf{P}_{\mathrm{xu}}\mathbf{u}_{\mathrm{dist}}(k+l) \leq \mathbf{P}_{\mathrm{ux}}\mathbf{x}_{\mathrm{dist}}(k+l), \tag{A.23}$$

$$\mathbf{u}_{\mathrm{dist\_out}}(k+l) = \mathbf{u}_{\mathrm{ret\_in}}(k+l), \tag{A.24}$$

$$\mathbf{P}_{\mathrm{d}}\mathbf{u}_{\mathrm{dist}}(k+l) = \mathbf{D}(k+l), \tag{A.25}$$

for $l = 0, \ldots, N_{\mathrm{p}} - 1$.

And the formulation for the distributor's echelon problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{A.26}$$

$$\text{s.t.} \quad \mathbf{x}_{\text{ret}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{ret}}(k+l) + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{ret}}(k+l), \tag{A.27}$$

$$\mathbf{x}_{\text{ret}}(k+1+l) \geq \mathbf{0}, \tag{A.28}$$

$$\mathbf{u}_{\text{ret}}(k+l) \geq \mathbf{0}, \tag{A.29}$$

$$\mathbf{P}_{\text{xx}}\mathbf{x}_{\text{ret}}(k+1+l) \leq \mathbf{X}_{\text{max}_{\text{ret}}}, \tag{A.30}$$

$$\mathbf{P}_{\text{uu}}\mathbf{u}_{\text{ret}}(k+l) \leq \mathbf{U}_{\text{max}_{\text{ret}}}, \tag{A.31}$$

$$\mathbf{P}_{\text{xu}}\mathbf{u}_{\text{ret}}(k+l) \leq \mathbf{P}_{\text{ux}}\mathbf{x}_{\text{ret}}(k+l), \tag{A.32}$$

$$\mathbf{P}_{\text{d}}\mathbf{u}_{\text{ret}}(k+l) = \mathbf{D}(k+l), \tag{A.33}$$

for $l = 0, \ldots, N_{\text{p}} - 1$.

The first six constraints for both problems have the same meaning as for the manufacturing echelon problem.

For the distribution echelon, the state vector $\mathbf{x}_{\text{dist}}$ consists on the sub-inventories levels and the quantity of overdue goods of both the distributor and retailer and the control action vector $\mathbf{u}_{\text{dist}}$ consists on their control actions, $\mathbf{u}_{\text{dist\_in}}$, $\mathbf{u}_{\text{dist\_out}}$, $\mathbf{u}_{\text{ret\_in}}$ and $\mathbf{u}_{\text{ret\_out}}$. In this case, the constraint (A.24) states that what leaves from the distributor needs to be equal to what reaches to the retailer. Constraint (A.25) guarantees the demand is fulfilled.

For the retailing echelon, the state vector $\mathbf{x}_{\text{ret}}$ consists on the sub-inventories and the quantity of overdue goods of the retailer and the control action vector $\mathbf{u}_{\text{ret}}$ consists on its control actions, $\mathbf{u}_{\text{ret\_in}}$ and $\mathbf{u}_{\text{ret\_out}}$. The last constraint, (A.33) guarantees that the demand is fulfilled. When there is only one player per echelon, the retailer's problem for both the Decentralized and Distributed strategies is essentially the same.

Appendix B

Model Predictive Control: Configuration 2

This appendix presents an explanation for the Centralized, Decentralized and Distributed strategies for the configuration represented in Figure 2.1(b). It also presents the formulation for the entire Supply Chain problem, each Supply Chain player's problem and each Supply Chain echelon's problem, for the Centralized, Decentralized and Distributed strategies, respectively.

B.1 Centralized Model Predictive Control

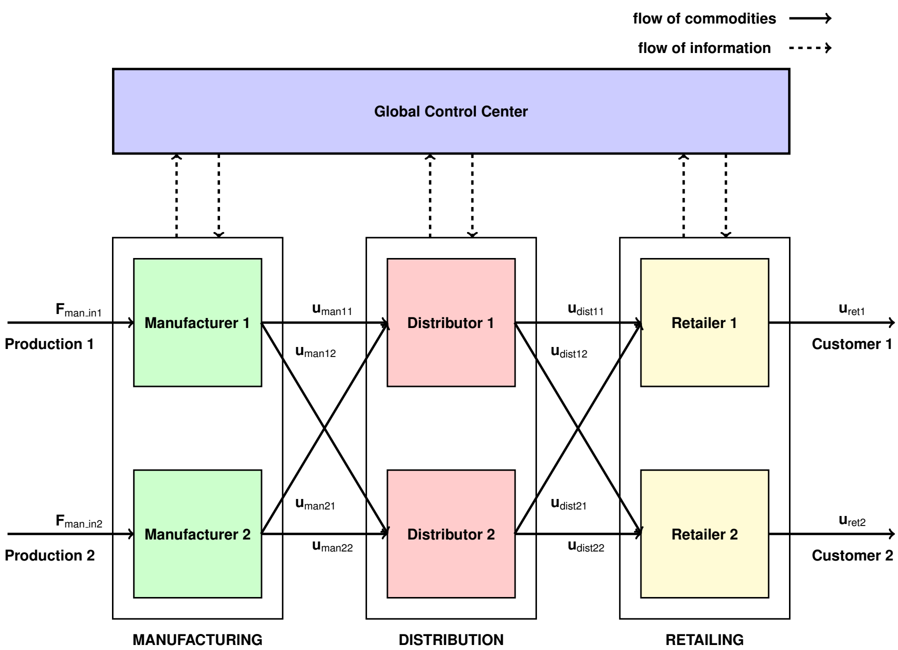


Figure B.1: Centralized Control Strategy for a Supply Chain configuration with three echelons and two players per echelon, considering the additional player - the Global Control Center.

