Route Optimization – The Case Study of Quinta do Arneiro

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Abstract

Logistics operations play an increasingly important role in companies, which translates into a growing pressure to reduce its costs and increase its efficiency. Of all these operations, transportation is the most relevant, since it is the one with a higher percentage of costs. Thus, in an increasingly competitive business environment, the development of an efficient distribution system becomes a real necessity. The present dissertation is developed in this context, analysing the case of Quinta do Arneiro (QA), a company that specializes in the production and sale of baskets of biological products, which are highly perishable. The company, located in Torres Vedras, delivers the baskets directly to customers, essentially covering the districts of Lisbon and Setúbal. QA intends to analyse and improve its current distribution system, in order to minimize the distance and duration of the routes. The minimization of distance has the purpose of reducing distribution costs the and optimization of the duration has the purpose of minimizing the loss of quality of the products, transported in non-refrigerated vehicles. After the characterization of the problem and an adequate literature review, it is concluded that the models that best suit the problem are two Vehicle Routing Problem (VRP) variants: the Capacitated VRP (CVRP) and the Time-Dependent VRP (TDVRP). Two CVRP models and one TDVRP model are developed and for each of them two scenarios are defined, which aim at the optimization of each route individually and of the daily clusters, respectively. The models are implemented in GAMS and the obtained s are analysed. The developed scenarios for the CVRP models allow significant reduction values of the total distribution costs, achieving a reduction of 2% when optimizing individual routes and 28% when optimizing daily clusters. The TDVRP model, although validated, does not allow to obtain solutions for instances of the dimension analysed in this work. For QA, these results imply the reduction of the fleet from 4 to 3 vehicles, which translates into a reduction of the distribution costs of 1942€ (28%) for the analysed month.

Keywords: logistics, perishable food, route optimization, capacitated vehicle routing problem, time-dependent vehicle routing problem

1. Introduction

Currently, market competitiveness has a growing tendency, regardless of the sector. New technologies that contribute to the increase of competitiveness are emerging daily, and that pressures companies to reach higher levels of performance to remain relevant. Thus, logistics plays an increasingly decisive role in the success of a company. Within the logistics operations, transportation is one of the main functions and the one that carries higher costs. A good transportation system can increase logistics efficiency, reduce operations costs and promote quality of service. In the specific case of perishable products, such as biological products, transport plays an even more important role because an inefficient distribution system results in the loss of value of the products transported and, consequently, in the reduction of the profitability of the company. Route optimization is a real concern in several companies nowadays, which is one of the reasons why the Vehicle Routing Problem (VRP) is one of the most popular combinatorial optimization problems.

This paper aims to analyse and optimise the distribution system of Quinta do Arneiro (QA). This company produces and sells biological products, its main business being the sale of biological baskets, which are delivered directly to the customer. To optimize the company's routes, VRP models are developed, with the purpose of minimizing distance (to reduce costs) and time (to reduce loss of product quality during transportation). The structure of this paper is as follows: section 2 presents the case study and describes the problem characteristics. Section 3 contains a relevant literature review about VRP: presents different types of VRP, resolution methods and mathematical formulations for the selected VRP variants to be used in this paper. In section 4, the developed mathematical models are characterised. Section 5 describes procedures of data collection and processing, the current distribution scenario and discusses the results of the different scenarios analysed and compared. Finally, in section 6, the main conclusions of the current work are presented, and possible future studies are proposed to improve the obtained results.

2. Case study description

QA is a company that specializes in the production and sale of baskets with biological products (highly perishable). The company is located in Torres Vedras and delivers the baskets directly to the customers. Transportation is the most expensive logistics operation for the company, representing a cost of approximately 80000€ per year. The distribution is carried out by 3 or 4 distributors, 3 of whom are employed full-time by QA and use company vehicles, and the fourth is an external distributor that uses its own vehicle, providing distribution services to QA only on Tuesdays and Wednesdays. The delivery areas vary according the day of the week (Table 1), covering mainly the districts of Lisbon and Setúbal.

Table 1 - Daily distribution areas

Day	Distribution areas
Tuesday	Mafra, Sintra, Oeiras, Sintra Oriental
Wednesday	Lisboa, Sintra Ocidental
Thursday	Amadora, Oeiras, Lisboa, Alcochete, Moita, Barreiro, Montijo, Almada, Seixal
Fridyay	Mafra, Oeiras, Amadora, Odivelas, Loures, Torres Vedras, Vila Franca de Xira, Lisboa

These areas are adjusted as the business expands, based on the experience of the distributors. There is no analysis performed or use of any system to define the distribution areas, and therefore they may need to be optimized. Optimizing the distribution areas and the sequence of deliveries for each vehicle may translate into a significant reduction in the route distances and, consequently, the associated fuel costs.
As for the duration of the routes, most of the customers are located in urban areas, so delivery times are dependent on the traffic intensity of each zone, and this is a factor that must be taken into account, since the distribution vehicles are not refrigerated, and organic products are highly perishable. Therefore, it is also relevant to optimize the duration of the company routes, in order to minimize the products quality loss during transportation.

