Fault-Tolerant Control for Terminal Rendezvous in Active Removal of Space Debris

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Abstract—This work addresses Passive Fault-Tolerant Control design techniques for close range maneuvers in Active Debris Removal (ADR) mission. The contributions of this work include: the development of a 6 Degree of Freedom (DoF) linear and nonlinear models that will be used for the control system design and analysis, respectively; the design of decoupled relative linear and angular motion controllers under nominal conditions, i.e., considering neither model uncertainties nor faults, by using $H_{\infty}$-control techniques; the in-depth assessment of the impact of the design parameters on the characteristics of the closed-loop system; the development of a mathematical model for parametric uncertainties and actuator faults; the design of controllers by using $H_{\infty}$-control techniques, for the uncertain models of the healthy and faulty plants; the evaluation of the robust stability and performance of the closed-loop system by means of $\mu$-analysis and of worst-case scenario simulations, in the presence of actuator faults; a thorough discussion on the benefits and limitations of each of the controller designs, as well as directions for future work.

I. INTRODUCTION

Recent studies indicate that the evolution of orbital debris population in low Earth orbit (LEO) has reached a critical point, such that the environment is unstable and population growth inevitable [1, 2]. Therefore, in the mid-term, collisions will be the main source of new debris, leading to a space debris chain reaction, rendering some orbital regions unfit to perform space activities. Thus, in order to protect the near-Earth environment for future space missions, ambitious measures must be considered, such as the ADR mission developed by European Space Agency (ESA), called e.deorbit, that would target an ESA-owned neglected satellite in LEO, the ENVISAT, capture it, and then safely burn it up in a controlled atmospheric reentry [3].

Since not every planned mission occurs as expected, one fundamental feature present in this type of mission is safety. In particular, if the chaser spacecraft collides either with the target or other debris, it creates more debris and accelerates the collision chain reaction.

Thus, a safety assessment is particularly important when considering Close Range Rendezvous (CRR) capture mechanisms, such as the robotic arm, due to the fact that the target is uncooperative and that there is a small distance between the two spacecraft. Hence, the controllers of such systems must be able to react to unusual events, such as external perturbations or component malfunctions, by stabilizing the system, while fulfilling the performance requirements, in order to guarantee overall safety. The motivation for this paper is therefore the study of fault-tolerant control systems for the CRR stage of ADR missions, with and without internal malfunctions, using robust control techniques and exploiting actuators redundancy.

The organization of this paper is as follows. In section II the 6 Degree of Freedom (DoF) model describing the relative dynamics and kinematics between two spacecraft is developed. Section III provides the formulation of the control design problem with 1 DoF and 2DoF controllers, and an in-depth analysis of the impact of the weight parameters in the overall performance of the controlled system. Section IV describes the framework adopted for parametric uncertainty and actuator fault modeling, and the process followed for the fault-tolerant controller design and its robust analysis by means of $\mu$-analysis. Section V compares in simulation the controllers designed.

II. RELATIVE DYNAMICS AND KINEMATICS

This work focuses on the orbital relative motion between two spacecraft, during the CRR stage of an ADR mission. The relevant coordinate or reference frames that will be used throughout this work are the Inertial Frame, the Local Vertical, Local Horizon (LVLH), the ENVISAT body-fixed frame, and the chaser body-fixed frame, for further information the reader is redirected to [4].

A. Relative Position Dynamics and Kinematics

This section provides a description of the relative position dynamics and kinematics between two spacecraft, following the approach provided in [5]. The assumption posed is that the target is only subject to the action of the gravity field of the Earth, while the chaser is under the action of the gravity force and also subject to forces from thrust actuators. The resulting system of linear time-varying differential equations, summarized in (1), is described in the LVLH frame and is valid for an arbitrary relative motion between the chaser and the target.