This problem can be formulated as follows:

$$\min_{\tilde{\mathbf{u}}_k} \quad J(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k) \tag{B.1}$$

$$\text{s.t.} \quad \mathbf{x}(k+1+l) = \mathbf{A}\mathbf{x}(k+l) + \mathbf{B}_\mathrm{u}\mathbf{u}(k+l), \tag{B.2}$$

$$\mathbf{x}(k+1+l) \geq \mathbf{0}, \tag{B.3}$$

$$\mathbf{u}(k+l) \geq \mathbf{0}, \tag{B.4}$$

$$\mathbf{P}_\mathrm{xx}\mathbf{x}(k+1+l) \leq \mathbf{X}_\mathrm{max}, \tag{B.5}$$

$$\mathbf{P}_\mathrm{uu}\mathbf{u}(k+l) \leq \mathbf{U}_\mathrm{max}, \tag{B.6}$$

$$\mathbf{P}_\mathrm{xu}\mathbf{u}(k+l) \leq \mathbf{P}_\mathrm{ux}\mathbf{x}(k+l), \tag{B.7}$$

$$\mathbf{P}_\mathrm{d}\mathbf{u}(k+l) = \mathbf{D}(k+l), \tag{B.8}$$

for $l = 0, \ldots, N_\mathrm{p} - 1$.

The constraints presented for this configuration are very similar to the ones presented in subsection 2.2.1 for the configuration with one player per echelon. The major differences are related to the number of entities per echelon. This results in vectors and matrices with higher dimensions, since the total number of sub-inventories and control actions increases. Similarly to the previous case, the first constraint (B.2) corresponds to the model dynamics. Constraints (B.3) and (B.4) impose that there are no negative states nor negative flows of goods. Constraints (B.5) and (B.6) impose that the storage and transport capacities cannot be exceeded. Constraint (B.7) imposes that the quantity of goods that is taken from a player does not exceed the quantity of goods stored in that player. Finally, the last constraint, (B.8), guarantees that the customer demand is fulfilled. In this case, the model decides also the best way to divide the customer demand between the two retailers.

B.2 Decentralized Model Predictive Control

In this strategy, more players per echelon means more flows and states, leading to state and control actions vectors with higher dimensions and also to more compatibility constraints between players.

Figure B.2 presents the Supply Chain configuration with two players per echelon, the controllers for each player and their respective inflows and outflows discriminated.

The problems are still solved sequentially, starting from retailer 2, in the most downstream echelon and ending in manufacturer 1, which belongs to the most upstream echelon of the Supply Chain.

When solving retailer's 2 problem, the outflow $\mathbf{u}_{\text{ret\_out2}}$ is equal to the demand, so the only control actions that need to be determined are the inflows $\mathbf{u}_{\text{ret\_in21}}$, from distributor 1 to retailer 2, and $\mathbf{u}_{\text{ret\_in22}}$, from distributor 2 to retailer 2. Retailer's 1 problem is solved following a similar logic.

Then, distributor's 2 problem is solved. The outflows $\mathbf{u}_{\text{dist\_out21}}$, from distributor 2 to retailer 1, and $\mathbf{u}_{\text{dist\_out22}}$, from distributor 2 to retailer 2, are already determined because they need to be equal to the retailers' inflows, so the only control actions left to determine are the inflows $\mathbf{u}_{\text{dist\_in21}}$, from manufacturer 1 to distributor 2, and $\mathbf{u}_{\text{dist\_in22}}$, from manufacturer 2 to distributor 2. Distributor's 1 problem is solved following a similar logic.

Finally, when reaching to the manufacturer's echelon, manufacturer's 2 problem is solved, where the only variable yet to determine is the quantity of goods that will enter in the Supply Chain, $\mathbf{F}_{\text{man\_in2}}$, since the outflows $\mathbf{u}_{\text{man\_out21}}$, from manufacturer 2 to distributor 1, and $\mathbf{u}_{\text{man\_out22}}$, from manufacturer 2 to distributor 2 are already fixed. Then, manufacturer's 2 problem is solved following a similar logic.

After solving all these problems, for a specific time instant, the flows are implemented backwards, from the manufacturing echelon to the retailing echelon and the process repeats itself until arriving to the last time instant of the simulation.