3. Literature Review

3.1. VRP

The study of the optimization of physical distribution routes started in the 1950s and was initially approached by (Dantzig et al., 1954). The article introduces the Truck Dispatching Problem, which seeks to model a homogeneous fleet (i.e. trucks with the same capacity) that can meet the demand for a set of fuel stations from a distribution centre, with the lowest distance travelled. In 1964, Clarke & Wright generalized this problem to a linear optimization problem: calculating the routes of a set of vehicles of varying capacity, starting from a common distribution centre and supplying a set of geographically dispersed customers. This problem is known in the Operational Research literature as VRP.
The classic version of VRP, also called Capacitated Vehicle Routing Problem (CVRP), is one of the most popular combinatorial optimization problems. It can be described as a problem of optimizing the routes of a fleet of identical vehicles departing from a distribution center (DC) and supplying a set of "customers" (shops, cities, schools, etc.) with a static and deterministic, and geographically dispersed.

3.2. Resolution methods for the VRP

VRP is classified as a NP-Hard (non-deterministic polynomial-time hard) problem. Over the last decades, several methods of solving the problem were proposed, always with the aim of reaching the best possible solutions. The three main groups of solution methods are described in the following paragraphs.

3.2.1. Exact methods

This type of algorithm tries to find an optimal solution of the problem, within the imposed restrictions. However, the methods in this category tend to have fairly long computational execution times, so they are only feasible for small problems (less than 200 customers, approximately).
The main algorithms of exact resolution can be grouped in: branch-and-bound, set-partitioning and branch-and-cut.

3.2.2. Heuristics

This type of method performs a relatively limited exploration of the search space and generally produces good quality solutions with reasonable computing times. Classical heuristic methods do not allow the intermediate solution to deteriorate during the search for better solutions, which may lead to the algorithm being restricted to local optimal solutions. Heuristics can be grouped in: constructive methods and methods of improvement.

3.2.3. Metaheuristics

These methods are the most used, currently, in the resolution of VRP's. They perform a deep exploration of the most promising regions of the solution space. The quality of solutions obtained is usually much higher than the quality of the solutions obtained through classical heuristics. One of its main characteristics is the ability to continue with the search for a feasible solution after finding a local optimum. In the last decades, there has been a significant growth in the development of VRP metaheuristics, and the algorithms can be grouped into 3 main groups: local search, population search and learning mechanisms.

3.3. VRP variants

The current literature includes several VRP variants, which include features such as travel durations over time, timing window delivery, or dynamic (i.e. evolving in real time) information. Two of these variants are discussed in this section, since they have specific characteristics that resemble those of the distribution system used in QA, and therefore, will be the ones in which the developed models on this paper will be based on.

3.3.1 CVRP

This is the classical VRP variant, as previously mentioned. In the CVRP, the vehicles depart from and return to the same DC. Customer demand is deterministic (i.e. known in advance) and it is not possible to subdivide deliveries (i.e. a customer being supplied by more than one vehicle). Given the simplicity of the case study distribution system (only 3 or 4 vehicles, 1 DC and deterministic customer demand), this variant is well suited to the problem and can be used to minimize the route distances.

3.3.2. Time-Dependent VRP (TDVRP)

The TDVRP assumes that the travel times are not constant, being a function of the real time. This variant of VRP is introduced in the literature by Malandraki & Daskin (1992), who developed a mixed-integer linear programming (MILP) formulation. The formulation, however, does not satisfy the non-passing (or first-in-first-out, FIFO) property, which ensures that if a vehicle leaves vertex node *i* to node *j* at a certain time, then if the departure time is later than the one previously set, the vehicle will always arrive later to *j*. Currently, most TDVRP models adopt a FIFO model based on the version presented by Ichoua et al. (2003). In this version, the work day is divided into several periods of time, with an average speed associated with each period. The duration of travel between two locations is highly dependent on the time of day, especially in urban areas, where traffic is usually more intense. Since QA customers are mainly located in urban areas, the TDVRP allows to model more realistic travel times, and therefore, this variant is well suited to model QA distribution system regarding route duration. As for the most efficient resolution methods, the adaptive memory search algorithm of Ichoua et al. (2003), which recombines and improves a set of good quality routes using tabu-search.

3.3.3 Mathematical Formulations

Since this paper aims to obtain optimal solutions, the developed models need to have a mathematical formulation.

