Entanglement-induced deviation from the geodesic motion in quantum gravity:

Gravity-matter entanglement and the weak equivalence principle

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Abstract

We study the derivation of the effective equation of motion for a pointlike particle in the framework of quantum gravity. Just like the geodesic motion of a classical particle is a consequence of classical field theory coupled to general relativity, we introduce the similar notion of an effective equation of motion, but starting from an abstract quantum gravity description. In the presence of entanglement between gravity and matter, quantum effects give rise to modifications of the geodesic trajectory, primarily as a consequence of the nonzero overlap between various coherent states of the gravity-matter system. Finally, we discuss the status of the weak equivalence principle in quantum gravity and its possible violation due to the nongeodesic motion. This article summarizes the results presented in [9].

1 Introduction

The formulation of the theory of quantum gravity (QG) is one of the most fundamental open problems in modern theoretical physics. In models of QG, as in any quantum theory, superpositions of states are allowed. In a tentative "theory of everything", which includes both gravity and matter at a fundamental quantum level, superpositions of product gravity-matter states are particularly interesting. Entangled states are highly nonclassical, and as such are especially relevant because they give rise to a drastically different behavior of matter from what one would expect based on classical intuition, as confirmed by numerous examples from the standard quantum mechanics (QM). Therefore, it is interesting to study such states in the context of a QG coupled to matter, in particular the Schrödinger cat-like states. Moreover, a recent study [8] suggests that physically allowed states of a gravity-matter system are generically entangled due to gauge invariance, providing additional motivation for our study.

In standard QM, entanglement is generically a consequence of the interaction. Nevertheless, there exist situations which give rise to entanglement even without interaction. For example, the Pauli exclusion principle in the case of identical particles generates entanglement without an interaction, giving rise to an effective force (also called the "exchange interaction"). We investigate in detail whether an entanglement between gravity and matter could also be described as a certain type of an effective interaction, and if so, what are its aspects and details. In order to study this problem, we analyze the motion of a free test particle in a gravitational field. In general relativity (GR), this motion is described by a geodesic trajectory. However, we show that in the presence of the gravity-matter entanglement, the resulting effective interaction causes a deviation from a classical geodesic trajectory. In particular, we generalize the standard derivation of a geodesic equation from the case of classical gravity to the case of a full QG model, and derive the equation of motion for a particle which contains a non-geodesic term, reflecting the presence of the entanglement-induced effective interaction. The effects we discuss are purely quantum with respect to both gravity and matter, unlike previous studies of quantum matter in classical curved spacetime [1,2,10,13]. As a consequence of the modified equation of motion for a particle, we also discuss the status of the equivalence principle in the context of QG, and a possible violation of its weak flavor.

The paper is organized as follows. In section 2 the multipole formalism is employed and a modified geodesic equation for a particle is derived from the covariant conservation of the entangled stress-energy tensor. Subsection 2.1 contains the general setup, the abstract quantum gravity framework that will be used, and the main assumptions. In subsection 2.2 we discuss the effective covariant conservation equation, which receives a correction to the classical one, due to the quantum gravity effects. In subsection 2.3 we put everything together and derive our main result — the effective equation of motion for a point particle, with the leading quantum correction. In subsection 2.4 we discuss the consistency of the assumptions that enter the approximation scheme used to derive the effective equation of motion. Section 3 is devoted to the discussion of the consequences of our results in the context of the weak equivalence principle. For the purpose of clarity, in subsection 3.1 we first provide the definitions of various flavors of the equivalence principle. Then, in subsection 3.2 we discuss the status of the equivalence principle in the context of quantum gravity and the results obtained in section 2. Subsection 3.3 discusses the universality and equality between inertial and gravitational masses, in the context of the Newtonian approximation. Finally, section 4 contains our conclusions, discussion of the results and possible lines of further research.

Our notation and conventions are as follows. We will work in the natural system of units in which $c = \hbar = 1$ and $G = l_p^2$, where l_p is the Planck length and G is the Newton's gravitational constant. By convention, the metric of spacetime will have the spacelike Lorentz signature $(-,+,+,+)$. The spacetime indices are denoted with lowercase Greek letters $\mu, \nu, \dots$ and take the values $0,1,2,3$. These can be split into the timelike index 0 and the spacelike indices denoted with lowercase Latin letters $i, j, k, \dots$ which take the values $1,2,3$. The Lorentz-invariant metric tensor is denoted as $\eta_{\mu\nu}$. Quantum operators always carry a hat, $\hat{\phi}(x)$, $\hat{g}(x)$, etc. The parentheses around indices indicate symmetrization with respect to those indices, while brackets indicate antisymmetrization:

$$A_{(\mu\nu)} \equiv \frac{1}{2}\left(A_{\mu\nu} + A_{\nu\mu}\right) , \qquad A_{[\mu\nu]} \equiv \frac{1}{2}\left(A_{\mu\nu} - A_{\nu\mu}\right) .$$

Finally, we will systematically denote the values of functions with parentheses, $f(x)$, while functionals will be denoted with brackets, $F[\phi]$.

2 Geodesic equation in quantum gravity

In this section we discuss the motion of a particle within the framework of quantum gravity. The exposition is structured into four parts — first, we introduce the abstract quantum gravity formalism, and give some technical details about the description of the states. In the second part, we discuss the quantum version of the covariant conservation equation of the stress-energy tensor. In the third part we obtain within the quantum formalism, the effective equation of motion for the particle. Finally, in the fourth part we discuss the self-consistency assumptions that go into the calculation.