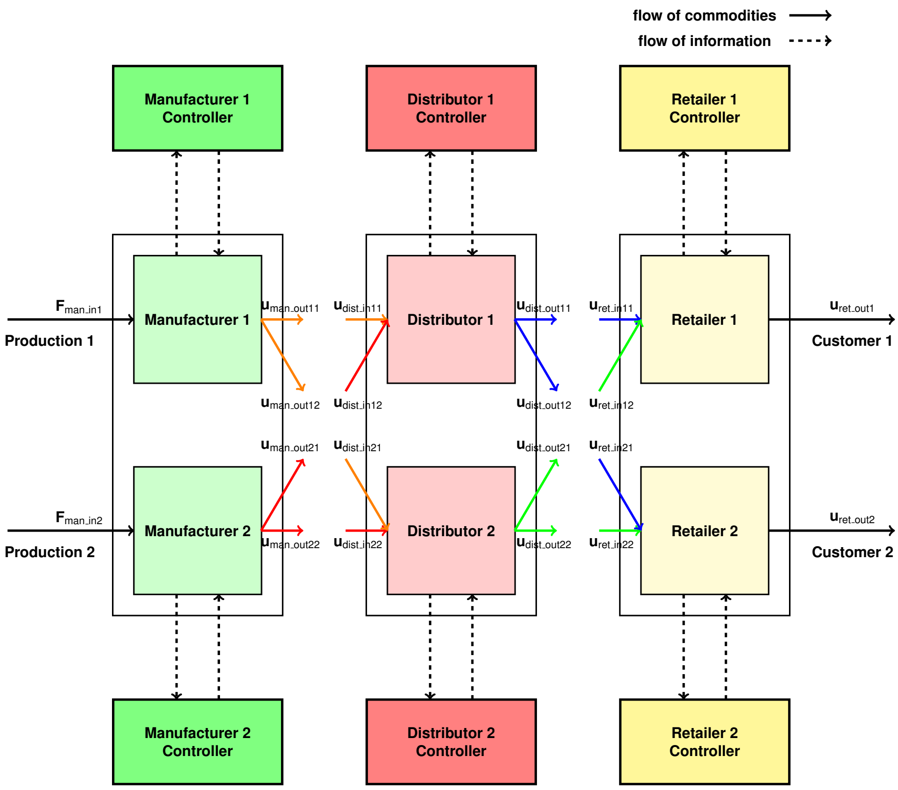


Figure B.2: Decentralized Control Strategy for a Supply Chain configuration with three echelons and two players per echelon.

Next, the formulation for each Supply Chain player problem is presented. Starting from manufacturer 1 and ending in retailer 2.

Figure B.3 represents manufacturer 1 problem, its controller and the limited information it has access to - the current quantity of goods stored in its sub-inventory, the storage capacity it is still available and the transport capacity of the links directly connected to it.

The formulation for this problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{B.9}$$

$$\text{s.t.} \quad \mathbf{x}_{\text{man1}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{man1}}(k+l) + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{man1}}(k+l), \tag{B.10}$$

$$\mathbf{x}_{\text{man1}}(k+1+l) \geq \mathbf{0}, \tag{B.11}$$

$$\mathbf{u}_{\text{man1}}(k+l) \geq \mathbf{0}, \tag{B.12}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}_{\mathrm{man1}}(k+1+l) \leq \mathbf{X}_{\mathrm{max}_{\mathrm{man1}}}, \tag{B.13}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}_{\mathrm{man1}}(k+l) \leq \mathbf{U}_{\mathrm{max}_{\mathrm{man1}}}, \tag{B.14}$$

$$\mathbf{P}_{\mathrm{xu}}\mathbf{u}_{\mathrm{man1}}(k+l) \leq \mathbf{P}_{\mathrm{ux}}\mathbf{x}_{\mathrm{man1}}(k+l), \tag{B.15}$$

$$\mathbf{u}_{\mathrm{man\_out11}}(k+l) = \mathbf{u}_{\mathrm{dist\_in11}}(k+l), \tag{B.16}$$

$$\mathbf{u}_{\mathrm{man\_out12}}(k+l) = \mathbf{u}_{\mathrm{dist\_in21}}(k+l), \tag{B.17}$$

for $l = 0, \ldots, N_{\mathrm{p}} - 1$.

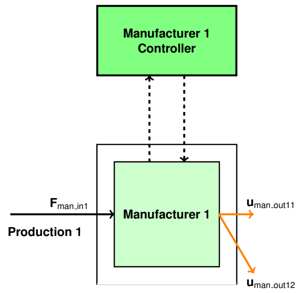


Figure B.3: Decentralized Control Strategy for a Supply Chain configuration with three echelons and two players per echelon - Manufacturer 1 Controller.

Constraint (B.10) represents the manufacturer 1 dynamics. Constraints (B.11) and (B.12) impose non-negative states and control actions. In this case, the state vector $\mathbf{x}_{\mathrm{man1}}(k+l)$ includes the goods' sub-inventory levels that manufacturer 1 has stored at each time instant and the control action vector $\mathbf{u}_{\mathrm{man1}}(k+l)$ consists in the inflows that enter from production - $\mathbf{F}_{\mathrm{man\_in1}}$ - and the outflows that leave for distributor 1 - $\mathbf{u}_{\mathrm{man\_out11}}$ - and distributor 2 - $\mathbf{u}_{\mathrm{man\_out12}}$. Constraints (B.13) and (B.14) impose that the storage and transport capacities cannot be exceeded. Constraint (B.15) imposes that what leaves from manufacturer 1, to distributors 1 and 2, $\mathbf{u}_{\mathrm{man\_out11}}$ and $\mathbf{u}_{\mathrm{man\_out12}}$, cannot exceed the quantity stored. And constraints (B.16) and (B.17) are compatibility constraints. constraints (B.16) imposes that what leaves from manufacturer 1 to distributor 1 is equal to what enters in distributor 1 from manufacturer 1. Constraint (B.17) imposes that what leaves from manufacturer 1 to distributor 1 is equal to what enters in distributor 1 from manufacturer 1. This means that there are no losses between players. This way, the only flow that yet to determine in this problem is the inflow $\mathbf{F}_{\mathrm{man\_in1}}$.
Figure B.4 presents manufacturer 2, its controller and inflows and outflows.