\[
\begin{aligned}
\ddot{x} - \omega^2 x - 2\omega \dot{z} - \dot{\omega} z + \frac{\mu}{r^3_t} x &= \frac{1}{m_c} F_x \\
\ddot{y} + \frac{\mu}{r^3_t} y &= \frac{1}{m_c} F_y \\
\ddot{z} - \omega^2 z + 2\omega \dot{x} + \dot{\omega} x - 2\frac{\mu}{r^3_t} z &= \frac{1}{m_c} F_z
\end{aligned}
\] (1)
Since the target orbital period is several times larger than the time window that will be studied, and due to the low eccentricity of the orbit, the target orbit will be considered nearly circular. Herewith, the orbital angular rate will be assumed to be constant, meaning that \( \dot{\omega} = 0 \), and that \( \frac{\mu}{r^3} = \omega^2 \), leading to:

\[
\begin{align*}
\ddot{x} - 2\omega \dot{y} &= \frac{1}{m_c} F_x \\
\ddot{y} + \omega^2 y &= \frac{1}{m_c} F_y \\
\ddot{z} + 2\omega \dot{z} - 3\omega^2 z &= \frac{1}{m_c} F_z
\end{align*}
\]

This formulation of the relative motion obtained is three simultaneous second-order, linear, coupled differential equations, also referred as Clohessy-Wiltshire equations [6]. The system can now be formulated in state space form, with the state vector \( \mathbf{x}_p = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T \) and the input vector \( \mathbf{F} = [F_x, F_y, F_z]^T \), as follows:

\[
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 2\omega & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3\omega^2 & -2\omega & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_p \\
\dot{x}_p \\
\dot{y}_p \\
\dot{z}_p \\
\ddot{x}_p \\
\ddot{y}_p \\
\ddot{z}_p
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{1}{m_c} \\
\frac{1}{m_c} \\
\frac{1}{m_c} \\
\frac{1}{m_c} \\
\frac{1}{m_c} \\
\frac{1}{m_c}
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_p \\
\dot{x}_p \\
\dot{y}_p \\
\dot{z}_p \\
\ddot{x}_p \\
\ddot{y}_p \\
\ddot{z}_p
\end{bmatrix}
\]

or, in compact form

\[
\dot{\mathbf{x}}_p = \mathbf{A}_p \mathbf{x}_p + \mathbf{B}_p \mathbf{F}
\]

Considering \( \mathbf{x}_a = [\phi, \theta, \psi, \omega_x, \omega_y, \omega_z]^T \), the following linear state space description for the attitude dynamics was obtained

\[
\begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix}
= \mathbf{J}_a^{-1} \mathbf{H} \mathbf{x}_a + \mathbf{J}_a^{-1} \mathbf{N},
\]

where, \( \mathbf{J} \) is the inertia matrix, \( \omega_0 \) is the target orbital angular velocity. This linear model can be further condensed into

\[
\begin{bmatrix}
\dot{\omega}_x \\
\dot{\omega}_y \\
\dot{\omega}_z
\end{bmatrix}
= \mathbf{A}_d \mathbf{x}_a + \mathbf{B}_d \mathbf{N}.
\]

The linear kinematics can be formulated as

\[
\dot{\mathbf{x}} = \mathbf{Q} \mathbf{u}
\]

that can be condensed as a state space description, assuming that \( \mathbf{x}_a = [\phi, \theta, \psi, \omega_x, \omega_y, \omega_z]^T \) in the form

\[
\begin{bmatrix}
\dot{\mathbf{x}}_a \\
\mathbf{F}
\end{bmatrix}
= \mathbf{A}_d \mathbf{x}_a + \mathbf{B}_d \mathbf{N}
\]

Considering a similar state vector, it is possible to define a linear attitude model by combining (4) and (10) to get

\[
\begin{bmatrix}
\dot{\mathbf{x}}_a \\
\mathbf{F}
\end{bmatrix}
= \mathbf{A}_d \mathbf{x}_a + \mathbf{B}_d \mathbf{N}
\]

### B. Attitude Dynamics and Kinematics

The two most widely used representations of the attitude problem are the quaternions and the Euler angles. If quaternions [7] are used, they would eliminate the known issues with the Euler angles singularities [8]. However, with the appropriate definition of reference frames, one can guarantee that the reference trajectory lies far from the singularities. Also, the Euler angles provide a direct physical interpretation of the attitude of the vehicle, while the quaternion description does not lend itself to a simple visualization of the attitude of the vehicle. Therefore, it is chosen to use Euler angles to define the chaser attitude.

