Analysis of Cryptocurrencies Exchange Rates

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June 2019

Abstract

Cryptocurrencies have drawn tremendous attention in the financial markets over the past few years. Analyzing the distributional properties and stylized facts of cryptocurrency exchange rates is beneficial for investors to assess their portfolio and it is relevant for regulatory agents to assess the market risk of financial institutions. Despite this, research on the properties of these exchange rates is relatively limited due to its recent manifestation. The nature of cryptocurrency prices, hence the associated exchange rates, are expected to be different from traditional currency exchange rates. The purpose of this project is to analyze statistical properties of cryptocurrency exchange rates through a new approach of recording price movements. An event-based time scale that captures periodic activities in the market helps to distillate a set of indicators capturing important information, and demonstrate that this indicators help construct directional change profiles of cryptocurrencies markets, considering that the use of physical time scales for studying financial time series runs the risk of missing important activities. The study confirms the usefulness of the price analysis approach through the use of an intrinsic time and the results corroborate that this method has advantages over time series sampling based on a fixed time interval.

Keywords: Directional Changes, Intrinsic Time, Time-Series Analysis, Stylized Facts, Cryptocurrency Market.

1. Introduction

The work developed links three main topics: the financial market, more specifically the cryptocurrency market, an event-based approach to observe the price movement as opposed to a fixed time interval normally used in time series and the concept of stylized facts.

During the research phase, it was found that some studies have coupled the world of the financial market, which includes not only the stock indexes but the also the forex market (foreign exchange market for trading currencies pairs), with the modern concept of directional changes. The application of this framework has yielded interesting results, therefore bringing people to analyse time series through the lens of the DC framework.

Likewise, the adoption of stylized facts, as opposed to bare facts, to explain a number of characteristics features common across stock indexes and exchange rates have been implemented as an instrument to expand the current knowledge in the field avoiding the numerous statistical descriptions.

Nevertheless, there is no academic study that blends the three topics all together. The idea with this endeavor is to analyse the statistical properties of the recently embraced cryptocurrency market within the scope of the directional change framework with the goal of extracting stylized facts that "possibly" helps on the understanding of the proclaimed: "coin of the future".

Today cryptocurrencies have become a global phenomenon. Due to the nature and technology behind it, it has caught the attention of investors, governments, regulatory agents and academics.

Emerging debates over the nature of cryptocurrency markets have been taking place all over social media and academic institutions. The first empirical analysis of eight forms of cryptocurrencies (70% of cryptocurrency market capitalization) (Zhang, Wang, Li, & Shen, 2018) concluded that: there are heavy tails, absence of autocorrelations, volatility clustering, leverage effect and long-range dependence in the returns of cryptocurrencies. There exists power-law correlation between price and volume.

(Phillip, Chan, & Peiris, 2018) The empirical data analysis shows Cryptocurrencies exhibit long memory, leverage effects, stochastic volatility and heavy tailedness.

Stylized facts refer to some important statistical properties of random variables of assets prices. (Bariviera, Basgall, Hasperué, & Naiouf, 2017) investigates some statistical properties of the bitcoin market and concluded that bitcoin presents a large volatility, it is reducing over time, long range memory is not related to market liquidity.

Recently, in the forex market of fiat currencies, a new approach has been adopted that revealed new insights. The concept of Directional Changes has allowed a new approach to summarize the price movements in financial markets, which consists of an event-based framework. The idea behind it focuses on the extracting a new intrinsic time unit, independent of the notion of physical time, defined by events. The directional change marks the market as downtrend or uptrend events based on a threshold defined the investor. The DC concept has been proved many times to be helpful in the study of the FX market, specially to establish new mathematical relations. (Guillaume et al., 1997) used the concept of DC to derive a scaling law that established a relation between the number of directional changes to the size of the threshold.

(Glattfelder, Dupuis, & Olsen, 2011) reported twelve new scaling laws by analysing 14 different currency pairs using the DC concept. This played an important role in describing complex systems, establishing invariance of scale in FX market.

(Bisig, Dupuis, Impagliazzo, & Olsen, 2012) measures the impact of political and economic events on the currency markets. Just like Richter Scale measures the intensity of earthquakes, the scale of market quakes (SMQ) quantifies the shock in the forex market caused by major economic and political events. DC concept has also helped created new indicators in the FX market as an alternative to common time indicators (E. P. K. Tsang, Tao, Serguieva, & Ma, 2015) and (E. Tsang & Ma, n.d.).