For the CVRP, there are several mathematical formulations in the literature. As such, only three of the most relevant formulations are presented in this paper. The Three-Index formulation proposed by Fisher & Jaikumar (1981), uses three indices: *i*, *j* and k, of which *i* represents the departure node, *j* represents the arrival node and *k* represents a vehicle allocated to the arc (*i,j*); A binary variable x_{ij} determines if the travel (*i,j*) is performed by vehicle, being $x_{ij}=1$ if the travel is made, and $x_{ij}=0$ otherwise. The Two-Index formulation proposed by Laporte et al. (1985), is an extension of the TSP formulation proposed by Dantzig et al. (1954). In this formulation, the variable x_{ij} is integer and represents the number of times that the arc (*i,j*) appears in the optimal solution. If *i,j* do not correspond to DC nodes, x_{ij} is binary; if $i = 0$, then x_{ij} can be equal to 0, 1 or 2, the latter case corresponding to a return trip between the deposit and the client *j*. Finke et al. (1984) introduce the Two-Commodity Flow formulation for a TSP-type problem and Baldacci et al. (2004) adapt it to the VRP. In this formulation, a copy of the DC is added. This copy corresponds to the point where all routes end. The binary variable x_{ij} assumes the value 1 if the arc (*i, j*) belongs to the optimal solution. As for the flow variables y_{ij} and y_{ji}, these represent, respectively, the load carried by the vehicle and the empty space in the vehicle when it travels in the arc (*i, j*), that is, $y_{ij} + y_{ji} = Q$ (where Q corresponds to capacity of the vehicle).

As for the TDVRP, there are not many mathematical formulations of TDVRP in the literature. Malandraki & Daskin (1992) introduce this variant of VRP and propose a MILP formulation in which each arc (*i, j*) is replaced by m arcs (*i, j, m*), where *m* is the number of time periods in which the day is divided. The duration of the arc (*i, j*) is a time-dependent piecewise function, which varies according to the period of the day (in the same time interval, the duration is constant). Franceschetti et al. (2013) propose a MILP formulation for the Time-Dependent Pollution Routing Problem, with the aim of optimizing vehicle speeds in order to minimize fuel consumption and vehicle emissions. The duration of travel is dependent on the time of day is modeled by a two-level speed function: in the first the speed is limited by the intensity of the traffic and in the second the speed is defined by the limits imposed by law.

4. Mathematical Models

The developed models in this chapter intend to optimize the distance and duration of QA's distribution routes. Two CVRP models (CVRP1 and CVRP2) are developed to minimize route distances and a TDVRP model is developed to minimize route durations. CVRP1, based on the Three-Index formulation of Fisher & Jaikumar (1981); CVRP2, based on the Two-Commodity Flow formulation of Baldacci et al. (2004). The developed TDVRP is adapted on the MILP formulation of Malandraki & Daskin (1992), . The outputs intended for the models developed are as follows:

- Delivery order of route (s)
- Total distance of the route (s) - CVRP models only
- Total duration of the route (s) - TDVRP model only

The mathematical formulations of the three models are described in the following sections.

4.1. CVRP1

Indices

i, j – indicate all the network nodes
k - indicates the vehicle

Sets

$I = \{0, ..., n\}$ - set of customers and DC(i=0)
$V = \{1, ..., v\}$ - set of vehicles

Parameters

$dist_{ij}$ - distance between nodes i and j

$proc_i$ - quantity to be delivered to client i

cap_k - vehicle capacity k

Variables

<u>Binary</u>

x_{ijk} = 1, if the vehicle k travels the arc (i, j) and 0 otherwise

<u>Non-negative</u>

u_i - auxiliary variable for the elimination of sub-routes

Objective Function

$$min\ z = \sum_{i=0}^{n} \sum_{\substack{j=0 \\ i \neq j}}^{n} \sum_{k=1}^{v} dist_{ij} x_{ijk} \qquad (4.1)$$

s.t:

$$\sum_{i=0}^{n} \sum_{k=1}^{v} x_{ijk} = 1, \quad \forall j \in I \backslash \{0\} \qquad (4.2)$$

$$\sum_{\substack{i=0 \\ i \neq j}}^{n} x_{ijk} = \sum_{\substack{i=0 \\ j \neq i}}^{n} x_{jik}, \quad \forall k \in V, j \in I \qquad (4.3)$$

$$\sum_{i=0}^{n} proc_i \sum_{\substack{j=0 \\ j \neq i}}^{n} x_{ijk} \leq cap_k, \quad \forall k \in V \qquad (4.4)$$

$$\sum_{j=1}^{n} x_{0jk} \leq 1, \quad \forall k \in V \qquad (4.5)$$

$$u_i - u_j + n \times x_{ijk} \leq n - 1, \quad i \neq j \wedge \forall i, j \in I \backslash \{0\}, \ k \in V \qquad (4.6)$$

$$x_{ijk} \in \{0,1\}, \quad \forall i, j \in I \wedge i \neq j, \quad k \in V \qquad (4.7)$$

$$u_i \geq 0, \quad \forall i \in I \qquad (4.8)$$

The objective function (4.1) minimizes the total distance. Equation (4.2) ensures that all customers are supplied only once. That is, each customer is visited only once and only by one vehicle. Equation (4.3) ensures the continuity of vehicle movement, meaning it ensures that when a vehicle reaches a node (client), it must leave the node. Inequality (4.4) defines that the total demand for the route does not exceed the capacity of the vehicle associated with it. The restrictions (4.5) ensure that each vehicle carries out at most one route. The restrictions (4.6) ensure that there is no creation of sub-routes, that is, there is no route formation that does not include the DC. The domains of the variables are formulated in constraints (4.7) and (4.8).