2.1 Preliminaries and the setup

We work in the so-called generic abstract quantum gravity setup, as follows. Starting from the Heisenberg picture for the description of quantum systems, we assume that gravitational degrees of freedom are described by some gravitational field operators $\hat{g}(x)$, while matter degrees of freedom are described by matter field operators $\hat{\phi}(x)$, where x represents the coordinates of some point on a 4-dimensional spacetime manifold $\mathcal{M}_4$. Both sets of operators have their corresponding canonically conjugate momentum operators, $\hat{\pi}_g(x)$ and $\hat{\pi}_\phi(x)$, such that the usual canonical commutation relations hold. The total (kinematical) Hilbert space of the theory is $\mathcal{H}_{\text{kin}} = \mathcal{H}_G \otimes \mathcal{H}_M$, where the gravitational and matter Hilbert spaces $\mathcal{H}_G$ and $\mathcal{H}_M$ are spanned by the bases of eigenvectors for the operators $\hat{g}$ and $\hat{\phi}$, respectively. The total state of the system, $|\Psi\rangle \in \mathcal{H}_{\text{kin}}$, does not depend on x, in line with the Heisenberg picture framework.

There are three points that are worth emphasizing about this setup. First, since we aim to present the analysis of geodesic motion which is model-independent, we refrain from specifying what are the fundamental degrees of freedom $\hat{g}$. Instead, we merely assume that the expected values of the operators describing the effective spacetime geometry, i.e., the metric, connection, curvature, etc., depend somehow on $g = \langle\Psi|\hat{g}|\Psi\rangle$ and $\pi = \langle\Psi|\hat{\pi}_g|\Psi\rangle$, and are expressible as operator functions in terms of them:

$$\langle\Psi|\hat{g}_{\mu\nu}|\Psi\rangle = g_{\mu\nu}(g,\pi_g)\,, \langle\Psi|\hat{\Gamma}^\lambda{}_{\mu\nu}|\Psi\rangle = \Gamma^\lambda{}_{\mu\nu}(g,\pi_g)\,,$$
$$\langle\Psi|\hat{R}^\lambda_{\mu\nu\rho}|\Psi\rangle = R^\lambda_{\mu\nu\rho}(g,\pi_g)\,,\dots$$

When discussing these geometric operators, for simplicity we will usually not explicitly write their (g,π_g)-dependence.

Second, in order for any operator function to be well defined, some operator ordering has to be assumed. However, since we aim to work in an abstract model-independent QG formalism, we do not choose any particular ordering, but merely assume that one such ordering has been fixed. In a similar fashion, we also simply assume that all operators and spaces are well defined, convergent, and otherwise specified in enough mathematical detail to have a well defined and unique QG model.

Third, we employ a natural distinction between gravitational and matter degrees of freedom, assuming that the separation between gravity and matter present in the classical theory, described by action of the form

$$S_{\text{total}}[g,\phi] = S_{\text{gravity}}[g] + S_{\text{matter}}[g,\phi] \,,$$

remains present also in the full quantum regime. After the introduction of the above conceptual setup, we turn to some more practical details. For the purpose of discussing geodesic motion, we are mostly interested in the classical theory of the abstract QG introduced above. To that end, the main objects of attention are *coherent states* of gravity and matter. Denoting them as $|g\rangle \in \mathcal{H}_G$ and $|\phi\rangle \in \mathcal{H}_M$, respectively, by definition they are assumed to saturate Heisenberg inequalities for the gravitational and matter field operators,

$$\Delta\hat{g}\Delta\hat{\pi}_g \approx \frac{\hbar}{2} \,, \qquad \Delta\hat{\phi}\Delta\hat{\pi}_\phi \approx \frac{\hbar}{2} \,.$$

Moreover, we want these coherent states to be *classical* in the sense that all four uncertainties $\Delta\hat{g}$, $\Delta\hat{\pi}_g$, $\Delta\hat{\phi}$ and $\Delta\hat{\pi}_\phi$, are simultaneously minimal, compared to some scale. In other words, we do not want $|g\rangle$ and $|\phi\rangle$ to be just arbitrary squeezed states, but rather only the squeezed states which are also coherent. Working with coherent states is convenient for the analysis of the classical theory because then the notion of a "point in a phase space" of the theory has minimal uncertainty, allowing one to reconstruct the classical theory from the full quantum description.

Given a full coherent state vector as

$$|\Psi\rangle = |g\rangle \otimes |\phi\rangle \,, \tag{1}$$

we wish to evaluate and study the "effective classical" values for the metric tensor and the matter stress-energy tensor as expectation values of the corresponding operators:

$$g_{\mu\nu} = \langle\Psi|\hat{g}_{\mu\nu}|\Psi\rangle \,, \qquad T_{\mu\nu} = \langle\Psi|\hat{T}_{\mu\nu}|\Psi\rangle \,. \tag{2}$$

However, in quantum gravity, one should not discuss only states of the form (1), but also linear combinations of such states. In particular, an arbitrary state $|\mathbf{\Psi}\rangle \in \mathcal{H}_{\text{kin}}$ can be written as a linear combination of coherent states of the form

$$|\mathbf{\Psi}\rangle = \sum_{i,j} c_{ij} |g_i\rangle \otimes |\phi_j\rangle \,, \tag{3}$$

where indices i and j count all possible coherent states in the theory. The coefficients c_{ij} are in general not unique, since coherent states typically form an overcomplete basis of the Hilbert space. However, while the state (3) is the most general state, it is also quite cumbersome to work with. Therefore, for the purpose of our paper, we will restrict to a *toy example state*, defined by having only two terms in the sum (3), as

$$|\mathbf{\Psi}\rangle = \alpha|\Psi\rangle + \beta|\tilde{\Psi}\rangle \,, \tag{4}$$

where $|\tilde{\Psi}\rangle \equiv |\tilde{g}\rangle \otimes |\tilde{\phi}\rangle$ is some other coherent state analogous to (1) but giving different expectation values for the classical metric and stress-energy tensors:

$$\tilde{g}_{\mu\nu} = \langle\tilde{\Psi}|\hat{g}_{\mu\nu}|\tilde{\Psi}\rangle \,, \qquad \tilde{T}_{\mu\nu} = \langle\tilde{\Psi}|\hat{T}_{\mu\nu}|\tilde{\Psi}\rangle \,. \tag{5}$$

One can see that our toy-example state (4) is a Schrödinger-cat type of state, describing a coherent superposition of two classical configurations of gravitational and matter fields.

