Real-Time Situational Awareness in Smart Grids by Regularization of Linear Inverse State-Estimation Problems

Inês Lopes Roça
ines.roca@ist.utl.pt

Instituto Superior Técnico, Lisboa, Portugal

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Abstract

This thesis proposes a state estimation methodology that combines (i) a regularization-based method for solving ill-conditioned load-estimation problems with (ii) a Markov model for restricting load-variations to probable time-varying load changes. The proposed solution relies upon the capability to solve sequences of linear inverse problems that are formulated as dependent on loads whose dynamics are parameterized in non-stationary Markov chains. As a first approach, Markov information is used to forecast the state vector one step ahead, allowing a simple forward dynamic estimation. Further on, a fully dynamic formulation of the estimation problem that takes grid loading states as a Hidden Markov process is presented and solved with the Viterbi Algorithm. Several monitoring contexts under different grid topologies are finally analyzed with the proposed state-estimation approach.
Keywords: Markov chains, Tikhonov regularization, Dynamic Estimation, Hidden Markov Models

I. Introduction

The intelligent management and control of the future Smart Grid must rely upon the accurate estimation of the present and future electrical states of the distribution network. Such estimation at the distribution level is very challenging, since the customary monitoring infrastructure inhibits access to real-time widespread information, precluding redundancy, whose existence is at the basis of classical state estimation approaches.

To solve this problem, forecast measurements derived from historical and/or statistical data about load consumption and power generation are commonly used. These so-called pseudo-measurements allow reaching network observabillity, but usually lead to poor accuracy of the state estimation (SE) results, since this information is not reliable when compared to the real-time measurements [1].

Despite these difficulties, nowadays there are new kinds of information that can be used for estimation. A few years ago, utilities started a massive deployment of metering infrastructures and nowadays one has already access to large volumes of historical metering data. The proper use of these measurements is key to enhance the performance of distribution system state estimators, allowing SE to play a key role also at distribution level [1, 2].

This thesis proposes to estimate the present and future electrical states of distribution grids by combining (i) a regularization-based method for solving ill-conditioned load-estimation problems with (ii) a Markov model for restricting load-variations to probable time-varying load changes. The solution of the proposed estimation problem relies upon the capability to solve sequences of linear inverse problems that are formulated as dependent on loads whose dynamics are parameterized in non-stationary Markov chains.

II. Background

1. State Estimation problem

In power systems, the State Estimation problem consists of identifying the state vector $\mathbf{x}$ from available information of measurements which have some inaccuracy:

$$\mathbf{z_{meter}} = \mathbf{h}(\mathbf{x}) + \mathbf{e_{meter}}. \tag{1}$$

The state vector is usually composed by the voltage magnitudes and voltage phase angles for each bus in the system, and its characterization is important since it allows to represent the entire state of the system at any given time. The measured variables $\mathbf{z_{meter}}$ can be written as a function of state variables through a set of non-linear functions $\mathbf{h}(\mathbf{x})$, dependent on Kirchhoff's laws and grid admittance matrix $\mathbf{Y}$ [3].

2. Weighted Least Squares

Power system SE commonly uses the Weighted Least Squares (WLS) method to estimate the state vector $\mathbf{x}$. This allows to minimize the weighted sum of squared residuals [4], according to the objective function:

$$\mathbf{J}(\mathbf{x}) = (\mathbf{z_{meter}} - \mathbf{h}(\mathbf{x}))^{\mathbf{T}} \cdot \mathbf{W} \cdot (\mathbf{z_{meter}} - \mathbf{h}(\mathbf{x}))\,, \quad (2)$$

where $\mathbf{W}$ is a diagonal matrix with the inverse of the variance of the error.

The minimization of Eq. 2 is usually performed applying iteratively the Gauss-Newton method, which leads to the following equations to be solved at each iteration:

$$\Delta\mathbf{x} = \underbrace{\left[\mathbf{H}^{\mathbf{T}}_{\mathbf{x^k}} \cdot \mathbf{W} \cdot \mathbf{H}_{\mathbf{x^k}}\right]}_{\mathbf{G}}^{-\mathbf{1}} \cdot \mathbf{H}^{\mathbf{T}}_{\mathbf{x^k}} \cdot \mathbf{W} \cdot (\mathbf{z_{meter}} - \mathbf{h}(\mathbf{x^k})), \quad (3)$$

$$\mathbf{x^{k+1}} = \mathbf{x^k} + \mathbf{\Delta x}. \quad (4)$$

In distribution systems, there is a lack of real time measurements, therefore the gain matrix $\mathbf{G}$ is ill-conditioned. For this reason, the vector $\mathbf{z_{meter}}$ contains mainly pseudo-measurements, and the weighting matrix is used to reflect the accuracy differences [5].

