ABSTRACT: This paper is to design and optimise a stiffened plate which serves as a part of a ship structure. Multi-Objective structural optimization of a stiffened plate subjected to a combined stochastic buckling and fatigue loads in minimising the structural section area, the maximum displacement and fatigue damage satisfying a predefined target reliability level is performed. The Pareto frontier solutions calculated by Non-Dominated Sorting Genetic Algorithm (NSGA-II) were used to define the feasible surface of the design variables. The first order reliability method is employed to identify the topology of the stiffened plate as a part of the Pareto frontier solutions in reducing the failure probability due to fatigue and buckling. Comparing with the original section area, the optimised section area decreased by 8%.

1 INTRODUCTION
Steel stiffened panels are mainly used for the structural design of ships. They are usually used in ship and ocean structures to withstand tensile or compressive axial load and lateral pressure, due to the effect of waves and hydrostatic pressure in the ocean. For the safety of ship structure, it is critical to predict its load carrying capacity. Under complex bending conditions, the effect of lateral pressure on the plate collapse strength of the plate depends on the interaction of axial and lateral loads [1]. The Pareto frontier, ultimate limit state, and target reliability, defined as additional constraints to determine the optimal design solution as demonstrated in [2].

Limit state method has been widely applied in ship design, presented by IACS, Common Structural Rules for Bulk Carriers and Oil Tankers [3]. Recent developments in structural reliability methods and optimisation tools allow design methods based on coupling the reliability analysis, in which the uncertainties related to design variables can be considered directly.

The FORM (first order reliability methods) approaches have been used for structural assessment as shown in [4-7], but it can also be used for probabilistic analysis of different practical applications [8].

The reliability analysis performed in this paper is FORM, which provides a method for evaluating reliability with reasonable accuracy and is sufficient for practical application.

Combining the reliability methods with the structural optimisation techniques, the three-step method of stiffened panel design is proposed. Once the structure topology is determined, the scantling of the structural components of the stiffened plate is performed and optimized, in which the design variables and the objective functions related to the minimum net section area, which satisfied the minimum weight, displacement fatigue damage and constraints requirements, including the ultimate compressive strength are defined in a purely deterministic manner.

Then the Pareto frontier method [9] is used to determine the optimal design solution, which satisfies all the constraints and minimises the three objective functions. The results can be used as a basis for the target reliability-based optimisation, which is required to guarantee the structural integrity. This step accommodates the uncertainties of the design variables, and the computational models are involved.

The primary objective is to optimise the dimensions of a stiffened plate of a ship. The calculation of some primary input data such as loads on the ship is based on empirical formulas and specification rules, not on actual records of sea state conditions. It is optimised without specific and detailed data from a ship, so the classical method is not applicable here because it is difficult to give criteria to determine whether the feasible solution is retained or not.

The objective is to perform a multi-objective optimisation of ship stiffened plates and to obtain a
complete method flow suitable for solving this kind of problems of various ships at the same time. In this paper, the author chooses NSGA-II [9] to ensure that the optimal solution can be obtained quickly with sufficient quantity and accuracy when only the ship’s main dimensions are known.

The Pareto frontier is applied for simultaneous minimisation of the net sectional area, structural displacement and fatigue damage. Employing the Pareto Frontier, an optimal solution, accounting for the existing constraints, may be chosen using a utility function to rank the different designs, or by using 2D or 3D scatter diagrams to identify the more attractive ones. In the present study, an additional constraint is introduced representing the target reliability level to determine the most appropriate design solution.

2 ULTIMATE STRENGTH OF SHI HULL

2.1 Main dimension of bulk carrier

A 175,000-ton bulk carrier is used as a target ship. The main dimensions of the bulk carrier are:

- Length between the perpendiculars: L = 289 m;
- Depth: D = 24.7 m;
- Breadth: B = 45 m;
- Design Draft: d = 18 m;
- Block Coefficient: $C_b = 0.79$.

