Multi-objective Optimization of an Electrical Machine Magnetic Core using a Vanadium-Cobalt-Iron alloy

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Innovation in the field of electrical machines has been focused on increasing efficiencies and power densities through the application of optimization techniques and new materials. The Vanadium-Cobalt alloy (VaCoFe) is a promising material that presents a high saturation point (2.2-2.4T) but has higher losses than classical Silicon-Iron alloys (FeSi). This work is presented in an attempt to clarify the conditions in which VaCoFe’s proprieties are advantageous on the design of electrical machines, in particular in the relations between efficiency, power and volume.

The application of VaCoFe in a simple magnetic circuit, a single-phase transformer, is analyzed. Each material’s $B-H$ and losses curves are experimentally determined. Electromagnetic and thermal models are developed in frequency domain, to be used in optimization tools. The electromagnetic model transformation to the frequency domain is done through an equivalent $B-H$ curve, independent of time. The thermal model is based on an equivalent circuit of thermal resistances. The models are validated experimentally and with finite element models. A multi-objective genetic optimization algorithm is applied to optimize the transformer’s geometry, maximizing its output power and minimizing its volume, for VaCoFe and FeSi cores.

Results show that, for low frequencies, transformers with VaCoFe cores allow higher power densities than with FeSi cores. However, for higher frequencies, the impact of higher core losses and the temperature limits imposed prevent reaching the saturation point of VaCoFe, such that the advantages of one material over the other are not clear. Therefore, the new VaCoFe alloy presents clear advantages when compared to FeSi for low frequency values, however, for higher ones these advantages are reduced.

*Keywords*— Cobalt Alloys, Electrical Machines, Electrical Performance, Optimization methods, Transformers, Vanadium Alloys.

I. INTRODUCTION

The search for higher power densities and efficiencies in electrical machines is a constant pursuit among research and industrial communities. These parameters are the source of considerable economic gains as they lead to lighter, more efficient machines that consume less material during production and less energy during their lifecycle. These characteristics are extremely advantageous in applications with limited energy storage such as electric mobility and transportation where weight, volume and efficiency are crucial limitations. In this sense, the development of machines has been made by a combined approach of applying new soft-magnetic materials in cores and optimizing existent topologies.

Besides the improvements made to classical Iron-Silicon alloy (FeSi) and different alloys such as Iron-Nickel (NiFe) and Iron-Cobalt (CoFe), new materials as soft magnetic composites and amorphous or nanocrystalline metals are being explored [1]. However, despite the increase in material variety, non oriented FeSi alloys are still the most commonly used given their lower price and good magnetic and mechanical properties compared to other materials [2].

Amorphous and nanocrystalline metals present very low core losses and high magnetic permeabilities but also low saturation points [3]. This results in high-efficiency machines with low power densities [4]. Soft magnetic composites have low core losses but present low saturation points and permeability [5]. Because composite cores are made by compactation of small particles of material, they allow the development of tridimensional flux geometries that, added to the material’s low losses, result in machines with increased efficiency but more fragile mechanically [6] [7]. The application of new materials is then limited to specific applications where their proprieties are the most suited and compensate the increased cost.

CoFe alloys with the adition of 2% Vanadium (VaCoFe) are one of the most promising materials. They present the highest saturation point (2.2T–2.4T) and are stronger mechanically than other crystalline alloys. The disadvantages are the increased core losses in relation to FeSi alloys, lower magnetic permeability and high material price [1]. Also the properties of these types of alloy are highly dependet on post-processing. They need a heat treatment in the form of an annealing at temperatures between 704ºC and 871ºC in dry hydrogen or vacuum to achieve optimal mehanic and magnetic proprieties [8] [9]. VaCoFe’s properties suggest the possibility of very high power dense machines with reduction of volumes but with lower efficiencies [1].