3. Tikhonov regularization

Tikhonov regularization is the most commonly used method of regularization of inverse ill-posed problems. Consider a linear problem of the form $\mathbf{M} \cdot \mathbf{x} = \mathbf{b}$, finding the unknown vector $\mathbf{x}$ is an ordinary least squared minimization problem. In an under-determined system, solving the inverse-problem is an ill-posed problem. For this reason, it is necessary to include a regularization term in order to give preference to a particular solution with desirable properties [4, 6]. Therefore, the objective function of the regularization problem is the one presented in Eq. 5:

$$\mathbf{J}(\mathbf{x}) = \|\mathbf{M} \cdot \mathbf{x} - \mathbf{b}\|^{\mathbf{2}} + \lambda \|\boldsymbol{\beta} \cdot \mathbf{x}\|^{\mathbf{2}}\,, \quad (5)$$

where λ is a positive constant chosen to control the solution vector [7].

The argument $\hat{\mathbf{x}}$ that minimizes the objective function is the one that verifies $\nabla_{\mathbf{x}}\mathbf{J} = \mathbf{0}$, which yields the Eq. 6:

$$\hat{\mathbf{x}} = (\mathbf{M^T} \cdot \mathbf{M} + \lambda \boldsymbol{\beta}^{\mathbf{T}} \cdot \boldsymbol{\beta})^{-\mathbf{1}} \cdot \mathbf{M^T} \cdot \mathbf{b}. \quad (6)$$

Tikhonov regularization emerges as an alternative to the WLS method, the traditional tool to solve ill-posed problems. In the first instance, this alternative allows to overcome the difficulty of assigning weights to different types of measurements, since the commonly known pseudo-measurements are incorporated into the regularization term.

4. Dynamic state estimation

Owing to the dynamic nature of system loads, the static state vector also varies dynamically. This justifies the use of an algorithm to estimate dynamically the state vector, and that defines the 'best' representation with time for the system evolution.

Several models were developed to define pseudo dynamics of the distribution system. Tracking State Estimation techniques and Kalman filter approaches are the most common dynamic solutions proposed in the literature [8, 9]. They rely on the assumption that time behaviour of the power system is a quasi-stationary process, where the non-linear system model is approximated to a linear model [10]. But whenever there are large changes in the load or production profile, the nonlinearities in the system become important, which results in the degradation of the performance of these techniques [11].

5. Markov models

The uncertainty of load and generation makes the dynamics of consumers and production very difficult to model, since a good characterization of their behaviour requires a deep knowledge on the existing correlations and time-dependencies, simultaneously [12]. The dynamics can be however represented as a statistical model, where the underlying assumption is that it can be well characterized as a parametric random process, and that the parameters of the stochastic process can be determined (estimated) in a precise, well-defined manner [5, 12].

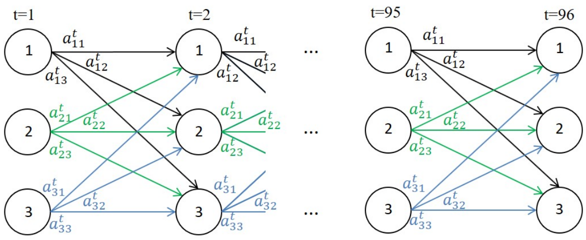


Figure 1: Trellis diagram for a non-stationary Markov process with 3 states (as a simplification) and 96 time periods that is used to represent the intra-day dynamics of an input aggregate daily profile with 15min resolution.

In [12], a method to characterize different types of consumers or energy production technologies as a Markov process was proposed. The method relies upon generic individual profiles (observed profiles) to create a discrete-time stationary Markov process to parameterize the average volatility and time-dependency of a particular group. Then, the aggregate metering data (aggregate profile) is also used to parameterize a non-stationary Markov process, related to previous stationary process, in a way that reflects the typical intra-day dynamics of that group of profiles (Fig. 1).

As a result, the dynamics of a given type of load is parameterized into a set of non-stationary Markov transition matrices $[a^t_{ij}]$, restricting load-variations to probable time-varying load changes.

6. **Hidden Markov Models**

The process previously presented results from a direct observation of the power states over the time. Despite loads being characterized by Markov chains, the state of the system cannot be observed directly in real-time. Instead, the only available data in real-time are measurement information, which are functions of the state (combination of every loads state). Due to measurements noise, the association of states and corresponding measurements is stochastic. Reconstructing the unknown sequence of states from a set of observations is in fact a SE problem, but since the loads are characterized by a Markov process, the SE problem is a Hidden Markov problem [13, 14].