The transformation between the two coordination systems, LVLH and chaser body frame, is represented by the sequence of rotations 2-3-1, which represents the Euler angle sequence pitch, yaw and roll (\( \theta, \psi, \phi \), respectively) around the axes \( y, z \) and \( x \) of the chaser body frame [9]. This rotation sequence is adopted instead of the more popular sequence 3-2-1, because the latter would reach a singularity when simulating one of the scenarios considered in this work. Since, in that case, the chaser revolves around the \( y \)-axis, and it would eventually reach a singularity at \( \theta = \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \).

### C. Overall Linear Time-Invariant Model

By combining the models described by (4) and (10), a state space model that describes the motion of the center of mass of the chaser with respect to the target, as well as the attitude of the chaser with respect to the target’s orbital frame is obtained. [11] formulates this state space description, which considers as state vector \( \mathbf{x} = [\mathbf{x}_p, \mathbf{x}_a, \phi, \theta, \psi, \omega_x, \omega_y, \omega_z]^T \), as input vector \( \mathbf{u} = [F_x, F_y, F_z, N_x, N_y, N_z]^T \), and as output vector \( \mathbf{y} = [x_p, \dot{x}_p, \phi, \theta, \psi, \omega_x, \omega_y, \omega_z]^T \), where \( I_{12 \times 12} \) is an 12×12 identity matrix:

\[
\begin{bmatrix}
\dot{\mathbf{x}}_a \\
\mathbf{y}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{A}_d & \mathbf{B}_d \\
\mathbf{A}_a & \mathbf{B}_a
\end{bmatrix}
\begin{bmatrix}
\mathbf{x} \\
\mathbf{u}
\end{bmatrix}
\]

which can be equivalent written as

\[
\begin{bmatrix}
\dot{\mathbf{x}}_a \\
\mathbf{y}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{A}_m & \mathbf{B}_m \\
\mathbf{C}_m & \mathbf{D}_m
\end{bmatrix}
\begin{bmatrix}
\mathbf{x} \\
\mathbf{u}
\end{bmatrix}
\]

### III. Nominal Controller

This section is devoted to the controller design under nominal circumstances, i.e., the plant without uncertainties nor faults, and for each of the motions separately, since the system can be decoupled. An assessment of the benefits and disadvantages of a 2 DoF controller vs a 1 DoF controller, as well as of the impact of the design weights on the performance of the closed-loop system is also provided.
Prior to the actual controller design it is necessary to establish the interconnection structure. The layout of this structure, as well as the definition of the elements contained within it, will determine the success of the controller. The structure used in this nominal (i.e., non-faulty) scenario is depicted in Figure 1 and it is valid for both the relative in-plane and out-plane position motion, as well as the attitude dynamics and kinematics. It is important to take into consideration that the shapes and parameter values of the elements may differ depending on the motion.

For the weight selection, the approach followed is that of noticing that the closed-loop characteristics are similar to a 2nd order system. These characteristics are chosen by taking into consideration the desirable performance of the closed-loop system. $W_{\text{ref}}$ represents the closed-loop transfer function with ideal handling qualities. Since the desired closed-loop behavior is similar to a 2nd order system, $W_{\text{ref}}$ has the following shape

$$W_{\text{ref}} = \frac{\omega^2}{s^2 + 2\zeta \omega s + \omega^2}. \quad (13)$$