Not only the DC concept was used to establish mathematical relationships and developing new indicators but also to generate new successful trading strategies and profitable forecasting. (Ye, Raju Chinthalapati, Serguieva, & Tsang, 2018), (Aloud, Tsang, Olsen, & Dupuis, 2012), (Bakhach, Tsang, & Chinthalapati, n.d.), (Kampouridis & Otero, 2017) developed trading strategies that joins the concept of DC concept with traditional indicators of technical analysis

The most relevant studies and papers are very recent, which demonstrates that this new field is far from being understood. This project hopes to deepen the knowledge about Directional Changes, Cryptocurrencies and see how this different concepts may help us in comprehend the true value of cryptocurrency market.

2. Background

2.1. Physical Time vs Intrinsic Time

A regular decision for an agent when looking at the financial market is to select the fixed time interval that he wishes to inspect the price, ranging from seconds through hourly, daily, monthly to annually changes. This is due to the inherent characteristics of a trade. A trade happens every time someone places an order and there is a consent on the present price of an asset. Therefore, the value is changing with time at an heterogeneous rate. This is commonly referred in the financial field as "tick-by-tick" data, where "tick" is a "trade". Consequently, the price is presented when a transaction is placed at a specific time.

The following figure, it is evident that there are considerable patterns being overlooked from the fact that the information is compacted by taking the mean value of the price during the time interval decided by the trader. This phenomenon is highlighted when working with high-frequency data. Hence, the point here is that with this analysis, there is actually a loss of information that may contain essential intelligence vital for the correct comprehension of the behaviour of the markets.

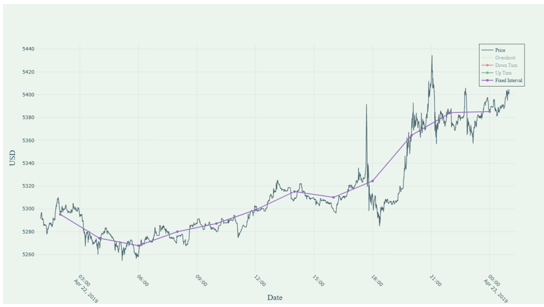

Figure 1: This graph shows the price curve of Bitcoin during one day: April 22, 2019 between 06:00 - 18:00. The blue line represents the value of the asset at an heterogeneous rate while the purple line illustrates the value of the stock averaged every hour and labelled to the right side. This second line typify the standard way an investor analyse the market, fixed-time interval

To deal with this issue, different solutions have been proposed among them is the study of price time series using intrinsic time.

In intrinsic time, time is defined by events. This event-based approach withdraws two single occurrences: uptrend and downturn events. An event is the basic unit of intrinsic time and characterized by a threshold θ of fixed magnitude, in contrast with the point-based system adopted by physical time scales. The physical time scale is homogeneous in which time scales equally spaced on any chosen time scale seconds, minute scale. In contrast, intrinsic time is inhomogeneous in time and independent of the notion of the physical time scales where any occurrence of an event represents a new intrinsic time.

Figure 2: This graph shows the price curve of Bitcoin during one day: April 22, 2019 between 06:00 - 18:00. The blue line represents the value of the asset at an heterogeneous rate while the other lines show the intrinsic time triggered only at periodic events across different magnitudes in the price curve.

2.2. Directional Changes

A directional-change (DC) event can take one of the two forms- a downturn event or an upturn event. An uptrend terminates when a Downturn event takes place. Similarly, a downtrend termintes an upturn C event takes place. Therefore, the DC event approach defines a price time series in FM as a sequence of:

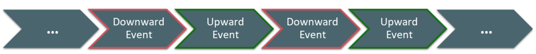

Figure 3: This figure represents the sequence of events in directional change. That is to say that after every down trend there is an up trend and after this the occurrence of another down trend. This loop goes on, simplifying the price movements either in and up or down direction.

Each event is composed of two steps: a directional change event and an overshoot event.

