Crew scheduling in a train operating company – Fertagus case study

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Abstract. Essential to any metropolitan area, public transportation services aim to meet the passenger demand. Lately, emergent and disruptive technologies are being used for the railway sector. In line with this, the initial aim of this dissertation is to conceive a mathematical model which outputs an optimal daily maintenance crew schedule for a train operating company, Fertagus. One of the major contributions of this model is related with the introduction of skillsets for each maintenance crew member. Later, it was decided to integrate the driving crew scheduling problem to the initial model, and thus, obtain an optimal daily schedule comprehending both crews. The present work follows the works by Mira (2018) and Méchain (2017). This way, a mixed-integer linear programming model, considering the preventive maintenance actions scheduled for each day of the week, is presented, assigning both the maintenance crew and drivers to a daily scheduling, while minimizing the associated costs. The model outputs a data file indicating where and when each worker carries out the respective maintenance action and something similar concerning drivers and tasks. The program is initially validated by an illustrative example and is then applied to a real case scenario. While it was possible to obtain an optimal schedule for the maintenance crew, concerning Fertagus case study, for the driving crew, only a medium-size problem was possible to solve, and so, larger size instances are left for further research.

Keywords: Railway Management, Maintenance Crew Scheduling, Driving Crew Scheduling, Mixed-Integer-Linear Programming, Optimization

1. Introduction

Lisbon, as Portugal's capital, is one of the main focus of population in the country whether it is due to professional or personal reasons furthermore is nowadays also a highly rated city for many travellers all across the world. It is in fact becoming a busier city, not only due to the many thousands of tourists that come and go every day, but also consequence of the slight local population growth over the years according to the *United Nations World Urbanization Prospects* (UN 2018). To satisfy passengers high demand across the whole Lisbon metropolitan area, several transportation systems had to be created. In fact, there is a wide range of options to choose from, naming buses, subway, boats and trains. All these different means of transport together, when articulated form an integrated mobility system, essential to any avant-garde city. In the present study the focus resides on a Portuguese train operating company, Fertagus, based in Lisbon.

In an era where technology evolves at a frenetic pace and consequently, new researches and approaches, arise to solve more complex problems, railway systems are not an exception. The *"rising traffic demand, congestion, security of energy supply and climate changes"* (EC, 2019) are some of the challenges that the European Union faces, so these new technologies can play a major role influencing the way future rail automation and maintenance are organised. A European rail initiative named Shift2Rail (S2R) was created in 2009, "when key European rail sector players, under the coordination of the Association of the European Rail Industry (UNIFE), began investigating a policy instrument that could facilitate a step change for the European rail system." (S2R, 2019). This initiative acts as a test bed for new technology developments, not only helping enhance the railway industry competitiveness edge, as helping railway systems to establish a new and broader part in transport markets. Nowadays, already under the Horizon 2020, a research and innovation program that promotes innovation, further researches have been conducted, for example, on arising and promising disruptive technologies. In line with these ideas, the current work aims to use such techniques to solve crew scheduling problem for the case study of a train operating company in Lisbon

1.1. Problem statement

The ultimate objective of this dissertation is to develop a model that minimizes the costs associated with crew scheduling for a railway company and that creates its daily planning for a given week. To achieve this goal, a company's rolling-stock timetable and previously scheduled maintenance activities must be considered. Thus, it is proposed to apply the decision model to the case study of Fertagus operating rolling-stock schedule, adapting a previously obtained maintenance scheduling plan from (Mira, 2018), that outputs a daily maintenance and driving crew scheduling, considering the maintenance actions that have to be performed for each day of the week with the lowest associated costs possible.

1.2. Document Structure

The present document is organized in the following order. In Section 2 a literature review is presented. In section 3, the mixed-integer linear programming model is exhibited and described. The Fertagus' case study is exposed in section 4 and its results and analysis are displayed in section 5. Lastly, in section 6, the conclusions and the future research are stated.

2. Related Literature

The following chapter summarizes the most relevant papers analysed during this thesis, on maintenance and crew scheduling in transportation companies.

Haghani and Shefali (2002) deals with the problem of scheduling bus maintenance. Its aim is to design a daily schedule that minimizes the number of unavailability hours for each vehicle. Hereby, it is desired that as many inspections as possible are carried out through idle time, this is when buses are out of service, and so maximize the usage of maintenance resources. Using an integer programming approach, the model outputs a maintenance schedule for each bus, in addition to the minimum number of maintenance lines that should be assigned for each type of action.

Méchain (2017) focuses on a problem of maintenance planning for a train operating company, Fertagus. A mixed integer linear programming (MILP) model is conceived, taking into account the many technical and infrastructure constraints regarding the company. The aim of this study is to develop a model that outputs a technical maintenance plan for a time horizon of 52 weeks, while minimizing the cost related with preventive maintenance. Besides this, the optimization model defines which maintenance actions need to be carried out each week, the maintenance line where the maintenance takes place and also the number of spare parts necessary to fulfil the technical plan.

Mira (2018) develops a mixed-integer linear programming decision model which provides a weekly, optimal rolling-stock schedule, capable of including the maintenance actions, considering a previously scheduled preventive maintenance plan for each of the different weeks of the year. This model is validated for a small scale illustrative example and later applied to the Fertagus train operating company. However, due to computational capacity limitations only a 3-day schedule was possible to execute instead of the weekly one. The results show that by rearranging the operating rolling-stock schedule, it is possible to reduce meaningfully the deadheading distance covered by train units. Moreover, maintenance actions were successfully included and the 3-day schedule was obtained, indicating which units and when they should carry out maintenance, as well as the maintenance type to be performed. Finally, some analysis is conducted and it is possible to conclude that the solution obtained is not sensitive to variations on the weight of the different components of the objective function.

M. Pour et al. (2018) addresses an hybrid Constraint Programming/Mixed Integer Programming framework to solve a signalling maintenance crew scheduling problem for a section of the Danish railway system. This hybrid framework is split in two parts. First, in the construction phase, initial feasible solutions are obtained through a Constraint Programming (CP) model. After, a Mixed Integer Programming (MIP) solver is used for further improvement of these initial solutions. Accordingly, this "*hybridised framework is a contribution to the development of integration between MIP and CP, where CP greatly reduces the time required by the MIP to produce a solution"* (M. Pour et al., 2018). The model is based on the problem faced by Banedanmark's planning team, a company responsible for most of the railway infrastructure in Denmark, which provided the model formulation. The main aim of this research is to find feasible solutions for larger instances of the maintenance crew scheduling problem. While a general purpose MIP solver can only deal with a maximum period of two weeks due to an extensive number of real-life attributes and constraints, and a Constraint Optimisation Problem (COP) model does not get improved solutions, this hybrid framework is capable of generating good results for planning horizons up to eight weeks. The results of this hybrid framework are later presented and then compared with both the results of modelling the problem as a Constraint Optimisation Problem (COP), and the results of solving the MIP directly. To sum up, it was possible to verify that the proposed hybrid CP/MIP framework outputs better results than both solving the problem as a MIP problem directly and using COP to improve the initial solutions found by CP.

