Phenomenology of a single right-handed neutrino seesaw model

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(Dated: September 2017)

The Standard Model is remarkably successful at describing the interactions of elementary particles. However, the experimental observation of neutrino oscillation calls for extensions of the model in some way to accommodate neutrino masses and mixing. In this thesis, we consider a particular extension, constrained by an A_4 discrete symmetry, with one right-handed neutrino and three Higgs doublets in a singlet and triplet of A_4, respectively. Neutrino masses arise due to a seesaw mechanism, and the resulting neutrino sector is studied taking into account the renormalization group running from high to low energies. It is shown that, in the A_4 symmetric case, the model is not compatible with neutrino data. The analysis is repeated with the inclusion of a term in the Higgs potential which breaks A_4 softly. We find that experimental measurements of neutrino mass-squared differences are only reproduced in the case of soft breaking of A_4.

Keywords: Standard Model, Neutrino masses and mixing, A_4 symmetry, Seesaw mechanism

I. INTRODUCTION

At the most fundamental level, elementary particles and their interactions are remarkably well described by the Standard Model (SM). However, the formulation of this theory is lacking in some particular aspects. One of these relates to neutrinos, which are massless in the SM. Over the last few decades, neutrino oscillation experiments have provided irrefutable proofs that these particles are massive, albeit with a small mass [1], contradicting the standard theory. As the SM only contains left-handed (LH) neutrinos and not their right-handed (RH) counterparts, no mechanism for neutrino mass can be defined without an extension of the SM field content of the SM. Since the SM is somehow incomplete, it is plausible to consider it as an effective theory valid at low energies. Under this premise, one should take into account (effective) operators with dim>4 which encode the effects of unknown physics lying at higher energy scales. Remarkably, in the SM case, there is a unique dimension-five operator which respects the symmetries of the theory, and gives rise to Majorana neutrino masses [2] after spontaneous symmetry breaking.

The most natural way to generate such an operator is to consider the existence of very heavy states which decouple from the theory, leading to effective neutrino mass operators. This idea is in the basis of the well-known seesaw mechanism, which may be realized in several ways. In the type I version, [3–7] heavy RH neutrinos are added to the SM. A minimum of two of these states is necessary to achieve compatibility with neutrino data. Rather than adding two or more RH neutrinos, one can also expand other sectors of the SM. In this thesis, we consider an additional A_4 symmetry of the theory and an expanded scalar sector, with 3 Higgs fields in an A_4 triplet and one RH neutrino in a singlet. It is our goal to conclude whether there is a region of the parameter space where all experimental measurements are reproduced after renormalization group running to the low energies of current experimental measurements.

II. THE STANDARD MODEL OF PARTICLE PHYSICS

The SM is based on the gauge group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, where c stands for color, L for left-handedness and Y for hypercharge. The matter content of the SM is [8]:

$$
\text{Fermions}\begin{cases}
\textbf{Quarks}\ q_{L\alpha} \equiv \begin{pmatrix} u_{L\alpha} \\ d_{L\alpha} \end{pmatrix} \sim (3, 2, \tfrac{1}{6})\,, \\
u_{R\alpha} \sim (3, 1, \tfrac{2}{3})\,, \\
d_{R\alpha} \sim (3, 1, -\tfrac{1}{3})\,, \\
\textbf{Leptons}\ \ell_{L\alpha} \equiv \begin{pmatrix} \nu_{L\alpha} \\ l_{L\alpha} \end{pmatrix} \sim (1, 2, -\tfrac{1}{2})\,, \\
l_{R\alpha} \sim (1, 1, -1)\,,
\end{cases}
\\
\textbf{Higgs}\ \phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1, 2, \tfrac{1}{2})\,,
\tag{1}
$$

where the numbers in brackets indicate how the fields transform under the SM gauge group (1 for singlet, 2 for doublet and 3 for triplet). The hypercharge Y of a given field is given by

$$Y = Q - T_3\,, \tag{2}$$

where Q is the electric charge and T_3 is the third component of weak isospin. The fields u_α and d_α correspond to up- and down-type quarks, and ν_α and l_α to neutrinos and charged leptons, respectively. The index α runs through the three generations of fermions, while the subscripts L and R denote the chirality of the field (LH and RH, respectively). Chirality is defined by a field transformation law under application of the Dirac matrix $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, where γ^μ are the Dirac matrices

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forming a Clifford Algebra. Namely,

$$\gamma^5 \psi_R = \psi_R \ , \ \gamma^5 \psi_L = -\psi_L \, . \tag{3}$$

Any field can be decomposed into its RH and LH components, $\psi = \psi_R + \psi_L = P_R \psi + P_L \psi$, by the application of the projection operators $P_{R,L}$:

$$P_{R,L} = \frac{1 \pm \gamma^5}{2} \, . \tag{4}$$

In the following, we will focus on the electroweak sector of the SM described by the group $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. The electroweak Lagrangian density (identified from now on as the Lagrangian) is

$$\begin{aligned} \mathcal{L}_{\mathrm{SM}} =& (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi) - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ &+ i \overline{\ell_{L\alpha}} \not{D} \ell_{L\alpha} + i \overline{q_{L\alpha}} \not{D} q_{L\alpha} + i \overline{l_{R\alpha}} \not{D} l_{R\alpha} + i \overline{u_{R\alpha}} \not{D} u_{R\alpha} + i \overline{d_{R\alpha}} \not{D} d_{R\alpha} \\ &- (\mathbf{Y}^l_{\alpha\beta} \overline{\ell_{L\alpha}} \phi l_{R\beta} + \mathrm{H.c.}) - (\mathbf{Y}^u_{\alpha\beta} \overline{q_{L\alpha}} \tilde{\phi} u_{R\beta} + \mathrm{H.c.}) - (\mathbf{Y}^d_{\alpha\beta} \overline{q_{L\alpha}} \phi d_{R\beta} + \mathrm{H.c.}) \, , \end{aligned} \tag{5}$$

Here, the Feynman slash notation, $\not{p} = \gamma^\mu p_\mu$, is used, as well as the definitions $\overline{\psi} \equiv \psi^\dagger \gamma^0$ and $\tilde{\phi} \equiv i \tau_2 \phi^*$. $\mathbf{Y}^l$, $\mathbf{Y}^u$ and $\mathbf{Y}^d$ are general 3×3 complex Yukawa matrices and H.c. denotes the Hermitian conjugate. Local gauge invariance requires replacing the ordinary derivative ∂_μ by a covariant one D_μ given by

$$D_\mu \equiv \partial_\mu - i g W^a_\mu \frac{\tau_a}{2} - i g' B_\mu Y \, . \tag{6}$$