In addition to discussing superpositions in general, one also needs to address the issue of gauge invariance. Namely, assuming that a theory of quantum gravity ought to obey local Poincaré invariance, which is a gauge symmetry, the proper Hilbert space of the theory cannot be the full space $\mathcal{H}_{\text{kin}}$, since it contains state vectors which may fail to be gauge invariant. The *physical* Hilbert space $\mathcal{H}_{\text{phys}}$ of gauge invariant states is therefore a proper subset of $\mathcal{H}_{\text{kin}}$, specified by the Gupta-Bleuler-like conditions which enforce gauge invariance at the quantum level. Therefore, one cannot simply assume that the coherent state (1) is gauge invariant. In fact, it was argued in [8] that $\mathcal{H}_{\text{phys}}$ actually does not contain any separable states of the form (1), rendering the particular choice (1) non-invariant and thus unphysical. This represents an additional argument to study states of type (3) and, as a particularly convenient toy example, state given by (4).

Before continuing, it is important to emphasize two points. First, we are assuming that the state (4) is gauge invariant, i.e., an element of the physical Hilbert space $\mathcal{H}_{\text{phys}}$. This assumption is benign, in the sense that all main results of the paper will continue to hold qualitatively even for the more general state of type (3), as long as it is chosen to belong to $\mathcal{H}_{\text{phys}}$. It will become evident later on that qualitative conclusions of the paper do not depend on the fact that (4) has precisely two terms in the sum. Choosing the state with three, four or more terms will lead to analogous conclusions, although quantitative details of the computation may become technically more involved.

Second, given that (4) is a Schrödinger-cat type of state, there are some phenomenological restrictions on the values of the coefficients α and β. Namely, in the ordinary experimental situations we basically never observe these kind of states, which means that the overall entangled state $|\mathbf{\Psi}\rangle$ looks pretty much like a classical coherent state, say the state $|\Psi\rangle$. In other words, we want the fidelity between these two states to be large, $F(|\boldsymbol{\Psi}\rangle, |\Psi\rangle) = |\langle \boldsymbol{\Psi}|\Psi\rangle| \approx 1$. We will therefore choose to call the state $|\Psi\rangle$ the *dominant state*, while the $|\tilde{\Psi}\rangle$ will be called the *sub-dominant state*. Since we want to study the case of a general classical sub-dominant states $|\tilde{\Psi}\rangle$, without restricting the value of $|\langle\Psi|\tilde{\Psi}\rangle|$, the assumption of large fidelity boils down to that of small β, and consequently large α. Therefore, we will be systematically working in the limit $\beta \to 0$. It should be clear that, from the point of view of the classical limit, the most natural choice would be to take $\alpha = 1$ and $\beta = 0$, i.e., the state (1). But, as argued in [8], there is a danger that such a state may fail to be gauge invariant, so we need to introduce at least a small sub-dominant state, in order to ensure the gauge invariance of the total state.

Having adopted the state in the form (4), let us now introduce some technical apparatus to study it efficiently. To begin with, coherent states in general have nonzero overlap. Therefore, we introduce the overlaps as follows:

$$S_G \equiv \langle g|\tilde{g}\rangle \,, \qquad S_M \equiv \langle \phi|\tilde{\phi}\rangle \,, \qquad S \equiv \langle \Psi|\tilde{\Psi}\rangle = S_G S_M \,.$$

Note that, since in (4) only the relative phase between $|\Psi\rangle$ and $|\tilde{\Psi}\rangle$ is important, we can reabsorb the phases of the coefficients α and β into $|\Psi\rangle$ and $|\tilde{\Psi}\rangle$, respectively. In this way, we have $\alpha, \beta \in \mathbb{R}$, while only the overlap S between the two coherent states carries the information about the relative phase, and is therefore complex. Moreover, since S is a product between S_G and S_M, the phase of S can be distributed between S_G and S_M in an arbitrary way. A convenient choice is to have the phase in the matter sector, so that $S_G \in \mathbb{R}$ and $S_M \in \mathbb{C}$.

Next, we can decompose $|\tilde{g}\rangle$ and $|\tilde{\phi}\rangle$ into parts proportional to and orthogonal to $|g\rangle$ and $|\phi\rangle$, respectively,

$$|\tilde{g}\rangle = S_G|g\rangle + \epsilon_G|g^\perp\rangle \,, \qquad |\tilde{\phi}\rangle = S_M|\phi\rangle + \epsilon_M|\phi^\perp\rangle \,, \quad (6)$$

where $\langle g|g^\perp\rangle \equiv 0$, $\langle \phi|\phi^\perp\rangle \equiv 0$, and

$$\epsilon_G \equiv \sqrt{1-(S_G)^2} \,, \qquad \epsilon_M \equiv \sqrt{1-|S_M|^2} \,. \quad (7)$$

Note that $\epsilon_G, \epsilon_M \in \mathbb{R}$.

