Optimization of Maintenance and Availability in a Train Operating Company – Fertagus case study

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June 2018

Abstract. Railway systems in Europe are facing, nowadays, a wide range of measures that will hopefully bring till 2050 a new competitiveness and a key strategic importance to the sector, revolutionizing its importance in transportation. Herewith a lot of research will be induced, to apply new approaches. An optimized rolling-stock planning, associated with a sustainable reduction of costs, improvement of service reliability and adapted to the current customer demand and maintenance requirements is of high potential, and therefore, one way to respect budget goals, while new costs in research and development are added. A mixed-integer linear programming decision model, which considers the preventive maintenance actions that must be performed for each week of the year, is presented and gives a weekly rolling-stock schedule. It minimizes the operational costs, while adding maintenance actions to the roster, and is validated by an illustrative example of a small sized problem. Afterwards several real instances applied to Portuguese Railway operating company Fertagus are solved to optimality. Only a 3-day schedule was possible to run due to computational capacity limitations. Moreover, sensitivity analysis on the weights of the different components of the objective function demonstrated that the optimal solution found is not sensitive to significant variations of the weights. An intermediate model that links the 1-day operational planning model proposed and the annual tactical plan, able to reduce the size of the problem, and a comprehensive crew scheduling that considers the different skill of maintenance technicians and their experience, are proposed for further research.

Keywords: Railway management, Rolling-stock planning, Maintenance Scheduling, Robustness, Optimization, Integer linear programming

1. Introduction

1.1. Background

Transportation systems in Europe are facing, nowadays, a wide range of profound structural reforms, related to an increasing strategic importance of this sector. Transportation is one of the cornerstones of the European economy. Moreover, there is a need for a more competitive and connected Europe, capable to face a growing competition in fast developing world transport markets.

Railway transportation can be considered a smart and efficient mean of transportation, and more strategies should be studied to transform it into a more popular mean of transportation. However, some circumstances, still prevent railway transport from being a more widely used mean of transportation, for instance the ideal of a Single Transport Area is not yet possible. This problem will have to be addressed if the railway industry wants to become more competitive in the future.

To context the problem that will be addressed in this research, the railway operation management is briefly summarized. Railway operations management can be divided in three major operations: i) timetabling, ii) rolling-stock circulation planning to cover timetable and iii) crew scheduling to operate the rolling-stock (Huisman et al. 2005). These operations are usually carried out separately by railway operators, but their interdependency is crucial for a proper management of the all operation.

The rolling-stock planning problem should desirably not be separated from maintenance planning problem and rail inspection scheduling problem (Peng et al., 2013). In transportation companies, maintenance has a critical impact on both safety and availability. A careful maintenance planning is meant to be a trade-off between cost reduction and overly performed approach. If a vehicle lacks maintenance, the failure of its components will occur more frequently and will be more unpredictable. More specifically, there are two kinds of maintenance. The ***preventive maintenance***, which aims to preserve the healthy condition of equipment and prevent failure, and the ***corrective maintenance***, which handles equipment failure and recovers it to operational conditions. A careful maintenance planning has naturally a positive impact on costs reduction, but also on the reliability of its components. Furthermore, a reliable component does not compromise the fulfilment of the operational task, because of its failure.

1.2. Problem statement

The objective of this research work is to conceive a decision model capable of looking at the operation in a train operating company and perform a rolling-stock planning, for each week of the year. It must take into account the timetable of the company's operation activities and a schedule relative to the preventive maintenance activities of each week of the year. The operative and maintenance costs are to be minimized and the availability of units to be maximized. Therefore, it is intended to apply the decision model to the case study of Fertagus train operating company, using a preventive maintenance technical planning from (Méchain, 2017), which outputs a weekly schedule, contemplating the preventive maintenance actions that have to be performed for that week and with the smallest maintenance cost possible.

1.3. Document Structure

The present document is structured in subsequent sections. The first section is an introduction. The second section presents a brief review of the related literature. The third section exposes the mathematical model. The forth section presents the Fertagus' case study. The fifth section displays the results of this research. The sixth section presents the conclusions of this research.

2. Related Literature

Huisman *et al.* (2005) give a wide overview of state-of-the-art on operations research models and techniques used by passenger railway operators. Planning problems are usually classified by its planning horizon and can be divided in three planning phases, namely: strategic, tactical and operational. Operational planning handles the details of the timetable, for instance, the rolling-stock and crew schedules are constructed. Rolling-stock circulation problem allocates rolling-stock units to the trips to be operated. Routing due to maintenance of rolling-stock addresses the maintenance visits to maintenance facilities. These visits of the rolling-stock units to may already be incorporated in the rolling stock circulation problem.

Méchain (2017) addresses the problem of maintenance planning for a Portuguese railway operating company, Fertagus. A mathematical model is formulated first, concerning the various constraints of the company, but viable to be adapted to fit to any company's specifications. Technical constraints associated with the maintenance yard configuration are introduced in the model. The adaptation to the company involved data collecting related to maintenance activity operations in the maintenance yard. A MILP (Mixed-Integer Linear Programming) optimization model is developed, that minimizes the total cost spent on preventive maintenance and adapted to the company context. The model successfully outputs a technical maintenance planning for all the 52 weeks of a given year. In a broad planning perspective, the study lacks an operational planning capable of taking the obtained maintenance technical plan as an input and verify if the solution found is feasible to be implemented within the operations of each week.

Tréfond et al., (2017) study a rolling-stock planning problem with a robustness perspective for French passenger trains. First, the concept of robustness is discussed. Some indicators are assessed for the evaluation of rolling-stock rosters. Homogenization of turning-times is the chosen method, to absorb potential delays and so introduce robustness to the roster. The paper proposes a method based on an integrated ILP (Integer Linear Programming) model to add robustness to a roster while maintaining low operating costs. Tests were carried out to validate the model and verify the relevance of the used construction robustness indicator. A significant improvement in robustness indicators was observed, while maintaining low operating costs and meeting maintenance requirements. However, the maintenance approach in not very in-depth, considering simply the introduction of maintenance slots.