Half cross-section of the hull girder of this bulk carrier is shown in Figure 1. The cross-section contains a total of 129 plates and 98 stiffeners. A longitudinal stiffened plate of a tee-bar profile, with a stiffener spacing of 860 mm and a frame span of 2,950 mm, is analysed in the present study.

![Figure 1 Half cross section of a bulk carrier](image1)

Considering the geometrical characteristics of the bulk carrier, the plates and stiffeners of the midship section are shown in Figure 1. The details of the longitudinal stiffeners are summarised in Table 1, and the material properties are listed in Table 2 respectively.

**Table 1 Dimensions of longitudinals**

<table>
<thead>
<tr>
<th>No.</th>
<th>Dimensions (mm)</th>
<th>Type</th>
<th>Y.S.(MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200 × 20</td>
<td>Flat bar</td>
<td>320</td>
</tr>
<tr>
<td>2</td>
<td>150 × 18</td>
<td>Flat bar</td>
<td>320</td>
</tr>
<tr>
<td>3</td>
<td>250 × 25</td>
<td>Flat bar</td>
<td>320</td>
</tr>
<tr>
<td>4</td>
<td>200 × 20</td>
<td>Flat bar</td>
<td>320</td>
</tr>
<tr>
<td>5</td>
<td>420 × 12 + 100 × 20</td>
<td>Tee-bar</td>
<td>320</td>
</tr>
<tr>
<td>6</td>
<td>420 × 12 + 100 × 30</td>
<td>Tee-bar</td>
<td>320</td>
</tr>
<tr>
<td>7</td>
<td>320 × 12 + 100 × 18</td>
<td>Tee-bar</td>
<td>320</td>
</tr>
<tr>
<td>8</td>
<td>300 × 12 + 100 × 12</td>
<td>Tee-bar</td>
<td>320</td>
</tr>
<tr>
<td>9</td>
<td>300 × 12 + 100 × 16</td>
<td>Tee-bar</td>
<td>320</td>
</tr>
<tr>
<td>10</td>
<td>350 × 12 + 100 × 20</td>
<td>Tee-bar</td>
<td>360</td>
</tr>
<tr>
<td>11</td>
<td>300 × 12 + 100 × 18</td>
<td>Tee-bar</td>
<td>360</td>
</tr>
<tr>
<td>12</td>
<td>300 × 30</td>
<td>Flat bar</td>
<td>360</td>
</tr>
<tr>
<td>13</td>
<td>200 × 20</td>
<td>Flat bar</td>
<td>360</td>
</tr>
<tr>
<td>14</td>
<td>350 × 30</td>
<td>Flat bar</td>
<td>360</td>
</tr>
<tr>
<td>15</td>
<td>300 × 12 + 100 × 24</td>
<td>Tee-bar</td>
<td>360</td>
</tr>
</tbody>
</table>

**Table 2 Material properties**

<table>
<thead>
<tr>
<th>No.</th>
<th>Young’s modulus(N/mm²)</th>
<th>Poisson ratio</th>
<th>Yielding stress (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1E5</td>
<td>0.3</td>
<td>235</td>
</tr>
<tr>
<td>2</td>
<td>2.1E5</td>
<td>0.3</td>
<td>320</td>
</tr>
<tr>
<td>3</td>
<td>2.1E5</td>
<td>0.3</td>
<td>360</td>
</tr>
</tbody>
</table>

2.2 Ultimate strength of ship hull

The software MARS2000 [10] is used to estimate the ultimate strength and geometrical descriptors of the midship section. Once the midship section is designed, which includes the position, shape and properties of all plates and stiffeners, the features, of the hull girder are estimated and shown in Figure 2.

![Figure 2 Ultimate strength of ship hull](image2)

3 BOTTOM STIFFENED PLATE

3.1 Descriptors of stiffened plate

For bulk carriers in hogging, the most critical loading is the alternate hold loading (AHL) condition with odd-numbered holds loaded with high-density cargoes and even numbered holds empty. The effect of the local lateral pressure should be considered in the assessment of the
ultimate hull girder strength in the hogging and AHL conditions. In the present study, the ultimate strength of a bulk carrier hull girder under combined global and local loads in the hogging and AHL condition is investigated following the guidelines presented in [11].