Since the state (4) must be normalized, $\langle \boldsymbol{\Psi}|\boldsymbol{\Psi}\rangle = 1$, we have

$$\alpha^2 + \beta^2 + 2\alpha\beta\,\mathrm{Re}(S) = 1 \,,$$

where $\mathrm{Re}(S)$ denotes the real part of S. This equation can be treated as a quadratic equation for α, and solved to give

$$\alpha = -\beta\,\mathrm{Re}(S) \pm \sqrt{1+\beta^2\left[\mathrm{Re}(S)^2 - 1\right]} \,.$$

Choosing the positive solution without loss of generality, we can expand α into power series in the limit $\beta \to 0$ as:

$$\alpha = 1 - \beta\,\mathrm{Re}(S) + \mathcal{O}(\beta^2) \,.$$

Additionally, one can use (6) to rewrite $|\tilde{\Psi}\rangle$ into the form

$$|\tilde{\Psi}\rangle = S|\Psi\rangle + \epsilon|\Psi^\perp\rangle \,, \quad (8)$$

where

$$\epsilon = \sqrt{\epsilon_M^2 + \epsilon_G^2 - \epsilon_M^2\epsilon_G^2} \,, \quad (9)$$

and

$$\begin{aligned} |\Psi^\perp\rangle = & \frac{\epsilon_M S_G}{\epsilon}|g\rangle \otimes |\phi^\perp\rangle + \frac{\epsilon_G S_M}{\epsilon}|g^\perp\rangle \otimes |\phi\rangle \\ & + \frac{\epsilon_G \epsilon_M}{\epsilon}|g^\perp\rangle \otimes |\phi^\perp\rangle \,. \end{aligned} \quad (10)$$

Finally, given all of the above, we can rewrite the total state (4) as:

$$|\boldsymbol{\Psi}\rangle = \Big[1 + i\beta S_G\,\mathrm{Im}(S_M)\Big]|\Psi\rangle + \beta\epsilon|\Psi^\perp\rangle + \mathcal{O}(\beta^2) \quad (11)$$

At this point we can evaluate the expectation values for the metric and stress-energy operators in the state (11). Taking into account that the operator $\hat{g}_{\mu\nu}$ depends only on the gravitational degrees of freedom, we use (10) and (11) to obtain

$$\boldsymbol{g}_{\mu\nu} = \langle \boldsymbol{\Psi}|\hat{g}_{\mu\nu}|\boldsymbol{\Psi}\rangle = g_{\mu\nu} + \beta h_{\mu\nu} + \mathcal{O}(\beta^2) \,, \quad (12)$$

where $g_{\mu\nu}$ is the dominant classical metric (2), while the correction term $h_{\mu\nu}$ is given as

$$h_{\mu\nu} = 2\epsilon_G\,\mathrm{Re}\left(S_M\langle g|\hat{g}_{\mu\nu}|g^\perp\rangle\right) \,. \quad (13)$$

We see that the correction term is of purely quantum origin, without a classical analog — it is a function of the off-diagonal matrix elements of the metric operator and of the overlap between the two coherent states in (4).

One can perform a similar calculation of the expectation value for the stress-energy operator, noting that $\hat{T}_{\mu\nu}$ depends on both gravitational and matter degrees of freedom, to obtain

$$\boldsymbol{T}_{\mu\nu} = \langle \boldsymbol{\Psi}|\hat{T}_{\mu\nu}|\boldsymbol{\Psi}\rangle = T_{\mu\nu} + \beta\bar{T}_{\mu\nu} + \mathcal{O}(\beta^2) \,, \quad (14)$$

where $T_{\mu\nu}$ is the dominant classical stress-energy (2), while the correction term $\bar{T}_{\mu\nu}$ is given as

$$\bar{T}_{\mu\nu} = 2\epsilon\,\mathrm{Re}\left(\langle\Psi|\hat{T}_{\mu\nu}|\Psi^\perp\rangle\right) . \quad (15)$$

Again we see that the correction term is of purely quantum origin, being a function of the off-diagonal matrix elements of the stress-energy operator and of the overlap.

Regarding the effective entangled metric and stress-energy tensors (12) and (14), it is important to stress that they do not satisfy classical Einstein's equations of GR.

Namely, we assume that Einstein's equations are separately satisfied by the dominant metric and stress-energy (2) coming from the coherent state $|\Psi\rangle$, and by the sub-dominant metric and stress-energy (5) coming from the other coherent state $|\tilde{\Psi}\rangle$, as two different classical solutions. However, due to the nonlinearity of Einstein's equations, and due to the presence of the overlap terms $h_{\mu\nu}$ and $\bar{T}_{\mu\nu}$ in (12) and (14), quantities $\boldsymbol{g}_{\mu\nu}$ and $\boldsymbol{T}_{\mu\nu}$ do not satisfy Einstein's equations, as long as $\beta \neq 0$.

This leads us to the following physical interpretation. Given that the sub-dominant classical solution $(\tilde{g}_{\mu\nu}, \tilde{T}_{\mu\nu})$ is quadratic in $|\tilde{\Psi}\rangle$ which enters (4) multiplied by β, in the limit $\beta \to 0$ the sub-dominant solution is of the order $\mathcal{O}(\beta^2)$ in the equations (12) and (14). Therefore, in our approximation the only nontrivial effect of its presence enters through the overlap terms (13) and (15). Consequently, the only classical spacetime-matter configuration which satisfies Einstein's equations and which is present in our description — is the dominant one $(g_{\mu\nu}, T_{\mu\nu})$. From a phenomenological point of view, therefore, it is natural to expand all quantities as corrections to the dominant classical configuration $(g_{\mu\nu}, T_{\mu\nu})$, including the equation of motion for a point particle. As we shall see in the remainder of the text, given that $(\boldsymbol{g}_{\mu\nu}, \boldsymbol{T}_{\mu\nu})$ contains quantum gravity corrections through the overlap terms, the presence of these quantum corrections in (12) and (14) will introduce an "effective force" term into the effective equation of motion for the particle, pushing it off the geodesic trajectory defined by the classical dominant metric $g_{\mu\nu}$.

2.2 Effective covariant conservation equation

The effective classical covariant conservation equation of (14) is given by:

$$\boldsymbol{\nabla}_\nu \boldsymbol{T}^{\mu\nu} = 0\,. \tag{16}$$

We also have the following effective classical expression

$$\boldsymbol{\Gamma}^\lambda{}_{\mu\nu} = \frac{1}{2}\boldsymbol{g}^{\lambda\sigma}\left(\partial_\mu \boldsymbol{g}_{\sigma\nu} + \partial_\nu \boldsymbol{g}_{\sigma\mu} - \partial_\sigma \boldsymbol{g}_{\mu\nu}\right)\,, \tag{17}$$

where $\boldsymbol{g}_{\mu\nu} \equiv \langle \boldsymbol{\Psi}|\hat{g}_{\mu\nu}|\boldsymbol{\Psi}\rangle$ is the effective classical metric and $\boldsymbol{g}^{\mu\nu}$ is its inverse matrix (see [9] for a more careful definition of (16) and (17)).