3. Mathematical model definition

The present mathematical model is an adaptation of the model presented by Tréfond et al (2017). To fit the Fertagus case study and integrate the information associated with the preventive maintenance model extracted from (Méchain, 2017). It is a decision model capable of using the preventive maintenance planning and the timetable activities from a train operating company to build a rolling-stock planning roster for each week of the year.

A **task** $\boldsymbol{T_i}$ is defined as a non-splittable trip to be realized between one **departure station** $\boldsymbol{Sd_i}$ and one **arrival station** $\boldsymbol{Sa_i}$. It is also characterized by **departure** and **arrival times,** $\boldsymbol{Dd_i}$ and $\boldsymbol{Da_i}$ respectively. The **demand** $\boldsymbol{DEM_i}$, corresponding to the number of train units needed to perform a task, and the **capacity** $\boldsymbol{CAP_i}$, corresponding to the maximal number of train units that can be used to cover that task, are also known. A **train or rolling-stock unit** $\boldsymbol{U_k}$ is a set of rail coaches that cannot be divided. Two or more units can be coupled to create a multiple unit, so that it can cover a higher demand task. A unit can be assigned to two **successive tasks** $\boldsymbol{T_i}$ **and** $\boldsymbol{T_j}$ if T_j starts from arrival station of T_i, and if the turning time between the two tasks is greater than a technical threshold (**minimal turning time** $\boldsymbol{TM_s}$, which is specific to each **station** $\boldsymbol{s}$). A **turning time** is the time between the arrival time of a task and the departure time of the next task covered by the same unit. More precisely, the turning time between tasks T_i and T_j is equal to $\boldsymbol{Dd_j - Da_i}$**.** For each **time-period** $\boldsymbol{p}$ (e.g. one day or one week) with an associated **length** $\boldsymbol{L}$, a train unit is assigned to a sequence of tasks called row of a **unit** $\boldsymbol{R_k}$**. A rolling-stock roster** is a sequence of rows. A unit can accomplish **two successive rows** $\boldsymbol{R_k}$ **and** $\boldsymbol{R_{k'}}$ during **two successive periods** $\boldsymbol{p}$ **and** $\boldsymbol{p+1}$ if R_k follows $R_{k'}$, i.e. if the unit can cover successively the last task of R_k in period p and the first task of $R_{k'}$ in period

$p + 1$. A **maintenance action** $\boldsymbol{KM_{k,m}}$ is defined as a preventive maintenance intervention to be realized between two successive tasks T_i and T_j, on a specific unit and at a specific station called depot. There is a limited number of kind of maintenance actions, which can be performed, and each kind of maintenance action has a characteristic **duration** $\boldsymbol{MT_m}$ and **working load** $\boldsymbol{AW_m}$. The working load and the duration relate as follows: $duration = \frac{working\ load}{working\ persons}$. Different kind of maintenance actions require a different number of working persons. Finally, for a given time-period, there is one or more predefined **days d**, when maintenance may occur. The number of days available for maintenance can be less than the number of days of the time-period. **Dead-headings** are trips with no passengers and can be added to the roster to move units from a station to another. These trips may be necessary to move units to or from the depot to perform maintenance actions, with an associated **duration** $\boldsymbol{DW_{s,depot}}$. Therefore, a unit can be assigned to a maintenance action (programmed in the maintenance plan) between two successive tasks if there is enough time to perform the maintenance action and the necessary dead headings, i.e. if $\boldsymbol{Dd_j - Da_i \geq DW_{s,depot} +}$ **duration** $\boldsymbol{MT_m + DW_{depot,s'}}$. Costs related to a unit are the number of kilometres that it travels. **Active costs** of a unit correspond to the number of kilometres travelled as an active unit, while **passive costs** correspond to the number of kilometres travelled as a passive unit. The total number of units used, and the active costs are called **primary costs**. Costs related to dead-headings and passive costs are called **secondary costs**. **Operating costs** include primary and secondary costs. Both are to be **minimized**. The impact of secondary costs is much lower than the impact of primary costs. However, the present work focusses on the secondary costs minimization, as a rolling-stock circulation planning problem, as the primary costs of the solution to be found remain unchanged. For a set of tasks and maintenance actions, a feasible solution to this rolling-stock planning problem consists of a roster related to one time-period, in which all tasks and maintenance actions are covered, and technical operating and maintenance constraints are respected. Moreover, the considered problem consists in building a robust roster. "A robust rolling-stock roster should anticipate operational disturbance possibilities in order to limit service quality deterioration and additional costs." (Tréfond et al., 2017). Improving robustness may be in conflict with operating costs minimization. In practice, it is unacceptable to degrade primary costs, since the obvious solution to improve robustness would be to use more train units. Then, the objective is a trade-off between secondary operating costs (dead-headings and passive units) and robustness, which is quantified by a robustness indicator explained further on. Therefore, secondary costs may be deteriorated to build robust solutions, provided that this deterioration is controlled.

The model computes on each task the number of active and passive units and creates dead-headings, so that all tasks are covered, while the operating costs are minimal. The maintenance actions are added, while building the rows of the roster and optimizing its robustness.

3.1 Indexes

$\boldsymbol{k}$	train unit
$\boldsymbol{s}$	station
$\boldsymbol{i}$	task
$\boldsymbol{j}$	task
$\boldsymbol{m}$	maintenance action
$\boldsymbol{d}$	day

3.2 Problem Data

General data:

$\boldsymbol{NU}$	number of train units and consequently of roster rows
$\boldsymbol{K}$	set of train units or roster rows, numbered $1..NU$, indexed by k

Data related to stations:

$\boldsymbol{NS}$	number of stations
$\boldsymbol{S}$	set of stations, numbered $1..NS$, indexed by s
$\boldsymbol{TM_s}$	minimal turning time at station s (parameter)

Data related to tasks:

$\boldsymbol{NT}$	number of real tasks to cover
$\boldsymbol{T}$	set of real tasks to cover, numbered $1..NT$, indexed by i, j
$\boldsymbol{Sd_i}$	departure station of task i (parameter)
$\boldsymbol{Sa_i}$	arrival station of task i (parameter)
$\boldsymbol{Dd_i}$	departure time of task i (parameter)
$\boldsymbol{Da_i}$	arrival time of task i (parameter)
$\boldsymbol{DEM_i}$	required number of units to cover task i (parameter)
$\boldsymbol{CAP_i}$	maximal number of units on task i (parameter)