The position of the stiffened plate selected for optimisation is on the bottom plate of the ship, which has coordinates of the weld position of the specified stiffener as (3.44, 0). The transverse distance from the middle of the ship is 3.44 m, and the height is 0 m. The stiffener type at this position is T-bar. The thickness of its adjacent bottom plates is 18 mm on both sides of it. The original geometric parameters of the stiffened plate are shown in Table 3. The geometry parameters of the specific plate are shown in Figure 3.

Table 3 Original geometric parameters of the stiffened plate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width of bottom plate, s</td>
<td>860mm</td>
</tr>
<tr>
<td>Web height, h_w</td>
<td>420mm</td>
</tr>
<tr>
<td>Web thickness, t_w</td>
<td>12mm</td>
</tr>
<tr>
<td>Flange Breadth, b_f</td>
<td>100mm</td>
</tr>
<tr>
<td>Flange thickness, t_f</td>
<td>20mm</td>
</tr>
</tbody>
</table>

Figure 3 T-type stiffened plate cross-section

3.2 Loads of stiffened plate

The wave-induced bending moments in hogging and sagging as given by DNV Rules are used here. The wave-induced bending moments in hogging and sagging conditions are estimated and in the case of the hogging condition as \( P_{w,CSR}^{hog} = 44 \text{ kPa} \). The inertia moment of the midship net section with respect to the neutral axis is \( I_{na} = 603.2 \text{ m}^4 \).

Moreover, the midship section modulus concerning the bottom line is \( W_b = 55.3 \text{ m}^3 \). The yield strength is \( \sigma_y = 315 \text{ MPa} \) and the Young’s modulus is \( E = 210 \text{ GPa} \).

The geometry parameters as presented in Table 3 will be redefined during the optimisation process. The studied longitudinal stiffener is subjected to an axial load resulting from the vertical still water and wave-induced bending moments as:

\[
\sigma_{global} = \frac{M_{sw} + \Psi M_{w}}{w_{bottom \text{ship}}} = 178,850 \text{ kPa} \tag{2}
\]

where \( \Psi \) is a combination factor between the still water and wave-induced loads ranging from 0.8 to 0.95 depending on the assumptions and it is assumed here to be a deterministic one of 0.9 [12].

The stiffener plate is also subjected to a lateral load, which is induced by the hydrostatic and dynamic local pressure. In the case of a full load condition:

\[
q_{local1} = (P_{sw1} + \Psi P_{w1})b_p = 191.9 \text{ kN/m} \tag{3}
\]

and in the case of the ballast load condition:

\[
q_{local2} = (P_{sw2} + \Psi P_{w2})b_p = 141.5 \text{ kN/m} \tag{4}
\]

The stiffened plate is assumed to be a simply supported beam subjected to a uniformly distributed lateral load, \( q_{local} \) and axial tensile force \( T = A(M_{sw,s} + \Psi M_{w,s})/W_{bottom \text{ship}} \) in the case of sagging loading and to an axial compressive force \( T^* = A(M_{sw,h} + \Psi M_{w,h})/W_{bottom \text{ship}} \) in the case of hogging respectively, where \( A \) is the net sectional area of the stiffened plate [2].

The maximum stresses at the middle of the beam are calculated as:

\[
\sigma_{max,x=0} = \sigma_{local} + \sigma_{global} \tag{5}
\]

where:

\[
\sigma_{local}(P_{sw}, P_{w}) = \frac{m_{x=0}(U')}{w_{stiffened \text{ plate}}} \tag{6}
\]

\[
\sigma_{global}(M_{sw,s}, M_{w,s}) = \frac{M_{sw,s} + \Psi M_{w,s}}{w_{bottom \text{ship}}} \tag{7}
\]

3.3 Optimisation considering weight and fatigue

The goal of the structural design is to find the optimal dimensions for the three-dimensional structures. Usually, this is regarded as a single objective optimisation problem. However, many design problems are multistate, multispecific or need to optimise multiple objectives simultaneously. There may be trade-offs between goals, and improving one feature requires compromising another. The challenge is to identify solutions that are part of the Pareto optimal set.
design, where no further improvement can be achieved without degrading one of the others.