A Hidden Markov Model (HMM) is characterized by:

- The N number of possible states;
- The M number of distinct observation categories $V = v_1, ..., v_M$;
- The state transition probability distribution $\mathbf{A} = [a_{ij}]$;
- The observation category probability distribution in state j,
  $\mathbf{B}(v_k, j) \ = \mathbf{P}\ [O(t) = v_k | s(t) = j]$;
- The initial state distribution π_i, such that $\pi_i = \mathbf{P}[s(1) = i]$, for $i = 1, \ldots, N$.

In order to find the "correct" state sequence associated with the given observation sequence, the most widely used criterion is to maximize the probability of a single state sequence (path). The Viterbi algorithm is the commonly used technique to find this path [14]. The complete procedure for finding the best state sequence is described later on in Section 3.

III. Implementation

1. **State Estimation by Regularization**

State Estimation with regularization methods relies on the assumption that a deviation in the observable variables corresponds to an update in the state vector, such that $\mathbf{\Delta z^m} = \mathbf{A_m} \cdot \mathbf{\Delta I}$, where matrix $[\mathbf{A_m}]$ depends on the power flow equations.

Every time new measurements values are gathered, the vector $\mathbf{\Delta z^m}$ is recalculated and the resulting $\mathbf{\Delta I}$ is determined, as an ordinary minimization problem, where $\mathbf{\Delta I}$ is the new state variable:

$$\text{Minimize} \qquad \|\mathbf{A_m} \cdot \mathbf{x} - \mathbf{\Delta z^m}\|^{\mathbf{2}}, \tag{7}$$

with,

$$\mathbf{x} = \mathbf{\Delta I} = \left[\overline{I_i} - \overline{I}_i^0\right], \ \forall i \in [1, n]. \tag{8}$$

Quadratic penalization term

One of the goals when using this method is to ensure that the adjustments in each state variable are, as much as possible, in accordance with their typical value. This corresponds to minimizing the relative norm of $\mathbf{\Delta I}$ instead. Designating by $\mathbf{I^0}$ the typical value of each load, i.e. the pseudo-measurements, the objective function should be:

$$\mathbf{J(x)} = \|\mathbf{A_m} \cdot \mathbf{x} - \mathbf{\Delta z^m}\|^{\mathbf{2}} + \lambda \left\|\mathbf{diag}\left(\frac{\mathbf{1}}{\mathbf{I^0}}\right) \cdot \mathbf{x}\right\|^{\mathbf{2}}. \tag{9}$$

The update step in each iteration is:

$$\mathbf{\Delta \hat{x}} = \mathbf{\Delta \hat{I}} = \left(\mathbf{A_m^T} \cdot \mathbf{A_m} + \lambda.\mathbf{D}\right)^{\mathbf{-1}} \mathbf{A_m^T} \cdot \mathbf{\Delta z^m}, \tag{10}$$

where λ is a number small enough to guarantee that the residual errors in the measurements are almost zero and $\mathbf{D}$ a diagonal matrix with the inverse of the Euclidean norm of each current.

$$\mathbf{D} = \begin{bmatrix} \frac{1}{||I_1^0||^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{||I_n^0||^2} \end{bmatrix} \tag{11}$$

Proposed penalization term

Although the previous regularization term is the most intuitive one when the purpose is to minimize the norm of the deviations relative to the expected value of the injected current, the resulting updating expression shows that the penalty factor is not proportional to the expected current, but to its squared value. The main consequence of this quadratic penalty factor is that the distribution of the measurement residuals is not made according to the intrinsic magnitude of the load, as it was intended.

To proceed as intended, an alternative approach is considered, that takes the regularization term simply as the Euclidean norm of the deviations, $\|\mathbf{\Delta I}\|^{\mathbf{2}}$. In order to penalize relative deviations from the expected value, the parameter λ is weighted as in the following:

$$\boldsymbol{\lambda} = \lambda \begin{bmatrix} \frac{1}{||I_1^0||} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{||I_n^0||} \end{bmatrix}. \tag{12}$$

The implemented iterative procedure for both regularization terms is represented in the following algorithm:

Data: $\mathbf{Y}$, $\mathbf{A_m}$, $\mathbf{I^{hist}}$, $\mathbf{z_{meter}}$, $\mathbf{h}$
for t=1,...,96 **do**
$\mathbf{I}(t) = \mathbf{I}^{hist}(t)$;
$[\mathbf{D}] = \mathbf{diag}\left(\frac{\mathbf{1}}{||\mathbf{I}||^2}\right)$;
$[\boldsymbol{\lambda}] = \lambda\mathbf{diag}\left(\frac{\mathbf{1}}{||\mathbf{I}||}\right)$;
while $|\mathbf{z_{meter}} - \mathbf{z^{new}}| \leq \tau$ **do**
$\mathbf{h} = \mathbf{h}(\mathbf{I}(\mathbf{t}))$;
$\mathbf{\Delta z^m} = \mathbf{z_{meter}}(\mathbf{t}) - \mathbf{h}$;
$\mathbf{\Delta I}(\mathbf{t}) = \begin{cases} (\mathbf{A_m^T A_m} + \lambda.[\mathbf{D}])^{-1}\mathbf{A_m^T \Delta z^m} \\ (\mathbf{A_m^T A_m} + [\boldsymbol{\lambda}])^{-1}\mathbf{A_m^T \Delta z^m} \end{cases}$;
$\mathbf{I}(\mathbf{t}) = \mathbf{I}(\mathbf{t}) + \mathbf{\Delta I}(\mathbf{t})$;
$\mathbf{V}(\mathbf{t}) = \mathbf{1} + \mathbf{ZI}(\mathbf{t})$;
$\mathbf{z^{new}} = \mathbf{h}(\mathbf{I}(\mathbf{t}))$
end
end

Algorithm 1: Algorithm implemented to estimate the injected currents $\hat{\mathbf{I}}$ with Static State Estimator with Tikhonov Regularization. The blue equations are only implemented in the approach with quadratic penalization term (TR_1). The red equations are only implemented in the approach with linear penalization term, i.e. the proposed solution (TR_2). In each time iteration the procedure must be repeated, and the first guess is simply the historical value for that time instant.

2. Dynamic State Estimation with Markov Information

As a first approach to make use of Markov information, one has to solve a tracking-like problem, where an *a priori* estimation of the state vector at instant t can be performed based on the estimation of the state at $t-1$ and the non-stationary transition probability matrix. Assuming that for a given instant of time $t-1$, each load has an estimated value of $\hat{I}_{1,t-1}, ..., \hat{I}_{n,t-1}$, the initial guess for time t is given by the system of Eqs. 13:

$$\begin{array}{l} \hat{I}^0_{1,t} = \sum_{j=1}^{S_{max}} \boldsymbol{\phi}_{1,t-1\to t}(\mathbf{S}(\hat{I}_{1,t-1}), j).I_{avg}(j) \\ \vdots \\ \hat{I}^0_{n,t} = \sum_{j=1}^{S_{max}} \boldsymbol{\phi}_{n,t-1\to t}(\mathbf{S}(\hat{I}_{n,t-1}), j).I_{avg}(j) \end{array} \tag{13}$$

where ϕ is a set of 95 transition matrices (Markov chain) and $I_{avg}(j)$ is the average value of each load state.

Once the *a priori* estimation is performed, the following step is to correct the prediction with the measurements gathered ($\mathbf{z_{t+1}}$). To achieve this, the state estimation problem must be solved as described previously [Section 1].

The main features of the iterative procedure implemented are summarized in Algorithm 2.

Data: $\mathbf{Y}$, $\mathbf{A_m}$, $\mathbf{I^{hist}}$,$\mathbf{z_{meter}}$,
$\mathbf{h}$,$\boldsymbol{\phi}_{1,t\to t+1}$,...,$\boldsymbol{\phi}_{n,t\to t+1}$, $I_{avg}(s)$
$\mathbf{I}_{t=1} = \mathbf{I^{hist}}_{t=1}$;
for t=1,...,95 **do**
while $||\mathbf{z_{meter}} - \mathbf{z^{new}}|| \leq \tau$ **do**
(...)
end
$\hat{I}^0_{1,t+1} = \sum_{j=1}^{S_{max}} \boldsymbol{\phi}_{1,t\to t+1}(\mathbf{S}(\hat{I}_{1,t}), j).I_{avg}(j)$;
$\hat{I}^0_{n,t+1} =$
$\sum_{j=1}^{S_{max}} \boldsymbol{\phi}_{n,t\to t+1}(\mathbf{S}(\hat{I}_{n,t}), j).I_{avg}(j)$;
end

Algorithm 2: Algorithm implemented to get an *a priori* estimation for the state vector, based on the estimation of the previous instant state and Markov information on loads. The prediction is then corrected with the measurements gathered, using regularization methods or WLS method.