Some comparative studies between FeSi and VaCoFe alloys in electrical machines have been made, with mixed results. In [10] a genetic algorithm based on finite element models (FEM) is used to optimize the geometry of synchronous reluctance machines (SRM). It is shown that the VaCoFe machine presents better efficiency and power density, until a certain rotation speed from which FeSi machine becomes the best option. In [11] high speed induction machines are built, and it is shown that a VaCoFe machine has slightly higher efficiency at the cost of greater operating temperatures. In permanent magnet synchronous machines (PMSM) considerable gains in power and torque may be achieved by substituting FeSi core for a VaCoFe one [12] [13] [14].

This work is presented to clarify the conditions in which VaCoFe alloys are advantageous in relation to classical FeSi ones in the design of electrical machines. To achieve that goal, the geometry of a simple magnetic circuit made of VaCoFe and FeSi is optimized using a genetic algorithm and analytical models. The results are then expanded to the context of
II. DETERMINATION OF B-H AND POWER CURVES

To model VaCoFe and FeSi alloys, their magnetizing B-H curves are determined experimentally. The curves are obtained by testing a magnetic circuit with a primary and secondary winding, with \( N_1 \) and \( N_2 \) turns respectively, and a core made of each material. The circuits are tested by applying current pulses in the primary and measuring the induced voltage in the secondary. Short pulses are used so that greater currents and magnetic field intensities (\( H \)) can be imposed without reaching the thermal limits of the windings' isolation.

A. Vanadium-Cobalt-Iron Magnetic Circuit

The VaCoFe alloy obtained is produced by Carpenter under the designation Hiperco® 50 and is made of 49% Cobalt, 49% Iron and 2% Vanadium with a saturation point between 2T and 2.4T. The material is sold as laminated sheets without a final heat treatment. The recommended treatment is annealing in dry hydrogen at 704°C to 871°C for 2h to 4h [15]. The choice of annealing temperature defines the final magnetic and mechanical properties such that greater temperatures increase the magnetic saturation point but lower yield and tensile strengths. In addition, the material is very sensitive to contaminants that impact its properties negatively [8]. This results in a sensitivity to the cutting method chosen to shape the sheets and its sequence with the heat treatment. In [8] two cutting methods were analyzed: stamping and wire-EDM (electric discharge machining) before and after annealing. It is shown that the process that results in the highest saturation (2.4T) is stamping followed by annealing. For economic reasons the material tested was cut by wire-EDM and then annealed which, according to [8], should result in a non-optimum saturation point of 2.0T.

The magnetic circuit tested is made of 19 rectangular laminations (55x35mm) of VaCoFe with windings located as shown in Figs. Fig. 1 and Fig. 2.

B. Iron-Silicon Magnetic Circuit

The magnetic circuit dimensions are shown in Fig. 3a). This circuit is made of forty (40) 2cm thick ‘E-I’ shaped lamination pairs. Primary and secondary windings are as shown in Fig. 3b).

C. Pulse Generator

The testing of the magnetic circuits is done with the circuit presented in Fig. 4. This circuit generates pulses by charging and discharging the capacitor C into the primary winding controlled by switch S.

An oscilloscope is used to measure the waveform of the current in the primary winding and the induced voltage in the secondary. The current, \( i_1 \), is used to compute the magnetic field intensity (\( H \)) form Ampère’s law (1.1). The H field for the VaCoFe circuit is given by (1.2) and by (1.3) for the FeSi
circuit. \( l \) is the length of the average path of the VaCoFe core and \((l_c + l_{lat})\) is the length of the characteristic length of the FeSi core. The induced voltage, \( u_2 \), is used to compute the magnetic flux density \((B)\) with the Induction law (2).

\[
\oint_c \mathbf{H}(t) \cdot dl = \oint_S \mathbf{J} \cdot \mathbf{n} \, dS \quad \text{(1.1)}
\]

\[
H_{VaCoFe}(t) = \frac{N_1 i_1(t)}{l} \quad \text{(1.2)}
\]

\[
H_{FeSi}(t) = \frac{N_1 i_1(t)}{(l_c + l_{lat})} \quad \text{(1.3)}
\]

\[
\oint_c \mathbf{E} \cdot dl = -\frac{d}{dt} \oint_S \mathbf{B} \cdot \mathbf{n} \, dS \Rightarrow \]

\[
\Rightarrow B(t) = \frac{1}{N_2 S} \int u_2(t) dt \quad \text{(2)}
\]

Fig. 4. Experimental setup used to generate pulses and test the magnetic circuits.