Data related to dead-headings:

$\boldsymbol{W_{s,s'}}$	pairs of stations s and s' between which there can exist a dead-heading (parameter)
$\boldsymbol{CW_{s,s'}}$	length of a dead-heading from station s to station s' in kilometres (parameter)
$\boldsymbol{DW_{s,s'}}$	duration of a dead-heading from station s to station s' in minutes (parameter)

Data related to maintenance:

$\boldsymbol{NM}$	number of maintenance actions
$\boldsymbol{MM}$	set of maintenance actions, numbered $1..NM$, indexed by m
$\boldsymbol{ND}$	number of days for maintenance
$\boldsymbol{D}$	set of days for maintenance, numbered $1..ND$, indexed by d
$\boldsymbol{KM_{k,m}}$	maintenance actions m that need to be performed on each unit k (parameter)

MT_m duration of maintenance action m in minutes (parameter)

AW_m working load of maintenance action m in minutes (parameter)

LN large number (parameter)

3.3 Data Pre-processing:

BVT set of beginning virtual tasks, numbered $NT + 1..NT + NS$, indexed by i, j

EVT set of ending virtual tasks, numbered $NT + NS + 1..NT + 2 * NS$, indexed by i, j

NTT number of total tasks (real + virtual tasks)

TT set of total tasks, numbered $1..NT + 2 * NS$, indexed by i, j

$R_{i,j}$ processed parameter to identify the set of all pairs of tasks i, j that can be chained up by the same unit

$DdU_{i,j}$ processed parameter for the departure time of a row of a unit

$DaU_{i,j}$ processed parameter for the arrival time of a row of a unit

$\Delta_{k,i,j}$ processed parameter for the turning times homogenization

Virtual tasks, as the name suggests, do not correspond to an actual action. Their function is only to identify the initial and the final stations for each row of a unit. Virtual tasks do not have a demand, a duration nor a capacity. To clarify, real tasks are numbered from 1 to NT and the stations from 1 to NS; and thus, NS beginning virtual tasks are numbered from $NT + 1$ to $NT + NS$ corresponding to each station at the beginning of the time-period. Similarly, NS ending virtual tasks are numbered from $\boldsymbol{NT + NS + 1}$ **to** $\boldsymbol{NT + 2NS}$ corresponding to each station at the end of the time-period. In this model, each unit starts at a station s with a beginning virtual task $\boldsymbol{NT + s}$, executes a sequence of real tasks, and arrives at a station s' with an ending virtual task $\boldsymbol{NT + NS + s'}$.

To build each row of the roster, we first need to identify the set of all pairs of real or virtual tasks $\boldsymbol{i, j}$ possible to chain up by the same unit. For this purpose, the variable $\boldsymbol{R_{i,j}}$ is used. More precisely:

$\forall\ (\boldsymbol{i, j}) \in (\boldsymbol{TT}, \boldsymbol{TT})$,

$$R_{i,j} \begin{cases} = 1 & \text{if the pair of tasks } (i, j) \\ & \text{can bechained up directly} \\ = 0 & \text{otherwise.} \end{cases}$$

The pair of tasks (i, j) can be **chained up directly** by the same unit if stations correspond and, for real tasks, if the turning time between i and j can be respected:

- any pair of real tasks i, j can be chained up if $Sd_j = Sa_i$ and $Dd_j \geq Da_i + TM_{Sa_i}$;
- any real task j can follow a beginning virtual task i, $NT + s$, if $Sd_j = s$;
- any ending virtual task j, $NT + NS + s$, can follow a real task i if $Sa_i = s$.

The parameter $\boldsymbol{W_{s,s'}}$ identifies the **set of all pairs of stations $\boldsymbol{s, s'}$ between which there can exist a dead-heading**. This parameter only presents two values, more precisely, if $W_{s,s'} = 1$ there can exist a dead-heading between s and s', and if $W_{s,s'} = 0$ it is not possible.

The pair of tasks (i, j) can be **chained up** by the same unit **using a dead-heading** if it is possible to insert a dead-heading between the stations that link i and j, and if the duration of the dead-heading respects the turning time between i and j:

- for any pair of real tasks (i, j) it is possible to insert a dead-heading from the arrival station of i to the departure station of j if $W_{Sa_i, Sd_j} = 1$ and $Dd_j \geq Da_i + TM_{Sa_i} + DW_{Sa_i, Sd_j}$;
- for any pair of beginning virtual task i and real task j, $(NT + s, j)$, it is possible to insert a dead-heading from s to the departure station of j if $W_{s, Sd_j} = 1$;
- for any pair of real task i and ending virtual task j, $(i, NT + NS + s)$, it is possible to insert a dead-heading from the arrival station of i to s if $W_{Sa_i, s} = 1$.

The departure and arrival times of a unit have also to be computed. The **departure time of a unit** starting at station s and whose first real task is i is denoted by $DdU_{NT+s,i}$. A unit starts at station s through a beginning virtual task j. Then, it executes a real task i, either directly from station s or from a different station s'. In the latter case, a dead-heading is performed from s to s' with duration $DW_{s,s'}$. Let s' be the departure station of i ($s' = Sd_i$):

- if $s = s'$, then $DdU_{NT+s,i} = Dd_i$ (the unit starts at the same time as task i);
- otherwise, $DdU_{NT+s,i} = Dd_i - DW_{s,s'}$ (the unit starts at the same time as the dead-heading).

Similarly, the **arrival time of a unit** is denoted by $DaU_{i,NT+NS+s'}$. A unit executes a last task i ending at station s. Then, it arrives at station s' through an ending virtual task j, either directly or by performing a dead-heading from s to s' with duration $DW_{s,s'}$. Let s be the arrival station of i ($s = Sa_i$):

- if $s = s'$, then $DaU_{i,NT+NS+s} = Da_i$ (the unit ends at the same time as task i);
- otherwise, $DaU_{i,NT+NS+s} = Da_i - DW_{s,s'}$ (the unit ends at the same time as the dead-heading).

To integrate robustness in the solution, a robustness indicator is used based on the statement that homogeneous turning times bring robustness to a rolling-stock plan. **Turning times homogenization indicator $\boldsymbol{\Delta_{k,i,j}}$** will discourage short turning times, and so**,** absorb potential delays.