Y stands for the generator of $\mathrm{U}(1)_Y$ and τ^a $(a = 1, 2, 3)$ are the Pauli matrices (generators of $\mathrm{SU}(2)_L$). B_μ and W^a_μ are the four vector fields introduced to ensure local gauge invariance under $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$, respectively. The invariant kinetic terms are constructed using the field strengths

$$\begin{aligned} B_{\mu\nu} &\equiv \partial_\mu B_\nu - \partial_\nu B_\mu \, , \\ W^a_{\mu\nu} &\equiv \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g \epsilon^{abc} W^b_\mu W^c_\nu \, . \end{aligned} \tag{7}$$

As for the potential associated to the scalar field ϕ, one has

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \, , \tag{8}$$

where μ has dimensions of mass and λ is a dimensionless parameter. Notice that there are no mass terms in the Lagrangian of (5), as no such term would be invariant under the gauge group. This is rather obvious for gauge fields. For fermions, a mass term would be of the form $-m \overline{\psi} \psi = -m(\overline{\psi_L} \psi_R + \overline{\psi_R} \psi_L)$, leading to a term that is not invariant under $\mathrm{SU}(2)_L$ or $\mathrm{U}(1)_Y$. There seems to be a mass term for the scalar Higgs ϕ field in the potential of (8), but this is contingent on having $\mu^2 > 0$, which, as we will see, is not interesting from the theoretical viewpoint.

The mass problem of the SM is solved by the Higgs mechanism. After spontaneous symmetry breaking (SSB) in the electroweak sector, mass terms arise for gauge bosons and fermions. The former acquire mass from the Higgs kinetic term $(D^\mu \phi)^\dagger (D_\mu \phi)$, the latter from the Yukawa terms, given in the last line of (5).

A. The Higgs mechanism

SSB occurs when the Lagrangian of a theory respects a certain symmetry but the vacuum state (or lowest-energy state) does not. In the SM, the vacuum is identified as the configuration of the field ϕ which minimizes (8). For $\mu^2 > 0$, the minimization condition is $|\phi| = 0$, while for $\mu^2 < 0$, it is $|\phi|^2 \equiv |v_{\mathrm{SM}}|^2 = -\mu^2/2\lambda$. In this case, the fields possess a nonzero value in the vacuum. If a charged field has a nonzero vacuum expectation value (VEV), then the vacuum has an electric charge, which cannot occur as electric charge is a conserved quantity of the SM. Since only neutral fields can acquire a nonzero value in the vacuum, we have

$$\langle \phi \rangle = \begin{pmatrix} \langle \phi^+ \rangle \\ \langle \phi^0 \rangle \end{pmatrix} = \begin{pmatrix} 0 \\ v_{\mathrm{SM}} \end{pmatrix} \, . \tag{9}$$

In general, the minimization condition sets $v_{\mathrm{SM}} = e^{i\theta} \sqrt{-\frac{\mu^2}{2\lambda}}$. Choosing a particular VEV corresponds to fixing a value for θ. We fix $\theta = 0$, thus realizing electroweak symmetry breaking (EWSB). The Higgs field can be parametrized as oscillations around the vacuum state as

$$\phi(x) = \exp\left(i \frac{\xi^a(x) \tau^a}{\sqrt{2} v_{\mathrm{SM}}} \right) \begin{pmatrix} 0 \\ v_{\mathrm{SM}} + \frac{H(x)}{\sqrt{2}} \end{pmatrix} \, , \tag{10}$$

where $H(x)$ and $\xi(x)$ are real fields with zero VEV. Before SSB, it is possible to apply a gauge transformation to ϕ, leaving the Lagrangian invariant. Choosing the unitary gauge, we absorb the exponential in (10) and obtain

$$\phi(x) = \begin{pmatrix} 0 \\ v_{\mathrm{SM}} + \frac{H(x)}{\sqrt{2}} \end{pmatrix} \, . \tag{11}$$

After EWSB, $H(x)$ will correspond to the the physical Higgs boson field. Replacing (11) in the Lagrangian of (5), the VEV v_{SM} gives rise to new terms. Starting with the scalar and gauge sectors, one has

$$D_\mu\phi = \begin{pmatrix} 0 \\ \frac{\partial_\mu H}{\sqrt{2}} \end{pmatrix} - i\frac{g'}{2}\begin{pmatrix} 0 \\ B_\mu \end{pmatrix}(v_{\rm SM} + H/\sqrt{2}) - i\frac{g}{2}\begin{pmatrix} W^1_\mu - iW^2_\mu \\ -W^3_\mu \end{pmatrix}(v_{\rm SM} + H/\sqrt{2}) \,, \tag{12}$$

$$V(\phi) = \frac{\lambda}{4}H^4 + \sqrt{2}\lambda v_{\rm SM} H^3 - \mu^2 H^2 + \text{const.}. \tag{13}$$

Defining the field combinations

$$\begin{aligned} W^+_\mu &= (W^-_\mu)^\dagger = \frac{1}{\sqrt{2}}(W^1_\mu - iW^2_\mu)\,, \\ A_\mu &= \sin\theta_W W^3_\mu + \cos\theta_W B_\mu\,, \\ Z_\mu &= \cos\theta_W W^3_\mu - \sin\theta_W B_\mu\,, \end{aligned} \tag{14}$$

where θ_W is the so-called weak mixing angle, we obtain the bilinear terms in the scalar and gauge fields,

$$(D^\mu\phi)^\dagger(D_\mu\phi) - V(\phi) = \frac{1}{2}(-2\mu^2)H^2 + \frac{g^2 v_{\rm SM}^2}{2}W^{\mu -}W^+_\mu + \frac{1}{2}\frac{g^2 v_{\rm SM}^2}{2\cos^2\theta_W}Z^\mu Z_\mu + \ldots \,. \tag{15}$$

Thus, upon EWSB the Higgs (H) and gauge bosons ($W^\pm, Z$) obtain masses

$$\begin{aligned} m_H &= \sqrt{-2\mu^2} = 2v_{\rm SM}\sqrt{\lambda}\ ,\ m_W = \frac{g v_{\rm SM}}{\sqrt{2}}\,, \\ m_Z &= \frac{g v_{\rm SM}}{\sqrt{2}\cos\theta_W} = \frac{m_W}{\cos\theta_W}\,. \end{aligned} \tag{16}$$

The remaining field A_μ is massless and is identified as being the photon.