4.2. CVRP2

Sets

$I' = \{0, ..., n+1\}$ – set of clients and depots (original and copy)

Parameters

cap – capacity of each vehicle
nv – number of vehicles

Variables

<u>Binary</u>

x_{ij} = 1 if the arc (i,j) is part of the solution, 0 otherwise

<u>Non-negative</u>

f_{ij}, f_{ji} – flow variables

Objective Function:

$$min \frac{1}{2}\sum_{i=0}^{n+1}\sum_{\substack{j=0\\j\neq i}}^{n+1} dist_{ij} * x_{ij} \quad (4.9)$$

s.t:

$$\sum_{\substack{j=0\\j\neq i}}^{n+1}(f_{ji} - f_{ij}) = 2 * proc_i\,, \quad \forall i \in I' \quad (4.10)$$

$$\sum_{j=1}^{n} f_{0j} = \sum_{j=1}^{n} proc_j \quad (4.11)$$

$$\sum_{j=1}^{n} f_{j0} \leq nv * cap - \sum_{j=1}^{n} proc_j \quad (4.12)$$

$$\sum_{j=1}^{n} f_{n+1,j} \leq nv * cap \quad (4.13)$$

$$f_{ij} + f_{ji} = cap * x_{ij}\,, \quad \forall i,j \in I' \wedge i \neq j \quad (4.14)$$

$$\sum_{\substack{j=1\\i<j}}^{n} x_{ij} + \sum_{\substack{j=1\\i>j}}^{n} x_{ji} = 2\,, \quad \forall i \in I' \quad (4.15)$$

$$x_{ij} \in \{0,1\}, \quad \forall i,j \in I', i \neq j \quad (4.16)$$

$$f_{ij}, f_{ji} \geq 0, \quad \forall i,j \in I' \wedge i \neq j \quad (4.17)$$

The objective function (4.9) corresponds to the minimization of half the total distance of the routes in both directions. Division by 2 is necessary to get the distance from just one route. Equation (4.10) defines that the difference between the output flow and the input flow in each client *i*, is equal to twice the demand of that same client. Equation (4.11) ensures that the total output flow of the warehouse corresponds to the total customer demand. Equation (4.12) ensures that the total inflow into the warehouse corresponds to the difference between total fleet capacity and total customer demand. Equation (4.13) determines that the total output flow of the copy warehouse corresponds to the total capacity of the fleet. Equation (4.14) ensures that the sum of the input and output flows on each client *i* is equal to the capacity of the vehicle (if the arc (*i, j*) is part of the solution). This constraint defines the arcs of an admissible solution. The constraints (4.15) guarantee that, in an admissible solution, each client must have 2 incident arcs. The domains of the variables are indicated in constraints (4.16) and (4.17).

4.3 TDVRP

Indices

m - time interval in which the vehicle crosses the arc (i, j)

Sets

I' = {1, ..., 2v + n} - set of clients and all DC's (initial and terminal)

I^c = {*v* + 1, ..., *v* + *n*} - set of clients

I^i = {1, ..., *v* + *n*} - set of clients and initial DC's

I^t = {*v* + 1, ..., 2*v* + *n*} - sets of clients and terminal DC's

T = {1, ..., *M*} - number of time intervals considered for each arc (*i, j*)

Parameters

$tviag_{ijm}$ - duration of travel between *i* and *j* at time interval *m*

$lsup_{ijm}$ - upper bound for the time interval m in the arc (*i, j*)

t - warehouse start time (same for all vehicles)

B1, B2 - large numbers

C - capacity of the largest vehicle

Objective Function:

$$\min z = \sum_{k=1}^{v} t_{v+n+k} \quad (4.18)$$

Subject to:

$$\sum_{\substack{i=v+1\\i\neq j}}^{v+n}\sum_{m=1}^{M} x_{ijm} = 1\,, \quad \forall j \in I^t \quad (4.19)$$

$$\sum_{\substack{j=v+1\\j\neq i}}^{v+n}\sum_{m=1}^{M} x_{ijm} = 1\,, \quad \forall i \in I^i/I^c \quad (4.20)$$

$$\sum_{\substack{j=v+n+1\\j\neq i}}^{2v+n}\sum_{m=1}^{M} x_{ijm} = 1\,, \quad \forall i \in I^c \quad (4.21)$$

$$t_j - t_i - B_1 x_{ijm} \geq tviag_{ijm} + tserv_j - B_1, \quad \forall i \in I^i \wedge \forall j \in I^t \wedge i \neq j, \quad \forall m \in T \quad (4.22)$$

$$t_i + B_2 * x_{ijm} \leq lsup_{ijm} + B_2, \quad \forall i \in I^i \wedge \forall j \in I^t \wedge i \neq j, \quad \forall m \in T \quad (4.23)$$

$$t_i - lsup_{ijm-1} * x_{ijm} \geq 0, \quad \forall i \in I^i \wedge \forall j \in I^t \wedge i \neq j, \quad \forall m \in T \quad (4.24)$$

$$w_i - w_j - C * \sum_{m=1}^{M} x_{ijm} \geq proc_j - C, \quad \forall i \in I^i \wedge \forall j \in I^t \wedge i \neq j \tag{4.25}$$