3. Dynamic State Estimation with Hidden Markov Model

The SE problem of a distribution network, whose dynamics of the loads is characterized by a Markov process, is formulated as a Hidden Markov problem. The (hidden) states in which one is interested are the combination of load states of all loads in the system, and the sequence of observations is the set of real-time measurements along the day.

Data: $\boldsymbol{\pi}$, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{O}$
$\boldsymbol{\delta}_1(i) = \mathbf{P}(O_1|i).\boldsymbol{\pi}_i$;
while $t = 2, ..., T$ **do**
for $j \in S$ **do**
$\boldsymbol{\delta}_t(j) = \max_{i\in\mathbf{S}}[\boldsymbol{\delta}_{t-1}(i).a_{i,j}]\mathbf{P}(O_t|j).$;
$\boldsymbol{\varphi}_t(j) = \mathbf{arg}\max_{i\in\mathbf{S}}[\boldsymbol{\delta}_{t-1}a_{i,j}]$;
end
end
$S^*(T = 96) = \mathbf{arg}\max_{i\in\mathbf{S}}[\boldsymbol{\delta}_T(i)]$;
for $t = 95, ..., 1$ **do**
$S^*_t = \boldsymbol{\varphi}_{t+1}(S^*_{t+1})$;
$\overline{I}^*_{1,t} = I_{1,avg}(S^*_t)$;
$\vdots$
$\overline{I}^*_{n,t} = I_{n,avg}(S^*_t)$
end
$\mathbf{I^{hist}_t} = \mathbf{I^*_t}$;
for $t = 1, ..., 96$ **do**
Apply Regularization or WLS method
end

Algorithm 3: Viterbi Algorithm implemented to search the most likely daily sequence of states, $\mathbf{S_t}$, given a sequence of measurements $\mathbf{O_t}$. The prediction is then corrected with the measurements gathered, using regularization methods or WLS method.

Attending to this formulation, the Viterbi Algorithm is used to predict the most likely sequence of states (path). This algorithm resorts to dynamic programming and allows a Forward / Backward estimation process. Recalling the trellis diagram presented before (Fig. 1), in the forward process, each path in the trellis is weighted using the likelihood of transitions, based on the observation probability of a given state. At the end, the path with the highest final cumulative probability is selected, and in the backward process the corresponding sequence of states is recovered (see Algorithm 3).

4. Verification and Illustration for comparison

In order to test the performance of state estimators previously presented, an exercise is considered over a radial grid with two loads (of different types) and a single measurement of the feeder-current. Two different cases for the loads are considered: (i) two loads with same average and standard deviation and (ii) two loads with different averages and standard deviations. To evaluate the performance of each method, the sum of the squared error (SSE) for each load is calculated. Based on the SSE, an average percentage error is computed as follows:

$$Error_{load_i}[\%] = \frac{\sqrt{SSE_i}}{Avg_{load_i} \times 96} \times 100. \quad (14)$$

Table 1 summarizes the average state estimation error obtained for each method and approach.

Table 1: Estimated error, average in time [%].

		Static			Markov			HMM		
		WLS	TR_1	TR_2	WLS	TR_1	TR_2	WLS	TR_1	TR_2
#1	Load 1	0.91	0.94	0.88	0.74	0.74	0.72	0.72	0.73	0.70
	Load 2	0.91	0.94	0.88	0.74	0.74	0.72	0.72	0.73	0.70
#2	Load 1	2.18	1.35	1.37	2.16	1.23	1.14	1.00	0.83	0.81
	Load 2	0.73	0.45	0.46	0.72	0.41	0.38	0.33	0.28	0.27

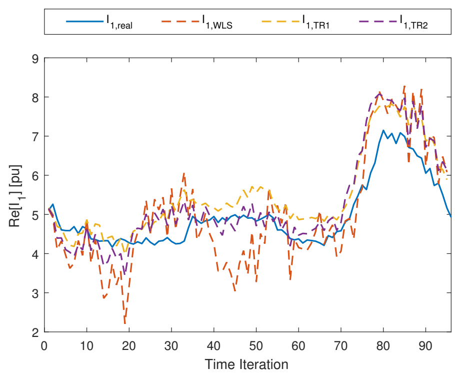
Although the test exercise was quite simple, it allowed us to draw some interesting conclusions.