B. Lepton electroweak currents

Before analyzing how EWSB grants mass to fermions, we study the lepton-gauge boson interaction terms in (5). We obtain

$$\begin{aligned} \mathcal{L}^l_{\rm G} =& i\overline{\ell_{L\alpha}}\left(-ig\not{W}^a\frac{\tau^a}{2} + ig'\not{B}\frac{1}{2}\right)\ell_{L\alpha} + i\overline{l_{R\alpha}}\left(0 + ig'\not{B}\right)l_{R\alpha} \\ =& \frac{1}{2}\overline{\ell_{L\alpha}}\begin{pmatrix} (gs_W - g'c_W)\not{A} + (gc_W + g's_W)\not{Z} & \sqrt{2}g\not{W}^+ \\ \sqrt{2}g\not{W}^- & -(gs_W + g'c_W)\not{A} - (gc_W - g's_W)\not{Z} \end{pmatrix}\ell_{L\alpha} - g'\overline{l_{R\alpha}}(c_W\not{A} - s_W\not{Z})l_{R\alpha}\,. \end{aligned} \tag{17}$$

We have used the fields of (14), together with the simplified notation $s_W \equiv \sin\theta_W$ and $c_W \equiv \cos\theta_W$. The diagonal interaction terms, combining particles with the same electric charge, correspond to the neutral-current (NC) Lagrangian $\mathcal{L}_{\rm NC}$, while the off-diagonal ones describe charged currents (CC) $\mathcal{L}_{\rm CC}$. Namely,

$$\begin{aligned} \mathcal{L}^\ell_{\rm NC} =& \frac{1}{2}\overline{\nu_{L\alpha}}\left[(gs_W - g'c_W)\not{A} + (gc_W + g's_W)\not{Z}\right]\nu_{L\alpha} - \frac{1}{2}\overline{l_{L\alpha}}\left[(gs_W + g'c_W)\not{A} + (gc_W - g's_W)\not{Z}\right]l_{L\alpha} \\ & - g'\overline{l_{R\alpha}}\left[c_W\not{A} - s_W\not{Z}\right]l_{R\alpha}\,, \end{aligned} \tag{18}$$

and

$$\mathcal{L}^\ell_{\rm CC} = \frac{g}{\sqrt{2}}\left(\overline{\nu_{L\alpha}}\not{W}^+ l_{L\alpha} + \overline{l_{L\alpha}}\not{W}^-\nu_{L\alpha}\right)\,. \tag{19}$$

By requiring that neutrinos do not couple with the photon, the condition $(gs_W - g'c_W) = 0$ is obtained, yielding $\tan\theta_W = g'/g$. Therefore,

$$\begin{aligned} \mathcal{L}^\ell_A =& -\frac{1}{2}2g'c_W\overline{l_{L\alpha}}\not{A}l_{L\alpha} - g'c_W\overline{l_{R\alpha}}\not{A}l_{R\alpha} \\ =& -g'c_W(\overline{l_{L\alpha}}\not{A}l_{L\alpha} + \overline{l_{R\alpha}}\not{A}l_{R\alpha}) \\ =& -g'c_W\bar{l}\gamma^\mu l A_\mu\,, \end{aligned} \tag{20}$$

$$\begin{aligned} \mathcal{L}^\ell_Z =& \frac{g}{2c_W}\overline{\nu_{L\alpha}}\not{Z}\nu_{L\alpha} + \frac{g}{c_W s_W^2}\overline{l_{R\alpha}}\not{Z}l_{R\alpha} \\ & - \frac{g}{2c_W}(c_W^2 - s_W^2)\overline{l_{L\alpha}}\not{Z}l_{L\alpha}\,. \end{aligned} \tag{21}$$

$\mathcal{L}_{\rm CC}$ and $\mathcal{L}_{\rm NC}$ describe the interactions between leptons and the physical gauge bosons.

C. Fermion masses

After EWSB, the Yukawa terms in the Lagrangian read

$$\mathcal{L}_{\text{Yuk.}} = -\left(v_{\text{SM}} + \frac{H}{\sqrt{2}}\right)\big(\mathbf{Y}^l_{\alpha\beta}\overline{l_{L\alpha}}l_{R\beta} + \mathbf{Y}^u_{\alpha\beta}\overline{u_{L\alpha}}u_{R\beta} + \mathbf{Y}^d_{\alpha\beta}\overline{d_{L\alpha}}d_{R\beta}\big) + \text{H.c.}\,. \tag{22}$$

Considering only the bilinear terms in the fermion fields, and defining the mass matrices for the ψ fields $\mathbf{M}^\psi = v_{\text{SM}}\mathbf{Y}^\psi$, we obtain

$$\mathcal{L}_{\text{Yuk.}} = -\big(\mathbf{M}^l_{\alpha\beta}\overline{l_{L\alpha}}l_{R\beta} + \mathbf{M}^u_{\alpha\beta}\overline{u_{L\alpha}}u_{R\beta} + \mathbf{M}^d_{\alpha\beta}\overline{d_{L\alpha}}d_{R\beta} + \text{H.c.}\big) + \dots\,, \tag{23}$$

where the missing terms describe fermion-Higgs interactions. In general, the fermion mass terms are not diagonal, meaning that they mix different generations. In order to bring these fields to the physical basis, where mass matrices are diagonal, we perform a "rotation" in flavor space. The bi-diagonalization of the matrices $\mathbf{M}$ is performed by unitary matrices $V^{l,u,d}_{L,R}$:

$$\begin{aligned}
V_L^{l\dagger}\mathbf{M}^l V_R^l &= \text{diag}(m_e, m_\mu, m_\tau) \equiv \mathbf{D}^l\,,\\
V_L^{u\dagger}\mathbf{M}^u V_R^u &= \text{diag}(m_u, m_c, m_t) \equiv \mathbf{D}^u\,,\\
V_L^{d\dagger}\mathbf{M}^d V_R^d &= \text{diag}(m_d, m_s, m_b) \equiv \mathbf{D}^d\,.
\end{aligned} \tag{24}$$

From these relations, we conclude that we can change to a basis where the fermion fields are mass eigenstates by performing the rotations

$$\begin{aligned}
&l_L \to V_L^l l_L\ ,\ u_L \to V_L^u u_L\ ,\ d_L \to V_L^d d_L\,,\\
&l_R \to V_R^l l_R\ ,\ u_R \to V_R^u u_R\ ,\ d_R \to V_R^d d_R\,.
\end{aligned} \tag{25}$$

These transformations operate differently on the two components of the $\text{SU}(2)_L$ doublet $q_{L\alpha}$. Considering the quark-gauge interaction terms, which are analogous to those of (17), we see that the NC term is unaffected by this rotation, while the CC term becomes

$$\begin{aligned}
\mathcal{L}^q_{\text{CC}} &= \frac{g}{\sqrt{2}}\overline{u_{L\beta}}(V_L^u)^*_{\beta\alpha}\gamma^\mu(V_L^d)_{\alpha\rho}d_{L\rho}W^+_\mu + \text{H.c.}\\
&= \frac{g}{\sqrt{2}}\overline{u_{L\beta}}\gamma^\mu(V_L^{u\dagger}V_L^d)_{\beta\rho}d_{L\rho}W^+_\mu + \text{H.c.}\,.
\end{aligned} \tag{26}$$

This defines the quark mixing matrix, or Cabibbo-Kobayashi-Maskawa (CKM) matrix [9, 10], $V_{\text{CKM}} \equiv V_L^{u\dagger}V_L^d$. An interpretation of the presence of the quark mixing matrix in the CC interaction is that massive up-type quarks do not interact with the massive down-type quarks of their generation individually, but rather with combinations of down-type quarks defined by the rotation $V_{\text{CKM}}d_L$.