With effective classical expressions (16) and (17) in hand, we can now employ (12) and (14) to expand them into the dominant and correction parts. First we use (12) and $\boldsymbol{g}_{\mu\lambda}\boldsymbol{g}^{\lambda\nu} = \delta^\nu_\mu$ to find the inverse entangled metric $\boldsymbol{g}^{\mu\nu} = g^{\mu\nu} - \beta g^{\mu\rho}g^{\nu\sigma}h_{\rho\sigma} + \mathcal{O}(\beta^2)$, and then substitute into (17) to obtain

$$\boldsymbol{\Gamma}^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} + \frac{\beta}{2} g^{\lambda\sigma}\left(\nabla_\mu h_{\sigma\nu} + \nabla_\nu h_{\sigma\mu} - \nabla_\sigma h_{\mu\nu}\right) + \mathcal{O}(\beta^2)\,, \tag{18}$$

where the Christoffel symbols in ordinary ∇_μ are defined with respect to the dominant classical metric $g_{\mu\nu}$. Then, expanding (16) into the form

$$\partial_\nu \boldsymbol{T}^{\mu\nu} + \boldsymbol{\Gamma}^\mu{}_{\sigma\nu}\boldsymbol{T}^{\sigma\nu} + \boldsymbol{\Gamma}^\nu{}_{\sigma\nu}\boldsymbol{T}^{\mu\sigma} = 0\,,$$

we substitute (14) and (18). For convenience, introduce the following shorthand notation (see our conventions from the last paragraph of the Introduction),

$$F^\mu{}_{\nu\sigma} \equiv \nabla_{(\sigma} h^\mu{}_{\nu)} - \frac{1}{2}\nabla^\mu h_{\nu\sigma}\,, \tag{19}$$

so that, dropping the term $\mathcal{O}(\beta^2)$, we can write:

$$\nabla_\nu\left(T^{\mu\nu} + \beta\bar{T}^{\mu\nu}\right) + 2\beta F^{(\mu}{}_{\nu\sigma}T^{\nu)\sigma} = 0\,. \tag{20}$$

This equation is the one we sought out — it represents the analog of the classical covariant conservation equation, while taking into account the overlap between the two coherent states in (4), approximated to the linear order in β. The equation (20) represents the effective classical covariant conservation law for the stress-energy tensor, with the included quantum correction, represented to first order in β. It is the starting point for the remainder of our analysis, and it will be used in the derivation of the equation of motion for a point particle.

2.3 Effective equation of motion

We are now ready to derive the equation of motion for a particle in the single pole approximation, using the technique first presented in [3] and [7]. However, instead of starting with the classical covariant conservation law, we start from the effective covariant conservation law (20), which contains the quantum correction terms. Throughout, we assume the following relation of scales, $\mathcal{O}(\beta) > \mathcal{O}_1 \geq \mathcal{O}(\beta^2)$. We begin by contracting (20) with an arbitrary test function $f_\mu(x)$ of compact support, and integrating over the whole spacetime. We then perform the partial integration to move the covariant derivative from the stress-energy tensors to the test function. The boundary term vanishes since the test function has compact support, giving

$$\int_{\mathcal{M}_4} d^4x\sqrt{-g}\Big[-\left(T^{\mu\nu} + \beta\bar{T}^{\mu\nu}\right)\nabla_\nu f_\mu + 2\beta F^{(\mu}{}_{\nu\sigma}T^{\nu)\sigma}f_\mu\Big] = 0. \tag{21}$$

Now we need to model the dominant and correction parts of the stress-energy tensor. For the dominant part, it is straightforward to assume the single pole approximation, as was done in the classical case:

$$T^{\mu\nu}(x) = \int_{\mathcal{C}} d\tau B^{\mu\nu}(\tau)\frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}}\,. \tag{22}$$

Regarding the correction term, we also use the single pole approximation,

$$\bar{T}^{\mu\nu}(x) = \int_{\mathcal{C}} d\tau \bar{B}^{\mu\nu}(\tau) \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} . \tag{23}$$

Following a procedure detailed in [9], we arrive to the following three equations:

$$B^{\mu\nu} + \beta\bar{B}^{\mu\nu} = (B + \beta\bar{B})u^\mu u^\nu \equiv m(\tau) u^\mu u^\nu , \tag{24}$$

where we emphasize that the parameter m may depend on the particle's proper time τ.

$$\nabla m = \beta m u^\sigma \left(u^\nu u_\lambda F^\lambda{}_{\nu\sigma} - F^\nu{}_{\nu\sigma} \right) , \tag{25}$$

establishing that, in contrast to the classical case, here the parameter m fails to be constant.

$$\nabla u^\mu + \beta u^\nu u^\sigma F^\mu_{\perp\nu\sigma} = 0 . \tag{26}$$

Where this is the equation of motion in its final form, with the shorthand notation $F^\mu_{\perp\nu\sigma} \equiv P^\mu_{\perp\lambda} F^\lambda{}_{\nu\sigma}$. The presence of the orthogonal projector in the second term should not be surprising. Namely, since acceleration must always be orthogonal to the velocity, the second term in the equation must also be orthogonal to velocity, and this is guaranteed by the presence of the orthogonal projector.