First of all, it should be pointed out that regularization methods allow more satisfactory results than the classic WLS method. Regularization methods in SE ensure that the adjustments in each state variable are, as much as possible, in accordance with its typical value. Regularization method with the proposed penalty factor allows us to change the load according to its expected value, so corrections are always proportional to the intrinsic magnitude of the load. This is a more plausible assumption since it guarantees some balance from the point of view of effort distribution by the two loads. This is not relevant in the first case, since a homogeneous system was considered. However, in the second case, the regularization methods show more accurate results, in particular the one with the proposed regularization term.

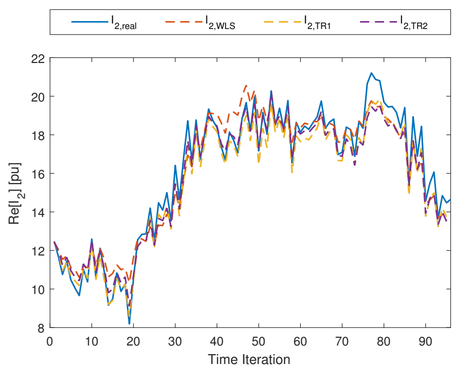
The Dynamic State Estimator with Markov Information allows to constantly update the vector of pseudo-measurements, which in addition to the typical dynamics of the load (assumed to be characterized in a set of Markov chains) also takes into account the present situation of the system. Results clearly show that this process can improve the SE problem, without greatly increasing the complexity of the method.

With respect to Dynamic State Estimator with HMM, the fact that the estimator resorts to the entire intra-day behavior and dynamics to predict the state of the system at a given moment provides more accurate estimation results. Nevertheless, the great improvement in the estimation accuracy comes with a much higher computational effort. The SE problem as formulated with HMM might become intractable for realistic sized grids.

Figure 2 and Figure 3 present a sample of the estimation results with Markov Information and with Hidden Markov process, respectively.

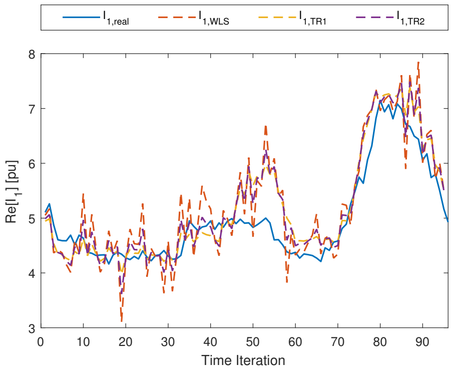


(a) Load 1

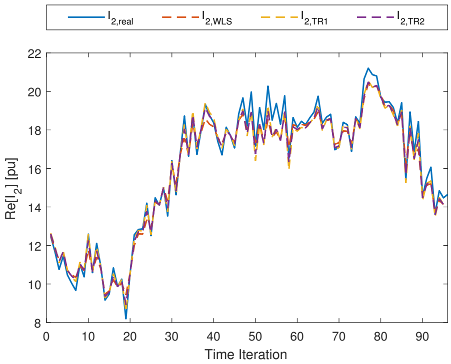


(b) Load 2

Figure 2: Dynamic State Estimator with Markov Information for a radial grid with two loads of different types and sizes (Case #2). The blue line represents the true injected current for each load. The dashed lines represent the estimated currents: the red dashed lines represent the solution for WLS method; the yellow and purple dashed lines represent the solution for the Tikhonov regularization with the quadratic and the proposed penalty term, respectively.



(a) Load 1



(b) Load 2

Figure 3: Dynamic State Estimator with Hidden Markov Model for a radial grid with two loads of different types and sizes (Case #2). The blue line represents the true injected current for each load. The dashed lines represent the estimated currents: the red dashed lines represent the solution for WLS method; the yellow and purple dashed lines represent the solution for the Tikhonov regularization with the quadratic and the proposed penalty term, respectively.

IV. Results

The proposed regularization method and the dynamic state estimation approaches were applied to a more complex grid. The grid was settled to reproduce some of the challenges posed to state estimation. Fig. 4 presents the grid topology and a sample of historical load profiles for each load is presented in Fig. 5.