The damping coefficient $\zeta$ in (13) is directly related to the desired overshoot, $M_P$, of the step response [10]. The model frequency can be found from the settling time $t_s$ and damping coefficient [11]. The control weight $W_u$ needs to limit high frequency activity and ensure low-frequency tracking. Hence, $W_u$ needs to be a high-pass filter. A first order weight is usually a low-pass filter (see (17)), and since in low frequencies it is desired that controlled system tracks exactly the reference signal, then $A$ is set to 1.

$$W_r = \frac{1}{s + A\omega} \quad (17)$$

$$W_n = \frac{As + \omega}{s + M\omega} \quad (16)$$

Since the measurements will be corrupted with noise, the weight $W_n$ is set to have its peak value at high-frequency, where the noise has an allowed magnitude $A$.

The interconnection diagram presented in Figure 1 will be referred to as the generalized plant and will be denoted by $P$. The inputs of the generalized plant can be separated into two groups, the commanded signal $u$ sent by the controller; and the normalized exogenous signals $w$, i.e. the reference command $w_r$, measurement noise $w_n$ and actuator disturbances $w_d$. Similarly, the outputs of the generalized plant can also be divided into two groups, the measurements $\nu$, i.e. the signals that will enter the controller, more specifically the control error signal $e$; and the performance outputs $z$. The latter category can be further divided into tracking performance error $z_r$ and actuator effort $z_u$. Hence, taking this, and the element interconnection of Figure 1 into consideration, one can formulate the generalized plant as

$$F_l(P, K) = \begin{bmatrix} -W_eGw_d & -W_eW_{\text{ref}}w_r & 0 & -W_eG \\ 0 & 0 & 0 & 0 \\ -Gw_d & -W_{\text{ref}}w_r & W_n & -G \end{bmatrix}$$

and to find $K$, one must minimize $\gamma$ such that

$$\|F_l(P, K)\|_\infty < \gamma \quad (20)$$

Thus, all the information about the system, weights, and interconnections is present in the closed-loop transfer function, and it is used by the $H_\infty$-synthesis to design the controller. Hence, the performance and robustness properties of the controller designed is directly related to the weighting functions used in the interconnection diagram.
A. 2 DoF Control Design

Due to demanding conditions on the CRR stage, a 2 DoF controller is now analyzed, since feedforward control generally has a positive contribution to the control performance. Hence, this controller has access not only to the tracking error, but also to the reference signal. Thus, the controller has two input channels and can be defined in the following form \[ K = [K_1 \quad K_2], \] 

where \( K_1 \) is the feedforward controller and \( K_2 \) is the stabilizing feedback controller. Although this controller structure brings benefits in terms of performance, the controller has higher complexity when compared with the 1 DoF counterpart.

The new controller input is added to the interconnection diagram as a measurement output, i.e. the measurement vector \( \nu \) is augmented into \( \nu = [r \quad e]^T \), leading to the diagram of Figure 2.

![Figure 2: Signal-based 2DoF interconnection diagram](image)

Considering the representations in Figure 1 and Figure 2, this 2 DoF control design can also be seen as being built upon the 1 DoF control design studied formerly. Hence, the previous evaluation of the impact of each weight represented in the diagram is still relevant, with the major difference being the reference weight \( W_r \). This weight \( W_r \) will not only serve the purpose of narrowing down the bandwidth of the reference signal, but also as a mechanism to manipulate the usage of the feedforward controlling term.

B. In-depth Parameter Analysis

The motion considered is the relative position motion on the out-plane, i.e. the relative motion on the perpendicular axis to the orbit plane, since this is the simpler motion, given that it is a SISO plant, and all the control setup will be performed in that context and later reused for the higher complexity motions.

Consider the tracking error \( W_e \), as in (15). There are three main parameters to be varied, namely \( A \), \( M \), and \( \omega \). Figure 3 shows that there is a correlation between \( A \) and the steady state error of the controlled system, a smaller \( A \) leads to a smaller steady state error. However, decreasing \( A \) increases \( \gamma \), i.e. the \( \mathcal{H}_\infty \)-norm, and it is desired that this value remains smaller than 1. If \( \gamma \) is larger than 1, then there are no guarantees that the performance requirements are fulfilled [5].