Boyer et al. (2018) presented an integrated approach for the Flexible Vehicle and Crew Scheduling Problem (FVCSP), common in urban bus companies. In fact, the development of model integrating both Vehicle Scheduling and Crew Scheduling problems in a single approach is one of the main contributions of this work. The aim of this problem focus on minimizing the costs related with vehicles usage and drivers wage. Besides dealing with several constraints related to both drivers and vehicles, the model also has the flexibility required to deal with scenarios where the number of available vehicles/drivers can change daily. First, a mixed-integer linear programming model is proposed, and then a Variable Neighbourhood Search (VNS) metaheuristic approach, capable of solving larger instances, closer to real-life situations. Results

showed that approaching the FVCSP with a commercial mixed-integer linear programming solver CPLEX 12.7, as expected, feasible solutions could only be returned, in reasonable time, for small instances. On the other hand, using the VNS approach, for much larger cases, good results were obtained in a practical time. It is finally suggested, for future work, the combined optimization of both the timetabling with the vehicle and crew scheduling problem.
Although the initial model (M. Pour et al., 2018) on the preventive signalling maintenance crew scheduling for a Danish railway company, was considered appropriate, the final version of this model was completely modified and adapted to meet Fertagus' case study requirements, as well as to fit information extracted from (Mira, 2018) and (Méchain, 2017) models.

3. Mixed-integer linear programming model to schedule maintenance crew and drivers

The present mathematical model is an adaptation of the model presented by (M. Pour et al., 2018) on the preventive signaling maintenance crew scheduling problem. However, to comprehend the Fertagus case study and to integrate the information associated with the maintenance model extracted from (Mira, 2018), the present model had to be basically built from scratch. In fact, this study is in a certain way a continuation of the researches carried out by Méchain, (2017) and Mira, (2018), as Figure 1 suggests. It is also important to mention that the workers' skills were added to the problem, meaning that a maintenance worker requires the right competence to carry out a given maintenance action. Furthermore, also contemplates an integrated driving crew scheduling approach. This model is intended to be initially validated using a small-size illustrative example, and later applied to a real case scenario, aiming to reduce the cost associated with both drivers and maintenance crew scheduling.

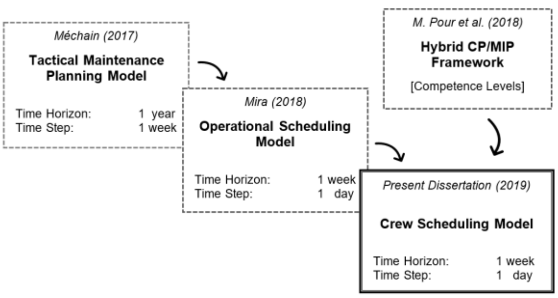

Figure 1 – Present work related with previous researches

3.1 Constants

NU	Number of units
NS	Number stations
NMW	Number of maintenance workers
NCC	Number of competences
NDW	Number of driving workers
NM	Number of different types of maintenance actions
NT	Number of tasks
ND	Number of days
TMS	Daily maximum service time for drivers
Δ_m	Gap between drivers' entry/exit hour and beginning of the first/end of the last task, respectively
t_{min}	Gap required by maintenance workers when changing unit in successive maintenance actions
t_{man}	Gap required by units to set up for maintenance after arriving and before departing the depot
LN	Large number

3.2 Sets

K	Set of units, k
S	Set of stations, s
T	Set of tasks, i
MM	Set of maintenance actions, m
CC	Set of competences, c
D	Set of days, d
MW	Set of driving workers, mw
DW	Set of driving workers, dw

3.3. Parameters

Cmw_{mw}	daily cost of each maintenance worker mw
Cdw_{dw}	daily cost of each driving worker dw
Sd_i	departure station of task i
Sa_i	arrival station of task i
Dd_i	departure time of task i
Da_i	arrival time of task i
MT_m	duration of maintenance action m
AWt_m	total amount of work required for each maintenance m
$AW_{m,c}$	amount of work per competence c, required for each maintenance m
$X_{k,i}$	tasks i, carried out on unit k
$Y_{k,i,j}$	pair of tasks (i,j) linked by unit k
$YM_{k,i,j,m}$	maintenance actions m performed on unit k, between pair of tasks (i,j)
$KM_{k,m}$	maintenance actions m that need to be performed on each unit k
$ZM_{k,d}$	units k that cover any maintenance action, on a given day d
$MWC_{mw,c}$	competences c, mastered by each maintenance worker mw

3.4. Variables

$wMd_{mw,d}$	binary variable set to 1 if maintenance worker mw works on day d, and set 0 otherwise

$\boldsymbol{wMd_{mw,k,m,d}}$	binary variable set to 1 if maintenance worker mw, performs maintenance action m on unit k, on day d
$\boldsymbol{t1m_{mw,k,m,d}}$	maintenance worker mw starting time, performing maintenance action m, on unit k, on day d
$\boldsymbol{t2m_{mw,k,m,d}}$	maintenance worker ending time, performing maintenance action m, on unit k, on day d
$\boldsymbol{wDd_{dw}}$	binary variable set to 1 if driving worker dw, works this day and set 0 otherwise.
$\boldsymbol{wDt_{dw,i}}$	binary variable set to 1 if driving worker dw performs task i
$\boldsymbol{eD_{dw,i}}$	binary variable set to 1 if driving worker dw enters service on task i
$\boldsymbol{sD_{dw,i}}$	binary variable set to 1 if driving worker dw exits service on task i

Two additional variables that are linear dependent on other decision variables were also defined:

$\boldsymbol{t1i_{dw,i}}$	driving worker dw, entry hour for task i
$\boldsymbol{t2i_{dw,i}}$	driving worker dw, exit hour for task i

3.5. Objective function

Minimize: $\sum_{mw\,\in\,MW}\sum_{d\,\in\,D} Cmw_{mw} \times wMd_{mw,d} \;+\; \sum_{dw\,\in\,DW} Cdw_{dw} \times wDd_{dw}$