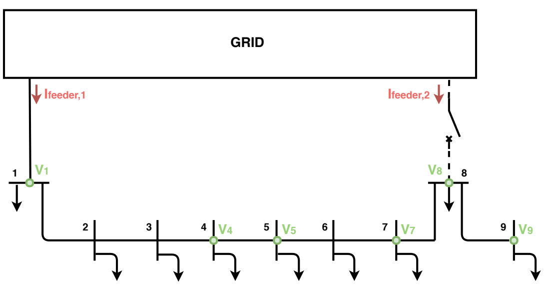
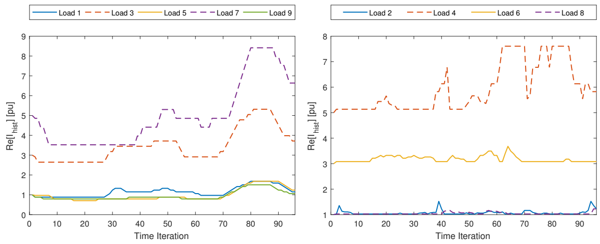


Figure 4: Application Grid topology. The grid consists in a 9-bus topology, that can be changed from a radial to a meshed configuration. Real time measurements are mainly of feeder-currents and bus voltages, and can be placed at several locations.



(a) Type 1 - Low volatility (b) Type 2 - High volatility

Figure 5: Historical profiles of the Application Grid loads.

Loads have been previously classified into two different typical load profiles. The intra-day dynamics of each type of loads was previously parameterized in a set of Markov chains, composed by 95 transition matrices (15min resolution). Besides type, each load is also characterized by a specific consumption magnitude: both average consumption and standard-deviation.

Subsection 1 and Subsection 2 present the results for the Dynamic State Estimator with Markov Information for radial and meshed configuration, respectively. In Subsection 3, a comparison between the fully dynamic formulation and a simpler representation of load dynamics is performed.

1. Results for the radial configuration

The Dynamic Estimator with Markov Information was applied for the radial configuration. Three different measurement sets (Feeder-current only, feeder-current and voltage at bus 8, feeder-current and voltage at bus 7) were considered.

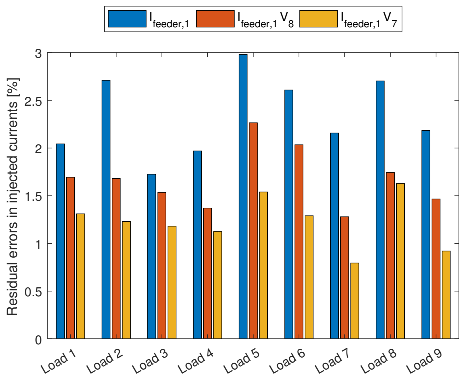


Figure 6: Residual error in percentage for each load obtained with Dynamic Estimator with Markov Information for the radial configuration. The figure compares error for different combination of real time measurements type and location. The blue bars corresponds to the results obtained with only feeder-current measurements. The red and yellow bars corresponds to the error obtained when measuring feeder-current and voltage at bus 8 and bus 7, respectively.

Note that the results obtained by only measuring the feeder-current in real-time have an appreciable accuracy (errors below 3% – see blue bars). Despite having nine degrees of freedom, the feeder-current being a measurement of the aggregation of the nine loads provides, together with the information on the dynamics, enough information to carry out a good estimation.

By adding an extra real-time measurement, observability of the system is increases and an additional constraint is added to the SE problem. Results show that this allows us a better perception of the state of the system, as expected. Consequently, the estimation of the state of all the loads is obtained with higher accuracy, and the error decreases to about 2%.

It should also be noted that the location of the voltage measurement has impact on the estimator performance. In this application, a measurement of voltage at bus 7 allows a better estimation than measurement at bus 8. This may be due to differences in size and type of the loads presented in the measurement bus. Results tend to be better when the measurement are set to be with larger and less volatile loads.

2. Results for the meshed configuration

The Dynamic Estimator with Markov Information was then applied for the meshed configuration. Four different measurement sets (feeder-current only, both feeder-currents, feeder-current and voltage at bus 8, both feeder-currents and voltage at bus 8) were considered.

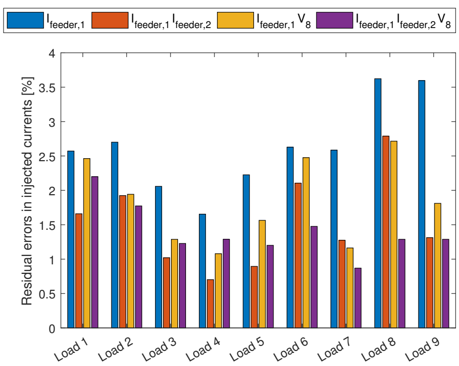


Figure 7: Residual error in percentage for each load obtained with Dynamic Estimator with Markov Information for the meshed configuration. The figure compares error for different combination of real time measurements type and location. The blue bars correspond to the errors obtained with only feeder-current measurements. The red bars correspond to the error obtained when measuring both feeder-current. The yellow bars correspond to the error obtained measuring feeder-current and voltage at bus 8. The purple bars correspond to the error obtained when measuring both feeder-currents and voltage at bus 8.