Pareto optimisation problems have been found in various research fields, and computational methods have been developed to identify the Pareto frontier.

3.3.1 Decision variables
In this study there are five decision variables considered that determine the shape of the cross-sectional area. Choosing the appropriate range of the decision variables is a fundamental issue. The appropriate range can make it easier to get results that meet the specific requirements in the subsequent Pareto frontier calculation.

The decision variables assumed here are:

\[ x_1 = t_p, \ x_2 = h_w, \ x_3 = t_w, \ x_4 = b_f, \ x_5 = t_f \]

The range is defined as:

\[ x_{i,\text{min}} \leq x_i \leq x_{i,\text{max}}, i \in [1, 5] \]

The original dimensions of the stiffened plate with its attached plate considered here is \( t_p = 0.018 \text{ m}, b_f = 0.1 \text{ m}, t_f = 0.02 \text{ m}, h_w = 0.42 \text{ m}, t_w = 0.012 \text{ m} \). Since the optimal design is based on this model, the dimensions of the decision variables will not change too much. So it can be used as a reference for the definition of the new ranges of the variables. Then after some trial operations, the final definitions of the variable ranges are as follows:

\[ x_{1,\text{min}} = 0.012 \text{ m}, \ x_{1,\text{max}} = 0.03 \text{ m} \]

\[ x_{2,\text{min}} = 0.4 \text{ m}, \ x_{2,\text{max}} = 0.5 \text{ m} \]

\[ x_{3,\text{min}} = 0.012 \text{ m}, \ x_{3,\text{max}} = 0.03 \text{ m} \]

\[ x_{4,\text{min}} = 0.1 \text{ m}, \ x_{4,\text{max}} = 0.2 \text{ m} \]

\[ x_{5,\text{min}} = 0.012 \text{ m}, \ x_{5,\text{max}} = 0.03 \text{ m} \]

3.3.2 Objective functions
Three critical factors need to be taken into consideration leading to three objective functions that need to be built. All of them need to meet the requirement of the Classification Society Rules.

The two-objective structural responses considered is minimising the weight, which leads to minimising of the net sectional area and minimising the structural displacement, which defines a multi-objective optimisation problem:

\[ F_1 = \min \{z_{x=0}(b,x)\} \]

\[ F_2 = \min \{A(b,x)\} \]

where \( z_{x=0}(b,x) \) is the displacement at the middle of the span and \( A(b,x) \) is the net-sectional area of the stiffened plate, \( b = \{\sigma_y, E\} \) is for the material properties. The third objective function is to minimizing the fatigue damage:

\[ F_3 = \min \{D_{x=0}(b,x)\} \]

3.3.3 Constraints
The dimensions of the flange, web and attached plate of the stiffened plate have to satisfy the following restrictions:

\[ G_1: x_1 \geq \frac{b_p}{c} \sqrt{\frac{\sigma_y}{235}} \]

\[ G_2: x_3 \geq \frac{h_w}{c_w} \sqrt{\frac{\sigma_y}{235}} > 0 \]

\[ G_3: x_5 \geq \frac{b_r}{c_r} \sqrt{\frac{\sigma_y}{235}} > 0 \]

where \( b_p \) is the space defined as a distance between the longitudinal stiffeners, \( C=100, \ C_w = 75, \ C_f = 12 \).

The type of load on the stiffened plate will induce the plate buckling since the stiffener is subjected to a tensile load and the attached plate to a compressive load in bending. Some variables for the optimisation are listed in Table 4 and 5. In the fatigue damage calculation, the S-N curve D as suggested in [3] is used.