$$w_{n+v+k} = 0, \quad \forall k \in V \tag{4.26}$$

$$w_k \leq cap_k, \quad \forall k \in V \tag{4.27}$$

$$x_{ijm} \in \{0,1\}, \quad \forall i,j \in I'' \wedge i \neq j \tag{4.28}$$

$$t_j \geq 0, \quad \forall j \in I'' \tag{4.29}$$

$$w_j \geq 0, \quad \forall j \in I'' \tag{4.30}$$

The objective function (4.18) optimizes the total time of the routes of all the vehicles, minimizing the sum of the arrival times to the terminal warehouses. Constraints (4.19), (4.20) and (4.21) ensure that the routes start in the initial depots and end up in the terminal depots, that each customer is visited only once and that a vehicle cannot travel directly from an initial depot to a terminal one. Equation (4.22) determines the start time of node j. Constraints (4.23) and (4.24) ensure that the corresponding time interval *m* to the arc path (*i,j*) is chosen according to the start time of node *i*.

Constraints (4.25) ensure that the load carried by the vehicle when departing from node *j* is equal to or greater than the load carried from node *i* minus the load carried from node *j*. Equation (4.26) ensures that all vehicles arrive empty at the terminal depots. Constraints (4.27) ensure that vehicle capacities are not exceeded. The domains of the variables are modelled in the constraints (4.28), (4.29) and (4.30).

5. Results

In this chapter, the results of the models are described and analyzed. All models are implemented in the GAMS programming language and solved through the ILOG CPLEX optimization software on the Network Enabled Optimization System (NEOS) server on an Intel Xeon E5-2698 @ 2.3GHz Dual CPU. The maximum memory allocated to models submitted on the server is 3 GB of RAM and the maximum time set for running the models is 10 800 seconds. All models are validated using small dimension instances with a known solution.

5.1. Data collection

Most of the data used in the models is obtained directly from the company, such as: clients locations (zipcodes), demand (1 unit for each client), fleet capacity (80 units for QA vehicles and 65 units for the external distributor vehicle), fuel consumption (9l/100km, which is converted into 0,122€/l using corresponding fuel price for January 2018) and the remaining distribution costs (leasing, maintenance, salaries and the service cost of the external distributor). Distances are obtained from the Distance Matrix API from Google Maps. As for times: service times are obtained by timing delivery times of distributors, whereas to obtain travel times, the Distance Matrix API and Google Maps website are used to define traffic factors for each time interval *m*, from the TDVRP formulation (Table 2). Times with no traffic (obtained through the API) are multiplied by the traffic factors to obtain the estimated times with traffic.

Table 2 - Traffic factors for the TDVRP

Day	Time period (*m*)		
	1 (9:30h-11h)	2 (11h-16h)	3 (16h-19h)
Tue	1,29	1,10	1,22
Wed	1,39	1,19	1,36
Thu	1,14	1,03	1,31
Fri	1,15	1,11	1,15

5.2. Current situation

This paper analyses QA routes for January of 2018. This time period corresponds to 4 weeks and 56 routes. The distances and corresponding fuel costs for each route of the current situation are shown in Table 3. The routes 3 for Tuesday and Wednesday correspond to the ones carried out by the external distributor. The route distance value is 9 665 km, which corresponds to a total cost of 1 179 € in fuel (this is the total cost of fuel, also the routes of the external distributor). As for the total distribution costs of the current situation, these are shown on Table 4. The total fuel costs are different than in Table 3 because Table 4 doesn't account for the fuel costs of the routes carried out by the external distributor, since that cost is paid by the distributor (because fuel costs are already included in the cost of the service). In the current situation, the total cost of distribution is € 6 803.6, of which € 4901 correspond to costs incurred with QA vehicles and full-time distributors, and € 1 903 is the cost of the external distributor service.