Equations (24), (25) and (26) are the main result of this paper, and we discuss them in turn. Equation (24) determines the structure of the stress-energy tensor describing the point particle, as a function of tangent vectors of its world line and a scalar parameter $m(\tau)$. Formally, it has the same form as its classical counterpart, and provisionally the parameter m may be even called *effective mass*. Namely, in the rest frame of the particle, integration of the $\boldsymbol{T}^{00}$ component of the entangled stress-energy tensor over the 3-dimensional spatial hypersurface can be interpreted as the total rest-energy of the kink configuration of fields that represents the particle. This terminology is of course provisional, since all these notions are merely a part of the semiclassical approximation of the full quantum gravity description.

Equation (25) determines the proper time evolution of the parameter $m(\tau)$. In contrast to the classical case, where $m(\tau)$ is a constant (see [9]), here we see that its time derivative is proportional to (covariant derivatives of) the overlap $h_{\mu\nu}$ between the dominant and sub-dominant classical geometry, via (19). If one puts $\beta = 0$, (25) reduces to the classical case, as expected. The overlap between the two geometries gives rise to an effective force that is responsible for the change in time of the particle's effective mass. Since the particle is (effectively) not isolated, its total energy is therefore not conserved, in the sense of equation (25).

Finally, and most importantly, equation (26) represents the effective equation of motion of the particle, determining its world line. It has the form of the classical geodesic equation with an additional correction term proportional to β and to the overlap term $h_{\mu\nu}$. This additional term represents an *effective force*, pushing the particle off the classical geodesic trajectory. It is analogous to the notion of the "exchange interaction" force in molecular physics, in the region where the wavefunctions of the two electrons in a molecule overlap.

In our case, however, the force term is determined by the overlap between the two classical spacetime and matter configurations superposed in the state (4), and in particular by the off-diagonal components of the metric operator $\hat{g}_{\mu\nu}$, see (13). It is thus a *pure quantum gravity effect*, a consequence of the nontrivial structure of the metric operator. Of course, the detailed properties and the magnitude of the force term depend on the choice of the two classical gravity-matter configurations and on the details of the quantization of the gravitational field.

2.4 Consistency of the approximation scheme

It can be shown (see [9] that the presented model obeys the following consitensy condition:

$$\begin{aligned} \mathcal{O}_1(T) \geq & 2\beta\epsilon \,\mathrm{Re} \left(\langle \Psi | \hat{T}_{\mu\nu} | \Psi^\perp \rangle \right) \\ & + T^{\mu\nu} - \int_{\mathcal{C}} d\tau \, m(\tau) u^\mu u^\nu \frac{\delta^{(4)}(x - z(\tau))}{\sqrt{-g}} . \end{aligned} \tag{27}$$

This inequality should be interpreted as follows. Given an explicit model of quantum gravity, and within it an explicit configuration of matter fields that make up a particle, one can estimate all three quantities on the right-hand side of (27), namely the off-diagonal components of the stress-energy operator, its expectation value in the dominant coherent state, and the total mass of the particle, respectively. Then, the consistency condition (27) gives a lower bound on the scale $\mathcal{O}_1$, which represents an estimate of the error when discussing the effective equation of motion for the particle. In other words, the equation of motion can be considered to be approximately valid only across scales much larger than the $\mathcal{O}_1$ scale, bounded from below by inequality (27).

3 Status of the weak equivalence principle

In light of the results of section 2, it is important to discuss the status of the equivalence principle (EP). Throughout the literature, one can find various different formulations of EP. In this section we opt to specify one particular definition of the weak and strong equivalence principles (WEP and SEP, respectively) and to use this definition in the rest of the text.

3.1 Definition and flavors of the equivalence principle

The purpose of the equivalence principle is to prescribe the coupling of matter to gravity [4]. Its precise formulation therefore depends on the particular choice of the gravitational and matter degrees of freedom which one uses to describe matter and gravity. For the purpose of this paper, we assume that the classical limit of quantum gravity corresponds to general relativity, which means that in this limit the fundamental gravitational degrees of freedom give rise to a nonflat spacetime metric. Given any choice of the gravitational degrees of freedom that belong to this class, in the classical framework one can formulate the equivalence principle as a two-step recipe to couple matter to gravity (we will discuss the quantum framework in subsection 3.2).

Start from the classical equation of motion for matter degrees of freedom in flat spacetime, written symbolically as

$$\mathcal{D}_{\text{flat}}[\phi, \eta_{\mu\nu}] = 0 \,, \tag{28}$$

where ϕ denotes the matter degrees of freedom, $\eta_{\mu\nu}$ is the Minkowski metric, while $\mathcal{D}_{\text{flat}}$ is an appropriate functional describing the equation of motion for ϕ in flat spacetime and is assumed to be local. Given this equation of motion, couple it to gravity as follows:

1. Rewrite the equation of motion in a manifestly diffeomorphism-invariant form, typically by a change of variables to a generic curvilinear coordinate system,

   $$\mathcal{D}_{\text{curvilinear}}[\phi, g^{(0)}_{\mu\nu}] = 0 \,,$$

   where $g^{(0)}_{\mu\nu}$ is still the flat spacetime metric, appropriately transformed from $\eta_{\mu\nu}$, and similarly for $\mathcal{D}_{\text{curvilinear}}$.

2. Promote the curvilinear equation of motion to the equation of motion in curved spacetime by replacing the flat spacetime metric $g^{(0)}_{\mu\nu}$ with an arbitrary metric $g_{\mu\nu}$,

   $$\mathcal{D}_{\text{curvilinear}}[\phi, g_{\mu\nu}] = 0 \,,$$

   thereby specifying the equation of motion for matter coupled to gravity.

The first step describes the matter equation of motion from a perspective of a generic curvilinear (or "arbitrarily accelerated") coordinate system, reflecting the principle of *general relativity*. The second step simply promotes that same equation to curved spacetime as it stands, with no additional coupling of any kind. This can be loosely formulated as a statement of *local equivalence between gravity and acceleration*, which is how the EP historically got its name. Also, note that these two steps operationally correspond to the standard *minimal coupling* prescription [4].