Subject to:

$$wM_{mw,k,m,d} \;=\; 0 \qquad \forall\; mw \in MW, k \in K, m \in M, d \in D \mid KM_{k,m} = 0 \qquad (1)$$

$$wM_{mw,k,m,d} \;=\; 0 \qquad \forall\; mw \in MW, k \in K, m \in M, d \in D \mid ZM_{k,m} = 0 \qquad (2)$$

$$wM_{mw,k,m,d} \;\le\; wMd_{mw,d} \qquad \forall\, mw \in MW, k \in K, m \in M, d \in D \qquad (3)$$

$$t1m_{mw,k,m,d} \;+\; t2m_{mw,k,m,d} \le LN \times wM_{mw,k,m,d} \qquad \forall\, mw \in MW, k \in K, m \in M, d \in D \qquad (4)$$

$$t2m_{mw,k,m,d} \;\ge\; t1m_{mw,k,m,d} \qquad \forall\, mw \in MW, k \in K, m \in M, d \in D \qquad (5)$$

$$t1m_{mw,k,m,d} \;\ge\; (Da_i + t_{man}) \times wM_{mw,k,m,d} \qquad \forall\, i \in T, j \in T, mw \in MW, k \in K, m \in M, d \in D \mid YM_{k,i,j,m} = 1 \qquad (6.1)$$

$$t2m_{mw,k,m,d} \;\le\; \left(Dd_j - t_{man}\right) \times wM_{mw,k,m,d} \qquad \forall\, i \in T, j \in T, mw \in MW, k \in K, m \in M, d \in D \mid YM_{k,i,j,m} = 1 \qquad (6.2)$$

$$t2m_{mw,k,m,d} \ge t1m_{mw,k,m,d} + MT_m - \left(1 - wM_{mw,k,m,d}\right) \times LN \qquad \forall\, i \in T, j \in T, mw \in MW, k \in K, m \in M, d \in D \mid YM_{k,i,j,m} = 1 \qquad (7)$$

$$t2m_{mw,k,m,d} - t1m_{mw,k,m,d} \le MT_m \times wM_{mw,k,m,d} \qquad \forall\, i \in T, j \in T, mw \in MW, k \in K, m \in M, d \in D \mid YM_{k,i,j,m} = 1 \qquad (8)$$

$$t1m_{mw,k,m2,d} \ge t2m_{mw,k,m1,d} - LN \times \left(1 - wM_{mw,k,m2,d}\right) \qquad \forall\, i \in T, j \in T, mw \in MW, k \in K, m \in M, d \in D \mid YM_{k,i,j,m1} = 1 \;\wedge YM_{k,i,j,m2} = 1 \;\wedge\; m_1 \ne m_2 \qquad (9)$$

$$t1m_{mw,k2,m,d} \ge t2m_{mw,k1,m,d} + t_{min} - LN \times \left(1 - wM_{mw,k2,m,d}\right) \qquad \forall\, mw \in MW, k_1 \in K, k_2 \in K, m \in M, d \in D \mid KM_{k1,m} = 1 \;\wedge KM_{k2,m} = 1 \;\wedge\; k_1 \ne k_2 \qquad (10.1)$$

$$t1m_{mw,k2,m2,d} \ge t2m_{mw,k1,m1,d} + t_{min} - LN \times \left(1 - wM_{mw,k2,m2,d}\right) \qquad \forall\, mw \in MW, k_1 \in K, k_2 \in K\,, m_1 \in M, m_2 \in M, d \in D \mid KM_{k1,m1} = 1 \;\wedge KM_{k2,m2} = 1 \;\wedge\; k_1 \ne k_2 \;\wedge\; m_1 \ne m_2 \qquad (10.2)$$

$$AW_{m,c} \le \sum_{mw\,\epsilon\,MW \,\mid\, MWC_{mw,c}=1} \left(t2m_{mw,k,m,d} - t1m_{mw,k,m,d}\right) \qquad \forall\, i \in T, j \in T, k \in Km \in M, c\,\epsilon\, CC, d \in D \mid YM_{k,i,j,m} = 1 \qquad (11)$$

$$\sum_{i\,\epsilon\,T} eD_{dw,i} \le 1 \qquad \forall\, dw \in DW \qquad (12.1)$$

$$\sum_{i \in T} sD_{dw,i} \leq 1 \qquad \forall\, dw \in DW \qquad (12.2)$$

$$\sum_{j \in T} sD_{dw,j} = \sum_{i \in T} eD_{dw,i} \qquad \forall\, dw \in DW \qquad (12.3)$$

$$eD_{dw,i} \leq wDt_{dw,i} \qquad \forall\, dw \in DW, i \in T \qquad (13.1)$$

$$sD_{dw,i} \leq wDt_{dw,i} \qquad \forall\, dw \in DW, i \in T \qquad (13.2)$$

$$t1i_{dw,i} = (Dd_i - \Delta_m) \times eD_{dw,i} \qquad \forall\, dw \in DW, i \in T \qquad (14.1)$$

$$t2i_{dw,i} = (Dd_i + \Delta_m) \times sD_{dw,i} \qquad \forall\, dw \in DW, i \in T \qquad (14.2)$$

$$\sum_{dw \in DW} wDt_{dw,i} = 1 \qquad \forall\, i \in T \qquad (15)$$

$$wDt_{dw,i} \leq wDd_{dw} \qquad \forall\, dw \in DW, i \in T \qquad (16)$$

$$\sum_{dw \in DW} eD_{dw,i} = 1 \qquad \forall\, i \in T \mid ft_i = 1 \qquad (17.1)$$

$$eD_{dw,i} = wDt_{dw,i} \qquad \forall\, dw \in DW, i \in T \mid ft_i = 1 \qquad (17.2)$$

$$\sum_{dw \in DW} sD_{dw,i} = 1 \qquad \forall\, i \in T \mid lt_i = 1 \qquad (18.1)$$

$$sD_{dw,i} = wDt_{dw,i} \qquad \forall\, dw \in DW, i \in T \mid lt_i = 1 \qquad (18.2)$$

$$LN \times (1 - eD_{dw,i}) \geq \sum_{j \in T \mid Dd_j \leq Da_i \wedge i \neq j} wDt_{dw,i} \qquad \forall\, dw \in DW, i \in T \qquad (19.1)$$

$$LN \times (1 - sD_{dw,i}) \geq \sum_{j \in T \mid Dd_j \geq Da_i \wedge i \neq j} wDt_{dw,i} \qquad \forall\, dw \in DW, i \in T \qquad (19.2)$$

$$t2i_{dw,j} - t1i_{dw,i} \leq TMS + LN \times \left(2 - eD_{dw,i} - sD_{dw,j}\right) \qquad \forall\, dw \in DW, i \in T, j \in T \qquad (20)$$

$$wDt_{dw,i} + wDt_{dw,j} \leq 1 \qquad \forall\, dw \in DW, i \in T, j \in T \mid Sim_{i,j} = 1 \wedge i \neq j \qquad (21)$$