![Figure 3](image)

A similar event occurs when changing the high-frequency gain, i.e. a larger \( M \) translates into a higher overshoot of the system in a time response to a unit step, as well as a decrease in \( \gamma \), as illustrated in Figure 4. The main parameters influenced by the modification of the filter crossover frequency are the rise and settling times.

![Figure 4](image)

Consider \( W_u \) as in (14). Hence, similarly to the previous weight, there are three main parameters that can be used to tune the controlled system. Changes in the low-frequency gain, i.e. parameter \( A \), influence mainly the maximum actuation commanded by the controller. A larger value of \( A \) leads to a larger maximum actuation, as depicted in Figure 5. However, one must take into consideration the maximum allowable actuation and that a larger actuation over a longer period of time (lower actuation frequencies) leads into higher velocities, which will create a larger overshoot. One must also notice that on the other side of the spectrum. On the other hand a actuation signals with smaller magnitudes contribute to slower responses, thus not being able to fulfill the performance requirements.

Consider now that the controlled system is subject to measurement noise, which introduces high-frequency content errors. When this error signal is feedback into the controller, it is seen as error leading to high-frequency actuation. During the controller design, the parameter that regulates this high-frequency actuation is \( M \) in weight \( W_u \). To analyze the
influence of this parameter, the controlled system is subject to a step and evaluated during steady state. By inspecting Figure 6, it can be concluded that a larger value of $M$ leads to a lower actuation variance.

The filter crossover frequency of $W_u$ sets the frequency at which the controller reduces its sensitivity to the errors, i.e. $\omega$ influences the barrier between low-frequency and high-frequency components, recalling that, at high frequencies, it is needed to penalize the control so as not to track noise. As seen in Figure 7, the actuation variance in the high-frequency region increases as $\omega$ also increases. This is due to the fact that, for larger values of $\omega$, the controller tries to minimize the error in a larger bandwidth. As a consequence, an increase effort is put on the actuation.

Considering again a controlled system subject to noise measurements, the main parameter to take into consideration when analyzing weight $W_n$ is gain $A$. As seen in Figure 8 with the increase of $A$, there is a higher noise rejection rate by the controller, based on the lower actuation variance of the high-frequency components. However, with this higher noise rejection rate, there is also an higher control effort, that can be seen through the maximum actuation.

C. Controller Comparison

From the time domain comparison, one can evaluate the performance of each output channel of the closed-loop system to the different reference inputs. As stated previously, one of the most important actions that the spacecraft has to perform in the out-plane motion is to avoid collision with the target, if needed. Hence, the controllers are compared to a step reference signal, and by considering sensors with and without measurement noise. Recalling that the design of the signal-based controllers have an element, $W_{ref}$, which models the closed-loop system ideal model, this weight will also be taken into account in the analysis.

Table I: Step responses characteristics of the systems interconnected with the different controllers proposed and comparison with the reference model

<table>
<thead>
<tr>
<th></th>
<th>Rise time</th>
<th>Overshoot</th>
<th>$\frac{1}{N} \sum e^2$</th>
<th>$t_{RMS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1DoF</td>
<td>19.4 s</td>
<td>4.63 %</td>
<td>1.78</td>
<td>0.0785</td>
</tr>
<tr>
<td>2DoF</td>
<td>21.1 s</td>
<td>3.97 %</td>
<td>1.56</td>
<td>0.0319</td>
</tr>
<tr>
<td>$W_{ref}$</td>
<td>17.1 s</td>
<td>5.00 %</td>
<td>——–</td>
<td>——–</td>
</tr>
</tbody>
</table>

Table I shows the step response characteristics obtained with the different controllers. Although the 1 DoF controller has the best performance when compared with the remaining controller, in terms of rise time, when one analyzes the when one analyzes the Root Mean Square (RMS) of the actuation signal in steady state zone, it can be concluded that a much larger control effort is request to the actuators,
due to propagation of the measurement noise, i.e. the 1 DoF-controller is more sensitive to external perturbations, at least for this case.