It is important to stress the *local* nature of EP, which manifests itself in the assumption that the initial equation of motion (28) is local, and that the EP essentially does not change it at all, at any given point in spacetime. This has one important implication — the gravitational degrees of freedom manifest themselves only through *nonlocal measurements*, as tidal effects induced by spacetime curvature. We will return to this point and comment more on it later in the text.

Depending on the further specification of the matter degrees of freedom, one can distinguish between various flavors of the EP. For example, if one talks about mechanics of point particles, one can start from the Newton's first law of motion, which states that in the absence of any forces, a particle has a straight-line trajectory in Minkowski spacetime. According to the step 1 above, the differential equation for a straight line in a generic curvilinear coordinate system is the geodesic equation,

$$\frac{d^2 z^\lambda}{d\tau^2} + \mathit{\Gamma}^{\lambda}_{(0)\mu\nu} \frac{dz^\mu}{d\tau} \frac{dz^\mu}{d\tau} = 0 \,,$$

where the index (0) on the Christoffel symbol indicates that it is calculated using the metric $g^{(0)}_{\mu\nu}$, which is obtained by a curvilinear coordinate transformation from the Minkowski metric $\eta_{\mu\nu}$. Then, according to step 2, one again writes the same equation, only dropping the requirement of flat spacetime metric,

$$\frac{d^2 z^\lambda}{d\tau^2} + \mathit{\Gamma}^{\lambda}{}_{\mu\nu} \frac{dz^\mu}{d\tau} \frac{dz^\mu}{d\tau} = 0 \,,$$

so that this time the Christoffel symbol is calculated using an arbitrary metric $g_{\mu\nu}$, and now encodes the interaction with the gravitational degrees of freedom. So one starts from the Newton's first law of motion for a particle in the absence of the gravitational field, and ends up with a geodesic equation in the presence of the gravitational field. We define this flavor of the EP as the *weak equivalence principle* (WEP).

Instead of mechanical particles, one can study matter degrees of freedom described by a field theory. For example, if one starts from the equation of motion for a single real scalar field,

$$\left(\eta^{\mu\nu}\partial_\mu\partial_\nu - m^2\right)\phi = 0\,,$$

according to the step 1 of the EP, one can rewrite it in a general curvilinear coordinate system as

$$\left(g^{\mu\nu}_{(0)}\nabla_\mu\nabla_\nu - m^2\right)\phi = 0\,,$$

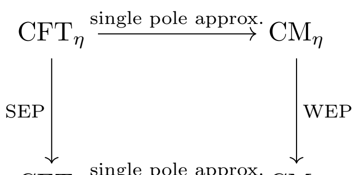
where the Christoffel symbol inside the covariant derivative is again calculated using the flat-space metric $g^{(0)}_{\mu\nu}$. Then, according to step 2 of the EP, this equation is promoted to curved spacetime as it stands, leading to

$$\left(g^{\mu\nu}\nabla_\mu\nabla_\nu - m^2\right)\phi = 0\,,$$

where now the covariant derivative is given with respect to an arbitrary metric $g_{\mu\nu}$ describing curved spacetime. Thus one arrives to the equation of motion for a scalar field coupled to gravity. We define this flavor of the EP as the *strong equivalence principle* (SEP).

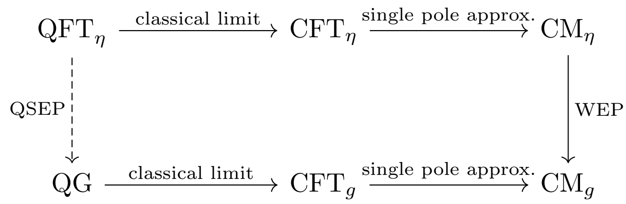
3.2 Equivalence principle and quantum theory

Adopting the above definitions of WEP and SEP, it is important to discuss their relationship. From the perspective of the classical field theory (CFT), the notion of a particle can be introduced as a localized kink-like configuration of matter fields, described as a solution of the (usually quite complicated) matter field equations. One can then employ the apparatus of multipole formalism and describe the evolution of this kink configuration in the single pole approximation, as it has been done in [3,7] (see [9]). Using this method, one can recover the equation of motion for a particle in classical mechanics (CM) as an approximation of the field theory. Moreover, all this can be done before or after the application of the EP, leading to the following diagram:



Here the indices η and g indicate that equations of motion in a given theory are written in flat and in curved spacetime, respectively. This diagram can be shown to commute in the classical case (see [9]).

Turning to the case of quantum theory. Starting first from some quantum field theory (QFT$_\eta$) which describes the fundamental matter fields in Minkowski spacetime, one can take its classical limit, giving rise to some effective classical field theory (CFT$_\eta$). Then, assuming that the latter features kink solutions, one can describe those using the single pole approximation, leading to classical mechanics (CM$_\eta$) of the corresponding particles. Finally, applying WEP one couples those particles to gravity. The resulting equation of motion will always be a geodesic equation, assuming that the initial QFT and all subsequent approximations respect the global Poincaré invariance of Minkowski spacetime. This symmetry guarantees the conservation of the stress-energy tensor of the matter fields throughout the sequence of approximations, leading invariably to the geodesic equation of motion for the particle. On the other hand, it is arguably more appropriate to take an alternative, more fundamental route — start from some fundamental quantum gravity (QG) model, and take the classical limit leading to some classical field theory (CFT$_g$) for both matter and gravity. Then, again assuming that this theory features kink solutions, employ the single pole approximation to obtain the classical mechanics for the particle in the gravitational field (CM$_g$). Note that this is in fact precisely the program that was performed in section 2, leading to the non-geodesic equation of motion (26) for the particle. In effect, one can conclude that the following diagram fails to commute:



As a side comment, note that the dashed QSEP arrow represents some hypothetical map leading from a QFT in Minkowski spacetime to a full-blown model of QG, according to a notion that might be called a "quantum strong equivalence principle". It is unclear whether such a principle exists or not, let alone what its formulation is supposed to be, even if one is given precisely defined models of QFT and QG in question. We introduce it here simply for completeness, speculating that such a notion should exist, as a generalization of SEP from classical to quantum physics. It is also convenient to introduce it, in order to close the diagram and discuss its commutativity.