$$wDt_{dw,i} \leq LN \times \sum_{j \in T \mid X_{k,j}=1} eD_{dw,j} \qquad \forall\, dw \in DW, k \in K, i \in T \mid X_{k,i} = 1 \wedge Nk_i = 1 \qquad (22.1)$$

$$wDt_{dw,i} \leq LN \times \sum_{j \in T \mid X_{k,j}=1} sD_{dw,j} \qquad \forall\, dw \in DW, k \in K, i \in T \mid X_{k,i} = 1 \wedge Nk_i = 1 \qquad (22.2)$$

$$wDt_{dw,i} \leq LN \times \sum_{j \in T \mid X_{k_1,j}=1 \vee X_{k_2,j}=1} eD_{dw,j} \qquad \forall\, dw \in DW, k_1 \in K, k_2 \in K, i \in T \mid X_{k_1,i} = 1 \wedge X_{k_2,i} = 1 \wedge k_1 \neq k_2 \wedge Nk_i = 2 \qquad (23.1)$$

$$wDt_{dw,i} \leq LN \times \sum_{j \in T \mid X_{k_1,j}=1 \vee X_{k_2,j}=1} sD_{dw,j} \qquad \forall\, dw \in DW, k_1 \in K, k_2 \in K, i \in T \mid X_{k_1,i} = 1 \wedge X_{k_2,i} = 1 \wedge k_1 \neq k_2 \wedge Nk_i = 2 \qquad (23.2)$$

$$wMd_{mw,d} = 0, 1 \qquad \forall\, mw \in MW, d \in D \qquad (24)$$

$$wM_{mw,k,m,d} = 0, 1 \qquad \forall\, mw \in MW, k \in K, m \in MM, d \in D \qquad (25)$$

$$t1m_{mw,k,m,d} \geq 0 \qquad \forall\, mw \in MW, k \in K, m \in MM, d \in D \qquad (26)$$

$$t2m_{mw,k,m,d} \geq 0 \qquad \forall\, mw \in MW, k \in K, m \in MM, d \in D \qquad (27)$$

$$wDd_{dw} = 0, 1 \qquad \forall\, dw \in DW \qquad (28)$$

$$wDt_{dw,i} \in \{0,1\} \qquad \forall\, dw \in DW, i \in T \qquad (29)$$

$$eD_{dw,i} \in \{0,1\} \qquad \forall\, dw \in DW, i \in T \qquad (30)$$

$$sD_{dw,i} \in \{0,1\} \qquad \forall\, dw \in DW, i \in T \qquad (31)$$

The objective function is sectioned in two terms, the first one addressing the maintenance crew costs and the second one related to the driving crew costs, while its aim is to minimize the costs associated with both crews. In order to facilitate understanding of the constraints, it was decided to divide them in two groups. The first group regarding maintenance crew and the second related to the driving crew.

a) Constraints related to the maintenance crew

Constraints (1) and (2) guarantee that if a unit k does not go to depot to perform maintenance actions on day d, , then no maintenance worker mw, will be performing any maintenance on that unit k. Furthermore, if a maintenance action m, is not previously scheduled to be performed on a unit k, then no maintenance worker mw, will be assigned to perform it. Constraint (3) establishes that if a maintenance worker performs any maintenance action on a day d, then he/she is assigned to work on that day.

Constraint (4) secures that if a maintenance worker mw is assigned, then its starting/finishing time must be greater than zero, meanwhile constraint (5) assures that the finishing time of a maintenance action, logically must be greater than its starting time.

Constraint (6.1) and (6.2) assure that maintenance tasks are performed within the right time gap. More precisely, constraint (6.1) guarantees that maintenance actions performed between pair of tasks (i,j) can only start after the arrival time of the unit Da_i to the depot, plus a gap required by units to set up for maintenance tman. On the other hand, constraint (6.2) assures that maintenance actions end before the departure time of the unit Dd_j from the depot, less tman. Constraint (7), guarantees that the finishing hour of a maintenance action m, $t2m_{mw,k,m,d}$, must be greater than the starting hour $t1m_{mw,k,m,d}$, plus the duration necessary to carry out the maintenance action, MT_m. Additionally, constraint (8) ensures that the previously defined duration of a maintenance action MT_m, cannot be exceeded

Constraints (9), (10.1) and (10.2) assure that if a maintenance worker is assigned to two different maintenance actions m_1 and m_2, he/she can only start another maintenance action m_2, after finishing the one that was started first. While constraint (9) guarantees that this happens for two maintenance actions performed on the same unit k, constraints (10.1) and (10.2) impose that this temporal coherence is established for two different units k_1 and k_2, whether the maintenance actions to be performed are the same or not.

Finally, constraint (11) states that the amount of work per competence required by each one of the maintenance actions, $AW_{m,c}$, must be satisfied by the maintenance workers assigned to the respective maintenance action. Logically maintenance worker mw, can only be assigned to a maintenance action m, in case he/she possesses at least one of the required competences to carry it out, i.e. $MWC_{mw,c} = 1$.

b) Constraints related to the driving crew

Constraints (12.1) and (12.2) assure that each driving worker dw, can only enter and exit to service once, this is one entry task and one exit task. Additionally, constraints (12.3) guarantee that if dw enters to service, then, logically he/she must also exit. Constraints (13.1) impose that each driving worker dw, must perform the task i, in which his/her shift starts, i.e. if he/she enters to service on that same task i. In line with (13.1), constraint (13.2) assures the same happens regarding the exit task. Constraints (14.1) and (14.2) define the entry and exit hour for each driver $t1i_{dw,i}$, $t2i_{dw,i}$, respectively, keeping in mind that the set up time required by drivers before starting his/her first task and after finishing his/her last task is Δ_m.

Constraints (15) establish that every task must be carried by only one driving worker, additionally, constraints (16) states that if a driver dw is assigned to any task, logically he/she works on that day.

Constraints (17.1) and (17.2) have to do with first tasks, fti. First, constraints (17.1) assure that every first task requires a driver dw to enter service, while constraints (17.2) assure that the driving worker who entered to service on that task i, also carries it out. Similarly, constraints (18.1) and (18.2) express parallel ideas regarding last tasks, i.e. every last task lti, implies that the driver dw who performed it, exits service after that task.

Constraints (19.1) assure that a driver dw cannot carry out any task j that occurs before his entry task i, while constraints (19.2) guarantee that no task j can be performed after the driver’s exit task i. Constraints (20) secure that each driver dw does not work longer than the established maximum service time TMS (540 minutes).

Constraints (21) state that two different tasks performed simultaneously (Figure 3.7) require two driving workers to carry them out.