The formulation for this problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k) \tag{B.18}$$

$$\text{s.t.} \quad \mathbf{x}_{\mathrm{man2}}(k+1+l) = \mathbf{A}\mathbf{x}_{\mathrm{man2}}(k+l) + \mathbf{B}_{\mathrm{u}}\mathbf{u}_{\mathrm{man2}}(k+l), \tag{B.19}$$

$$\mathbf{x}_{\mathrm{man2}}(k+1+l) \geq \mathbf{0}, \tag{B.20}$$

$$\mathbf{u}_{\mathrm{man2}}(k+l) \geq \mathbf{0}, \tag{B.21}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}_{\mathrm{man2}}(k+1+l) \leq \mathbf{X}_{\mathrm{max}_{\mathrm{man2}}}, \tag{B.22}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}_{\mathrm{man2}}(k+l) \leq \mathbf{U}_{\mathrm{max}_{\mathrm{man2}}}, \tag{B.23}$$

$$\mathbf{P}_{\mathrm{xu}}\mathbf{u}_{\mathrm{man2}}(k+l) \leq \mathbf{P}_{\mathrm{ux}}\mathbf{x}_{\mathrm{man2}}(k+l), \tag{B.24}$$

$$\mathbf{u}_{\mathrm{man\_out21}}(k+l) = \mathbf{u}_{\mathrm{dist\_in12}}(k+l), \tag{B.25}$$

$$\mathbf{u}_{\mathrm{man\_out22}}(k+l) = \mathbf{u}_{\mathrm{dist\_in22}}(k+l), \tag{B.26}$$

for $l = 0, \ldots, N_{\mathrm{p}} - 1$.

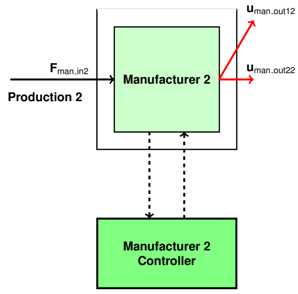


Figure B.4: Decentralized Control Strategy for a Supply Chain configuration with three echelons and two players per echelon - Manufacturer 2 Controller.

This problem's formulation is identical to the formulation presented for manufacturer 1. The difference between problems consists in the variables included in the state vector $\mathbf{x}_{\mathrm{man2}}(k+l)$, that now includes the goods' sub-inventory levels that manufacturer 2 has stored at each time instant, and the control action vector $\mathbf{u}_{\mathrm{man2}}(k+l)$, which now includes the inflows that enter from production - $\mathbf{F}_{\mathrm{man\_in2}}$ - and the outflows that leave for distributor 1 - $\mathbf{u}_{\mathrm{man\_out21}}$ - and distributor 2 - $\mathbf{u}_{\mathrm{man\_out22}}$.

Figure B.5 presents a representation of distributor 1, its controller and inflows and outflows of goods.

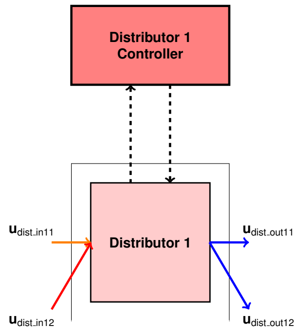


Figure B.5: Decentralized Control Strategy for a Supply Chain configuration with three echelons and two players per echelon - Distributor 1 Controller.

The formulation for this problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{B.27}$$

$$\text{s.t.} \quad \mathbf{x}_{\mathrm{dist1}}(k+1+l) = \mathbf{A}\mathbf{x}_{\mathrm{dist1}}(k+l) + \mathbf{B}_{\mathrm{u}}\mathbf{u}_{\mathrm{dist1}}(k+l), \tag{B.28}$$

$$\mathbf{x}_{\mathrm{dist1}}(k+1+l) \geq \mathbf{0}, \tag{B.29}$$

$$\mathbf{u}_{\mathrm{dist1}}(k+l) \geq \mathbf{0}, \tag{B.30}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}_{\mathrm{dist1}}(k+1+l) \leq \mathbf{X}_{\mathrm{max}_{\mathrm{dist1}}}, \tag{B.31}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}_{\mathrm{dist1}}(k+l) \leq \mathbf{U}_{\mathrm{max}_{\mathrm{dist1}}}, \tag{B.32}$$

$$\mathbf{P}_{\mathrm{xu}}\mathbf{u}_{\mathrm{dist1}}(k+l) \leq \mathbf{P}_{\mathrm{ux}}\mathbf{x}_{\mathrm{dist1}}(k+l), \tag{B.33}$$

$$\mathbf{u}_{\mathrm{dist\_out11}}(k+l) = \mathbf{u}_{\mathrm{ret\_in11}}(k+l), \tag{B.34}$$

$$\mathbf{u}_{\mathrm{dist\_out12}}(k+l) = \mathbf{u}_{\mathrm{ret\_in21}}(k+l), \tag{B.35}$$

for $l = 0, \ldots, N_{\mathrm{p}} - 1$.

This problem's formulation is essentially the same as seen for the manufacturers' problems. Now the state vector $\mathbf{x}_{\mathrm{dist1}}(k+l)$ includes the information related to the quantity of good's stored by distributor 1 at each time instant and the control action vector $\mathbf{u}_{\mathrm{dist1}}(k+l)$ includes information related to the flows of distributor 1. The last two constraints are now compatibility constraints between distributor 1 and retailers 1 and 2.

Figure B.6 presents distributor 2, its controller and inflows and outflows.

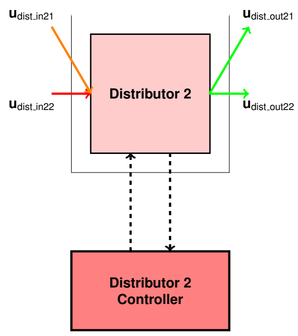


Figure B.6: Decentralized Control Strategy for a Supply Chain configuration with three echelons and two players per echelon - Distributor 2 Controller.