Table 3 - Current situation route distances and fuel cost

Week	Day-route	Distance (km)	Cost(€)	Week	Day-route	Distance (km)	Cost(€)
	Tue - 1	219	26,7		**Tue - 1**	209,4	25,5
	Tue - 2	178,9	21,8		**Tue - 2**	169,4	20,7
	Tue - 3	146	17,8		**Tue - 3**	142,6	17,4
	Tue - 4	185,3	22,6		**Tue - 4**	253	30,9
	Wed - 1	138,5	16,9		**Wed - 1**	160,8	19,6
	Wed - 2	141,6	17,3		**Wed - 2**	136,2	16,6
1	**Wed - 3**	157	19,2	**2**	**Wed - 3**	168,2	20,5
	Wed - 4	154,3	18,8		**Wed - 4**	160	19,5
	Thu - 1	169,5	20,7		**Thu - 1**	151,1	18,4
	Thu - 2	136,3	16,6		**Thu - 2**	148,7	18,1
	Thu - 3	237,7	29		**Thu - 3**	260	31,7
	Fri - 1	167,6	20,4		**Fri - 1**	164	20
	Fri - 2	179,5	21,9		**Fri - 2**	186,3	22,7
	Fri - 3	73,1	8,9		**Fri - 3**	122,8	15
Week	**Day-route**	**Distance (km)**	**Cost(€)**	**Week**	**Day-route**	**Distance (km)**	**Cost(€)**
	Tue - 1	225,1	27,5		**Tue - 1**	214,7	26,2
	Tue - 2	200,2	24,4		**Tue - 2**	191,4	23,3
	Tue - 3	142,4	17,4		**Tue - 3**	133,5	16,3
	Tue - 4	218,7	26,7		**Tue - 4**	192,3	23,5
	Wed - 1	146,1	17,8		**Wed - 1**	154,2	18,8
	Wed - 2	146,4	17,9		**Wed - 2**	146,9	17,9
3	**Wed - 3**	237,2	28,9	**4**	**Wed - 3**	189,6	23,1
	Wed - 4	145,1	17,7		**Wed - 4**	146,4	17,9
	Thu - 1	156,1	19		**Thu - 1**	157,8	19,2
	Thu - 2	138,1	16,8		**Thu - 2**	133,3	16,3
	Thu - 3	303,4	37		**Thu - 3**	371,5	45,3
	Fri - 1	174,5	21,3		**Fri - 1**	182,4	22,3
	Fri - 2	156,3	19,1		**Fri - 2**	169,9	20,7
	Fri - 3	75,4	9,2		**Fri - 3**	99	12,1
				Average		**172,6**	**21,1**
				Total		**9 664,80**	**1179,1**

Table 4 - Total distribution costs for the current situation

Type of cost		Value(€)
QA Vehicles	Fuel	1 018,5
	Others	3 882,3
Total QA Vehicles		4 900,8
External distributor		1 902,8
Total		**6 803,6**

5.3. Scenarios

For each one of the three models, the process is the same, composed of two phases: first, the routes are optimized individually to see if there is room for improvement in the order of distribution. At this stage, the daily distribution clusters are maintained, i.e. the customers allocated to each vehicle are the same as in the current situation, and there may only be changes in the order of distribution (scenario a). In a second phase, each model is applied to the set of all the daily delivery points, with no pre-defined allocation of each truck to a set of customers (scenario b). The objective is to redefine and optimize the daily distribution clusters by allocating customers of a given day to each vehicle in order to minimize the distance or the total duration (depending on the model) of the routes. Scenarios 1a and 1b refer to CVRP1, scenarios 2a and 2b refer to CVRP2; scenarios 3a and 3b refer to TDVRP.

5.3.1. Scenarios 1a and 2a

Scenarios using the CVRP models are 1a and 1b (for CVRP1); 2a and 2b (for CVRP2). This means that both CVRP models are used to optimize each route individually (a scenarios) and to optimize the daily clusters and subsequent order of delivery of each vehicle (b scenarios). For scenario 1a, there is a distance reduction in all instances, in comparison to the current situation. On average, there is a daily reduction of 17,3%. The smallest reduction value is 1,7% (week 1, Friday -1) and the highest value is 58,8% (week 3, Wednesday – 3). In this scenario, for 17 of the 56 instances executed, the optimal solution is not reached, there being some instances whose solution presents a significant gap (maximum value of 36, 3%) - gap is the percentage difference between the solution obtained and the best possible solution for a particular instance. However, for routes 4 on Monday, weeks 1 and 3, the solution allows a substantial reduction of the distance traveled (29.3% and 38.9%, respectively), to a corresponding gap of more than 30%. The distance and the total cost of fuel in this scenario correspond to 7916 km and 966 €, respectively.

As for scenario 2a, it corresponds to the application of the CVRP2 to each route individually. Only route 1 on Friday, week 1, shows a distance increase of 0.1%. The average reduction value is 14.5%, the largest reduction being observed on route 4 of Tuesday in week 2 (38.7%). In this scenario, all instances reach the optimal solution, being, therefore, the 0% gap in all cases. In view of this information, the increase in distance observed in one of the routes is explained by a characteristic of the formulation used, which considers paths in both directions and therefore minimizes the distances based on the path in which the average distance (*i, j*) and (*j, i*) is smaller. Thus, if the distance matrix used is asymmetric (as is the case), the optimal solution of the model may not correspond to the actual optimal solution. For this scenario, the total values of distance and cost of fuel correspond to 8152 km and 995 €, respectively. The distance (and fuel cost) reduction percentages of scenarios 1a and 2a are compared to the current situation are shown on Table 5.