Finally, for single unit tasks, i.e. tasks with $Nk_i = 1$, constraints (22.1) assure that all tasks that require a train unit k, can only be carried out by a driving worker dw, that has entered to service on that same unit k, i.e. his first task has to be done on the same unit. Similarly, constraints (22.2) assure that a driver dw must perform tasks carried out on the same unit k, that he/she is finishing service, i.e. his/her last task. Analogously, constraints (23.1) and (23.2) guarantee that this also occurs for tasks that require two units k_1 and k_2, i.e. tasks with $Nk_i = 2$. These tasks can only be carried out by a driver dw who enters and exits service on one of those train units k_1 or k_2. These last four constraints altogether assure that drivers do not change unit.

4. Case study of Fertagus

Section 4 explores the case study under analysis in the present research, in which the mathematical model is applied to the Fertagus case study.

4.1. Fertagus train operating company

Fertagus is a private train operating company, a branch of the group Barraqueiro, which links Setúbal, to Roma-Areeiro. Besides operating the railway line, the company is also accountable for the maintenance of the rolling-stock units as well as the maintenance of some railway stations. The railway line has an extension of 54 kilometres, split into 3 different routes *"Linha de Cintura"*, *"Linha do Sul"* and *"Linha do Sado"* and serves 14 different stations. The company's maintenance yard has several lines with different functions. Hereby it is important to know that there are 3 lines where maintenance may be executed.

4.2. Parameters for the case study

In this case study, 17 train units are supposed to cover 196 daily tasks, in which some of them have to perform maintenance actions previously scheduled. There are 14 different types of maintenance activities and each one requires a certain set of competences/skills, so the amount of work required to perform it depends not only on the maintenance action but also on the competences needed, $AW_{m,c}$. There are 10 distinct competences which a worker can master, and each worker may use several competences at once. Finally, the maintenance crew team is formed by 16 workers, whereas the number of drivers is equal to 29. The ultimate aim is to obtain the best operational crew scheduling possible for one day of the week under a short computational time. One that minimizes the costs related to employing both maintenance and driving crew. All the necessary data is exposed in the following tables.

Tables 1 and 2 display the maintenance and driver workers daily cost respectively. Table 3 introduces the constants used on this model. The skillset of each maintenance worker is defined in Table 4. Table 5 presents the duration of each maintenance action and the respective amount of work *per* competence. From Table 6, it is possible to know which units go to the depot to perform maintenance action on this day. Table 7 exposes which units are performing maintenance and the scheduled activity. Table 8 contains information regarding stations, and Table 9 defines all the scheduled tasks present in the rolling-stock schedule. Table 10 establishes which tasks are assigned to each unit. Table 11 specifies between which pair of tasks, each train unit, performs the scheduled maintenance actions. Table 12 defines which unit links to consecutive tasks.

Table 1 – Cost of employing a maintenance worker

mw	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Cmw_{mw}	43	43	43	46	40	40	43	43	43	43	43	36	66	60	66	60

Table 2 – Cost of employing a driving worker

dw	1	2	3	4	(…)	26	27	28	29
Cdw_{dw}	54	54	54	54		54	54	54	54

Table 3 – Constants used

Constants	Unit	Value
NU	-	17
NS	-	15
NMW	-	16
NCC	-	10
NDW	-	29
NM	-	14
NT	-	196
ND	day	1
TMS	min	540
Δ_m	min	15
t_{min}	min	5
t_{man}	min	5
LN	-	10000

Table 4 – Maintenance crew competences

$MWC_{mw,c}$		c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
mw	1	1	1	1	1	1	1	0	0	1	1
	2	1	1	1	1	1	1	0	0	1	1
	3	1	0	0	0	0	1	0	0	1	1
	4	1	1	1	0	0	1	0	0	1	1
	5	1	0	0	0	0	0	0	0	1	1
	6	0	0	0	0	0	0	0	0	0	1
	7	1	0	1	1	1	1	0	0	1	1
	8	1	1	1	1	1	1	0	0	1	1
	9	1	0	0	0	0	0	0	0	1	1
	10	1	1	1	1	1	1	0	0	1	1
	11	1	0	0	0	0	0	0	0	1	1
	12	1	1	1	1	1	0	0	0	1	1
	13	1	1	1	1	1	1	1	1	1	1
	14	1	0	1	1	1	1	1	1	1	1
	15	1	1	1	1	1	1	1	1	1	1
	16	1	0	1	1	1	1	1	1	1	1

Table 5 – Amount of work and duration of maintenance actions

m	MT (min)	Amount of Work *per* Competence, $AW_{m,c}$ (min)										AWt_m (min)
		c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	
1	150	744	0	0	0	0	0	0	0	300	250	744
2	420	1680	0	0	0	0	0	0	0	420	210	1680
3	210	840	0	0	0	0	0	0	0	175	420	840
4	210	840	0	0	0	0	0	0	0	525	100	840
5	276	840	0	0	0	0	420	0	0	200	100	840
6	186	744	0	0	0	0	0	0	0	300	400	744
7	186	744	0	0	0	0	0	0	0	250	150	744
8	186	744	0	0	0	0	0	0	0	150	70	744
9	186	744	0	0	0	0	0	0	0	125	75	744
10	186	744	0	700	150	150	0	0	0	200	100	744
11	420	840	840	0	100	100	0	210	0	200	100	840
12	53	210	0	0	0	0	0	210	0	100	50	210
13	53	210	0	0	200	0	0	0	100	100	0	210
14	60	60	0	0	0	60	0	0	20	20	0	60

Table 6 – Units going to depot to perform maintenance

$ZM_{k,d}$		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
d	1	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0

Table 7 – Maintenance actions scheduled for each unit

$KM_{k,m}$		1	2	3	4	5	6	7	8	9	10	11	12	13	14
k	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0
	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	9	1	0	0	0	0	0	0	0	1	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 8 – Information concerning stations

Station Name	Station Number
Roma-Areeiro	1
Entrecampos	2
Sete-Rios	3
Campolide	4
Pragal	5
Corroios	6
Foros de Amora	7
Fogueteiro	8
PMC (Depot)	9
Coina	10
Penalva	11
Pinhal-Novo	12
Venda do Alcaide	13
Palmela	14
Setúbal	15

Table 9 – Information about tasks

Task	Sd_i	Sa_i	Dd_i	Da_i
1	9	1	309	332
2	1	15	343	401
3	15	1	418	476
4	1	10	483	516
5	10	1	523	556
6	1	10	563	596
(…)				
192	10	1	1033	1066
193	1	10	1073	1106
194	10	1	1143	1176
195	1	15	1183	1242
196	15	9	1258	1280