For each material, pulses are applied with increasing amplitude until saturation is reached. The magnetization \( B-H \) curve is obtained by plotting \( B \) and \( H \) values for each pulse.

D. Experimental Results

The magnetization curves obtained experimentally are presented in Fig. 5 for VaCoFe and FeSi. The ideal curve obtained in [8] for the optimum processing is also shown.

The VaCoFe curve obtained experimentally has lower saturation and permeability than the ideal one because of the non-optimal processing of the acquired material.

Between VaCoFe and FeSi, the former’s saturation point (2T) is higher than the latter’s (1.6T). Ideally, the permeability would also be higher, however the VaCoFe experimental curve shows similar permeability to FeSi.

The magnetic core losses curves are presented in Fig. 6. It is shown that VaCoFe presents higher losses than the FeSi, for 50Hz and 400Hz. It is then expected that the use of the VaCoFe alloy results in electrical machines with greater power density but with lower electrical efficiency than FeSi ones.

![Magnetic losses graph](image)

Fig. 6. Magnetic losses for VaCoFe (red) and FeSi (black) for 50Hz (continuous lines) and 400Hz (dotted lines).

III. ELECTROMAGNETIC AND THERMAL MODELS OF A SINGLE-PHASE TRANSFORMER

In order to identify the advantages of the use of VaCoFe in electrical machines, the geometry of simple magnetic circuit, a single-phase transformer, is optimized for VaCoFe and FeSi cores. This approach allows the comparison between the best geometries for each material for the same power range. This allows the identification of the ranges in which each material is the best option.

To optimize the transformer geometry, a multi-objective optimization algorithm is used. An accurate model of the relevant physics of the transformer is needed to evaluate objective functions and restraints. Because the algorithms are iterative, the computing time of the models has a high impact on the total running time. For this reason, analytical electromagnetic and thermal models are developed. Because the analysis is focused on steady-state operation the models are based on lumped parameters in the frequency domain. This avoids the computing cost of time dependent models. In the case of the electromagnetic model, the frequency domain transformation is achieved by computing and applying a non-time dependent equivalent \( B-H \) effective curve.

The type of transformer geometry analysed is shown in Fig. 7. It is formed by two horizontal parts and three vertical columns of magnetic material and primary and secondary windings in the central column. The height, width and depth of the magnetic core are \( h_{core} \), \( w_{core} \) and \( w_c \), respectively. It is...
assumed that the central column has a square-section, $S_c$, ($w_c \times w_c$) and the lateral ones have a rectangular cross-section, with half the width of the central column ($w_c/2 \times w_c$). This assures that the magnetic flux density is the same in all columns, $B_c = B_{lat}$. The available area for copper windings is $h_{cu}$ by $w_{cu}$.

![Diagram](image)

**Fig. 7.** Magnetic core geometry and dimensions with the considered magnetic flux directions and characteristic lengths.

### A. Electromagnetic Model

1) **Equivalent Circuit**

The lumped parameter electromagnetic model (LPM) is based on the Steinmetz ‘T’ equivalent circuit of a transformer [16], presented in Fig. 8. $\bar{U}_1$ and $\bar{I}_1$ are the primary voltage and current phasors, $\bar{U}_2$ and $\bar{I}_2$ are the secondary voltage and current seen form the primary side, and $\bar{U}_m$ and $\bar{I}_m$ are the magnetization voltage and current. The winding resistances are $r_1$ and $r_2$. As a simplification, the leakage flux of the transformer is considered to be approximately zero, $\lambda_1 \approx 0$ e $\lambda_2' \approx 0$. $L_{meff}$ is the effective magnetizing inductance. This parameter models the relation between voltage and current of the magnetic core as given by the effective $B-H_{eff}$ curve as described in section III.A.2). $R_m$ is the core resistance and models core losses. These are dependent on $B$ and $R_m$ is computed based on the material’s core losses curve (3) where $V_{core}$ is the core volume and $\bar{p}_m$ is the losses density given by the curve.