By adding an extra branch to close the loop, the state estimation complexity increases. However, with only one feeder-current measurement, the estimation accuracy is considerable and comparable with the ones obtained for the radial configuration. Yet, results get significantly better if an extra real-time measurement is added to the estimator (see red and yellow bars).

Attending to the results obtained when an extra measurement is added (comparing red bars with yellow bars), the second feeder-current allows a better estimation, for most of the loads, than an extra voltage measurement. Results suggest that feeder-currents are more aggregating variables than bus voltages, highlighting the importance of measuring feeder-currents at distribution grids.

3. Forward Estimation vs. Forward/Backward Estimation

The SE problem was formulated as a Hidden Markov process and then solved using the Viterbi Algorithm, both for the radial and meshed configuration of the Application grid (see Section 3). The results are shown in Fig. 8 and Fig. 9, for the radial and meshed configuration respectively.

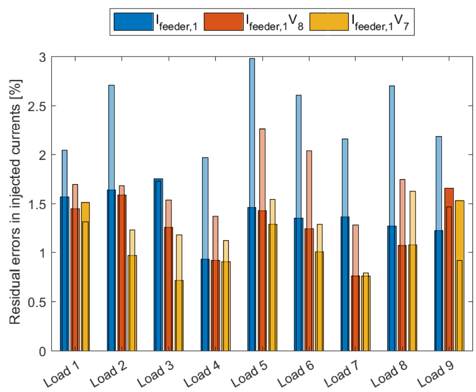


Figure 8: Residual error in percentage for each load obtained using Dynamic Estimator with HMM for the meshed configuration. The figure compares the error of HMM formulation (dark colors) with the error obtained with a simpler representation of load dynamics (Estimation with Markov Information is shown in light colors). The blue bars corresponds to the results obtained with only feeder-current measurements. The red and yellow bars corresponds to the error obtained when measuring feeder-current and voltage at bus 8 and bus 7, respectively.

Since each estimation takes into account not only the full dynamics of the system, but also the daily behavior of the observations, it is expected to achieve a better estimation with Hidden Markov approach than with Markov information approach. In general, the results obtained corroborate with the expectations.

Nevertheless, the estimation improvement comes with a much higher computational effort. As the grid size increases, the dimension of the state space explodes and with it the estimation effort. To avoid such curse on dimensionality, a simplified version of the problem was proposed that used Markov information to forecast the state vector one step ahead only, allowing a simple forward dynamic estimation. Tests performed with the simplified approach revealed that the accuracy of the estimation could still be appreciable.

Other aspect to be noted is that, the advantage of forward-backward over a simple forward estimation become less important as more measurement variables are added.

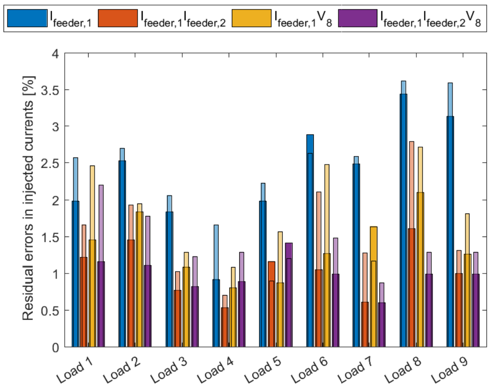


Figure 9: Residual error in percentage for each load obtained with Dynamic Estimator with HMM for the meshed configuration. The figure compares the error of HMM formulation (dark colors) with the error obtained with a simpler representation of load dynamics (Estimation with Markov Information is shown in light colors). The blue bars corresponds to the results obtained with only feeder-current measurements. The red bars correspond to the error obtained when measuring both feeder-current. The yellow bars correspond the error obtained measuring feeder-current and voltage at bus 8. The purple bars correspond the error obtained when measuring both feeder-currents and voltage at bus 8.

V. Conclusions

The main conclusion to be drawn from the work done is that getting a good representation of system dynamics is an effective way to improve state estimation in distribution networks. Markov chains allow synthesizing realistic system dynamics, and that is very important since it allows obtaining much better pseudo-measurements, essential to improve the solution accuracy of the state estimation solution.

Another important results of the research conducted is that the regularization method used has a critical role on the accuracy of estimation. The proposed regularization method contributes significantly to the estimation quality since it leads the solution to be in accordance to the intrinsic magnitude of each load.

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