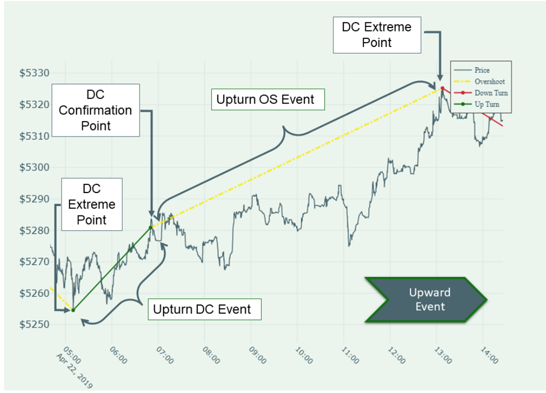

Figure 4: An upward event is characterized by an upturn directional change event and an upturn overshoot event. At the extremities a trend is confined by two extreme points and in the transition exists the directional change confirmation point.

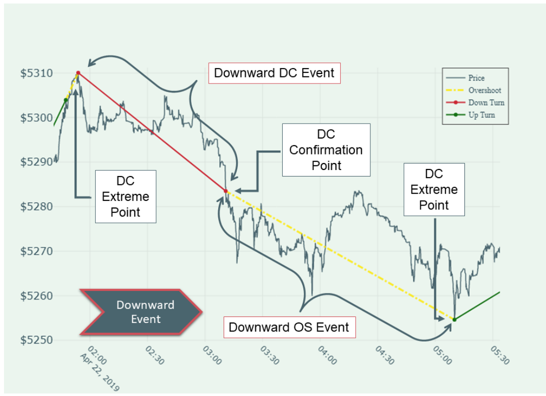

Figure 5: An downward event is also characterized by a downturn directional change event and a downturn overshoot event. At the extremities a trend is confined by two extreme points and in the transition exists the directional change confirmation point.

The directional change event is determined by a price variation considered by the observer. Here the threshold is a percentage that the user considers significant for the specific situation. One observer may consider 0.05% a significant change, while another observer may consider 5% is significant. Observers who use different thresholds will observe different DC events and trends.

2.3. Spectral Analysis

Studying the price series under physically fixed time intervals require division of time into periods of equal length which have the drawback of missing major price movements. On the other hand, directional change events are significant events as they capture periodic major change in a price time series in which the magnitude of an event is defined by the observer. Therefore, diverse time periods in a price time series possibly will enclose a different number of DC (NDC) events of a different magnitude. If such statement hold, one can state that the price evolution is independent of physically fixed time changes. To verify this the following exercise was made:

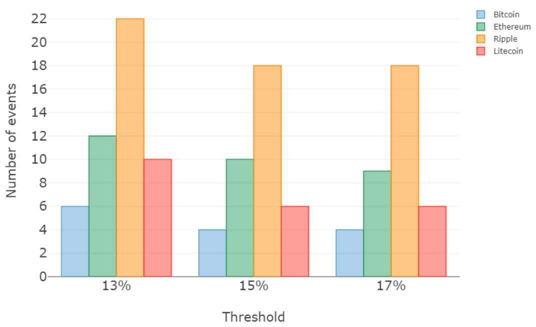

Figure 6: The bar graph represents the number of directional changes events using a threshold of 13%,15% and 17% during the month of January 2018

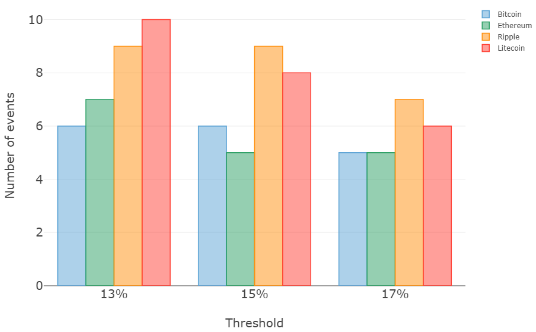

Figure 7: The bar graph represents the number of directional changes events using a threshold of 13%,15% and 17% during the month of February 2018

The reported draw attention to two vital observations:

1. For different cryptocurrencies indices the same time periods with the same threshold magnitude on different days may possibly enclose a different NDC events

2. With the same threshold magnitude, some time periods on the same day have more DC events than others

These two observations point out those DC events of different magnitude are independent of physical time changes.

3. Implementation

3.1. Data Set

The cryptocurrency data are from a website https://www.cryptodatadownload.com, which includes the timestamp and the close price every hour from July 2017 to May 2019 (not included). In particular, we focus on the top 4 cryptocurrencies (according to their market value) capitalization. The selected four forms of cryyptocurrencies account for 76,08% of the total capitalization, therefore the sample is an appropriate representation of the cryptocurrency market. The following cryptocurrencies were used in the study: Bitcoin (BTC), XRP (Ripple), ETH (Ethereum), LTC (Litecoin). The number of ticks is 15370. The data is filtered as missing data from certain timestamps get the median price of the previous and following consecutive prince.