$$R_m = \frac{(U_m)^2}{V_{core} \bar{p}_m} \quad (3)$$

![Diagram](image)

**Fig. 8.** Electromagnetic lumped parameter model of the single-phase transformer.

2) **$B-H$ Effective Curves**

$B-H$ magnetizing curves that describe magnetic materials are non-linear and present a saturation effect of $B$ with the increase of $H$. Given that the electric current relates to $H$ by Ampère’s law and the voltage relates to $B$ by the Induction law, an AC excitation that causes de core to saturate results in non-sinusoidal current waveforms. Because of this, a constant induction coefficient can’t be defined as it varies in time with the instantaneous excitation.

A constant coefficient is needed to use the LPM. This is achieved by converting the real material, described by its $B-H$ curve, to a linear material, described by a point in its effective $B-H_{eff}$ curve. Each point in the $B-H_{eff}$ curve represents a linear material that approximates the real material for a given RMS value of the AC excitation. This way, a sinusoidal voltage and flux density with RMS value $B$, results in sinusoidal effective magnetic field intensity with RMS value $H_{eff}$ and sinusoidal effective currents, that approximate non-sinusoidal $H(t)$ in the material.

There are several methods to obtain a $B-H_{eff}$ curve. The one chosen is presented in [17] as the Average Energy Method. This method computes $H_{eff}$ by considering the conservation of energy. Given a $B-H$ curve, the average value of energy density is given by (4). Using a $B-H_{eff}$ curve, this quantity is given by (5) where $\mu_{eff}$ is the effective permeability. Equaling (4) and (5) we obtain (6). $H_{eff}$ is computed with (7) for increasing values of $B$, and a $B-H_{eff}$ curve is built from the $B-H$ curve.

$$\langle \Phi_m \rangle = \frac{4}{T} \int_0^T \left( \int_{B(0)}^{B(t)} H(B) dB \right) dt \quad (4)$$

$$\langle \Phi_m \rangle = \frac{1}{2} \langle H_{eff} B \rangle = \frac{1}{2} \frac{B^2}{\mu_{eff}} \quad (5)$$

$$\mu_{eff} = \frac{B}{\bar{H}} = \frac{B}{\frac{T}{4} \int_0^T \int_{B(0)}^{B(t)} H(B) dB dt} \quad (6)$$

$$H_{eff} = \frac{B}{\mu_{eff}} = \frac{1}{B} \frac{T^2}{4} \int_0^T \int_{B(0)}^{B(t)} H(B) dB dt \quad (7)$$

In Fig. 9 are presented examples of $B-H_{eff}$ curves computed from the ideal VaCoFe and experimental FeSi $B-H$ curves.

![Graph](image)

**Fig. 9.** VaCoFe (black) and FeSi (red) $B-H_{eff}$ curves obtained from ideal VaCoFe and experimental FeSi $B-H$ curves.
law (9) where $U_m$ is the RMS value of the magnetizing voltage. With (8) and (9) an effective induction coefficient can be computed (10).

$$i_{m,eff}(t,B) = \sqrt{2}H_{eff}(B)\sin(\omega t) \cdot \frac{(l_c + l_{lat})}{N_i}$$

$$B = \frac{U_m}{\omega N_i S_c}$$

$$L_{m,eff} = \frac{N_i S_c \sqrt{2}B \sin(\omega t)}{i_{m,eff}(t,B)} = \frac{N_i U_m}{H_{eq}(B) \cdot (l_c + l_{lat}) \cdot \omega}$$

B. Thermal Model

One important limitation of a transformer is its operating temperature. A thermal analytical model, based on a lumped parameter circuit of thermal resistances, is developed to compute the transformer’s steady-state surface temperature for a given core geometry.

Because the transformer is made of metallic materials with high thermal conductivities, convection with the surrounding air is the process that most impacts the surface temperature. Conductive heat transfer in the core and windings is neglected and only convective thermal resistances are considered in the model circuit.