As explained before, the turning time between two successive tasks i and j equals $Dd_j - Da_i$. By default, all turning times lower than 1 minute are considered as 1-minute-turning times. Conversely, turning times higher than 60 minutes are not considered.

For a turning time between real tasks i and j **chained up directly** by a unit k:

$$\Delta_{k,i,j}\begin{cases}=\dfrac{1}{\max(1,Dd_j-Da_i)} & if\ Dd_j-Da_i\le 60;\\ =0 & otherwise.\end{cases}$$

For any pair of real tasks i and j **linked by a dead-heading**, there are two turning times: one between i and $W_{s,s'}$, and one between $W_{s,s'}$ and j. By default, $W_{s,s'}$ is placed in the middle, so that both turning times are equal. So, two equal turning times are considered:

$$\Delta_{k,i,j}=\frac{2}{\max\left(1,\dfrac{Dd_j-Da_i-DW_{Sa_i,Sd_j}}{2}\right)}$$

3.4 Decision Variables

$\forall\ k\in K,\ i\in TT,$

$$x_{k,i}\begin{cases}=1 & \text{if unit k covers task i;}\\ =0 & \text{otherwise.}\end{cases}$$

$\forall\ k\in k,\ i\in TT,\ j\in TT,\ (i,j)|R_{i,j}=1,$

$$y_{k,i,j}\begin{cases}=1 & \text{if unit k covers successively tasks i and j;}\\ =0 & \text{otherwise.}\end{cases}$$

$\forall\ k\in K,\ i\in TT,\ \ j\in TT,\ m\ \in MM,\ (i,j)|R_{i,j}=1,\ (k,m)|KM_{k,m}=1,$

$$yM_{k,i,j,m}\begin{cases}=1 & \text{if maintenance action m is performed on unit k, between the pair of tasks (i, j);}\\ =0 & \text{otherwise.}\end{cases}$$

$\forall\ k\in K,\ d\in D,$

$$zM_{k,d}\begin{cases}=1 & \text{if unit k covers any maintenance action on day d; action on day d;}\\ =0 & \text{otherwise.}\end{cases}$$

3.5 Objective function

This MILP model is based on costs of an existing cost-optimal solution, computed to improve robustness. Robustness is considered by optimizing the turning times homogenization robustness indicator. However, the resulting criteria may conflict with operating costs minimization. In practice, it is unacceptable to degrade primary costs, and so, the objective function has to be a trade-off between robustness and secondary costs. It is a weighted sum of three terms related to operating costs, robustness indicator and shuntings for maintenance purpose, as described further on.

Objective function to be minimized:

$$PW*\sum_{k\in K}\sum_{i\in TT|R_{i,j}=1}\sum_{j\in TT|R_{i,j}=1}CW_{Sa_i,Sd_j}*y_{k,i,j}+PTHOM*\sum_{k\in K}\sum_{i\in T|R_{i,j}=1}\sum_{j\in TT|R_{i,j}=1}\Delta_{k,i,j}*y_{k,i,j}+PTZM*\sum_{k\in K}\sum_{d\in D}zM_{k,d} \quad (1)$$

Secondary Costs

The first term of (1) corresponds to the secondary operating costs. Secondary costs are composed of passive trips and dead-headings. Passive trips are usually negligible compared to dead-headings, and therefore, they are not accounted for in the model. In the objective function, costs related to a dead-heading linking two tasks i and j have a specific penalty, in particular its length CW_{Sa_i,Sd_j}, which is the number of kilometers of a dead-heading between station Sa_i and station Sd_j.

Robustness Indicator

The second term of (1) is the value of the robustness indicator based on turning times. As mentioned before, there is a need to homogenize turning times in the roster, so the turning times homogenization indicator $\Delta_{k,i,j}$ is to be minimized.

Shuntings for Maintenance

The last term of (1) takes into account the number of shuntings to the depot needed to be executed, to fulfil the maintenance actions. It is desirable to run shuntings as lower as possible due to two reasons: On one hand, it is a considerable expense to the company. On the other hand, minimizing the number of shuntings leads to the maximization of the availability of the train units, since they cannot run service tasks while parked at the depot.

Weights of the Objective Function

As described above, the objective function is a weighted sum of three terms. We define the following weights:

PW	weight associated with dead-heading in the objective function
$PTHOM$	weight associated with turning times in the objective function
$PTZM$	weight associated with shuntings for maintenance in the objective function

These parameters have to be set according to a trade-off between robustness and costs. Dead-headings generate the most important costs, then the weight PW should be high enough to limit the increase of corresponding costs. Then, the robustness weight should reflect the trade-off between robustness and costs. Shuntings also generate major costs, then $PTZM$ should be high enough to avoid more shuntings to the depot than necessary

3.6 Constraints

To implement the various specifications of the model, the objective function presented in the previous chapter must be subjected to a few constraints.

Existence of a Roster

The existence of a rolling-stock roster of NU units without maintenance requires the verification of the following constraints:

$$\sum_{i \in BVT} x_{k,i} = 1 \quad \forall\ k \in K \tag{1}$$

$$\sum_{j \in TT | R_{i,j}=1} y_{k,j,i} = \sum_{j \in TT | R_{i,j}=1} y_{k,i,j} \quad \forall\ k \in K, i \in T \tag{2}$$

$$\sum_{k \in K} x_{k,i} \geq DEM_i \quad \forall\ i \in T \tag{3}$$

$$\sum_{k \in K} x_{k,i} \leq CAP_i \quad \forall\ i \in T \tag{4}$$

$$x_{k,i} = \sum_{j \in TT | R_{i,j}=1} y_{k,i,j} \quad \forall\ k \in K, i \in T \cup BVT \tag{5}$$

$$x_{k,i} = \sum_{j \in TT | R_{i,j}=1} y_{k,j,i} \quad \forall\ k \in K, i \in EVT \tag{6}$$