The formulation for this problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{B.36}$$

$$\text{s.t.} \quad \mathbf{x}_{\mathrm{dist2}}(k+1+l) = \mathbf{A}\mathbf{x}_{\mathrm{dist2}}(k+l) + \mathbf{B}_{\mathrm{u}}\mathbf{u}_{\mathrm{dist2}}(k+l), \tag{B.37}$$

$$\mathbf{x}_{\mathrm{dist2}}(k+1+l) \geq \mathbf{0}, \tag{B.38}$$

$$\mathbf{u}_{\mathrm{dist2}}(k+l) \geq \mathbf{0}, \tag{B.39}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}_{\mathrm{dist2}}(k+1+l) \leq \mathbf{X}_{\mathrm{max}_{\mathrm{dist2}}}, \tag{B.40}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}_{\mathrm{dist2}}(k+l) \leq \mathbf{U}_{\mathrm{max}_{\mathrm{dist2}}}, \tag{B.41}$$

$$\mathbf{P}_{\mathrm{xu}}\mathbf{u}_{\mathrm{dist2}}(k+l) \leq \mathbf{P}_{\mathrm{ux}}\mathbf{x}_{\mathrm{dist1}}(k+l), \tag{B.42}$$

$$\mathbf{u}_{\mathrm{dist\_out21}}(k+l) = \mathbf{u}_{\mathrm{ret\_in12}}(k+l), \tag{B.43}$$

$$\mathbf{u}_{\mathrm{dist\_out22}}(k+l) = \mathbf{u}_{\mathrm{ret\_in22}}(k+l), \tag{B.44}$$

for $l = 0, \ldots, N_{\mathrm{p}} - 1$.

This problem's formulation is identical to the formulation presented for distributor 1. The difference between problems consists in the variables included in the state vector $\mathbf{x}_{\mathrm{dist2}}(k+l)$, that now includes the goods' sub-inventory levels that distributor 2 has stored at each time instant, and the control action vector $\mathbf{u}_{\text{dist2}}(k+l)$, which now includes the inflows and outflows of distributor 2.

Figure B.7 presents retailer 1, its controller and inflows and outflows.

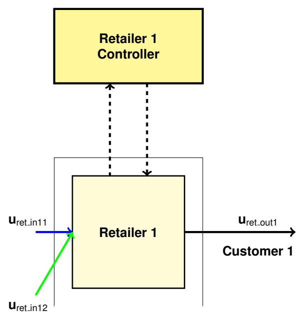


Figure B.7: Decentralized Control Strategy for a Supply Chain configuration with three echelons and two players per echelon - Retailer 1 Controller.

The formulation for this problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{B.45}$$

$$\text{s.t.} \quad \mathbf{x}_{\text{ret1}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{ret1}}(k+l) + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{ret1}}(k+l), \tag{B.46}$$

$$\mathbf{x}_{\text{ret1}}(k+1+l) \geq \mathbf{0}, \tag{B.47}$$

$$\mathbf{u}_{\text{ret1}}(k+l) \geq \mathbf{0}, \tag{B.48}$$

$$\mathbf{P}_{\text{xx}}\mathbf{x}_{\text{ret1}}(k+1+l) \leq \mathbf{X}_{\text{max}_{\text{ret1}}}, \tag{B.49}$$

$$\mathbf{P}_{\text{uu}}\mathbf{u}_{\text{ret1}}(k+l) \leq \mathbf{U}_{\text{max}_{\text{ret1}}}, \tag{B.50}$$

$$\mathbf{P}_{\text{xu}}\mathbf{u}_{\text{ret1}}(k+l) \leq \mathbf{P}_{\text{ux}}\mathbf{x}_{\text{ret1}}(k+l), \tag{B.51}$$

$$\mathbf{P}_{\text{d}}\mathbf{u}_{\text{ret1}}(k+l) = \mathbf{D}_1(k+l), \tag{B.52}$$

for $l = 0, \ldots, N_{\text{p}} - 1$.

The first six constraints, (B.46)-(B.51) have the same meaning as seen for the problems presented previously. The last constraint, (B.52) is related to customer demand satisfaction. Retailer 1 guarantees that half of the customer demand is fulfilled. The state vector $\mathbf{x}_{\text{ret1}}(k+l)$ includes the goods' sub-inventory levels that retailer 1 has stored at each time instant and the control action vector $\mathbf{u}_{\text{ret1}}(k+l)$ includes the inflows and outflows of retailer 1.

Figure B.8 presents retailer 2, its controller and inflows and outflows.

The formulation for this problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{B.53}$$

$$\text{s.t.} \quad \mathbf{x}_{\text{ret2}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{ret2}}(k+l) + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{ret2}}(k+l), \tag{B.54}$$

$$\mathbf{x}_{\text{ret2}}(k+1+l) \geq \mathbf{0}, \tag{B.55}$$

$$\mathbf{u}_{\text{ret2}}(k+l) \geq \mathbf{0}, \tag{B.56}$$

$$\mathbf{P}_{\text{xx}}\mathbf{x}_{\text{ret2}}(k+1+l) \leq \mathbf{X}_{\max_{\text{ret2}}}, \tag{B.57}$$

$$\mathbf{P}_{\text{uu}}\mathbf{u}_{\text{ret2}}(k+l) \leq \mathbf{U}_{\max_{\text{ret2}}}, \tag{B.58}$$

$$\mathbf{P}_{\text{xu}}\mathbf{u}_{\text{ret2}}(k+l) \leq \mathbf{P}_{\text{ux}}\mathbf{x}_{\text{ret2}}(k+l), \tag{B.59}$$

$$\mathbf{P}_{\text{d}}\mathbf{u}_{\text{ret2}}(k+l) = \mathbf{D}_{2}(k+l), \tag{B.60}$$

for $l = 0, \ldots, N_{\text{p}} - 1$.

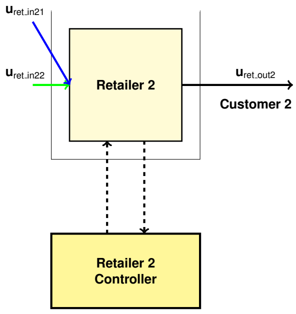


Figure B.8: Decentralized Control Strategy for a Supply Chain configuration with three echelons and two players per echelon - Retailer 2 Controller.