The transformer is modeled by a hollow cuboid. The surfaces of the cuboid dissipate the total joule losses $P_j$, made of the sum of windings’ copper and material’s core losses, (Fig. 10a). Each surface is modeled by a convective resistance, Fig. 10b). The thermal equation is given by (11) where $T_s$ and $T_{amb}$ are the surface temperature an ambient temperature. The convective thermal resistances are computed given the heat coefficient $h$ and area $A$ of each surface (12) [18].

![Thermal model of the transformer.](image)

$$T_s - T_{amb} = [R_{lat} \parallel R_{lat} \parallel R_{fr} \parallel R_{fr} \parallel R_{top}] P_j = R_{total} P_j$$

$$R_{lat} = \frac{1}{h_{lat} \cdot A_{lat}}, R_{fr} = \frac{1}{h_{fr} \cdot A_{fr}}, R_{top} = \frac{1}{h_{top} \cdot A_{top}}$$

Convection processes are modeled with experimental formulas. Therefore, $h$ is computed with the Nusselt, $Nu$, and Rayleigh, $Ra$, numbers for vertical and horizontal surfaces (13)-(18). $\nu$ is the fluid viscosity and $\beta$ is the volumetric expansion coefficient. The Prandtl number characterizes proprieties of the air and is given by (19), where $\mu_v$ is air’s kinematic viscosity, $k$ the thermal conductivity and $C_p$ its specific heat.

$$h_{lat} = h_{fr} = h_{vert} = k \cdot \frac{Nu_{vert}}{L_{vert}}, L_{vert} = h_{core}$$

$$h_{top} = h_{hor} = k \cdot \frac{Nu_{hor}}{L_{hor}}, L_{hor} = \frac{w_{core} \cdot d_{core}}{2(w_{core} + d_{core})}$$

$$Nu_{vert} = \left\{0.825 + \frac{0.387 \cdot (Ra)^{\frac{1}{8}}}{[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}]^\frac{8}{127}}\right\}$$

$$Nu_{hor} = 0.54 \cdot (Ra)^{\frac{1}{4}}$$

$$Ra_{hor} = Pr \cdot \frac{g \beta \rho^2}{\nu^2} (T_s - T_{amb})$$

$$Ra_{vert} = Pr \cdot \frac{g \beta \rho^2}{\nu^2} (T_s - T_{amb})$$

$$Pr = \frac{\mu_v C_p}{k}$$

Because of the dependency of $Ra$ with $T_s$, the convective resistances also depend on $T_s$, so the method of computing them is iterative (20).

$$T_s^{(n+1)} = R_{total}(T_s^{(n)}) P_j + T_{amb}$$

C. Model Validation

The proposed analytical models are validated against FEM based models and experimental results. To validate the electromagnetic LPM model a real transformer, Fig. 11, and a FEM electromagnetic model are tested for different primary voltages and resistive loads.

![Transformer used to validate the proposed model and its dimensions.](image)

Results of active power in primary and secondary windings and apparent power in the primary are shown in Fig. 12. For the active power in the primary and secondary winding, Fig. 12a), the average error deviation between LPM and experimental results is about 10%. For the apparent power, Fig. 12b), this average deviation is 5%. These deviations are mainly caused by
neglection of flux dispersion in the LPM. Results show that
despite FEM giving more accurate results, LPM model is a
reasonable approximation for the intended application in
optimization algorithms.

IV. OPTIMIZATION OF A SINGLE-PHASE TRANSFORMER

The optimization of a single-phase transformer’s core
gometry is made for VaCoFe and FeSi cores. The type of
ometry optimized is the one presented in section III. The
results of this analysis allow the drawing of conclusion about
the use of VaCoFe in more complex electrical machines. The
relevant objectives are maximizing output power and
minimizing core volume while keeping surface temperature
below safety limits. This combination of objectives also results
in the maximization of efficiency as the input power is limited
by the range of the decision variables. The chosen multi-
objective/multi-restrain genetic algorithm used is the NSGA-II
[19] because of the complex multi-physics equations and
constraints involved. An advantage of this algorithm compared
to others is its built-in elitist and explicit mechanisms that
preserve the best solutions from generation to generation and
promote an even spread of solutions across the objective
functions’ space. This results in faster convergence and avoids
local minimum solutions.