<table>
<thead>
<tr>
<th>Load Condition</th>
<th>Fraction of time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full load</td>
<td>Sagging</td>
</tr>
<tr>
<td>Ballast</td>
<td>Hoggling</td>
</tr>
</tbody>
</table>

Table 5 Weibull shape factor and the reference period of wave

<table>
<thead>
<tr>
<th>Wave</th>
<th>( T_{\text{reference}} )</th>
<th>( T_{\text{wave}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 year</td>
<td>8 sec</td>
</tr>
<tr>
<td>( h_{\text{Weibull}} )</td>
<td>0.931</td>
<td>-</td>
</tr>
</tbody>
</table>

3.4 Optimisation considered reliability
The empirical formula for the assessment of load carrying capacity of the stiffened panel would be more useful for the design [13] and for the reliability analysis of ship structure, although the factors of safety in association with uncertainties and deviations should be considered carefully [2]. The reliability analysis performed here is using the FORM techniques that identify a set of primary
random variables, which influence the limit-state 
under consideration. 
The limit-state function defines a failure surface 
when equals to 0, which is, in fact, an (n-1) 
dimensional surface in the space of n primary 
variables. The formation of RBDO is similar to 
the one of the optimisations where the objective limits 
state function, g (b, x) is minimised, and it is subject 
to constraints, where b is the vector of the 
deterministic design variables and x is the vector of 
the random variables. The limit state function here 
is defined as [2]:
\[ g(b, x) = \sigma_u(b, x) - \sigma_{\text{max}}(b, x) \]  
(22) 
where
\[ \sigma_{\text{max}}(b, x) = \sigma_{\text{global, max}}(b, x) + \sigma_{\text{local, max}}(b, x) \]  
(23) 
\[ \sigma_{\text{global, max}}(b, x) = k_1(X_{m,w}M_{sw} + \Psi X_{m,w}M_u)/W_b \]  
(24) 
\[ \sigma_{\text{local, max}}(b, x) = k_2(X_{p,w}M_{sw} + \Psi X_{p,w}P_w)l/W_{b,\text{stiff}} \]  
(25) 
This surface divides the primary variable space 
in a safe region, where g(b,x) > 0 and an unsafe 
area where g(b,x) < 0. The failure probability of a 
structural component concerning a single failure 
mode can formally be written as:
\[ P_f = P[g(b, x) \leq 0] = P_f = P[g(b, x) \leq 0] \]  
(26) 
where \( P_f \) denotes the probability of failure. In 
practical applications, the FORM methods provide a 
way of evaluating the reliability efficiently with 
reasonably good accuracy [2]. 
The required safety index is defined here as \( \beta_{\text{target}} \), the Beta indexes of all feasible design 
solution, which based on the sets of section sizes 
corresponding to the Pareto frontier solutions, are 
compared to the required target safety index, where 
the min{\( \beta_{\text{target}} - \beta_i \)} is the best reliability based 
design solution. 
Seven deterministic variables are considered 
here as \( b_1 = t_p, \ b_2 = h_u, \ b_3 = t_w, \ b_4 = b_f, \ b_5 = t_f, \ b_6 = \sigma_y, \ b_7 = E \), and ten random 
variables \( x_1 = M_{w,\text{BL,hog}}, \ x_2 = P_{w,\text{BL,h}}, \ x_3 = M_{sw,\text{BL,h}}, \ x_4 = P_{sw,\text{BL,h}}, \ x_5 = \sigma_u, \ x_6 = X_u, \ x_7 = X_{p,sw}, \ x_8 = X_{m,sw}, \ x_9 = X_{p,w}, \ x_{10} = X_{m,w}, \) 
are considered here. 
The local lateral load is defined as \( q_{\text{local}} = (X_{p,sw}P_{sw,\text{BL,h}} + \Psi X_{p,w}P_{w,\text{BL,h}})b \) and the net 
sectional stresses, resulting from the global bending 
load, is:
\[ \sigma_{\text{global}} = (X_{m,sw}M_{sw,\text{BL,h}} + \Psi X_{m,w}P_{w,\text{BL,h}})/W_b. \]  
(27) 
\( \sigma_u \) is the ultimate stress capacity with a model 
uncertainty factor \( X_u \), which is assumed to be 
described by the Normal probability density function, \( N_{X_u}(0.5,0.1) \). 
The model uncertainty factor \( X_{m,w} \) accounts for 
the uncertainties in the wave induced vertical 
bending moment calculation. Resulting in \( X_{m,w} \sim N_{X_{m,w}}(1,0.1) \) and the model uncertainty 
factor with respect to the still water load is \( X_{m,w} \sim N_{X_{m,w}}(1,0.1) \) and with respect to the local 
pressure load are modelled by \( X_{p,sw} \sim N_{X_{p,sw}}(1,0.1) \) and \( X_{p,w} \sim N_{X_{p,w}}(0.95,0.095) \). 
The fraction of time spent in each load condition 
may be estimated based on the statistical analysis of 
the operational profile of the bulk carrier ship. The 
assumed operational profile here is a full load, \( P_{\text{FL}} = 0.5 \), ballast load, \( P_{\text{BL}} = 0.35 \). The vertical 
wave-induced bending moment is in sagging in the 
full loading condition and in hogging in ballast and 
partial loading conditions. The still water bending 
moment is in sagging in full loading condition and 
in hogging in ballast and partial loading conditions. 
The ballast loading case is used in the present 
analysis since it transmits a compressive load to the 
stiffened plate at the bottom of the ship. 
The still water bending moment is fitted to the 
Normal distribution. The regression Eqn define the 
statistical descriptors of the still water bending 
moment as a function of the length of the ship, \( W= \) 
(DWT/Full load) as proposed in [14, 15] and the 
loads are taken as prescribed by the Classification 
Societies Rules [3]. 
The 5% confidence level value of the ultimate 
bending moment \( M_{u}^{5\%} = M_{u}^{5\%} \) is calculated by 
MARS2000 software and it is assumed that COV 
equals to 0.08 and it is fitted to the Lognormal 
probability density function:
\[ f_{M_u} = \frac{1}{M_u^{\sigma M_u}/2\pi} \epsilon^{-\frac{(\ln(M_u)/\mu M_u)}{2\sigma M_u}} \]  
(28) 
\[ \sigma M_u = \sqrt{\ln(COV^2 + 1)} \]  
(29) 
\[ \mu M_u = F_{M_u}^{-1}(0.05,\mu M_u,\sigma M_u) = M_u^{5\%} \]  
(30) 
The ultimate bending moment statistical 
descriptors are given in Table 6. 