This problem's formulation is identical to the formulation presented for retailer 1. The difference between problems consists in the variables included in the state vector $\mathbf{x}_{\text{ret2}}(k+l)$, that now includes the goods' sub-inventory levels that retailer 2 has stored at each time instant, and the control action vector $\mathbf{u}_{\text{ret2}}(k+l)$, which now includes the inflows and outflows of retailer 2. The last constraint guarantees that the customer demand that is not fulfilled by retailer 1 is fulfilled by retailer 2.

B.3 Distributed Model Predictive Control

In this strategy, each echelon is managed by one controller. The best control actions for players belonging to the same echelon are found simultaneously. However, echelons' problems are still solved sequentially, starting from the retailing echelon and ending in the manufacturer echelon. Figure B.9 presents the controllers per echelon and the discriminated flows per player for the Supply Chain configuration with two players per echelon.

When solving the retailers problem, the controller has only access to the information regarding both retailers - their storage and transport capacities, their sub-inventory levels and demand for each time instant and their predictions for future time instants.

The controller of the distribution echelon has access to the information regarding both distributors plus the information regarding both retailers, but it does not have access to the manufacturers' information.

The controller of the manufacturing echelon has now visibility over the entire Supply Chain.

Next, the formulation for each echelon's problem is presented, starting from the manufacturing echelon

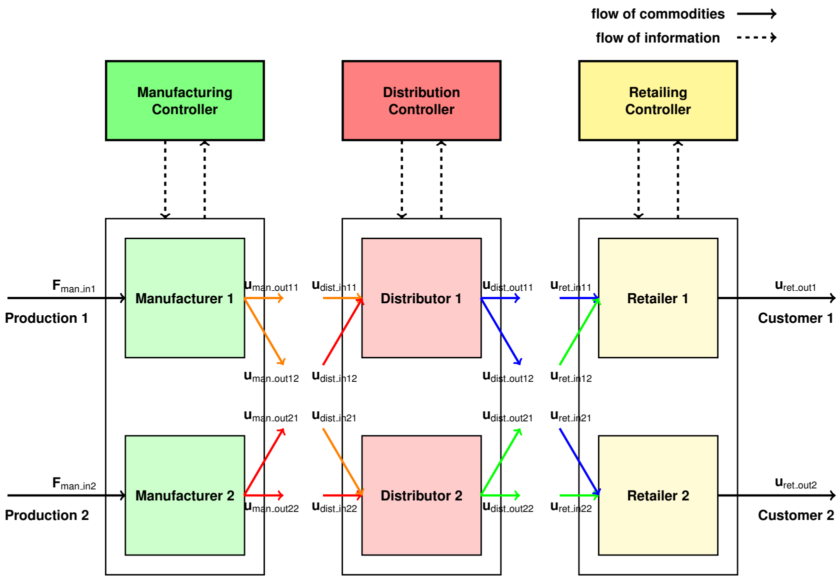


Figure B.9: Distributed Control Strategy for a Supply Chain configuration with three echelons and two players per echelon.

and ending in the retailing echelon.

Figure B.10 presents the manufacturing echelon's controller and the information it has access to.

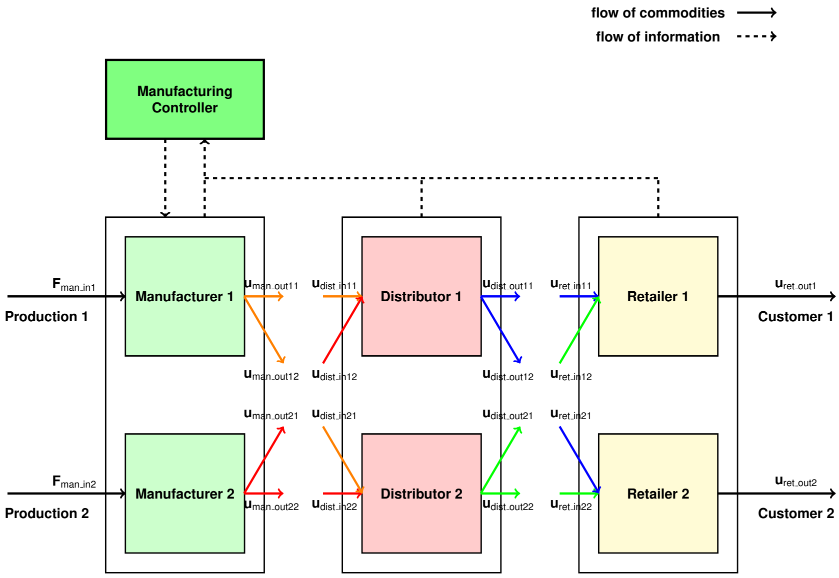


Figure B.10: Distributed Control Strategy for a Supply Chain configuration with three echelons and two players per echelon - Manufacturing Controller.