Table 5 - Percentages of distance reduction of scenarios 1a and 2a compared to the current situation

Week	Day-route	1a	2a	Week	Day-route	1a	2a
1	Tue - 1	28,20%	28,20%	2	Tue - 1	22,80%	22,00%
	Tue - 2	9,90%	9,90%		Tue - 2	10,90%	7,30%
	Tue - 3	17,00%	17,00%		Tue - 3	11,50%	10,50%
	Tue - 4	21,70%	21,70%		Tue - 4	39,30%	38,70%
	Wed - 1	13,30%	13,30%		Wed - 1	26,40%	22,10%
	Wed - 2	8,00%	8,00%		Wed - 2	19,30%	14,30%
	Wed - 3	13,10%	13,10%		Wed - 3	16,80%	13,70%
	Wed - 4	17,10%	17,10%		Wed - 4	26,20%	19,00%
	Thu - 1	11,30%	11,30%		Thu - 1	9,70%	7,60%
	Thu - 2	6,60%	6,60%		Thu - 2	18,60%	14,20%
	Thu - 3	5,70%	5,70%		Thu - 3	11,10%	10,00%
	Fri - 1	-0,10%	-0,10%		Fri - 1	4,60%	2,20%
	Fri - 2	8,10%	8,10%		Fri - 2	12,80%	11,80%
	Fri - 3	4,70%	4,70%		Fri - 3	10,20%	8,70%
Week	**Day-route**	**1a**	**2a**	**Week**	**Day-route**	**1a**	**2a**
3	Tue - 1	27,60%	25,80%	4	Tue - 1	25,80%	23,90%
	Tue - 2	16,70%	13,00%		Tue - 2	14,30%	10,80%
	Tue - 3	11,00%	10,10%		Tue - 3	9,80%	8,50%
	Tue - 4	28,00%	27,30%		Tue - 4	15,50%	14,70%
	Wed - 1	19,80%	16,30%		Wed - 1	22,30%	19,40%
	Wed - 2	19,40%	13,80%		Wed - 2	21,50%	17,10%
	Wed - 3	37,00%	35,50%		Wed - 3	17,70%	14,80%
	Wed - 4	21,60%	14,50%		Wed - 4	20,60%	13,60%
	Thu - 1	11,40%	8,70%		Thu - 1	14,80%	13,00%
	Thu - 2	10,90%	5,10%		Thu - 2	8,90%	4,70%
	Thu - 3	24,20%	23,90%		Thu - 3	30,90%	30,20%
	Fri - 1	5,50%	4,30%		Fri - 1	8,20%	6,80%
	Fri - 2	9,40%	7,10%		Fri - 2	13,20%	12,50%
	Fri - 3	22,80%	22,10%		Fri - 3	35,00%	33,90%
				Average		**17,30%**	**14,60%**

5.3.2. Scenarios 1b and 2b

Scenarios 1b (CVRP1) and 2b (CVRP2) optimize the daily clusters, by redefining the allocation of the clients to each vehicle and the subsequent order of distribution of each vehicle.

In Scenario 1b, for all instances that have a solution, the execution time limit of the model is reached, and in all cases, the gap is quite high, varying between 25.3 % (Week 1 - Thursday) and 60.4% (Week 3 - Monday). Exactly for this reason, the results are generally negative. Of the 16 analysed instances, there is an improvement compared to the current situation in only 4, and only one of them is significant (19.3% in Week 2, Friday). The high number of clients compared to the instances analysed in scenario 1a explains the difference in results obtained. On average, there is an increase of 8.5% compared to the current situation. For this scenario, it isn't relevant to discuss total values, since there are instances for which no solution is obtained.

Scenario 2b, on the other hand, produces good results for cluster optimization. This scenario uses the CVRP2 formulation, so it does not allow the distinction of vehicle capacity. Thus, for all instances, a capacity of 80 units is defined for vehicles. However, in all instances of this scenario, there is always a route that does not exceed 65 units, meaning that the capacity constraints of the fleet are always met. For this scenario, 9 of the 16 executed instances reach the computational time limit. However, the gaps associated with these instances are much lower than those observed in scenario 1b, ranging from 0.2% (Week 3 - Thursday) to 9.9% (Week 3 - Monday). Thus, even the instances for which an optimal solution is not proven, have very low gaps which represent results very close to the optimum. The reduced (or non-existent) gap values explain the reduction percentages obtained, with a significant reduction in all instances. The lowest reduction is 9.7% (Week 1 - Thursday) and the highest is 40.8% (Week 1 - Monday). In terms of overall figures, this scenario represents an average reduction of 23.6% compared to the current situation, with a total distance of 8 142 km and an associated fuel cost of € 993. The percentage of distance/ fuel cost reduction of scenarios 1b and 2b compared to the current situation are shown on Table 6.

Table 6 - Percentages of distance reduction of scenarios 1b and 2b comparing to the current situation

Week	Day-route	1b	2b
1	Tue	0,60%	40,80%
	Wed	-23,80%	18,40%
	Thu	-12,40%	9,70%
	Fri	-33,30%	14,30%
2	Tue	*	35,70%
	Wed	-7,20%	23,60%
	Thu	-20,40%	12,60%
	Fri	19,30%	33,10%
3	Tue	-8,00%	37,80%
	Wed	-13,40%	26,00%
	Thu	-0,30%	16,40%
	Fri	0,40%	11,60%
4	Tue	*	40,00%
	Wed	-14,80%	19,40%
	Thu	*	21,60%
	Fri	3,00%	17,40%
Average		**-8,50%**	**23,60%**

* memory limit exceeded

5.3.3. Comparison of all CVRP scenarios

A comparison of all scenarios regarding total distance and total distribution costs allows to better understand the improvement compared to the current situation. However, scenario 1b does not take part in this comparison, since there are instances whose execution exceeds the memory limit, and no solution is obtained.