| Table 6 Statistical descriptors of ultimate bending moments |
|-----------------|--------------|-------------|----------|
| Load Conditions | Distribution | Mean\(_{M_u}\) | StdDev\(_{M_u}\) | 5%       |
| M\(_u\)(sag)    | Lognormal    | 9.699       | 0.08      | 1.4289   |
| M\(_u\)(hog)    | Lognormal    | 9.648       | 0.08      | 1.3578   |

Table 7 Wave-induced dynamic pressure statistical descriptors (Gumbel distribution)
The Gumbel distribution, for the extreme values of the vertical wave-induced bending moment, over the reference period $T_r$ is derived based on the shape, $h$ and scale, $q$ factors of the Weibull distribution function as [16]:

$$\alpha_m = q(ln(n))^h$$  \hspace{1cm} (31)

$$\beta_m = \frac{q}{h}(ln(n))^{(1-h)/h}$$  \hspace{1cm} (32)

where $\alpha_m$ and $\beta_m$ are the parameters of the Gumbel distribution, $n$ is the mean number of load cycles expected over the reference time period $T_r$ for a given mean value wave period $T_w$. It is assumed here that $T_r = 1$ year and $T_w = 8$ sec. The mean number of load cycles $n$ is calculated as:

$$n = \frac{pT_r(365)(24)(3600)}{T_w}$$  \hspace{1cm} (33)

where $p$ is the partial time in which the ship is in seagoing conditions (full, ballast, partial loads).