As V_{CKM} is a unitary $n\times n$ matrix, where n is the number of generations, it can be described by n^2 parameters: $n(n-1)/2$ moduli that correspond to mixing angles and $n(n+1)/2$ complex phases. Not all of these phases are physical since the Lagrangian, aside from the CC terms, is invariant under

$$u_{L,R\alpha} \to e^{i\varphi^u_\alpha}u_{L,R\alpha}\ ,\ d_{L,R\alpha} \to e^{i\varphi^d_\alpha}d_{L,R\alpha}\,. \tag{27}$$

Applying these transformations would correspond to performing a global rotation of all quarks followed by a rephasing with $2n-1$ distinct phases. Therefore, $2n-1$ phases in V_{CKM} are unphysical. For the case of $n = 3$, there are three mixing angles θ_{12}, θ_{13}, θ_{23}, and one phase δ. The mixing matrix is commonly parametrized [11] as

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta}\\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}\\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}\end{pmatrix}, \tag{28}$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$. In the case of leptons, due to the absence of RH neutrino fields in the SM, there is no term in $\mathcal{L}_{SM}$ leading to Dirac neutrino masses upon EWSB. Thus, **neutrinos are massless in the SM**. This means that performing a rotation of the neutrino fields such as those in (25) will not affect the mass matrices, which remain diagonal. Then the transformation

$$\nu_{L\alpha} \to (V_L^l)_{\alpha\beta}\nu_{L\beta}\,, \tag{29}$$

ensures that both the NC and the CC terms remain unchanged by the diagonalization of the charged-lepton mass matrix. Consequently, no lepton mixing matrix arises.

III. NEUTRINO MASSES AND MIXING

Motivated by the experimental observation that neutrinos produced in weak interactions are always LH [12], RH neutrinos were not introduced in the SM. However, experimental results of neutrino oscillations have revealed that neutrinos have mass, with three distinct mass-squared differences indicating that at least two massive neutrinos exist [13]. Thus, extensions of the SM must be considered in order to account for neutrino masses.

Neutrino mass terms can be accommodated in the SM by adding sterile RH neutrinos $\nu_{R\alpha}$, which are $\mathrm{SU}(2)_L$ singlets with null hypercharge, so that they do not interact with the gauge fields. Yukawa terms like those of (5) can then be constructed, leading to neutrino mass matrices of the form $\mathbf{M}^\nu = v_{\mathrm{SM}}\mathbf{Y}^\nu$. However, because neutrino masses are significantly smaller than those of other charged fermions (an upper limit of 2 eV has been determined from tritium decay experiments [1]), the common approach is to consider a distinct mechanism for neutrino mass generation, one which can account for such a deviation from the pattern followed by the charged fermions. One way to justify such a distinct mass for neutrinos is to introduce a neutrino Majorana mass term [2].

Since neutrinos are neutral fermions, they can be their own antiparticles,

$$\nu = \nu^C \,, \tag{30}$$

where the antiparticle of a fermion ψ is defined as $\psi^C \equiv C\overline{\psi}^T$. C is the charge conjugation matrix, which obeys $C{\gamma^\mu}^T C^{-1} = -\gamma^\mu$. This property characterizes a Majorana particle. No charged fermion can be a Majorana particle, as particles and antiparticles have opposite charge. Given that

$$\gamma^5(\psi_L)^C = (\psi_L)^C \,, \tag{31}$$

the antiparticle of a LH field is a LH field. This leads to a new mass term for the LH neutrinos of the SM, called the Majorana mass term, defined as

$$-\frac{1}{2}m\overline{\nu_L}\nu_L^C + \mathrm{H.c.}\,. \tag{32}$$

Such a term will break any U(1) symmetry under which ν_L is charged, due to the fact that neutrino fields can no longer be rephased at will. Since $\nu = \nu^C$, the rephasing must be the same for ν and for ν^C, which is inconsistent with the definition of ν^C as an antiparticle. In the SM, this breaks the symmetry of lepton number L, and the Majorana mass term exhibits $\Delta L = 2$.

LH neutrinos are part of an $\mathrm{SU}(2)_L$ doublet. As such, there must be additional fields in the mass term (32) in order to form an invariant of the gauge group. Given the field content of the SM, there is no way to form such an invariant that is also renormalizable. Waiving the requirement of renormalization, the lowest dimensional term which induces Majorana mass terms under EWSB is the five-dimensional Weinberg operator [14],

$$\mathcal{L}_{\mathrm{Wein.}} = c_{\alpha\beta}\frac{1}{\Lambda}\left(\overline{\ell_{L\alpha}^C}\tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger\ell_{L\beta}\right) + \mathrm{H.c.}\,, \tag{33}$$

where the $c_{\alpha\beta}$ are complex coefficients. The presence of a non-renormalizable term such as the Weinberg operator in the effective SM Lagrangian suggests that the SM is not a complete theory, but rather an effective theory which is only valid at low enough energies. We would expect that the full theory manifests itself only at energies of the order of a high-energy scale Λ, considered in (33) as an energy cutoff. Upon EWSB, the Weinberg operator becomes

$$\mathcal{L}_{\mathrm{Wein.}} = \frac{v_{\mathrm{SM}}^2}{\Lambda}\left(c_{\alpha\beta}\overline{\nu_{L\alpha}^C}\nu_{L\beta} + \mathrm{H.c.}\right) + \mathcal{L}_{\mathrm{int.}}^{H\nu}\,, \tag{34}$$

where $\mathcal{L}_{\mathrm{int.}}^{H\nu}$ contains neutrino-Higgs interaction terms. Thus, comparison with (32) yields the Majorana mass matrix

$$\mathbf{M}_{\alpha\beta}^\nu = -\frac{2v_{\mathrm{SM}}^2}{\Lambda}c_{\alpha\beta}\,. \tag{35}$$

A. Lepton mixing

The presence of a neutrino mass matrix, as in the case of quarks, will lead to mixing between generations in electroweak currents upon diagonalization. Because the Majorana mass matrix is symmetric, it can be diagonalized by a single unitary transformation, V_L^ν. As such, in addition to the charged-lepton transformations of (25), we define

$$\nu_{L\alpha} \to (V_L^\nu)_{\alpha i}\nu_{Li}\,, \tag{36}$$

where the fields ν_i have definite masses m_i. The CC term of (19) becomes, under this transformation,

$$\mathcal{L}_{\mathrm{CC}}^\ell = \frac{g}{\sqrt{2}}\left[\overline{\nu_{Li}}\gamma^\mu({V_L^\nu}^\dagger V_L^l)_{i\alpha}l_{L\alpha}W_\mu^+ + \mathrm{H.c.}\right]\,. \tag{37}$$

The lepton mixing matrix, known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [15, 16], is thus defined by

$$U_{\mathrm{PMNS}} \equiv {V_L^l}^\dagger V_L^\nu\,. \tag{38}$$

As aforementioned, the neutrino fields cannot be rephased due to their Majorana nature. This means that, whereas we could remove up to $2n-1$ phases from the CKM matrix, in this case only n phases can be removed. For $n = 3$ generations, the PMNS matrix is then parametrized by three mixing angles and three phases. Namely [1],

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{-i\alpha_1/2} & 0 & 0 \\ 0 & e^{-i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{39}$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$. θ_{ij} are the lepton mixing angles, δ is a Dirac-type phase and $\alpha_{1,2}$ are Majorana phases, arising due to the Majorana character of neutrinos. We conclude that the low-energy neutrino sector of the (effective) SM Lagrangian contains nine parameters: three mixing angles, three phases and three neutrino masses.