Maintenance

Regarding the optimal technical planning that is used as an input to this model and the related maintenance actions that need to be inserted in the pairs of service tasks, the following constraints were formulated to encompass the planned maintenance actions:

$$yM_{k,i,j,m} \leq y_{k,i,j} \quad \forall\ k \in K,\ i \in TT,\ j \in TT,\ m \in MM \mid R_{i,j} = 1 \wedge KM_{k,m} = 1 \tag{7}$$

$$yM_{k,i,j,m} * \left(Dd_j - Da_i - DW_{Sa_i,depot} - DW_{depot,Sd_j}\right) \geq \sum_{m1 \in MM} yM_{k,i,j,m1} * MT_{m1} + 5 * \left(\left(\sum_{m1 \in MM} yM_{k,i,j,m1}\right) - 1\right) \quad \forall\ k \in K,\ i \in TT,\ j \in TT,\ m \in MM \mid KM_{k,m} = 1 \wedge R_{i,j} = 1 \tag{8}$$

$$\sum_{i \in T} \sum_{j \in T} \sum_{d \in D} yM_{k,i,j,m} = KM_{k,m} \mid R_{i,j} = 1 \wedge Da_i + DW_{Sa_i,depot} \geq 9 * 60 + (d-1) * 24 * 60 \wedge Dd_j - DW_{9depot,Sd_j} \leq 18 * 60 + (d-1) * 24 * 60 \quad \forall\ k \in K,\ m \in MM \mid KM_{k,m} = 1 \tag{9}$$

$$\sum_{i \in TT} \sum_{j \in TT} yM_{k,i,j,m} = 0 \mid i > NT \vee j > NT \quad \forall\ k \in K,\ m \in MM \mid KM_{k,m} = 1 \tag{10}$$

$$\sum_{k \in K} \sum_{m \in MM} AW_m * yM_{k,i,j,m} \leq 5 * 8 * 60 \mid KM_{k,m} = 1 \quad \forall\ i \in T,\ j \in T,\ d \in D \mid R_{i,j} = 1 \wedge Da_i + DW_{Sa_i,depot} \geq 9 * 60 + (d-1) * 24 * 60 \wedge Dd_j - DW_{9depot,Sd_j} \leq 18 * 60 + (d-1) * 24 * 60 \tag{11}$$

$$\sum_{m \in MM} \sum_{i \in T} \sum_{j \in T} yM_{k,i,j,m} \leq zM_{k,d} * LN \mid Da_i + DW_{Sa_i,depot} \leq d * 24 * 60 \wedge Da_i + DW_{Sa_i,depot} \geq (d-1) * 24 * 60 \wedge Dd_j - DW_{depot,Sd_j} \leq d * 24 * 60 \wedge Dd_j - DW_{depot,Sd_j} \geq (d-1) * 24 * 60 \quad \forall\ k \in K,\ d \in D \tag{12}$$

$$yM_{k,i,j,m} = 0 \quad \forall\ k \in K,\ i \in TT,\ j \in TT,\ m \in MM \mid R_{i,j} = 0 \vee KM_{k,m} = 0 \tag{13}$$

Decision Variables

$$x_{k,i} \in \{0,1\} \quad \forall\, k \in K,\ i \in TT \tag{14}$$

$$y_{k,i,j} \in \{0,1\} \quad \forall\, k \in K,\ (i,j) | R_{i,j} = 1 \tag{15}$$

$$yM_{k,i,j,m} \in \{0,1\} \quad \forall\, k \in K,\ i \in TT,\ j \in TT,\ m \in MM \tag{16}$$

$$zM_{k,d} \in \{0,1\} \quad \forall\, k \in K, d \in D \tag{17}$$

Constraints (2) guarantee that any unit starts with a beginning virtual task. Constraints (3) assure spatio-temporal coherence. A unit assigned to a task i, which arrives at station Sa_i, can either be assigned to a next task j, whose departure station $Sd_j = Sa_i$, or it can stay at station Sa_i. In the latter case, its next task will be an ending virtual task. This is modelled by the following formulation: for any real task i and any unit k, if there exists a task $j1$ so that unit k chains up $j1$ and i, then there exists a task $j2$ so that a unit k chains up i and $j2$. According to constraints (4), a real task i must be covered by at least DEM_i units. Constraints (5) assure that at most CAP_i units cover i. Constraints (6) express variables $x_{k,i}$ depending on the variables $y_{k,i,j}$ for any real or beginning virtual task i. Ending virtual tasks do not have successors. Then, constraints (7) define variables $x_{k,i}$ for each ending virtual task i. Constraints (8) guarantee coherence between each pair of tasks that is performed and the associated maintenance actions. In other words, a unit k covering a maintenance action m between the pair of tasks (i,j) also covers (i,j). Constraints (9) express that for a train unit k, the amount of time spent on the various (or single) maintenance actions $m1$, which are performed between the pair of tasks (i,j), cannot exceed the amount of time indeed available for those maintenance actions. The time spent on dead headings to the depot is accounted for. It is assumed that only one maintenance action can be performed at a time on the same unit and a 5-minutes interval of change between two consecutive maintenance actions. Constraints (10) assure that a maintenance action m associated with a train unit k will only be performed, if it was previously introduced in the technical plan, and also forces a maintenance action that is in the plan to be realized. Constraints (11) forbid a maintenance action to occur after a beginning virtual task or before an ending virtual task. Otherwise, the purpose of the virtual tasks would not be respected. Constraints (12) ensure that the sum of working loads AW_m related to all maintenance actions to be performed on a given day does not exceed the maximum working load available for one day of work: 5 men working 8 hours per day. Furthermore, it forces units to arrive and leave the depot within the operating hours of the workers (between 9:00 and 16:00). The goal is to maximize the availability of units. A unit parked in the depot without benefitting from any maintenance action implies a reduction of the resources available. Constraints (13) assure that if there is a maintenance action on a given day d and a given unit k, the variable $zM_{k,d}$, relative to a specific unit and day cannot be zero. In other words, it assures a coherence between the variables $yM_{k,i,j,m}$ and $zM_{k,d}$. Constraints (14) guarantee that if two tasks i and j cannot be chained or if a maintenance action m associated with a train unit k was not previously introduced in the technical plan, the variable $yM_{k,i,j,m}$ must be zero. The variables relative to constraints (14), (15), (16) and (17) are all binary variables.

4. Case study of Fertagus

In this section the Fertagus railway operating company is briefly described and the case study specifications are presented.