Figure 1 shows the total distance values for scenarios 1a, 2a, 2b and the current situation. For all scenarios there is a significant reduction compared to the current situation (9665 km). Scenario 1a is one that allows a greater reduction of distance, with a value of 7916 km (representing a reduction of 18%). Scenarios 2a and 2b show very similar values, with a total distance of 8153 km and 8142 km, respectively (representing a reduction of 15.6% and 15.8%).

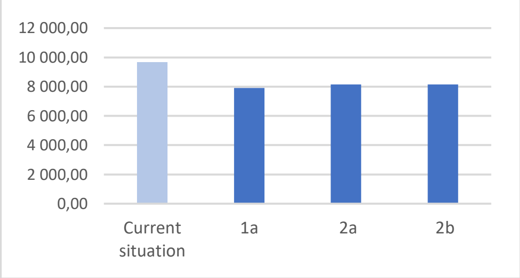

Figure 1 - Total distance of scenarios 1a, 2a, 2b and current situation

As for total distribution costs, values of scenarios 1a (6 620 €) and scenario 1b (6 646 €) are very similar to the value of the current situation (6 804 €) because they only reduce fuel costs, which doesn´t have much impact on the overall distribution costs (only 2% reduction in both cases). There is no reduction in the number of vehicles, as these two scenarios optimize each route individually.

On the other hand, scenario 2b shows a reduction of the number vehicles utilized for 14 of the 16 instances, and for this scenario, the maximum number of vehicles used per day is 3, which means that one of the vehicles is not required (QA uses 4 vehicles currently). This means that there are two options for this scenario: using only two of QA's vehicles and the vehicle of the external distributor (option 1); using the 3 vehicles owned by QA and stop requesting distribution services. Option 1 has a cost of € 5 266 (representing a reduction of 22.6%) and option 2 represents a cost of € 4 786, the latter being the option that allows the greatest reduction of costs compared to the current situation (reduction of 28.3%). Figure 2 shows total distribution costs comparison between scenarios 1a, 2a, 2b and the current situation

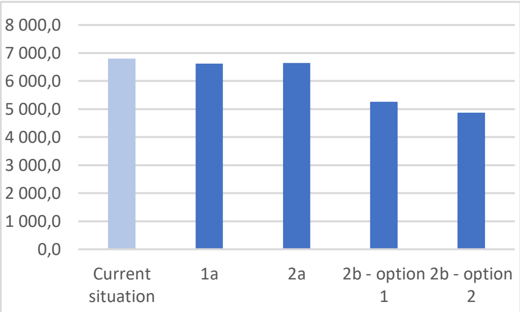

Figure 2 - Total distribution cost of scenarios 1a, 2a, 2b and current situation

5.3.4. TDVRP Scenarios

Scenarios 3a and 3b use the TDVRP model formulation to optimize individual routes in Scenario 3a and to optimize the clusters in

Scenario 3b. However, it was not possible to obtain results for these scenarios, since the executed instances reach the memory limit before a solution is obtained for the model. Thus, this formulation is tested in smaller instances to understand the maximum size of the instances for which a solution is obtained under the described conditions (Table 7).

Instance	Number of clients	Time intervals (M)	Number of equations	Number of variables	Gap (%)	CPU(s)
A	5	2	574	188	0	0,2
B	5	3	574	188	0	0,2
C	9	2	934	296	0	73,6
D	9	3	934	296	0	131,5
E	10	2	1136	358	0	688,0
F	10	3	1136	358	0	1898,2
G	11	2	1358	426	0	8284,2
H	11	3	1358	426	42,8	10800
I	12	2				*
J	12	3	1600	500	59,3	10800
L	15	2				*
M	15	3	2446	758	76,0	10800

*memory limit exceeded

Figure 3 - Tested instances for the TDVRP model

It can be observed that the largest instance for which the model returns an optimal solution is instance G, with 11 clients. This instance already has a high execution time (8284.2s). Instances I and L reach the memory limit (3GB), so no solution is obtained. The M instance, of 15 clients, does not reach the memory limit, however, the solution obtained for the instance in question has a gap too high (76%), thus it is not a viable solution. It is also interesting to note that the reduction of the number of time intervals (*M*) may not be translated into a better solution, since that for the instances of 12 and 15 clients, only a solution is obtained for those with $M = 3$.

6. Conclusions and future work

In this paper, three mathematical models are developed to optimize the routes of QA. The two developed CVRP models for distance (and cost) minimization allow a significant reduction in total distance (18% in scenario 1a) and in total distribution costs (28,3% in scenario 2b, option 2). Scenarios 3a and 3b, relative to the TDVRP model, are not analysed, since the formulation only allows to obtain solutions for very small instances. In the tests performed on the model, the largest instance for which the optimal solution is obtained has only 11 clients.

As future work, it is suggested the study of heuristics that can be applied in the development of a TDVRP model, since the model developed in the present dissertation does not allow to evaluate routes of the dimension analysed in this dissertation. It may be interesting to analyse if the optimized routes with respect to the distance are similar to those optimized for the duration.

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