A. Optimization Problem Definition

The objectives of the analysis are to maximize output power
(secondary winding power) and minimize volume. As such the
objective functions are $f_1$ and $f_2$ (21).

$$f_{1,2} = \begin{cases} \max (P_{out}) \\ \min (V_{core}) \end{cases}$$  \hspace{1cm} (21)

The decision variables are the independent design variables
that define the transformer. Together they form the optimization
vector $\mathbf{x}$. During the genetic optimization, each individual is
represented by a single vector that give values of $f_1$ and $f_2$. These individuals are represented in the 2D objective
function space. The final solutions are the most optimum and are said to
belong to the Pareto front represented in this space. In Table 2
the decision variables are presented and described as well as their
considered ranges.

Table 2 - Decision variables, their description and range.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_{1 \text{ rms}}$ [kV]</td>
<td>Primary winding voltage</td>
<td>$[1 \ 10]$</td>
</tr>
<tr>
<td>$R_{2 \text{ load}}$ [$\Omega$]</td>
<td>Secondary winding load</td>
<td>$[0 \ 100000]$</td>
</tr>
<tr>
<td>$A_{cu}$ [mm$^2$]</td>
<td>Copper wire cross-section</td>
<td>$[0.005 \ 3.3$</td>
</tr>
<tr>
<td>$h_{\text{core}}$ [cm]</td>
<td>Height of magnetic core</td>
<td>$[0.1 \ 100]$</td>
</tr>
<tr>
<td>$w_{\text{core}}$ [cm]</td>
<td>Width of magnetic core</td>
<td>$[0.1 \ 100]$</td>
</tr>
<tr>
<td>$S_c$ [cm$^2$]</td>
<td>Central column cross-section</td>
<td>$[0.01 \ 100000]$</td>
</tr>
</tbody>
</table>

To ensure the feasibility of the solutions a temperature
constraint must be added as well as constraints that avoid
impossible geometries. The safety limit temperature is
considered to be 90ºC. The constraints imposed are then (22)
The presented analytical models are used to compute the objective functions and restraints. The external input of the models (B-H\textsubscript{eff} and power loss curves and thermal constants) and the computation of objective functions and constraints for an element n from generation t of the genetic algorithm is presented in Fig. 13.

\[ w_{Cu} > 0, h_{Cu} > 0 \quad (22) \]
\[ T_s \leq 90^\circ C \quad (23) \]

Optimizations were made for VaCoFe and FeSi cores at 50Hz and 400Hz. For each optimization the Pareto front solutions are ordered by increasing output power. Several quantities are shown in the same order in function of core volume. These results are presented in Fig. 14.

The output power achieved with VaCoFe is in average 25% higher than with FeSi for the same volumes at 50Hz, Fig. 14a1). The efficiency is higher for VaCoFe until \( P_{out} = 20\text{kW} \) and \( V_{core} = 0.007 \text{m}^3 \), point from which FeSi becomes the more efficient material, Fig. 14b1). This behaviour is explained by the relations between core volume and core losses (24) and copper losses in the windings (25) and their impact on the transformer’s surface temperature (26).

\[ P_m = k B^2 f V_{core} \quad (24) \]
\[ P_{Cu} = \rho_{Cu} J^2 V_{Cu} \quad (25) \]
\[ T_s = \frac{1}{hA} (P_m + P_{Cu}) + T_{amb} \quad (26) \]

Core losses are proportional to the electrical frequency \( f \), to the squared of flux density \( B \) and the cube of the core’s linear length, \( l \) (27). Copper losses vary with the square of the current density \( J \) and the cube of \( l \) (28). Combining these relations in (26) we get that the surface temperature depends linearly with \( l \) and the square of \( B \) and \( J \) (29).

\[ T_s \propto (B^2 f + J^3) l \quad (29) \]

With the increase in the circuit dimensions and volume related to the increase in output power, result in an increase of temperature for constant \( B \) and \( J \). All solutions obtained are very close to the temperature limit imposed. This was achieved by decreasing either \( B \) or \( J \) while increasing the core volume.