4.1 Fertagus, a train operating company

Fertagus is a Portuguese train operating company. Fertagus' concession includes the operation of the railway line, the safety and maintenance of the trains and the maintenance of some railway stations. There are always less then seventeen trains in service, and rotatively, one train is in the PMC (depot).

4.2 Specific input parameters

During this work, input data for the model was collected. In the current case study, the 17 train units that are available for operational services are intended to cover a set of tasks and preventive maintenance actions for the 5 working days of a week. Although the research work by Méchain (2017) outputs a technical plan for 16 kinds of maintenance actions, only 14 are considered in the present research. The reason is that 2 of the 16 maintenance actions are meant to be performed during the weekend in specific periods of the year. The final goal is to find the best feasible solution that outputs a rolling-stock plan to a given week. Week 28 was chosen out of the 52 weeks of the year to perform this study.Tables 1 to 8 provide values for the parameters used relative to the case study.

Table 1 - Information concerning stations

Station Name	Station Number, s	Minimal Turning Time, TM_s (min)
Roma-Areeiro	1	1
Entrecampos	2	1
Sete-Rios	3	1
Campolide	4	1
Pragal	5	1
Corroios	6	1
Foros de Amora	7	1
Fogueteiro	8	1
PMC (depot)	9	1
Coina	10	1
Penalva	11	1
Pinhal-Novo	12	1
Venda do Alcaide	13	1
Palmela	14	1
Setúbal	15	1

In Table 1, the stations name, their corresponding number and their associated minimal turning time (in minutes) are given.

Table 2 - Constants

Constant	Unit	Value
NU	---	17
NS	---	15
NT	---	790
ND	day	5
NM	---	14
LN	---	10000
PW	---	850
$PTHOM$	---	300
$PTZM$	---	850

In Table 2, all the constants are shown.

Table 3 - Pairs of stations between which there can exist dead-endings

$W_{s,s'}$		s'														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	0	0	0	0	1	0	0	1	1	1	0	0	0	0	1
	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
s	8	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1
	9	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1
	10	1	0	0	0	0	0	0	1	1	0	0	0	0	0	1
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	15	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0

In Table 3, s and s' are respectively the departure and arrival stations of a possible dead-heading. If the value of W(s,s') equals zero, a dead-heading between stations s and s' is not possible. Otherwise, its value would be equal to one.

Table 4 - Lenght of dead-headings

$CW_{s,s'}$ (km)		s'														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	0	1.13	2.84	4.04	11.68	16.78	19.38	22.12	25.6	27.32	32.62	41.42	45.23	49.33	54.16
	2	1.13	0	1.71	2.91	10.55	15.65	18.25	20.99	0	26.19	31.49	40.29	44.1	48.2	53.03
	3	2.84	1.71	0	1.2	8.84	13.95	16.55	19.28	0	24.49	29.79	38.59	42.39	46.49	51.32
	4	4.04	2.91	1.2	0	7.65	12.75	15.35	18.09	0	23.29	28.59	37.39	41.19	45.29	50.12
	5	11.68	10.55	8.84	7.65	0	5.1	7.7	10.44	0	15.64	20.94	29.74	33.54	37.64	42.47
	6	16.78	15.65	13.95	12.75	5.1	0	2.6	5.34	0	10.54	15.84	24.64	28.44	32.54	37.37
	7	19.38	18.25	16.55	15.35	7.7	2.6	0	2.74	0	7.94	13.24	22.04	25.84	29.94	34.77
s	8	22.12	20.99	19.28	18.09	10.44	5.34	2.74	0	0	5.2	10.5	19.3	23.1	27.2	32.03
	9	25.6	0	0	0	0	0	0	0	0	1.7	0	0	0	0	28.6
	10	27.32	26.19	24.49	23.29	15.64	10.54	7.94	5.2	1.7	0	5.3	14.1	17.9	22	26.83
	11	32.62	31.49	29.79	28.59	20.94	15.84	13.24	10.5	0	5.3	0	8.8	12.6	16.7	21.53
	12	41.42	40.29	38.59	37.39	29.74	24.64	22.04	19.3	0	14.1	8.8	0	3.8	7.9	12.73
	13	45.23	44.1	42.39	41.19	33.54	28.44	25.84	23.1	0	17.9	12.6	3.8	0	4.1	8.93
	14	49.33	48.2	46.49	45.29	37.64	32.54	29.94	27.2	0	22	16.7	7.9	4.1	0	4.83
	15	54.16	53.03	51.32	50.12	42.47	37.37	34.77	32.03	28.6	26.83	21.53	12.73	8.93	4.83	0

Table 4 shows the distance in kilometres between stations s and s'.

Table 5 - Duration of dead-headings

$DW_{s,s'}$ (min)		s'														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	1	0	0	0	0	16	0	0	22	24	26	0	0	0	0	45
	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
s	8	22	0	0	0	0	0	0	0	0	5	0	0	0	0	21
	9	24	0	0	0	0	0	0	0	0	2	0	0	0	0	21
	10	26	0	0	0	0	0	0	5	2	0	0	0	0	0	17
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	15	45	0	0	0	0	0	0	21	21	17	0	0	0	0	0

Table 5 shows the duration of a dead-heading between stations s and s' (in minutes).

Table 6 - Information about tasks

Task (T_i)	DEM_i	CAP_i	Sd_i	Sa_i	Dd_i (min)	Da_i (min)
1	1	2	1	15	343	401
2	2	2	15	1	418	476
3	2	2	1	10	483	516
4	2	2	10	1	523	556
5	2	2	1	10	563	596
6	2	2	10	1	1053	1086
7	2	2	1	15	1093	1151
8	1	2	15	1	1168	1226
9	1	2	1	10	1233	1266
10	1	2	10	1	1283	1316
11	1	2	1	10	1333	1366
12	1	2	15	15	1152	1258
13	1	2	15	1	388	446
14	1	2	1	10	453	486
15	1	2	15	1	508	566
16	1	2	1	10	573	606
17	2	2	10	1	963	996
18	2	2	1	15	1003	1061
19	2	2	15	1	1078	1133
20	2	2	1	10	1143	1176
(...)						
817	0	0	12	12	0	0
818	0	0	13	13	0	0
819	0	0	14	14	0	0
820	0	0	15	15	0	0

Table 6, is as small excerpt of the table used to gather all information concerning all tasks of the study. The complete table is not presented, because of its extensive size. This table identifies the various tasks on the first column. The next columns give the required number of units, the maximal number of units, the departure station, the arrival station, the departure time and arrival time of a task. Tasks 1 to 790 are real tasks. The last 30 tasks are virtual tasks.