The formulation for this problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{B.61}$$

$$\text{s.t.} \quad \mathbf{x}_{\text{man}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{man}}(k+l) + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{man}}(k+l), \tag{B.62}$$

$$\mathbf{x}_{\text{man}}(k+1+l) \geq \mathbf{0}, \tag{B.63}$$

$$\mathbf{u}_{\mathrm{man}}(k+l) \geq \mathbf{0}, \tag{B.64}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}_{\mathrm{man}}(k+1+l) \leq \mathbf{X}_{\mathrm{max}_{\mathrm{man}}}, \tag{B.65}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}_{\mathrm{man}}(k+l) \leq \mathbf{U}_{\mathrm{max}_{\mathrm{man}}}, \tag{B.66}$$

$$\mathbf{P}_{\mathrm{xu}}\mathbf{u}_{\mathrm{man}}(k+l) \leq \mathbf{P}_{\mathrm{ux}}\mathbf{x}_{\mathrm{man}}(k+l), \tag{B.67}$$

$$\mathbf{u}_{\mathrm{man\_out11}}(k+l) + \mathbf{u}_{\mathrm{man\_out21}}(k+l) = \mathbf{u}_{\mathrm{dist\_in11}}(k+l) + \mathbf{u}_{\mathrm{dist\_in12}}(k+l), \tag{B.68}$$

$$\mathbf{u}_{\mathrm{man\_out12}}(k+l) + \mathbf{u}_{\mathrm{man\_out22}}(k+l) = \mathbf{u}_{\mathrm{dist\_in21}}(k+l) + \mathbf{u}_{\mathrm{dist\_in22}}(k+l), \tag{B.69}$$

$$\mathbf{u}_{\mathrm{dist\_out11}}(k+l) = \mathbf{u}_{\mathrm{ret\_in11}}(k+l), \tag{B.70}$$

$$\mathbf{u}_{\mathrm{dist\_out12}}(k+l) = \mathbf{u}_{\mathrm{ret\_in21}}(k+l), \tag{B.71}$$

$$\mathbf{u}_{\mathrm{dist\_out21}}(k+l) = \mathbf{u}_{\mathrm{ret\_in12}}(k+l), \tag{B.72}$$

$$\mathbf{u}_{\mathrm{dist\_out22}}(k+l) = \mathbf{u}_{\mathrm{ret\_in22}}(k+l), \tag{B.73}$$

$$\mathbf{P}_{\mathrm{d}}\mathbf{u}_{\mathrm{man}}(k+l) = \mathbf{D}(k+l), \tag{B.74}$$

for $l = 0, \ldots, N_{\mathrm{p}} - 1$.

Constraints (B.62) - (B.67) have the same meaning as seen before for the Centralized and Decentralized strategies. The state vector $\mathbf{x}_{\mathrm{man}}(k+l)$ and the control action vector $\mathbf{u}_{\mathrm{man}}(k+l)$ now include the sub-inventories levels and the control actions of the entire Supply Chain, respectively. The last constraint, (B.74), guarantees that the customer demand is fulfilled.

Constraints (B.68) - (B.73) are compatibility constraints that reflect the communication dynamics that exists for the Distributed strategy. Constraints (B.70) - (B.73) guarantee that what leaves from a player of the distribution echelon is equal to what reaches to a player of the retailing echelon. These control actions are fixed and equal to the ones found for the distribution echelon's problem. However, constraints (B.68) - (B.69) are softer than the ones presented for the Decentralized strategy. In this case, constraint (B.68) imposes that what leaves from the manufacturing echelon to distributor 1, needs to be equal to what enters distributor 1. This way, the manufacturing controller has more freedom to decide the best control actions to apply. Constraint (B.69) imposes the same relation between the manufacturing echelon and the distributor 2.

Figure B.11 presents the distribution echelon's controller and the information it has access to.

The formulation for this problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{B.75}$$

$$\text{s.t.} \quad \mathbf{x}_{\mathrm{dist}}(k+1+l) = \mathbf{A}\mathbf{x}_{\mathrm{dist}}(k+l) + \mathbf{B}_{\mathrm{u}}\mathbf{u}_{\mathrm{dist}}(k+l), \tag{B.76}$$

$$\mathbf{x}_{\mathrm{dist}}(k+1+l) \geq \mathbf{0}, \tag{B.77}$$

$$\mathbf{u}_{\mathrm{dist}}(k+l) \geq \mathbf{0}, \tag{B.78}$$

$$\mathbf{P}_{\mathrm{xx}}\mathbf{x}_{\mathrm{dist}}(k+1+l) \leq \mathbf{X}_{\mathrm{max}_{\mathrm{dist}}}, \tag{B.79}$$

$$\mathbf{P}_{\mathrm{uu}}\mathbf{u}_{\mathrm{dist}}(k+l) \leq \mathbf{U}_{\mathrm{max}_{\mathrm{dist}}}, \tag{B.80}$$

$$\mathbf{P}_{\mathrm{xu}}\mathbf{u}_{\mathrm{dist}}(k+l) \leq \mathbf{P}_{\mathrm{ux}}\mathbf{x}_{\mathrm{dist}}(k+l), \tag{B.81}$$

$$\mathbf{u}_{\text{dist\_out11}}(k+l) + \mathbf{u}_{\text{dist\_out21}}(k+l) = \mathbf{u}_{\text{ret\_in11}}(k+l) + \mathbf{u}_{\text{ret\_in12}}(k+l), \tag{B.82}$$

$$\mathbf{u}_{\text{dist\_out12}}(k+l) + \mathbf{u}_{\text{dist\_out22}}(k+l) = \mathbf{u}_{\text{ret\_in21}}(k+l) + \mathbf{u}_{\text{ret\_in22}}(k+l), \tag{B.83}$$

$$\mathbf{P}_{\text{d}}\mathbf{u}_{\text{dist}}(k+l) = \mathbf{D}(k+l), \tag{B.84}$$

for $l = 0, \ldots, N_{\text{p}} - 1$.