The Gumbel distribution function is described as:

$$F_{MW} = \exp \left\{ -\exp \left( -\frac{M_{MW,e}-\alpha_m}{\beta_m} \right) \right\}$$  \hspace{1cm} (34)

where $M_{MW,e}$ is a random variable that represents the extreme value of the vertical wave-induced bending moment over the reference time period, $T_r$.

The selected target ship is a bulk carrier larger than Panamax with 175,000 tones. For simplifying the calculation, the Alternate conditioned the Homogenous condition are catalogued into the full load condition. That is Full load condition: $p_1 = 0.5$;Ballast condition: $p_1 = 0.35$.

The wave-induced vertical bending moment and local dynamic pressure statistical descriptors are given in Table 7 and Table 8.

The still water bending moment is fitted to a Normal distribution. Regression Eqn defines the statistical descriptors of the still water bending moment as a function of length, $L$ and dead-weight ratio, $W= (DWT/Full\ load)$, which coefficients are given in Table 9 and the calculated mean and standard deviation of still water bending moment are listed in Table 10.

Table 8 Wave-induced vertical bending moment, statistical descriptors (Gumbel distribution)

<table>
<thead>
<tr>
<th>Load conditions</th>
<th>Fraction of time</th>
<th>$n_0$ cycles</th>
<th>$\alpha_m$ MN.m</th>
<th>$\beta_m$ MN.m</th>
</tr>
</thead>
<tbody>
<tr>
<td>FL(sag)</td>
<td>0.5</td>
<td>1971000</td>
<td>0.0248</td>
<td>0.00269</td>
</tr>
<tr>
<td>BL(hog)</td>
<td>0.35</td>
<td>1379700</td>
<td>0.0242</td>
<td>0.00268</td>
</tr>
</tbody>
</table>

Table 9 Mean value and standard deviation of still water bending moment

<table>
<thead>
<tr>
<th></th>
<th>FL(sag)</th>
<th>BL(hog)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean($M_{SW,max}$)</td>
<td>-24.846</td>
<td>49.074</td>
</tr>
<tr>
<td>StDev($M_{SW,max}$)</td>
<td>21.215</td>
<td>26.115</td>
</tr>
</tbody>
</table>

$$\text{Mean}(M_{SW}) = \frac{\text{Mean}(M_{SW,max})M_{SW,CS}}{100}$$  \hspace{1cm} (35)

$$\text{StDev}(M_{SW}) = \frac{\text{StDev}(M_{SW,max})M_{SW,CS}}{100}$$  \hspace{1cm} (36)

Table 10 Still water bending moment

<table>
<thead>
<tr>
<th>Load conditions</th>
<th>$W=\text{DWT}$/Full Load</th>
<th>Distribution</th>
<th>Mean, (MN.m)</th>
<th>StDev, (MN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FL (Sagging)</td>
<td>0.9</td>
<td>Normal</td>
<td>968.939</td>
<td>827.338</td>
</tr>
<tr>
<td>BL (Hogging)</td>
<td>0.2</td>
<td>Normal</td>
<td>2186.468</td>
<td>1163.541</td>
</tr>
</tbody>
</table>

The statistical descriptions of the uncertainty coefficients involved in the limit state function are assumed and listed in Table 11.

Table 11 Uncertainty coefficients

<table>
<thead>
<tr>
<th>Uncertainty factors</th>
<th>Distribution</th>
<th>Mean</th>
<th>StDev</th>
<th>COV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_u$</td>
<td>Normal</td>
<td>1.05</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$X_{SW}$</td>
<td>Normal</td>
<td>1.00</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$X_W$</td>
<td>Normal</td>
<td>1.00</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$X_S$</td>
<td>Normal</td>
<td>1.00</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Where denotes the normal distribution function and the first and second indicator inside of the brackets refer to the mean value and standard deviation respectively.

3.5 Analysis

3.5.1 Multi-objective optimisation

The Pareto frontier [17] is employed here allowing for the optimisation of the three criterion, as they are defined in the present study as the minimisation of net sectional area, displacement and the fatigue damage factor $D$, verifying all trade-offs among the optimal design solutions of the three criterion.