Table 7- Maintenance actions that need to be performed on week 28

$KM_{k,m}$		m													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
k	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	1	0	0	0	0	0	0	0	0	0	0	1	1	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	1	0	0	0	0	0	0	0	0	0	0	0	0	1
	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0

In Table 7, k and m are respectively the train units and maintenance actions. $KM_{k,m}$ equals one when a maintenance action must be performed on a specific unit.

Table 8 - Duration and working load of maintenance actions

Maintenance Action, m	MT_m (min)	AW_m (min)
1	150	744
2	420	1680
3	210	840
4	210	840
5	276	840
6	186	744
7	186	744
8	186	744
9	186	744
10	186	744
11	420	840
12	53	210
13	53	210
14	60	60

InTable 8, the first column identifies the different kinds of maintenance actions. The second column exhibits their duration and the third column their working load. Since the working load and the duration have a known relation $(duration = \frac{working\ load}{working\ persons})$, the number of workers needed for on each maintence task can be calculated. The maximum number of working persons available for preventive maintenance in Fertagus is 5.

5. Results

Through this section, the results for the case study exposed in the previous section are displayed, and analysed. The present study can be divided into two problems: i) the construction of a robust rolling-stock plan and ii) the inclusion of preventive maintenance actions into that plan.

5.1. Results Comparison with Fertagus' Plan

In order to compare the results of the model with the current plan used by Fertagus, it was decided to run the model without the maintenance actions first. Only the row of units 1 and 2 were chosen to perform a proper analysis and comparison of results. 2 dead-heading were introduced to fulfil all tasks of unit 2. For unit 1, no dead-headings were needed.

In, the first row of information Figure 1 gives the tasks that are covered by unit 1. In total 17 service

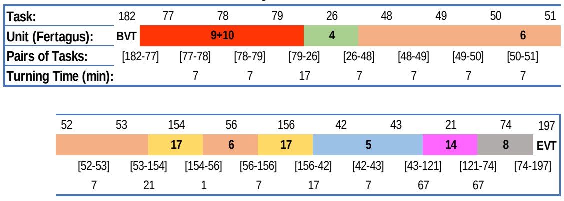

Figure 1- Row of unit 1, constructed with the information from ***Error! Reference source not found.*** *and* ***Error! Reference so***

tasks were covered. The second row of information shows the units that cover the same tasks as unit 1, in Fertagus' plan. As showed, the tasks now assigned to unit 1 are covered by six different units. Although only one row is compared, it is enough to clarify that the results differ a lot from Fertagus' plan. One reason is that the model avoids the use of dead-heading for a more economical solution. The model uses almost the same number of dead-headings as Fertagus', 22 and 23, respectively. However, the total distance covered by all units is 64% less, which has a big impact on costs. Another reason is the concern of the model for a robust solution.

5.2. Analysis of the Total Cost as Function of the Weight associated with Turning Times

A sensitivity analysis of the weight associated with turning times in the objective function ($PTHOM$) was carried out. the value of the weight associated with dead-heading (PW) was fixed in 1500 and $PTHOM$ varied from 12 to 7500, as 9 shows.

*Table 9 - Relation between the variation of the weight associated with turning times (**$PTHOM$**) and the variation of total cost of the objective function*

$PTHOM$	$PTHOM$ variation (%)	Total Cost	Total Cost variation (%)
12	-96	196914	-3.9
36	-88	197596	-3.6
60	-80	198268	-3.3
150	-50	200808	-2.0
300	---	204961	---
600	+100	213243	+ 4.0
1500	+400	238086	+16.2
7500	+2400	403712	+97.0

From the relation between $PTHOM$'s variation in percentage and the total cost variation in percentage, it clear that the variation of total cost is damped with respect to the turning times variation. It means that the avoidance of the dead-headings is prioritized to the homogenization of turning times. This is the desired result, which guarantees that turning times do not deteriorate the secondary costs.

5.3 Results Comparison with Fertagus' Plan

A version of the model only relative to the construction of the robust rolling-stock plan was run for the 5 working days of the week, to analyse its costs, size and computational performance.

The minimum cost is 372 319. As expected, the term relative to secondary costs has by far the biggest impact, contributing to 94% of the total cost. The robustness indicator term represents 6% of the total cost. It took about 52 minutes (3125.3 seconds) for the model run to be completed. The gap presented is considered as null, because its value is negligible (7.6 x 10^{-11} %), having no impact on the results. It means that the best bound for the objective function that was found corresponds to the best solution. The matrix has 5 273 315 variables. In fact, this computational time is quite acceptable, given the number of variables of the problem. It is also due to the data pre-processing, which compares all potential pairs of tasks and eliminates the pairs that do not present the requirements to be linked. It reduces considerably the size of the problem, making the model quite robust on a computational perspective.

5.4. Inclusion of the Preventive Maintenance Actions – 1 week

Several attempts were carried out to run the model for the time period of 5 working days, however the size of the problem becomes too large and the computational capacity available is not enough to run the analysis. 159 159 015 variables have to be processed, about 30 times more than the number of variables of the problem without maintenance actions. In fact, the problem is the decision variable $yM_{k,i,j,m}$, which is responsible for 93,3% of the number of variables. Computer gets out of memory. As an alternative, it was chosen to present the results for a 3-day plan, reducing in this way the size of the problem (but not its complexity). The rosters obtained for the units that must perform maintenance actions were analysed and unit 2 performs all its maintenance actions on Tuesday (day 2) and both unit 13 and 14 perform their maintenance actions on Monday (day 1).
It took about 20 minutes for the model run to be completed, with a total of 62 363 412 variables. Once again, the solution presented is the optimal solution. The best solution is a roster with a cost of 223 991. As usual, the term relative to secondary costs has by far the biggest impact, contributing to 93.7% of the total cost. The robustness indicator term represents 6% of the total cost and the shuntings for maintenance 0.3%.