Table 10 – Information on tasks carried out by each unit

$X_{k,i}$		*i* 1	2	3	4	5	6		191	192	193	194	195	196
k	1	0	**1**	**1**	**1**	**1**	**1**	(…)	0	0	0	0	0	0
	2	0	0	**1**	**1**	**1**	**1**		0	0	0	0	0	0
	3	0	0	0	0	0	0		0	0	0	0	0	0
	4	0	0	0	0	0	0		0	0	0	0	0	0
	5	0	0	0	0	0	0		0	0	0	0	0	0
	6	0	0	0	0	0	0		0	0	0	0	0	0
	7	0	0	0	0	0	0		0	0	0	0	0	0
	8	0	0	0	0	0	0		0	0	0	0	0	0
	9	0	0	0	0	0	0		0	0	0	0	0	0
	10	0	0	0	0	0	0		0	0	0	0	0	0
	11	0	0	0	0	0	0		0	0	0	0	0	0
	12	0	0	0	0	0	0		0	0	0	0	0	0
	13	0	0	0	0	0	0		0	0	0	0	0	0
	14	0	0	0	0	0	0		0	0	0	0	0	0
	15	0	0	0	0	0	0		0	0	0	0	0	0
	16	0	0	0	0	0	0		0	0	0	0	0	0
	17	0	0	0	0	0	0		**1**	**1**	**1**	**1**	**1**	**1**

Table 11 - Information on the unit and pair of tasks between each maintenance action is completed

$YM_{k,i,j,m}$		*m* 1	2	3	4	5	6	7	8	9	10	11	12	13	14
$Y_{k,i,j} = 1$	…								*(…)*						
	$Y_{1,7,8}$	0	0	**1**	0	0	0	0	0	0	0	0	0	**1**	0
	…								*(…)*						
	$Y_{3,23,24}$	0	0	0	0	0	**1**	0	0	0	0	0	0	0	0
	…								*(…)*						
	$Y_{9,99,100}$	0	0	0	0	0	0	0	0	**1**	0	0	0	0	0

Table 12 – Pair of tasks linked by each unit

$Y_{k,i,j}$			*j* 1	2	3	4	5	6	7		190	191	192	193	194	195	196
k	1	*i* 1		**1**	0	0	0	0	0		0	0	0	0	0	0	0
		2	0		**1**	0	0	0	0		0	0	0	0	0	0	0
		3	0	0		**1**	0	0	0		0	0	0	0	0	0	0
		4	0	0	0		**1**	0	0		0	0	0	0	0	0	0
		5	0	0	0	0		**1**	0		0	0	0	0	0	0	0
		6	0	0	0	0	0		**1**		0	0	0	0	0	0	0
		7	0	0	0	0	0	0			0	0	0	0	0	0	0
		(…)															
		190	0	0	0	0	0	0	0			0	0	0	0	0	0
		191	0	0	0	0	0	0	0		0		0	0	0	0	0
		192	0	0	0	0	0	0	0		0	0		0	0	0	0
		193	0	0	0	0	0	0	0		0	0	0		0	0	0
		194	0	0	0	0	0	0	0		0	0	0	0		0	0
		195	0	0	0	0	0	0	0		0	0	0	0	0		0
		196	0	0	0	0	0	0	0		0	0	0	0	0	0	
	2	*i* 1		0	0	0	0	0	0		0	0	0	0	0	0	0
		2	0		0	0	0	0	0		0	0	0	0	0	0	0
		3	0	0		**1**	0	0	0		0	0	0	0	0	0	0
		4	0	0	0		**1**	0	0		0	0	0	0	0	0	0
		5	0	0	0	0		**1**	0		0	0	0	0	0	0	0
		6	0	0	0	0	0		**1**		0	0	0	0	0	0	0
		7	0	0	0	0	0	0			0	0	0	0	0	0	0
		(…)															
		190	0	0	0	0	0	0	0			0	0	0	0	0	0
		191	0	0	0	0	0	0	0		0		0	0	0	0	0
		192	0	0	0	0	0	0	0		0	0		0	0	0	0
		193	0	0	0	0	0	0	0		0	0	0		0	0	0
		194	0	0	0	0	0	0	0		0	0	0	0		0	0
		195	0	0	0	0	0	0	0		0	0	0	0	0		0
		196	0	0	0	0	0	0	0		0	0	0	0	0	0	
	(…)																
	17	*i* 1		0	0	0	0	0	0		0	0	0	0	0	0	0
		2	0		0	0	0	0	0		0	0	0	0	0	0	0
		3	0	0		0	0	0	0		0	0	0	0	0	0	0
		4	0	0	0		0	0	0		0	0	0	0	0	0	0
		5	0	0	0	0		0	0		0	0	0	0	0	0	0
		6	0	0	0	0	0		0		0	0	0	0	0	0	0
		7	0	0	0	0	0	0			0	0	0	0	0	0	0
		(…)															
		190	0	0	0	0	0	0	0			**1**	0	0	0	0	0
		191	0	0	0	0	0	0	0		0		**1**	0	0	0	0
		192	0	0	0	0	0	0	0		0	0		**1**	0	0	0
		193	0	0	0	0	0	0	0		0	0	0		**1**	0	0
		194	0	0	0	0	0	0	0		0	0	0	0		**1**	0
		195	0	0	0	0	0	0	0		0	0	0	0	0		**1**
		196	0	0	0	0	0	0	0		0	0	0	0	0	0	

5. Results

In this section, the results of crew scheduling for the Fertagus case study are presented and analysed. The whole problem is here divided in two models and run separately, one for the maintenance crew scheduling and the other for the driving crew scheduling.