It is important to stress the reason why this diagram does not commute. Recalling the details of section 2, the local Poincaré symmetry is assumed to be respected at the fundamental level of QG and onwards, just like in the classical case. Moreover, the single pole approximation is used, avoiding any nonminimal coupling of the tidal forces that may be present. And yet, in spite of all that, the resulting equation of motion is not a geodesic. Looking at the equation of motion (26), the reason for this is the nontrivial overlap between coherent states describing two classical configurations of matter, and more importantly, of gravity. In other words, the deviation from the geodesic motion is a *pure quantum gravity effect* — it is not present in the classical case, nor in the case of quantum matter in classical Minkowski spacetime. A testimony of this fact is the quantum correction term for the metric (13), which features off-diagonal matrix elements of the metric operator $\hat{g}_{\mu\nu}$, $h_{\mu\nu} = 2\epsilon_G \operatorname{Re}\left(S_M \langle g|\hat{g}_{\mu\nu}|g^\perp\rangle\right)$. In this sense, due to the noncommutativity of the above diagram, one can argue that (within the discussed framework) *quantum gravity violates the weak equivalence principle.*

3.3 Universality, gravitational and inertial mass

It can be shown, by studying the Newtonian limit of the equation of motion (26), that both universality (i.e., the fact that gravitation interacts with all types of particles in the same way) and the equality between gravitational and inertial masses are violated in our context, corroborating the conclusions of the abstract analysis of the EP given in subsection 3.2 (see [9] for details). Moreover, it can be shown that the ratio between the gravitational and the inertial mass is:

$$\frac{m_G}{m_I} \equiv \left(1 - \frac{1}{3}\beta h^i{}_i\right) . \tag{29}$$

4 Conclusions

4.1 Summary of the results

In this paper, we have discussed the effective motion of a point particle within the framework of quantum gravity, in particular the case where both matter and gravity are in a quantum superposition of the Schrödinger cat type. Section 2 was devoted to the generalization of these results to the realm of the full quantum gravity. In subsection 2.1 we introduced the abstract quantum gravity framework, discussed the model of the superposition of two coherent classical states, and established the main assumptions for the derivation of the effective equation of motion. In subsection 2.2 we have analyzed in detail the quantum version of equation for the covariant conservation of stress-energy tensor, which is a crucial ingredient in the derivation of the effective equation of motion. The explicit derivation of the equation of motion itself was then given in subsection 2.3, giving rise to the main results of the paper — the equation for the stress-energy kernel (24), the equation for the time-evolution of the particle's mass (25), and the effective equation of motion for the particle (26). Most importantly, the effective equation of motion turns out to contain a non-geodesic term, giving rise to an effective force acting on the particle, as a consequence of the overlap terms between the two coherent states of the gravity-matter system. The last subsection 2.4 discusses the self-consistency of the assumptions used in the above analysis, giving rise to the equation (27) for the error estimate of the single pole approximation scale. In light of the nongeodesic motion established in section 2, it is important to discuss it in the context of the equivalence principle. This topic was taken up in section 3. After we have defined various flavors of the equivalence principle in subsection 3.1, the main analysis was presented in subsection 3.2, establishing the violation of (various forms of) the weak equivalence principle, as a consequence of the nongeodesic correction to the equation of motion (26). Finally, in subsection 3.3 we mentioned that both universality and the equality between gravitational and inertial masses are violated in our context, corroborating the conclusions of the abstract analysis of the EP given in subsection 3.2.

4.2 Discussion of the results

By far the most interesting topic to discuss in the context of the equation of motion (26) is how to estimate the magnitude of the nongeodesic term. As far as the analysis of this paper goes, we can only say that this term is very small, given that it is proportional to β, which is in turn bounded from above by phenomenological argument that we do not observe superpositions of the gravitational field in nature. However, aside from this qualitative argument, in order to estimate the actual magnitude of the nongeodesic term one would need to go beyond the abstract quantum gravity formalism, and construct an explicit quantum gravity model coupled to matter fields, find some explicit kink solutions of the matter sector, and then calculate the overlap terms and the off-diagonal terms of the metric operator. Of course, any estimate obtained in such a way would be model-dependent. We consider this to be a feature of the abstract quantum gravity approach, since the magnitude of the nongeodesic term represents one way to operationally distinguish between different QG models. In other words, one could use equation (26) to experimentally test and compare these models, at least in principle. Probably the most obvious such test would employ equation (29) which relates the gravitational and inertial mass of the particle.

One result that was not discussed in detail is the nonconservation law for the effective mass of the particle, (25). However, it is not really surprising that the particle's total rest energy fails to be constant in the presence of gravity-matter entanglement. As (25) tells us, the nonconservation is actually a consequence of the additional effective force, which is itself a consequence of the quantum overlap between two classical geometries and matter states. It is also important to discuss the generalization of our results from the case of the superposition of two coherent states to many coherent states. As long as we maintain the assumption that $\alpha \gg \beta$ in (4), it is straightforward to see that all our results and conclusions still hold in the generic case.

4.3 Future lines of research

One of the main lines of future work would be to perform a similar analysis as was done in this paper, but keeping the β^2 terms. Alternatively, one could repeat the analysis of this paper, but in a pole-dipole approximation. Also, given that the multipole formalism is also applicable to Riemann-Cartan spacetimes [5, 6, 11, 12], the analysis of this paper could be generalized to include coupling of quantum interference terms to spacetime torsion and the spin of the particle. In addition to all of the above, one important line of research would be to study possible connections to experiments. First, one shoud study the counterpart of the so-called *geodesic deviation equation*. Second, one could also test our results by measuring the violation of the universality and of the equality of the gravitational and the inertial mass in the semiclassical Newtonian limit.

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