In the case of VaCoFe the core losses reach much higher values than those of FeSi. As seen on Fig. 14c1) and d1), VaCoFe’s core losses increase faster than copper losses, so core losses have a greater impact than copper losses on temperature. The temperature is maintained constant with the increase of output power and volume from 20kW and 0.007m\(^3\) by making core losses constant resulting from decreasing \( B \) from the saturation point, Fig. 14e1). From 0.010m\(^3\) onward, the copper losses also stabilize resulting from a decrease in \( J \).

In the case of FeSi cores losses are lower than VaCoFe’s and also grow slower than copper losses, Fig. 14c1) and d1). Copper losses are the ones with the greater impact on temperature. This allows \( B \) to be close to saturation point Fig. 14e1). However, the temperature limit is reached at 0.008m\(^3\) and so copper losses are forced to be constant at the cost of decreased \( J \).

For higher volumes, copper losses for VaCoFe and FeSi are approximately constant and equal while core losses remain higher in the former’s case than the latter. This allows the efficiency of FeSi to overtake VaCoFe with the increase in volume.

From the results at 400Hz, the same phenomenon are observed. VaCoFe shows core loss impact on temperature, Fig. 14c2), which results in a decrease of \( B \) from the saturation point for greater volumes, Fig. 14e2). For FeSi copper losses are dominant resulting in the decrease of \( J \) however it is noted that \( B \) decreases slightly to limit core losses that reach high values because of the high frequency.

For a more direct comparison between FeSi and VaCoFe solutions, some points are chosen with same output power. Output power of 5kW and 25kW at 50Hz and 400Hz are compared in Table 3 and Table 4, respectively.

### Table 3 - Characteristics of the VaCoFe and FeSi magnetic circuit at 50Hz.

<table>
<thead>
<tr>
<th></th>
<th>FeSi</th>
<th>VaCoFe</th>
<th>FeSi</th>
<th>VaCoFe</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{out} \text{ [kW]} )</td>
<td>5</td>
<td>5</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>( V_{core} \text{ [dm}^3\text{]} )</td>
<td>2.763</td>
<td>1.932</td>
<td>18.66</td>
<td>15.37</td>
</tr>
<tr>
<td>(-30.0%)</td>
<td>(-17.6%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight [kg]</td>
<td>21.0</td>
<td>15.7</td>
<td>141.8</td>
<td>124.7</td>
</tr>
<tr>
<td>( \eta \text{ [%]} )</td>
<td>97.4</td>
<td>97.7</td>
<td>98.9</td>
<td>98.4</td>
</tr>
<tr>
<td>( P_{Cu} \text{ [W]} )</td>
<td>122.0</td>
<td>69.1</td>
<td>199.2</td>
<td>234.0</td>
</tr>
<tr>
<td>( P_{core} \text{ [W]} )</td>
<td>12.8</td>
<td>49.0</td>
<td>77.8</td>
<td>167.4</td>
</tr>
<tr>
<td>( B_0 \text{ [T]} )</td>
<td>1.25</td>
<td>2.28</td>
<td>1.17</td>
<td>1.16</td>
</tr>
<tr>
<td>( H_{core} \text{ [cm]} )</td>
<td>48.4</td>
<td>46.2</td>
<td>82.9</td>
<td>100.0</td>
</tr>
<tr>
<td>( W_{core} \text{ [cm]} )</td>
<td>30.3</td>
<td>27.6</td>
<td>31.3</td>
<td>42.8</td>
</tr>
<tr>
<td>( S_c \text{ [cm}^3\text{]} )</td>
<td>23.7</td>
<td>17.3</td>
<td>105.7</td>
<td>68.1</td>
</tr>
</tbody>
</table>
At 50Hz a transformer with VaCoFe core is clearly advantageous in relation to a FeSi one. At 5kW output power there is a 30% reduction in core volume and at 25kW a 17.6% reduction. For higher volumes the volume reduction is lower because of the lower $B$ to 1.16T in order to respect the temperature limit, as explained before.