The multi-objective optimisation was performed and the solution contains a series of optimal results, each of them includes the five design variables which determine the shape and area of the stiffened plate with the corresponding results of the three objective functions which are a sectional area,
displacement at the middle of the span and the fatigue damage factor $D$.

Figure 4 shows the minimisation of the two objective functions, $F_1$ (net sectional area) and $F_3$ (fatigue damage) simultaneously. Figure 5 shows the minimization of the two objective functions, $F_1$ (net sectional area) and $F_2$ (displacement) simultaneously.

Figure 4 indicates that the Pareto optimal frontier, whereby any improvement concerning $F_1$ comes at the bigger value of $F_2$. Each design solution, allocated at that frontier represents unique design solution parameters. The Pareto optimal solution collected here 100 optimal design solutions that are going to be verified with respect to the target reliability in the next section, leading to an additional constraint in the optimization process.

After that the points that do not meet the regulations were deleted, one can move on to the next step, the reliability design.

The optimization result of the stiffened plate with reliability index $3.72$ is that the thickness of the plate is $0.012$ m (12 mm), $h_{w}=0.496$m(496mm), $t_{w} = 0.0173$ m (17 mm), $b_{f} = 0.1526$ m (153 mm), $t_{r} = 0.012$ m (12 mm), the statistical nature of the constraints and design problems are defined in the objective function including the probabilistic constraints. The probabilistic constraints can specify the required reliability target level.

The reliability is performed based on the FORM [18, 19], and all random variables are considered as non-correlated ones. Applying FORM as a decision tool, the estimated probability of failure needs to be compared to an accepted target level. The target levels depend on different factors as reported in [20]. The target level adapted here, which may result in a redundant structure in $P_f = 10^{-3}$ ($\beta = 3.09$) for less serious and $P_f = 10^{-4}$ ($\beta = 3.71$) for serious consequences of failure values of the acceptable annual probability of failure [11].

During the buckling check step, the input values of the random variables, which describe the two loading conditions were taken into consideration, and so the two kinds of results were obtained. The Beta index of the buckling check is the combination of the two states using the fraction of time of the load condition of the bulk carrier as the weighting coefficient. After that, the result of the buckling check is combined with the results obtained by fatigue check again. At this point, the probability of the two outcomes is assumed for both as 0.5.

The final Beta index and its corresponding objective function values were calculated. The reliability index $\beta$, as a function of the net section area, is shown in Figure 6. The range of the Beta index of all design solutions at the Pareto frontier is from 2.737 to 4.11.

The design solution n° 6, $\beta = 3.72$ fits all constrains of the two objective functions and the required safety target level, as defined to be here, $\beta_{target} = 3.7$.

The optimization result of the stiffened plate with reliability index 3.72 is that $t_p = 0.012$ m (12 mm), $h_{w}=0.496$m(496mm), $t_{w} = 0.0173$ m (17 mm), $b_{f} = 0.1526$ m (153 mm), $t_{r} = 0.012$ m (12 mm), the
section area equals to 0.02075 (m\(^2\)).

Comparison with the original design section area = 0.0225 m\(^2\), the optimised section area is reduced by 8%.

4 CONCLUSIONS
The objective of this work was to perform a multi-objective nonlinear structural optimisation of a stiffened plate subjected to combined stochastic compressive loads accounting for the ultimate strength and reliability based constraints in the design. The solution of the three-objective structural responses, in minimising the weight, structural displacement and fatigue damage, was considered. The Pareto frontier solution was used to define the feasible surface solution of the design variables.

The reliability, index which defines the shortest distance from the origin to the limit-state boundary, was employed to identify the topology of the stiffened plate as a part of the Pareto frontier solution. Comparing with the original section area, the optimised section area is reduced by 8%. The presented methodology is flexible and demonstrated an excellent capacity to be used in the structural design of complex systems.

REFERENCE