Neutrinos are produced and detected in definite flavor states, but they evolve in time according to the values of their masses. Since flavor neutrino states are superpositions of neutrino mass eigenstates with distinct masses, neutrinos may oscillate between different flavors. A determination of the oscillation probability, such as that in [2], concludes that it is not dependent on the neutrino masses m_i, but rather on the mass-squared differences $\Delta m_{ij}^2 = m_i^2 - m_j^2$. As the sign of Δm_{31}^2 is indeterminate (while Δm_{21}^2 is positive), there are two possible orderings of neutrino masses, normal ordering (NO) or inverted ordering (IO), corresponding to

$$\begin{aligned} &\text{NO: } m_1 < m_2 < m_3\,, \\ &\text{IO: } m_3 < m_1 < m_2\,. \end{aligned} \tag{40}$$

Current global fits to all presently available oscillation data [13] are summarized in Table I.

B. The seesaw mechanism

If the SM is an effective theory, then the complete theory which describes particle physics must have additional degrees of freedom, described by fields that are decoupled at low energy. As such, these fields have typically large masses, comparable to the scale Λ in (33).

Extensions of the SM using the seesaw mechanism consider a tree-level interaction between lepton and Higgs fields mediated by new heavy particles. At low energy, this interaction reduces to a four-point vertex of the form $\ell\ell\phi\phi$ (bottom diagram of Figure 1), such as that of (33). Upon EWSB, this term will generate neutrino Majorana masses. For the seesaw mechanism to be possible, the extra fields ψ can be added in two ways. Type I [3–7] and III [17] seesaw mechanisms consider fields with interaction terms $\psi\phi\ell$ (top left diagram in Figure 1), while the type II [18–21] seesaw mechanism considers fields with $\psi\ell\ell$ and $\psi\phi\phi$ terms (top right diagram in Figure 1).

Focusing on the $\psi\phi\ell$ interaction terms, the condition of gauge invariance, given the representation assignments of ϕ and ℓ_L, imposes that ψ must have null hypercharge and must transform as either an $\mathrm{SU}(2)_L$ singlet (type I seesaw) or triplet (type III seesaw). The fields ψ must be fermionic due to angular momentum conservation.

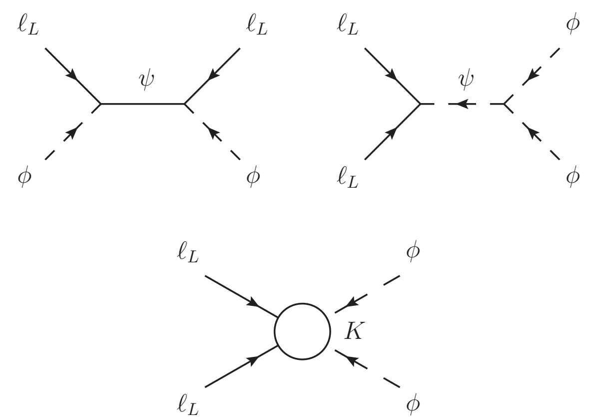

FIG. 1. Tree-level interactions mediated by new heavy particles, in type I and III seesaw mechanisms (top left) and type II seesaw mechanisms (top right), which give rise to the Weinberg operator of (33) at low energies (bottom). Notice that $K \propto c/\Lambda$ [see Eq.(33)].

We consider the type I seesaw mechanism in particular. In this case, the fields ψ are singlets of zero hypercharge. Thus, n_R sterile RH neutrinos ν_{Ri} are added to the SM. Using these fields, we can form a neutrino Yukawa term which gives rise to Dirac masses upon EWSB. We can also build a Majorana mass term for the RH neutrinos. The Lagrangian for this extended theory will therefore be

$$\begin{aligned} \mathcal{L}_{\rm I} =& \mathcal{L}_{\rm SM} + \frac{i}{2}\left[\overline{\nu_R}\gamma^\mu\partial_\mu\nu_R - \overline{\nu_R}\gamma^\mu\overleftarrow{\partial_\mu}\nu_R\right] \\ &- \left[\mathbf{Y}^\nu_{\alpha i}\overline{\ell^0_{L\alpha}}\tilde{\phi}\nu_{Ri} + \frac{1}{2}(\mathbf{M}_R)_{ij}\overline{\nu^C_{Ri}}\nu_{Rj} + \text{H.c.}\right], \end{aligned} \tag{41}$$

where the first term in brackets is the kinetic term for right-handed neutrinos, ℓ^0_L denotes the left-handed lepton fields in the weak basis, as opposed to the mass basis, and $\mathbf{M}_R$ is the Majorana mass matrix. Because the Majorana mass term is invariant under the action of the gauge symmetries of the theory, the value of $\mathbf{M}_R$ is not protected by these gauge symmetries and is free to be arbitrarily large. After EWSB, the neutrino terms in (41) are

$$\begin{aligned} \mathcal{L}^\nu_{\rm I} =& \frac{g}{2c_W}\overline{n^0_L}\gamma^\mu Z_\mu \mathbf{d}^\nu_{\rm NC} n^0_L \\ &+ \left[\frac{g}{\sqrt{2}}\overline{l^0_L}\gamma^\mu W^-_\mu \mathbf{d}_{\rm CC} n^0_L + \frac{i}{2}\overline{n^0_L}\gamma^\mu\partial_\mu n^0_L + \text{H.c.}\right] \\ &- \left[\frac{1}{2}\overline{n^0_L}\mathcal{M}(n^0_L)^C + \text{H.c.}\right]. \end{aligned} \tag{42}$$

Parameter	Best fit $\pm 1\sigma$ (3σ range)	
	Normal	Inverted
$\frac{\Delta m^2_{21}}{10^{-5}\mathrm{eV}^2}$	$7.50^{+0.19}_{-0.17}$ $(7.03 \to 8.09)$	
$\frac{\Delta m^2_{3\ell}}{10^{-3}\mathrm{eV}^2}$	$+2.524^{+0.039}_{-0.040}$ $(+2.407 \to +2.643)$	$-2.514^{+0.038}_{-0.041}$ $(-2.635 \to -2.399)$
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$ $(0.271 \to 0.345)$	
$\sin^2\theta_{23}$	$0.441^{+0.027}_{-0.021}$ $(0.385 \to 0.635)$	$0.587^{+0.020}_{-0.024}$ $(0.393 \to 0.640)$
$\frac{\sin^2\theta_{13}}{100}$	$2.166^{+0.075}_{-0.075}$ $(1.934 \to 2.392)$	$2.179^{+0.076}_{-0.076}$ $(1.953 \to 2.408)$
$\delta/^\circ$	261^{+51}_{-59} $(0 \to 360)$	277^{+40}_{-46} $(145 \to 391)$

TABLE I. Best-fit values and 3σ allowed ranges of the three-neutrino oscillation parameters in the NO and IO cases, obtained from a global fit of current neutrino oscillation data [13]. Note that $\Delta m^2_{3\ell} \equiv \Delta m^2_{31} > 0$ ($\Delta m^2_{3\ell} \equiv \Delta m^2_{32} < 0$) for NO (IO).