At 400Hz the advantages are not clear since for 5kW there is a 14.2% volume reduction and for 25kW there is an increase of 13.2%. At this frequency and at higher output powers, core losses are so high that it becomes infeasible the taking advantage of VaCoFe’s high saturation point because of the temperature constraint.

Fig. 14. Optimization results for FeSi and VaCoFe cores at 50Hz and 400Hz. a) Output power, b) electrical efficiency, c) magnetic core losses $P_{\text{core}}$, d) copper losses $P_{\text{Cu}}$, e) magnitude of magnetic flux density in the central column $B_c$.

<p>| Table 4 - Characteristics of the VaCoFe and FeSi magnetic circuit at 400Hz. |
|-----------------|--------|--------|--------|--------|
|                 | FeSi   | VaCoFe | FeSi   | VaCoFe |
| $P_{\text{out}}$ [kW] | 5      | 5      | 25     | 25     |
| $V_{\text{core}}$ [dm$^3$] | 0.571  | 0.490  | -14.2% | 2.43   | 2.75   | +13.2% |
| Weight [kg]      | 4.34   | 3.97   | 18.5   | 22.3   |
| $\eta$ [%]       | 99.3%  | 98.8%  | 99.6%  | 99.3%  |
| $P_{\text{Cu}}$ [W] | 29.3   | 29.5   | 62.5   | 91.9   |</p>
<table>
<thead>
<tr>
<th>$P_{\text{core}}$ [W]</th>
<th>8.3</th>
<th>32.5</th>
<th>35.7</th>
<th>75.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_r$ [T]</td>
<td>1.23</td>
<td>1.61</td>
<td>1.24</td>
<td>0.99</td>
</tr>
<tr>
<td>$H_{\text{core}}$ [cm]</td>
<td>31.8</td>
<td>59.2</td>
<td>56.2</td>
<td>86.2</td>
</tr>
<tr>
<td>$W_{\text{core}}$ [cm]</td>
<td>16.4</td>
<td>15.5</td>
<td>29.8</td>
<td>28.5</td>
</tr>
<tr>
<td>$S_e$ [cm$^2$]</td>
<td>7.67</td>
<td>3.76</td>
<td>18.2</td>
<td>14.2</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

This work presents a comparative analysis between VaCoFe and FeSi alloys in the context of their application on an optimized single-phase transformer.

The magnetization $B-H$ curves of each alloy were determined experimentally and compared. Some comments were made about VaCoFe alloy’s final properties’ sensitivity to the manufacturing processing.

Analytical electromagnetic and thermal models with low computation times were developed to be used in an optimization genetic algorithm. The electromagnetic model was based on a lumped parameter equivalent circuit in the frequency domain. The transformation from time to frequency domain is achieved by converting the material’s $B-H$ curve into a $B-H_{\text{eff}}$ curve based on the conservation of energy. The thermal model is based in an equivalent circuit of convective thermal resistances since this is the most relevant heat transfer mechanism. These models are validated against finite element models and experimental results.

The optimization was made with the NSGA-II algorithm and the developed models for each alloy at 50Hz and 400Hz. Results show that VaCoFe is better than FeSi at low frequencies as it results in significant decrease in core volumes for the same output power levels. However, at higher frequencies the advantages are not clear as there are mixed results. In addition, FeSi appears to be more efficient than VaCoFe for greater volume and output power.

These conclusions may vary when applied to different electric machines, however it is suggested that VaCoFe is superior to FeSi in applications where core losses are not critical in comparison to weight and volume reduction. However, at higher frequencies/rotation speeds the greater core losses causes higher temperature increases which limits the maximum flux density, efficiency and output power attainable. These results are aligned with [20] where, synchronous reluctance machines made of VaCoFe show greater operating temperatures than ones made of FeSi, specially at higher rotation speeds where the performance of FeSi machines becomes better.

The suggestion that VaCoFe is suited for low weight and high-power density applications is confirmed. However, the temperature problem is a barrier to the performance of VaCoFe machines at higher frequencies. This might be resolved with more advanced cooling systems, but this analysis must be made in a case by case basis.

REFERENCES