3.2. Indicators Summary

Directional Changes is a new way of summarizing price changes. In this section, it is used a set of indicators which are useful for extracting information. With this indicators, the aim is to construct profiles for price changes summarized under the DC framework. Consequently this indicators can be used in different ways: financial decision makers can use the DC indicators as a toll to filter the significance of the price dynamics in a time series, analytical results can be used as an input to forecasting or automated trading models to identify investment opportunities and adjust the input factors. These DC indicators can be computed from Algorithm 1.

Resuming the all the indicators presented in the previous section, the following table dissects one possible way to feature extract information from the time series in directional changes.

Directional Change Indicator:	Acronym:	Description:
Number of directional change events	N_{DC}	measures the frequency of DCs
Overshoot Values at Extreme Points	OSV	measures the magnitude of an overshoot
Time for completion of a trend	TT	measures the time that it takes to complete a trend
Total Price Movements Value at Extreme Points	TM	measure the scale of prices changes
Number of directional change events in Sub-threshold	$Sub - N_{DC}$	measures the frequency of DCs in each DC trend
Undershoot Value at Extreme Points	USV	measures the scale price changes in each DC trend
Time independent Coastline	$LenC$	maximum possible returns over the profiled period
Time-adjusted return of DC	R_{DC}	measures the return in each upturn or downturn

Table 1: Directional change indicators summary

3.3. Implementation of Directional Changes - Algorithm

Algorithm 1 Defining directional-change (DC) and overshoot (OS) events

if *event is Upturn Event* **then**
  if $p(t) \leq p^h \times (1 - \Delta x_{dc})$ **then**
    $event \leftarrow Downturn\ Event$
    $p^l \leftarrow p(t)$
    $t_1^{dc} \leftarrow t$ // End time for a Downturn Event
    $t_0^{os} \leftarrow t + 1$ // Start time for a Downward Overshoot Event
  else
    if $p^h < p(t)$ **then**
      $p^h \leftarrow p(t)$
      $t_0^{dc} \leftarrow t$ // Start time for a Downturn Event
      $t_1^{os} \leftarrow t + 1$ // End time for am Upward Overshoot Event
    end if
  end if
else
  if $p(t) \geq p^l \times (1 + \Delta x_{dc})$ **then**
    $event \leftarrow UpturnEvent$
    $p^h \leftarrow p(t)$
    $t_1^{dc} \leftarrow t$ // End time for an Upturn Event
    $t_0^{os} \leftarrow t + 1$ // Start time for an Upward Overshoot Event
  else
    if $p^l > p(t)$ **then**
      $p^l \leftarrow p(t)$
      $t_0^{dc} \leftarrow t$ // Start time for an Upturn Event
      $t_1^{os} \leftarrow t + 1$ // End time for an Upward Overshoot Event
    end if
  end if
end if

3.4. The Price Curve Coastline Comparison

With the aim of assessing the performance of the two different concepts: intrinsic time (directional-change events) and physical time changes (fixed time intervals), it is used a method for estimating the length of the price-curve coastline based on the price distance between fixed points. Basically, using the ***LenC*** indicator explained in the previous section. Assuming perfect foresight (meaning the ability to predict future events), the length of the price-curve coastline over a define time period T, represents the potential profit.

It is therefore plausible to expect the same behaviour from the cryptocurrency market. Regarding this, it is used tick-by-tick data from www.suissebank.com of the bitcoin price in US dollars from April 22,2019.

The measurement of the length of the price-curve coastline over a time period T, as defined by intrinsic time, is the average upwards and downards price moves, considering the number of events. Both the upwards and the dowards price movements are defined by a fixed threshold size θ. Under the intrinsic time, the length of the price-curve coastline (PCC) is measured by:

$$\boldsymbol{PCC_\theta} = \frac{\sum_{i=1}^{\boldsymbol{N_{DC}}} |P_{EXT_{i+1}} - P_{EXT_i}|}{\boldsymbol{N_{DC}}} \quad (1)$$

where θ is a fixed threshold (%), $\boldsymbol{N_{DC}}$ is the total number of events on which the length of the price-curve coastline is measured. Hence, the number of events is determined by the threshold size, θ, that it is used. P_{EXT_i} is the price of the $i - th$ turning point, whether upturn or downturn point.