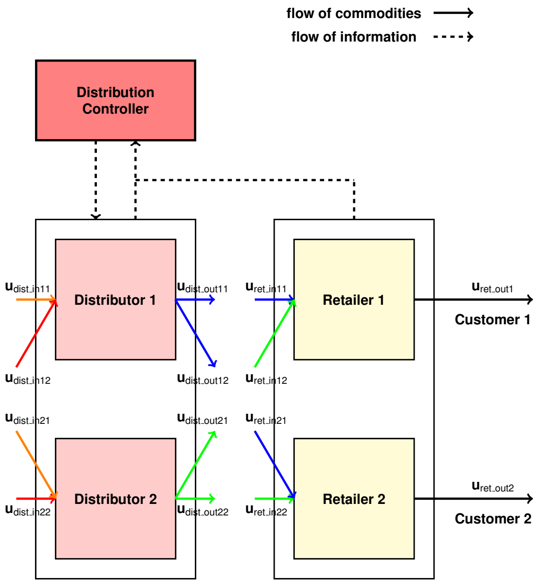


Figure B.11: Distributed Control Strategy for a Supply Chain configuration with three echelons and two players per echelon - Distribution Controller.

The problem of the distribution echelon is essentially the same as the one presented for the manufacturing echelon. In this case, the state vector $\mathbf{x}_{\text{dist}}(k+l)$ and the control action vector $\mathbf{u}_{\text{dist}}(k+l)$ include the quantity of goods stored and the flows of both distributors and retailers, respectively. The compatibility constraints, (B.82) and (B.83) are now only applied between the distribution and the retailing echelon, since the distribution echelon does not have access to the information regarding the manufacturing echelon.

Lastly, Figure B.12 presents the retailing echelon's controller and the information it has access to.

The formulation for this problem is given by:

$$\min_{\tilde{\mathbf{u}}_k} \quad J\left(\tilde{\mathbf{x}}_k, \tilde{\mathbf{u}}_k\right) \tag{B.85}$$

$$\text{s.t.} \quad \mathbf{x}_{\text{ret}}(k+1+l) = \mathbf{A}\mathbf{x}_{\text{ret}}(k+l) + \mathbf{B}_{\text{u}}\mathbf{u}_{\text{ret}}(k+l), \tag{B.86}$$

$$\mathbf{x}_{\text{ret}}(k+1+l) \geq \mathbf{0}, \tag{B.87}$$

$$\mathbf{u}_{\text{ret}}(k+l) \geq \mathbf{0}, \tag{B.88}$$

$$\mathbf{P}_{\text{xx}}\mathbf{x}_{\text{ret}}(k+1+l) \leq \mathbf{X}_{\text{max}_{\text{ret}}}, \tag{B.89}$$

$$\mathbf{P}_{\text{uu}}\mathbf{u}_{\text{ret}}(k+l) \leq \mathbf{U}_{\text{max}_{\text{ret}}}, \tag{B.90}$$

$$\mathbf{P}_{\text{xu}}\mathbf{u}_{\text{ret}}(k+l) \leq \mathbf{P}_{\text{ux}}\mathbf{x}_{\text{ret}}(k+l), \tag{B.91}$$

$$\mathbf{P}_{\text{d}}\mathbf{u}_{\text{ret}}(k+l) = \mathbf{D}(k+l), \tag{B.92}$$

for $l = 0, \ldots, N_\mathrm{p} - 1$.

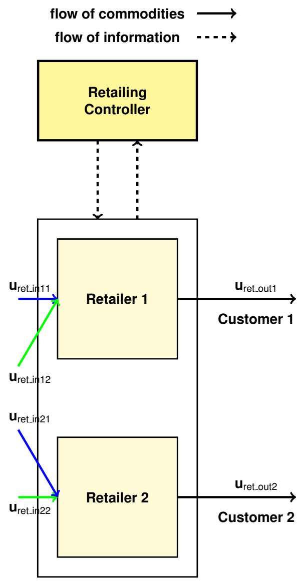


Figure B.12: Distributed Control Strategy for a Supply Chain configuration with three echelons and two players per echelon - Retailing Controller.

The formulation for this problem is essentially the same as seen for the other two echelons. The state vector $\mathbf{x}_\mathrm{ret}(k+l)$ and the control action vector $\mathbf{u}_\mathrm{ret}(k+l)$ include the sub-inventories levels and the inflows and outflows of both retailers, respectively. For this problem, there are no compatibility constraints, since the retailing echelon does not have information regarding both the manufacturing and the distribution echelons.

Another set of compatibility constraints was tested for the manufacturing and distribution echelons.

For the manufacturing echelon, constraints (B.68)-(B.69) were substituted by:

$$\begin{aligned}\mathbf{u}_\mathrm{man\_out11}(k+l) + \mathbf{u}_\mathrm{man\_out21}(k+l) + \mathbf{u}_\mathrm{man\_out12}(k+l) + \mathbf{u}_\mathrm{man\_out22}(k+l) = \\ \mathbf{u}_\mathrm{dist\_in11}(k+l) + \mathbf{u}_\mathrm{dist\_in12}(k+l) + \mathbf{u}_\mathrm{dist\_in21}(k+l) + \mathbf{u}_\mathrm{dist\_in22}(k+l)\end{aligned} \tag{B.93}$$

For the distribution echelon, constraints (B.82)-(B.83) were substituted by:

$$\begin{aligned}\mathbf{u}_\mathrm{dist\_out11}(k+l) + \mathbf{u}_\mathrm{dist\_out21}(k+l) + \mathbf{u}_\mathrm{dist\_out12}(k+l) + \mathbf{u}_\mathrm{dist\_out22}(k+l) = \\ \mathbf{u}_\mathrm{ret\_in11}(k+l) + \mathbf{u}_\mathrm{ret\_in12}(k+l) + \mathbf{u}_\mathrm{ret\_in21}(k+l) + \mathbf{u}_\mathrm{ret\_in22}(k+l)\end{aligned} \tag{B.94}$$

This way, the controllers have more freedom to find the best control actions. However, for this Supply Chain configuration, although there were some slight differences related to the distribution of overdue products between the Supply Chain players, the total quantity of overdue products was equal to the one obtained using the first set of compatibility constraints.