5.1. Maintenance crew scheduling problem

The maintenance crew scheduling model was executed for a specific day of a given week, adapting the scheduled maintenance actions from Mira (2018) model and using the actual Fertagus rolling-stock schedule. The minimum cost obtained for this problem was 454 monetary units. It is possible to observe that from the whole crew of 16 maintenance workers, only 10 are required to successfully carry out all maintenance actions and so, 6 of them are not assigned to work on this day. The maintenance actions performed by each worker, the respective unit, starting and finishing times are also presented here (Figure 2). Since the objective function focuses on minimizing the cost of employing workers, logically, the ones with an associated lower cost will be assigned, if they have the required competences.

```
[MAINTENANCE CREW]

Maintenance Worker #1 works on Day [1]
Maintenance Activity: (3); Unit: [1] Starting Hour (min): 778; Finishing Hour (min): 988
Maintenance Activity: (1); Unit: [9] Starting Hour (min): 623; Finishing Hour (min): 773

Maintenance Worker #2 works on Day [1]
Maintenance Activity: (3); Unit: [1] Starting Hour (min): 778; Finishing Hour (min): 988
Maintenance Activity: (1); Unit: [9] Starting Hour (min): 623; Finishing Hour (min): 773

Maintenance Worker #3 doesn't work on Day [1]

Maintenance Worker #4 doesn't work on Day [1]

Maintenance Worker #5 works on Day [1]
Maintenance Activity: (6); Unit: [3] Starting Hour (min): 624; Finishing Hour (min): 810
Maintenance Activity: (9); Unit: [9] Starting Hour (min): 815; Finishing Hour (min): 1001

Maintenance Worker #6 doesn't work on Day [1]

Maintenance Worker #7 works on Day [1]
Maintenance Activity: (6); Unit: [3] Starting Hour (min): 624; Finishing Hour (min): 810

Maintenance Worker #8 works on Day [1]
Maintenance Activity: (6); Unit: [3] Starting Hour (min): 625; Finishing Hour (min): 811
Maintenance Activity: (9); Unit: [9] Starting Hour (min): 816; Finishing Hour (min): 1002

Maintenance Worker #9 works on Day [1]
Maintenance Activity: (3); Unit: [1] Starting Hour (min): 778; Finishing Hour (min): 988
Maintenance Activity: (1); Unit: [9] Starting Hour (min): 623; Finishing Hour (min): 773

Maintenance Worker #10 works on Day [1]
Maintenance Activity: (13); Unit: [1] Starting Hour (min): 980; Finishing Hour (min): 1033
Maintenance Activity: (1); Unit: [9] Starting Hour (min): 623; Finishing Hour (min): 773
Maintenance Activity: (9); Unit: [9] Starting Hour (min): 789; Finishing Hour (min): 975

Maintenance Worker #11 doesn't work on Day [1]

Maintenance Worker #12 works on Day [1]
Maintenance Activity: (3); Unit: [1] Starting Hour (min): 770; Finishing Hour (min): 980
Maintenance Activity: (13); Unit: [1] Starting Hour (min): 980; Finishing Hour (min): 1033

Maintenance Worker #13 doesn't work on Day [1]

Maintenance Worker #14 works on Day [1]
Maintenance Activity: (13); Unit: [1] Starting Hour (min): 980; Finishing Hour (min): 1033
Maintenance Activity: (6); Unit: [3] Starting Hour (min): 624; Finishing Hour (min): 810

Maintenance Worker #15 doesn't work on Day [1]

Maintenance Worker #16 works on Day [1]
Maintenance Activity: (13); Unit: [1] Starting Hour (min): 980; Finishing Hour (min): 1033
Maintenance Activity: (1); Unit: [9] Starting Hour (min): 623; Finishing Hour (min): 773
Maintenance Activity: (9); Unit: [9] Starting Hour (min): 773; Finishing Hour (min): 959
```

Figure 2 – Results for the maintenance crew scheduling

As it can be observed in the results, it is important to note that while the same maintenance worker cannot perform two different maintenance activities simultaneously, there are cases where, for the same unit, two different activities are carried out at the same time. Different maintenance workers can start the same maintenance activity m, with some minutes of interval, and so, that action m may have several starting and finishing times, as it happens for action m_3 executed on train unit k_1.

5.2. Driving crew scheduling problem

After running the model it was possible to observe that the results obtained do not present enough quality to be presented, i.e. there are some specific cases where the driver assignment do not replicate the desired solution, namely the coupling/decoupling of units.