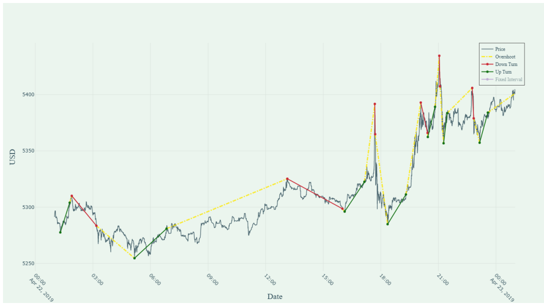

Figure 8: Price activities for BTC/USD on April 22, 2019. The blue line shows the tick-by-tick value of Bitcoin for that day. While the other line is the event-based summary using a threshold of 0.5%. This period is characterized by 13 events by the intrinsic time framework.

In opposition, the measurement of the length of the price-curve coastline defined by physical times changes at fixed time intervals is the average price movement between fixed points over a time period T, in which the time interval between these fixed points is equivalent. Under physical time, the length of the price-curve coastline $\boldsymbol{PCC_t}$ is measured by:

$$\boldsymbol{PCC_t} = \frac{\sum_{i=1}^{n} |P_{i+1} - P_i|}{n} \quad (2)$$

where P_i is the price at point i and n is the total number of fixed points which is equal to the number of events in the intrinsic time framework. For a better comparison it is used the same number of points both in the physical time and in the intrinsic time.

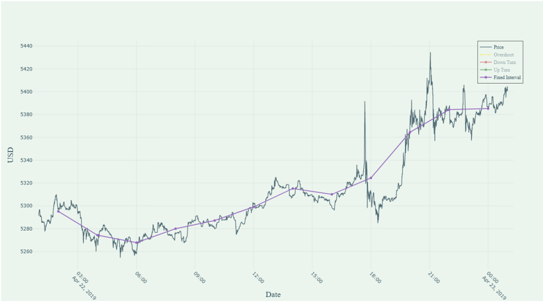

Figure 9: Price activities for BTC/USD on April 22, 2019. The blue line shows the tick-by-tick value of Bitcoin for that day. While the purple line is the fixed interval summary sampling the price value every two hours by means of averaging the asset value between a two hour window and stamping it to the right. This period is characterized by 11 intervals by the fixed physical time framework.

Taking into account the two possible ways to outline the price movements in the financial market, a conceivable question is raised: Which one, effectively, provides the foremost insights about future price evolution? Whether we are interpreting this question from the perpective of an investor, that is trying to asses the potential profit, from regulatory agents that try to reckon the risk of the financial institutions or an economics that is attempting yo evaluate the chance of a future big crises to help politician in their decision making

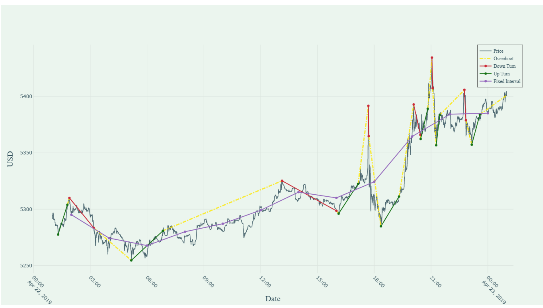

Figure 10: Price activities for BTC/USD on April 22, 2019. The blue line shows the tick-by-tick value of Bitcoin for that day. The purple line is the fixed interval summary sampling the price value every two hours. The other line is the event-based summary using a threshold of 0.5%. Both profiles have on average 12 events. The insight is that the usual way to observe the prices in the financial market culture overlooks important price activities. Look at what happens moments before 18:00 o'clock.

Intrinsic Time	Physical Time	⟨Intervals⟩	PCC(θ)	PCC(t)
1%	$5H$	5	181.6%	50.45%
0.5%	$2H$	12	118.3%	26.63%
0.1005%	$10min$	144	28.64%	9.96%

Table 2: Comparison of the price curve coastline between the intrinsic time and physical time

4. Results

4.1. Analysis of Returns in Directional Change

Additional investigation was conducted in dissecting the potential profit and risk an individual can expect between the use of a fixed-interval time frame and an event-based frame.