5.5. Analysis of the weights associated with Secondary costs and Shuntings for Maintenance

Since the last term of the objective function related to the shuntings for maintenance is an actual contribution of this research, a sensitivity analysis was performed relative to the relation between the weights of the secondary costs (PW) and the shuntings for maintenance ($PTZM$). The methodology followed consisted of keeping the sum of the values of all weights fixed, as well as the weight of the robustness indicator ($PTHOM$), and changing only the values of PW and $PTZM$. Four sets of values were tested (scenarios I, II, III and IV):

Table 10 – Sets of tested values for the analysis of $\boldsymbol{PW}$ *and* $\boldsymbol{PTZM}$

	I	II	III	IV
$\boldsymbol{PW}$	1500	850	200	30
$\boldsymbol{PTHOM}$	300	300	300	300
$\boldsymbol{PTZM}$	200	850	1500	1670
Sum	**2000**	**2000**	**2000**	**2000**

The results from the tests are presented in Table 11 and Figure 2 and refer to values of each cost component of the objective function.

Table 11 – Values of the different cost components of the objective function along the four tests

	I	II	III	IV
Secondary Costs	210240	119136	28032	4205
Robustness Indicator	13151	13151	13151	13151
Shuntings for Maintenance	600	2550	4500	5010
Total Cost	223991	134837	45683	22366

First, the value of the robustness indicator cost remained unchanged along the tests, which indicates that the solutions found are the same. Moreover, the growth of the cost relative to shuntings for maintenance increased proportionally to the variation of its weight, just like the value relative to the secondary costs that decreased along the tests. These relations are linear. Furthermore, the value of total cost follows the reduction of the secondary costs. The results are in line to what was expected, i.e. the secondary costs are not deteriorated with the introduction of robustness and the secondary costs and shuntings for maintenance are lined up to the same solution. Altogether, it means that the solution of the model was not affected by the change of the weights, which indicates that the optimal solution is robust to different preferences of the decision maker between the three components of the objective function.

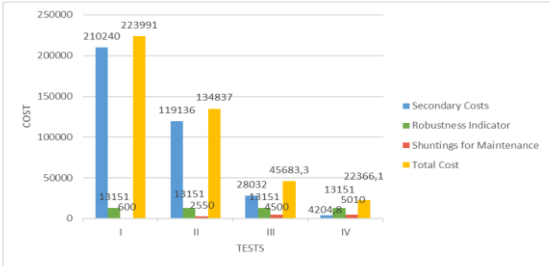

Figure 2 – Graph concerning the variation of costs components along the tested solutions.

6. Conclusion

6.1 Contributions

The goal of the present dissertation was to create a mathematical model that would provide an optimal robust rolling-stock plan, capable of including the maintenance tasks from a technical plan relative to the different weeks of the year. Moreover, at the same time reduce the overall costs of the operation to a minimum. The mathematical model developed is successfully adapted to the specific case of Fertagus railway company and it is flexible enough to be modified and fit to the conditions of a different train operating company. The proposed model is an operational model with a planning horizon of a week and a time step of a minute.

The results showed that the optimal solution for rolling stock plan, without maintenance actions, provides a reduction of total deadheading distance of around 64%, which represents an important research opportunity to test whether or not current train schedules of Fertagus train operating company are suboptimal according to the preferences of the decision maker. Such a reduction in the total deadheading distance may collide with slots made available to other train operators by the infrastructure manager, but the assessment of such impacts is outside the scope of the present research.
In fact, the increase in the number of decision variables, because of the inclusion of the maintenance, makes a 5-day schedule impossible to run for the computer used (out of memory). Only a 3-day schedule was possible to run, and results were presented for that case. Moreover, sensitivity analysis on the weights of the different components of the objective function showed that the optimal solution found is not sensitive to significant variations of the weights.
The mathematical model is considered to answer its purpose and solve to optimality several real instances.

6.1 Limitations

This mathematical model enables to find an optimized rolling stock roster, but the size of the problem is limited by the computational capacity. A way to work around this problem could be to add even more restrictions in the phase of data pre-processing and implement them through successive steps. They would work as filters, progressively reducing the size of the problem before the phase of minimization of the objective function and computations of its constraints. This method requires that possible solutions of the problem (but of no interest) are excluded before the start of the actual optimization process and associated linear programming relaxation.
Moreover, having in mind that Fertagus has a small fleet (with only 18 train units), the adaptation of the present model to larger fleets would require the use of metaheuristics to be able to reduce the computational time. Finally, the interface relative to the model could be more user friendly and accordingly an automatically generated chart could be implemented for the presentation of the obtained rosters. The analysis of the roster would be much faster and facilitated.

6.1 Future Research

A considerable improvement to this research would be the proposal of a method to promote a balanced usage of Fertagus fleet and consequently a balanced wear. The usage of the units could be measured by the travelled kilometres and a new term introduced in the objective function, concerning this number, but they are not the best indicator of the usage of a unit. A good method to guarantee an equal wear is to assign similar services to all units. With the cyclic rooster proposed by Tréfond et al (2017), after a specific number of cycles, all units were subjected to the same services. However, it does not fit the actual case study, due to a non-repeatability of the maintenance tasks scheduled along the time. It would be wise to align the repeatability of maintenance tasks with the repeatability of service tasks along a time period of interest for the company.
Regarding maintenance planning, it should be pointed that the costs of preventive maintenance only represent about a half of the costs of corrective maintenance. Corrective maintenance actions cannot be scheduled for an entire year, like preventive maintenance ones. Nevertheless, it would make sense to include corrective maintenance slots in the rolling-stock planning problem, at least for the time period of a week, and inform the decision maker on the robustness of such a plan with the corrective maintenance slots.
Comprehensive crew scheduling that takes into account the different skill of maintenance technicians and their experience is still missing in the current version of the model, and further research should include it. Finally, an intermediate model is missing that links the 1-day operational planning model proposed in this dissertation and the annual tactical plan proposed in Méchain (2017). Such intermediate model might allocate the weekly maintenance tasks into the different days using some criterion, minimizing or maximizing a certain objective function.

7. References

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