In the last equation we can find, in order of appearance, the NC and CC terms, the kinetic terms and the mass terms. We use the definitions

$$\begin{aligned} n_L^0 &\equiv \begin{pmatrix} \nu_L^0 \\ \nu_R^C \end{pmatrix} \ , \ \mathcal{M} = \begin{pmatrix} \mathbf{0} & \mathbf{m}_D \\ \mathbf{m}_D^T & \mathbf{M}_R \end{pmatrix} \ , \\ \mathbf{d}^\nu_{\mathrm{NC}} &= \begin{pmatrix} \mathbf{1}_{3\times 3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \ , \ \mathbf{d}_{\mathrm{CC}} = \begin{pmatrix} \mathbf{1}_{3\times 3} & \mathbf{0} \end{pmatrix} \ , \end{aligned} \tag{43}$$

where $\mathbf{m}_D$ is the $3\times n_R$ Dirac mass matrix and $\mathbf{M}_R$ is the $n_R \times n_R$ Majorana mass matrix. Since the mass matrix $\mathcal{M}$ is symmetric, it can be diagonalized through a unitary transformation

$$\mathbf{U}^\dagger \mathcal{M} \mathbf{U}^* = \mathbf{d}_{n_L} \ , \ \mathbf{d}_{n_L} = \begin{pmatrix} \mathbf{d}_\nu & \mathbf{0} \\ \mathbf{0} & \mathbf{d}_M \end{pmatrix} \ , \tag{44}$$

with $\mathbf{d}_\nu \equiv \mathrm{diag}(m_1, m_2, m_3)$ and $\mathbf{d}_M \equiv \mathrm{diag}(M_1, \ldots, M_{n_R})$, which will correspond to the masses of the three light components and the n_R heavy components, respectively, in the limit $\mathbf{M}_R \gg \mathbf{m}_D$. The mass eigenstates are defined as

$$n_L^0 = \mathbf{U} n_L \equiv \begin{pmatrix} \mathbf{V} & \mathbf{S} \\ \mathbf{R} & \mathbf{T} \end{pmatrix} \begin{pmatrix} \nu_{lL} \\ \nu_{hL} \end{pmatrix} \ , \tag{45}$$

where the matrix $\mathbf{U}$ has been written in block form. Using this form of $\mathbf{U}$ and (44), we are able to determine, for $\mathbf{M}_R \gg \mathbf{m}_D$,

$$\mathbf{d}_\nu \simeq -\mathbf{V}^\dagger \mathbf{m}_D \mathbf{M}_R^{-1} \mathbf{m}_D^T \mathbf{V}^* + \mathcal{O}\left(\frac{v_{\mathrm{SM}}^4}{M^3}\right) \ . \tag{46}$$

The Lagrangian with only the light fields, $\mathcal{L}_\nu^m$, is given by

$$\begin{aligned} \mathcal{L}_\nu^m =& \frac{g}{2c_W} \overline{\nu_{lL}} \gamma^\mu Z_\mu \mathbf{V}^\dagger \mathbf{V} \nu_{lL} \\ &+ \frac{g}{\sqrt{2}} \left[\overline{l_L^0} \gamma^\mu W_\mu^- \mathbf{V} \nu_{lL} + \mathrm{H.c.} \right] \\ &+ \left[-\frac{1}{2} \overline{\nu_{lL}} \left(-\mathbf{V}^\dagger \mathbf{m}_D \mathbf{M}_R^{-1} \mathbf{m}_D^T \mathbf{V}^* \right) \nu_{lL}^C \right. \\ &\left. + \frac{i}{2} \overline{\nu_{lL}} \gamma^\mu \partial_\mu \nu_{lL} + \mathrm{H.c.} \right] \ . \end{aligned} \tag{47}$$

Up to $\mathcal{O}\left(v_{\mathrm{SM}}^2/M^2\right)$, the matrix $\mathbf{V}$ is unitary and we can rotate the light mass eigenstates ν_{lL} by $\mathbf{V}^{-1}$ without adding flavor-changing neutral currents or off-diagonal elements in the kinetic term. We obtain, in this basis, the light-neutrino mass matrix

$$m_\nu \simeq -\mathbf{m}_D \mathbf{M}_R^{-1} \mathbf{m}_D^T \ . \tag{48}$$

The same procedure can be followed in the heavy neutrino sector, to obtain the heavy-neutrino effective mass matrix

$$\mathbf{M}_{\mathrm{eff}} \simeq \mathbf{M}_R + \frac{1}{2} \left[(\mathbf{M}_R^*)^{-1} \mathbf{m}_D^\dagger \mathbf{m}_D + \mathbf{m}_D^T \mathbf{m}_D^* (\mathbf{M}_R^*)^{-1} \right] \ . \tag{49}$$

IV. SINGLE RIGHT-HANDED NEUTRINO A$_4$ MODEL

In this thesis, we explore an extension of the SM constrained by an additional symmetry, invariance under the action of the discrete A$_4$ group, with only one RH neutrino assigned to an A$_4$ singlet and three Higgs doublets assigned to an A$_4$ triplet, $\Phi = (\phi_1, \phi_2, \phi_3) \sim \mathbf{3}$. The A$_4$-invariant Higgs potential $V(\Phi)$ has global minima for four distinct VEV configurations [22],

$$\begin{aligned} h_1 = v(1,0,0) \ , \ h_2 = v(1,1,1) \ , \\ h_3 = v(\pm 1, \eta, \eta*) \ , \ h_4 = v(1, e^{i\alpha_4}, 0) \ , \end{aligned} \tag{50}$$

where $\eta = e^{i\pi/3}, \alpha \in \Re$ and v is a normalization defined by $|h_i|^2 = v_{\mathrm{SM}}^2$. By studying the charged-lepton mass matrix with each of these VEV configurations, we determine $\ell_L \sim \mathbf{3}$ and obtain the Dirac mass matrix

$$\mathbf{m}_D \equiv v_i \mathbf{Y}^{\nu i} = y_\nu \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \ , \tag{51}$$

and from the field products in the neutrino Yukawa term,

$$-\mathbf{Y}^{\nu i} \overline{\ell_L} \tilde{\phi}_i \nu_R + \mathrm{H.c.} \ , \tag{52}$$