For this purpose, the Bitcoin price from July 7, 2017 to April 2, 2019 was submitted to the two different profile syntheses, while mindfully guaranteeing the same number of points of information for the sake of trustworthiness of comparison.

Taking this into account, it was found that using a threshold of 2.8% ensures the same number of intervals as the daily fixed interval for prices display: 642 points.

The following histogram, reveals that the probability of finding lower price changes is higher through the observation in daily returns, while with the use of a threshold of 2.8% the presence of returns lower than 2.2% is smaller. Accordingly, from the figure it is possible to recognize that from using the user-defined threshold, an agent contains an higher probability of assisting to larger returns both in positive or negative direction.

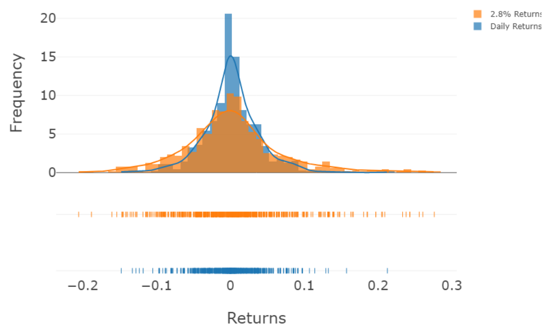

Figure 11: This graph displays the frequency at which different magnitudes of returns happen in the profiled period for Bitcoin price from July 7, 2017 to April 2, 2019. The comparison is performed summarizing the price daily and 2.8% return.

Apart from visual inspection, some statistics descriptors are presented in table so that a better understanding is performed. Kurtosis, Skewness and Jarque-Bera normality test were used:

Skewness measures the degree of asymmetry of a distribution around its mean value. Positive skew-

	⟨Intervals⟩	Kurtosis	Skewness	Jarque-Bera
Daily Returns	642	2.993	0.209	243.589
2.8% Returns	642	2.052	0.710	166.712

Table 3: Statistical descriptors to compare the returns obtain with physical time vs intrinsic time ensuring the same number of intervals.

ness indicates a distribution with an asymmetric tail extending toward more positive values, putting into the finance context this means there is more chances of an upside potential price change. While negative skewness indicates a distribution with an asymmetric tail extending towards more negative values, which suggests that there is more probability of a downside potential price move. As consequence, zero skewness means the tail are symmetric.

The daily returns present a skewness value of 0.209 while the 2.8% returns have a value of 0.710, from this difference it is confirmed that using a threshold the agent has more chances of an upside potential price change than a downside price movement.

Kurtosis measures the 'tail-heaviness'. In other words, fat tails is a statistical phenomenon in which extreme values (low and high values) are more frequent than the Gaussian distribution. Therefore, a low kurtosis indicates fewer outliers whereas a high kustosis indicates greater presence of extreme values. With high kustosis, one will have 'fat' tails, higher frequency of outcomes at the extreme negative and positive ends of the distribution curve. Carrying this observation into finance, it means that for investors, high kurtosis of the return distribution implies that the investor will experience occasional extreme returns. This phenomenon is known as kurtosis risk.

The daily returns present a kurtosis value of 2.993 while the 2.8% returns have a value of 2.052, which indicates that there is a higher risk of extreme values with the use of daily returns compared to a threshold of 2.8%.

The Jarque-Bera Test is a test for normality. The test relies on the skewness and kustosis of the data to see if it matches a normal distribution. In general a large Jarque-Bera values indicate that the data is not normally distributed.

The results show that the daily returns are further way from being normal than the 2.8% threshold.

By way of conclusion, an important aspect of the financial market is that an agent can profit when prices are rising or are expected to rise (bull market), likewise there is also gain when the market is characterized by falling prices (bear market). The origin of these terms suggests that the use of "bull" and "bear" to describe markets comes from the way the animals attack their opponents. A bull thrusts its horns up into the air, while a bear swipes its paws downward. These actions are metaphors for the movement of a market. If the trend is up, it's a bull market. If the trend is down, it's a bear market.

The figure highlights the fact using an event-based framework for the analysis of price can lead to a higher potential profit assuming perfect foresight.