Consequently, a medium size model problem was approached, containing part of the actual Fertagus rolling-stock schedule. The total number of tasks was reduced to 84, with the attention that previously task T85, is now T38. All the other tasks remained unchanged matching the ones from the complete planning. Only 4 rolling stock units are considered and the driving crew is now formed by 8 members. The inputs related with the linkage between tasks by a given train unit, Yk,i,j, the assignment of tasks to units, Xk,i, and the maintenance slot assigned to each unit, YMk,i,j,m, were also reduced. From Figure 3 it is possible to assess that only 7 driving workers are necessary to carry out all of the tasks, resulting in a total cost of 378 monetary units. Additionally, it can be observed that, as desired, drivers successfully carry out consecutive tasks, without changing units.

```
Driver Worker #1
Driving Worker [1] works on this day.
Task8;[1038-1040]; Unit: (1); Entry Hour (min):1023 (9); Exit Hour (min):0 (10)
Task8;[1038-1040]; Unit: (2); Entry Hour (min):1023 (9); Exit Hour (min):0 (10)
Task9;[1053-1086]; Unit: (1); Entry Hour (min):0 (10); Exit Hour (min):0 (1)
Task9;[1053-1086]; Unit: (2); Entry Hour (min):0 (10); Exit Hour (min):0 (1)
Task10;[1093-1151]; Unit: (1); Entry Hour (min):0 (1); Exit Hour (min):0 (15)
Task10;[1093-1151]; Unit: (2); Entry Hour (min):0 (1); Exit Hour (min):0 (15)
Task11;[1168-1226]; Unit: (1); Entry Hour (min):0 (15); Exit Hour (min):0 (1)
Task12;[1233-1266]; Unit: (1); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task13;[1283-1316]; Unit: (1); Entry Hour (min):0 (10); Exit Hour (min):0 (1)
Task14;[1333-1366]; Unit: (1); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task38;[1377-1379]; Unit: (1); Entry Hour (min):0 (10); Exit Hour (min):1394 (9)

Driver Worker #2
Driving Worker [2] works on this day.
Task1;[309-332]; Unit: (1); Entry Hour (min):294 (9); Exit Hour (min):0 (1)
Task2;[343-401]; Unit: (1); Entry Hour (min):0 (1); Exit Hour (min):0 (15)
Task3;[418-476]; Unit: (1); Entry Hour (min):0 (15); Exit Hour (min):0 (1)
Task3;[418-476]; Unit: (2); Entry Hour (min):0 (15); Exit Hour (min):0 (1)
Task4;[483-516]; Unit: (1); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task4;[483-516]; Unit: (2); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task5;[523-556]; Unit: (1); Entry Hour (min):0 (10); Exit Hour (min):0 (1)
Task5;[523-556]; Unit: (2); Entry Hour (min):0 (10); Exit Hour (min):0 (1)
Task6;[563-596]; Unit: (1); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task6;[563-596]; Unit: (2); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task7;[607-609]; Unit: (1); Entry Hour (min):0 (10); Exit Hour (min):624 (9)
Task7;[607-609]; Unit: (2); Entry Hour (min):0 (10); Exit Hour (min):624 (9)

Driver Worker #3
Driving Worker [3] works on this day.
Task37;[637-962]; Unit: (4); Entry Hour (min):622 (10); Exit Hour (min):977 (10)

Driver Worker #4
Driving Worker [4] works on this day.
Task24;[918-920]; Unit: (3); Entry Hour (min):903 (9); Exit Hour (min):0 (10)
Task25;[963-996]; Unit: (3); Entry Hour (min):0 (10); Exit Hour (min):0 (1)
Task25;[963-996]; Unit: (4); Entry Hour (min):0 (10); Exit Hour (min):0 (1)
Task26;[1003-1061]; Unit: (3); Entry Hour (min):0 (1); Exit Hour (min):0 (15)
Task26;[1003-1061]; Unit: (4); Entry Hour (min):0 (1); Exit Hour (min):0 (15)
Task27;[1078-1133]; Unit: (3); Entry Hour (min):0 (15); Exit Hour (min):0 (1)
Task27;[1078-1133]; Unit: (4); Entry Hour (min):0 (15); Exit Hour (min):0 (1)
Task28;[1143-1176]; Unit: (3); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task28;[1143-1176]; Unit: (4); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task29;[1187-1189]; Unit: (3); Entry Hour (min):0 (10); Exit Hour (min):1204 (9)
Task29;[1187-1189]; Unit: (4); Entry Hour (min):0 (10); Exit Hour (min):1204 (9)

Driver Worker #5
Driving Worker [5] works on this day.
Task17;[355-373]; Unit: (2); Entry Hour (min):340 (9); Exit Hour (min):0 (15)
Task17;[355-373]; Unit: (3); Entry Hour (min):340 (9); Exit Hour (min):0 (15)
Task18;[388-446]; Unit: (3); Entry Hour (min):0 (15); Exit Hour (min):0 (1)
Task19;[453-486]; Unit: (3); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task20;[487-502]; Unit: (3); Entry Hour (min):0 (10); Exit Hour (min):0 (15)
Task21;[508-566]; Unit: (3); Entry Hour (min):0 (15); Exit Hour (min):0 (1)
Task22;[573-606]; Unit: (3); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task23;[617-619]; Unit: (3); Entry Hour (min):0 (10); Exit Hour (min):634 (9)

Driver Worker #6
Driving Worker [6] works on this day.
Task30;[320-322]; Unit: (4); Entry Hour (min):305 (9); Exit Hour (min):0 (10)
Task31;[333-366]; Unit: (4); Entry Hour (min):0 (10); Exit Hour (min):0 (1)
Task32;[373-406]; Unit: (4); Entry Hour (min):0 (1); Exit Hour (min):0 (10)
Task33;[423-456]; Unit: (4); Entry Hour (min):0 (10); Exit Hour (min):0 (1)
Task34;[463-521]; Unit: (4); Entry Hour (min):0 (1); Exit Hour (min):0 (15)
Task35;[538-596]; Unit: (4); Entry Hour (min):0 (15); Exit Hour (min):0 (1)
Task36;[603-636]; Unit: (4); Entry Hour (min):0 (1); Exit Hour (min):651 (10)

Driver Worker #7
Driving Worker [7] works on this day.
Task15;[1152-1257]; Unit: (2); Entry Hour (min):1137 (15); Exit Hour (min):0 (15)
Task16;[1258-1280]; Unit: (2); Entry Hour (min):0 (15); Exit Hour (min):1295 (9)

Driver Worker #8
Driving Worker [8] doesn't work on this day.
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Figure 3 – Results for the driving crew scheduling

A sensitivity analysis of the weight associated with maintenance and driving workers employment cost in the objective function is not carried out since these are values that have the same units, and both were established by the Fertagus company. Moreover, as it could be noticed, the computational time required to run both models is really low, so there is no point in presenting an optimality gap analysis for neither case. However, it would be possible to run models for an increasing number of units and study the evolution of the computational time and the respective optimality gap.

6. Conclusion and future research

This final chapter exhibits the conclusions of the research, identifies some limitations and points out steps for further improvements the research here conducted.

Following the reviewed work, with a higher focus on Mira (2018) dissertation, it was decided to follow some thoughts exposed on his future research section, namely the "crew scheduling that takes into account the different skills of maintenance technicians". This idea was in fact the first main objective for the present dissertation: a maintenance crew scheduling model that considers different skillsets for each worker and that could be applied to a train operating company, Fertagus. In later stages of the conception of such model, it was also suggested to integrate the company's driver crew scheduling with the previous maintenance crew plan, so that a broader model would provide an optimal daily schedule concerning both crews. For the maintenance crew, it was possible to create and successfully solve the model for the Fertagus case study and therefore respect its constraints, some of them previously defined by Mira (2018) and Méchain (2017) and integrated in some of the inputs of the current study. On the other hand, concerning the driving crew a model was conceived outputting an optimal driving crew schedule for a medium-size problem, containing part of the actual Fertagus rolling-stock schedule. This medium-size driving crew scheduling model is another contribution of this research. It can even be used as groundwork for future work, keeping in mind the need to solve the identified problems concerning the units' coupling and decoupling. Additionally, the computational time required to run both models is significantly small, as expected, since the output consists on a daily schedule.

Firstly, one of the major limitations of this model is the fact that it is very dependent on the user inputs, i.e. the inputs defined by the user must accurately characterize the real-world situation. Secondly, since the objective function is a cost-minimization linear function, only focused on the financial variables related with the workers' wages, which is previously established by the company, no sensitivity analysis on the weight of these parameters was carried out. Additionally, as the computational time required to run the model is generally reduced, it did not make sense to do an optimality gap analysis function of time, as it is not large enough. Since the maintenance crew scheduling problem is a continuation of researches carried out by Mira (2018) and Méchain (2017), some limitations were already established beforehand as it happens for the number of units that can go to depot on a single day. Lastly, regarding the drivers scheduling, only a medium-size problem was successfully solved and its solution validated. In line with this, situations where units couple or decouple might have some incoherencies and so it is a matter left for further research, mentioned next in section.
Due to some limitations presented above, the initial model for the whole crew scheduling had to be split in two, so it would make sense to integrate both models in a single one, so that all the crew scheduling can be obtained by running a single model. Moreover, the output obtained is a daily schedule. As the computational time is really low, it is possible to obtain a weekly schedule if the model is run for several days. However, it is thought it may be useful to model a program that is able to output a weekly schedule in a single run.

Furthermore it might be interesting to define and obtain values for the amount of work per competence carried out by each maintenance worker, i.e. skills would be interpreted as "sub-tasks" executed by maintenance workers, so that the model would output, the time spent by each worker for each competence. This way it would be possible to analyse the most and least required skills, and so, a specialization of the working crew could be carried out. The main suggested improvement, however, is related with the coupling and decoupling of units, understandably concerning the drivers scheduling problem. It is believed that by solving this detail it would be possible to assign drivers in a more efficient way and so optimal solutions for even larger cases than the present Fertagus case study could be obtained.

7. References

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