for a general VEV configuration (v_1, v_2, v_3). Using (48), and given that in this case $\mathbf{M}_R \to M_R$ (there is only one RH neutrino), we obtain the light neutrino mass matrix at the high energy scale of operation of the seesaw mechanism,

$$m_\nu = -\frac{y_\nu^2}{M_R}\begin{pmatrix} v_1^2 & v_1 v_2 & v_1 v_3 \\ v_1 v_2 & v_2^2 & v_2 v_3 \\ v_1 v_3 & v_2 v_3 & v_3^2 \end{pmatrix}, \tag{53}$$

with two zero eigenvalues. We use the renormalization group equations (RGE) [23] to obtain the neutrino masses at the low energy scale of experimental measurements. The RGE are a set of first-order differential equations which determine the evolution of the couplings of a model relative to the energy scale. For the flavor coupling matrices κ^{ij}, defined by

$$m_\nu = -v_i v_j \kappa^{ij}, \tag{54}$$

we have the RGE

$$\begin{aligned}16\pi^2 \frac{d\kappa^{ij}}{dt} =& -3g^2\kappa^{ij} + 4\sum_{k,l=1}^{n_H} \lambda_{kilj}\kappa^{kl} + \sum_{k=1}^{n_H}\left[T_{ki}\kappa^{kj} + T_{kj}\kappa^{ik}\right] + \kappa^{ij}P + P^T\kappa^{ij} \\ &+ 2\sum_{k=1}^{n_H}\left\{\kappa^{kj}Y_i^{l\dagger}Y_k^l - \left[\kappa^{ik}+\kappa^{ki}\right]Y_j^{l\dagger}Y_k^l + Y_k^{l^T}Y_j^{l^*}\kappa^{ik} - Y_k^{l^T}Y_i^{l^*}\left[\kappa^{kj}+\kappa^{jk}\right]\right\},\end{aligned} \tag{55}$$

where λ_{ijkl} are the couplings of the scalar potential, obtained from the parameters of $V(\phi)$, $t = \log\mu$, μ is the energy scale, and

$$T_{ij} \equiv \mathrm{Tr}\left(Y_i^l Y_j^{l^\dagger}\right), \quad P \equiv \frac{1}{2}\sum_{k=1}^{n_H} Y_k^{l^\dagger} Y_k^l. \tag{56}$$

Y_i^l are the charged-lepton Yukawa matrices. We perform the running of the couplings numerically, in a C++ program which generates random points in the parameter space of $V(\phi)$ then applies the RGE, using a leading-log approximation. The results are tested by comparing the value obtained for

$$R = \frac{\Delta m_{21}^2}{|\Delta m_{3\ell}^2|} \approx 0.03. \tag{57}$$

We consistently obtain $R \sim 10^{-12}$ which, given the limited precision of the program, is compatible with $R = 0$. Thus, the degeneracy between eigenvalues is not sufficiently lifted in our model. This is not surprising, as it has been shown [24] that it is not possible to obtain three non-degenerate neutrino masses in a model with A_4 symmetry, three scalar doublets in a triplet, and three lepton families.

V. SOFTLY-BROKEN A_4 MODEL

The degeneracy between eigenvalues of the neutrino mass matrix seems to be protected by the A_4 symmetry imposed on the model, so that the masses remain nearly degenerate at low energy, after the action of the RGE. Thus, we consider a modified version of the model introducing soft breaking of A_4 at high energies, which should result in a larger range of available values for R. We break A_4 in the scalar sector, with the addition of an A_4 breaking term in the Higgs potential. This is achieved by adding an A_4 triplet of scalar (flavon) fields Ψ which interact with the Higgs multiplet [25]. Considering the scalar potential of Ψ decoupled from the Higgs scalars, the Higgs potential is altered upon SSB of Ψ. In the case $\langle\Psi\rangle \sim (1,0,0)$, the interaction terms lead to the potential

$$V_{\mathrm{SB}}(\Phi) = V(\Phi) + M_S^2\left(2\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2 - \phi_3^\dagger\phi_3\right). \tag{58}$$

This potential has distinct VEV configurations from (50). We determine them by performing a minimization of $V_{\mathrm{SB}}(\Phi)$. Beginning with the VEV

$$h_5 = v(1+2\varepsilon, 1-\varepsilon, 1-\varepsilon), \tag{59}$$

we obtain valid results for the ratio R over a wide range of values of ε, all belonging to the case of normal ordering. We move on to the study of the mixing parameters, namely the three mixing angles θ_{ij} of (39). In this case, we determine that only complex VEV configurations can reproduce experimental results. For $V_{\mathrm{SB}}(\Phi)$, we have

$$h_6 = v(1, \pm\varepsilon e^{i\theta}, \pm\varepsilon e^{-i\theta}), \tag{60}$$

where $\theta \in [-\pi, \pi]$, ε is a function of α, θ, and α is a parameter of $V(\phi)$, and

$$h_7 = v(1, \pm\varepsilon_1 e^{i\theta_1}, \pm\varepsilon_2 e^{i\theta_2}), \tag{61}$$

where $\theta_{1,2} \in [-\pi, \pi]$ and $\varepsilon_{1,2}$ are functions of $\alpha, \theta_1, \theta_2$. h_7 results from fine tuning of $V_{\mathrm{SB}}(\phi)$ and is under an additional conditions which fixes the value of a parameter of the potential. For these VEV configurations, we obtain the results presented in FIG. 2 and 3, where we plot s_{ij}^2

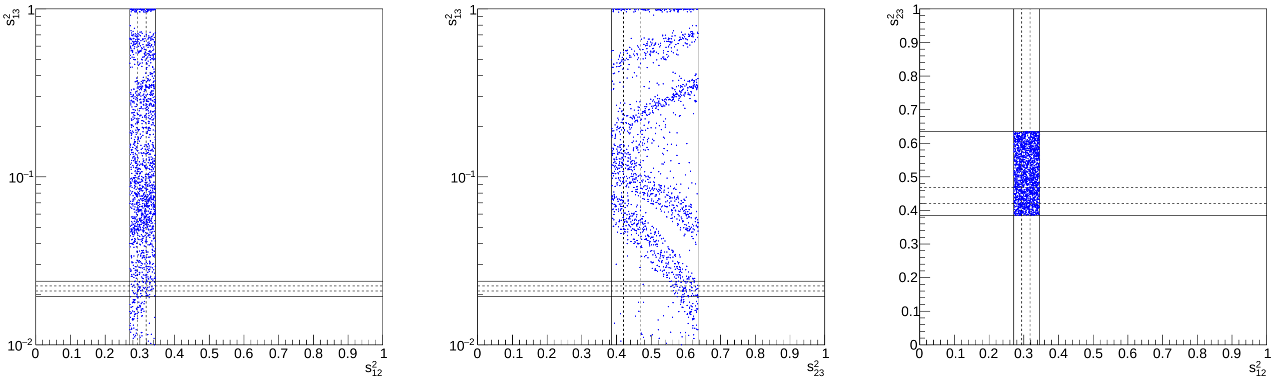

FIG. 2. Values obtained for s_{ij}^2 after RGE running of κ^{ij} and using h_6 for $\langle\Phi\rangle$. The axis corresponding to s_{13}^2 is in logarithmic scale. The solid (dashed) black lines enclose the 3σ (1σ) allowed interval for each variable. In all cases, R, s_{12}^2, and s_{23}^2 are within the 3σ experimental range.