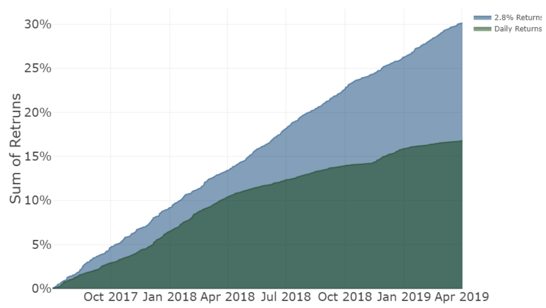

Figure 12: This graph displays the accumulation of returns for the profiled period. In other words can be seen as a maximum potential profit assuming perfect foresight.

5. Conclusions

History recorded by events, not snapshots at fixed intervals, so should market prices too. Directional Change events capture 'significant changes' useful for summarising price movements and gives new perspectives in price movements. DC enables discovery of regularities not captures by interval-based summaries. It is different from the traditional time series It provides a different angle for capturing and analysing price changes. The coastline captures the potential profit in trading in a time period under the observed threshold. This is useful information that we do not often find in the time series analysis. This technique or framework can have important application in mechanics. For example, in predictive maintenance one can use the directional change to detect regularities and gain new insights. As market prices, machine operate and are damaged by events. Recording those events can be of major importance.

5.1. Achievements

The major achievements of the present work...

- Found some interesting statistical observations
- The coastlines measured by the DC framework are bigger than the coastlines measures with time-series
- The reported results in this paper emphasize the significance of considering DC and OS

events in studying the price curve, rather than the physical fixed time intervals knowing the long coastline of price changes under the DC framework.

5.2. Future Work

Future deliberations may evolve in the direction of creating other indicators that can be extracted from the directional change framework. Therefore creating new useful insights that can be used for profiling a time series and a deeper understanding of the behaviour of any time signal. The more useful indicators one defines, the more information we can extract from the data. Another direction is to combine directional change analysis with the traditional time series analysis to explore synergy. Since they are two different methods to observe the same market, they may provide complementary market information.

Time series data as showed in this work, is also commonly encountered in the work of an Mechanical , for example, this framework can be used in predictive maintenance. Predictive maintenance is a method of preventing asset failure by analysing production data to identify patterns and predict issues before they happen. It is not a surprise therefore, that predictive maintenance has quickly emerged as a leading Industry 4.0. Implementing industrial IoT technologies to monitor asset health, optimize maintenance schedules, and gaining real-time alerts to operational risks, allows manufactures to lower service costs, maximize uptime, and improve production throughput. In addition, studying the time series under DC framework reduces the computational load and the complexity of time series known the small number of price points for evaluation. The computational costs (which consist of the cost of evaluating data rows) necessity not to be ignited. Therefore, computational costs have to be part of the criteria when it comes to choosing a method for studying price time series. The DC indicators can facilitate the construction of DC profiles from sensors data, because the sensors data displays diverse periodic patterns, revealed by the power of the data process in the high-frequency domain.

Potentially DC concept can be used to make feature extraction from high frequency data, reducing its complexity but maintaining characterizing patterns present in sequential time series. This can be used to feed the Machine Learning algorithms

In this paper offers purpose of this work was also to bring awareness to the field of Mechanical Engineering of the usefulness of Directional Change framework which can bring potential high benefits in the groundwork for the analysis of raw time series signals.

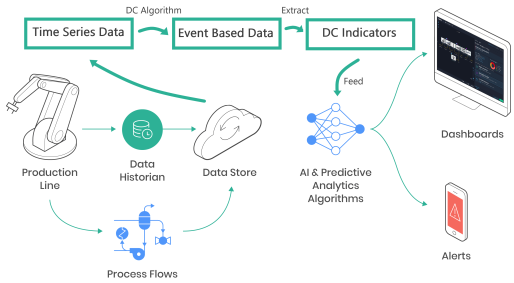

Figure 13: This figure presents a possible use of the directional change framework in the field of mechanic engineering. Concretely in predictive maintenance where big amounts of data from sensors could possibly be simplified to feed the machine learning algorithms without the loss of relevant information.

The DC indicators are different from traditional time series analysis indicators and the valuable tolls for risk management, volatility modelling and for creating automated trading models, the same ways these DC indicators can be good weapons to treat signal coming from sensors and be applied in mechanical engineering problems. I believe that the DC indicators can improve our study of dynamic behaviour of time signal, used a lot by mechanical engineering, and advance the quality of the predictions and inference we formulate regarding predictive maintenance.

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