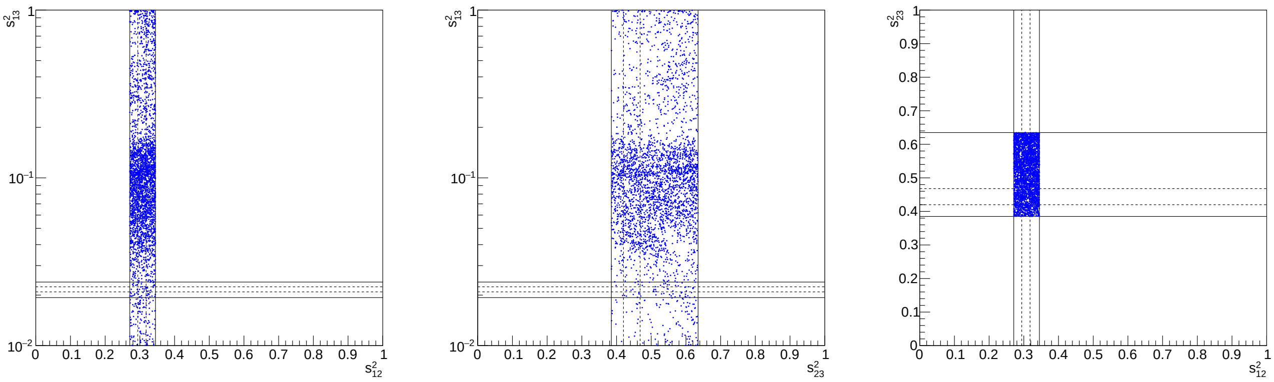

FIG. 3. Values obtained for s_{ij}^2 after RGE running of κ^{ij} and using h_7 for $\langle\Phi\rangle$. The axis corresponding to s_{13}^2 is in logarithmic scale. The solid (dashed) black lines enclose the 3σ (1σ) allowed interval for each variable. In all cases, R, s_{12}^2, and s_{23}^2 are within the 3σ experimental range.

for the three mixing angles, considering only cases with valid R. It can be seen that, in the case of the VEV configurations h_6 and h_7, it is possible to obtain results which are compatible with experimental observations for the lepton mixing angles.

In order to validate the results of the program, we obtain the scalar mass matrices, with eigenvalues corresponding to the squared masses of the scalars of the theory, both neutral and charged. These matrices are obtained from the scalar potential and their eigenvalues are functions of the parameters of $V_{\mathrm{SB}}(\phi)$, so that there will be a region of the parameter space where real scalar masses are obtained, corresponding to the region of validity of the VEV configuration considered. Only results of the program obtained in this region are valid, so we must consider the intersection of the region of validity of the VEV configuration with the region where experimental measurements are reproduced. Taking into account scalar masses, we must also consider the mass of the neutral boson H^0 observed at the LHC [1],

$$m_{\mathrm{H}^0} = 125 \pm 0.24 \text{ GeV}\,, \tag{62}$$

associated with the Higgs boson of the SM, which should correspond to the lightest neutral scalar of the theory. We fit the parameters of our model to these combined conditions and find that a valid region can be found for the lepton mixing angles and for m_{H^0} individually, but not simultaneously. For example, by lifting the constraint on m_{H^0} for h_6, we obtain the results presented in FIG. 4, where all mass values fall below 60 GeV. The results are similar for h_7. Finally we consider a distinct soft breaking of A_4, achieved with a different VEV configuration for the flavon field, $\langle\Psi\rangle \sim (1,1,0)$, to obtain a new Φ VEV and new scalar mass matrices. The results are similar, as the conditions for the mixing angles and the neutral scalar mass continue to fall on incompatible regions of the parameter space of $V_{\mathrm{SB}}(\Phi)$.

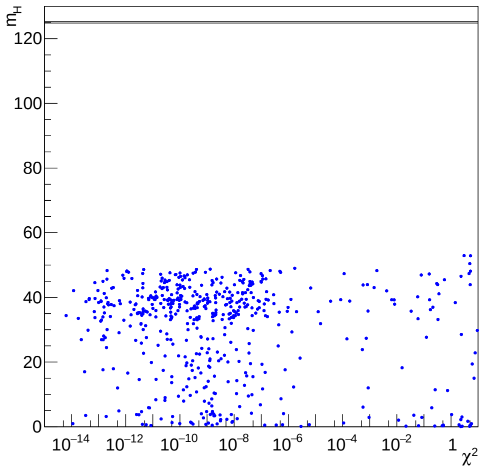

FIG. 4. Values obtained for m_{H^0} as a function of χ^2 (scattered points) using h_8. All neutrino parameters are within the experimental 3σ allowed range. The full black lines enclose the 1σ allowed interval for m_{H^0}.

VI. CONCLUSIONS

In this thesis, we considered an A_4 model with a minimal fermionic extension of one RH neutrino in an A_4 singlet, as well as three Higgs doublets in an A_4 triplet. We verified that this model could not reproduce measured neutrino mass-squared differences and altered it by adding a term in the Higgs potential which breaks A_4 softly. In this case, we were able to reproduce the experimental value of R, defined in (57), but could not find a region of the parameter space where the VEV configuration used was valid and both lepton mixing angles θ_{ij} and the Higgs mass m_{H^0} were compatible with experimental results. We were only able to satisfy these conditions individually, which suggests that the regions where each condition is satisfied do not intersect.

Although the results obtained indicate that the model must be discarded, they are not exhaustive as some avenues of exploration remain. As was indicated, we considered the softly-broken potential $V_{\mathrm{SB}}(\Phi)$ obtained with the flavon VEV configuration $\langle\Psi\rangle \sim (1,0,0)$, as well as $\langle\Psi\rangle \sim (1,1,0)$. In each case, the VEV configurations for Φ and the scalar mass matrices were distinct. Although these cases did not obtain valid results, we can still explore the remaining VEV configurations of (50). Given that the issue is an incompatibility between different conditions on the parameter space, and each condition can be satisfied individually, we can see that by taking a distinct scalar potential, leading to distinct conditions, these may overlap in some way and lead to valid results.

Conclusions regarding the validity of the model of this thesis are thus not definitive, as the discussion above reveals further work to be performed on the topic before the model presented must be discarded.

ACKNOWLEDGEMENTS

The author would like to thank her supervisor, Prof. Dr. Filipe Rafael Joaquim, for all his support and guidance, as well as her family, her friends Mariana, Martim and Rita, and especially Zelda and Madalena, who all helped in different and equally important ways.

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