IV. FAULT-TOLERANT CONTROLLER

This section is devoted to the controller design that takes into consideration the model uncertainty, more specifically parametric uncertainties, as well as actuator faults. Thus, in order to use a similar controller design methodology to the one described in the previous section, it is required to model both the parametric uncertainties, and the actuator faults.

A. Model Uncertainty

The unstructured uncertainties used to described unmodelled or neglected system dynamics usually occur in the high-frequency range (although not exclusively) and may include time delays, parasitic coupling and other nonlinearities. However, dynamic perturbations may also be caused by inaccurate description of component characteristics, torn-and-worn effects on plant components, or event shifting of operating points. Such perturbations may be represented by deviations of certain system parameters over either complex or real values. This type may have a significant component in the low-frequency range [15].

Consider the model mass, $m_c$, as a parametric uncertainty. It is only known that $m_c \in [m_{\text{min}}, m_{\text{max}}]$, where $m_{\text{min}}$ and $m_{\text{max}}$ are the minimum and maximum values of the parameter, respectively. Let the nominal value $\bar{m}_c$ be chosen as $\bar{m}_c = \frac{m_{\text{min}} + m_{\text{max}}}{2}$ and the scaled error $\Delta$ as

$$\Delta = \frac{m_c - \bar{m}_c}{m_{\text{max}} - \bar{m}_c}$$

Implying that

$$m_c = \bar{m}_c + W\Delta, \quad \text{with} \quad W = m_{\text{max}} - \bar{m}_c,$$

and the class of uncertainties

$$\Delta := \{\Delta \in \mathbb{R} | -1 < \Delta < 1\}$$

In order to proceed with the adopted design methodology, one has to rewrite this model in the form illustrated in Figure 9. This is most easily performed by introducing auxiliary signals that form and into the uncertainty blocks.

B. Actuator Fault Model

Although faults may impact different system-components, this dissertation will focus on the study of actuator faults, which will require an appropriate fault model. Consider that an actuator fault can be interpreted as a modification of the system input vector $u$. Through the reviewed literature [16], four main types of faults can be identified: loss-of efficiency, lock-in place, hard-over, and floating around the trim fault, and with the exception of the latter, the remaining fault types can be fully modeled with the integration of two scalar parameters.

The input generated by a faulty actuator can be described by $u_a$, as

$$u_a = \mathcal{E}u + \mathcal{B},$$

where $u$ is the plant input received from the controller, and $\mathcal{E}$ and $\mathcal{B}$ characterizes the effectiveness and the bias parameter, respectively. This was performed for a single output. However, it can be readily adapted to multiple outputs, by transforming $\mathcal{E}$ and $\mathcal{B}$ into diagonal matrices, i.e. $\mathcal{E} = \text{blkdiag}\{\mathcal{E}_1, ..., \mathcal{E}_n\}$ and $\mathcal{B} = \text{blkdiag}\{B_1, ..., B_n\}$.

By analyzing (25) and Figure 10, the bias parameter can be interpreted as a constant perturbation in the actuation. Although not explicitly stated before, this type of fault was already taken into consideration in previous control designs, such as in Figures 2, through the inclusion of the exogenous input $w_d$. Thus, from this point forward, the main focus will be on actuator efficiency faults, i.e. $u_a = \mathcal{E}u$.

Furthermore, the efficiency fault can be interpreted as a parametric uncertainty. Hence, $\mathcal{E}$ can be rewritten as $\tilde{\mathcal{E}} = \mathcal{E} + W_{\mathcal{E}}\Delta_{\mathcal{E}}$, where $\mathcal{E}$ is the nominal value, $\Delta_{\mathcal{E}}$ is a nominal uncertainty, and $W_{\mathcal{E}}$ sets the maximum limits for $\mathcal{E}$.

Due to the physical limitations of thrusters, the associated operating region is comprised in $[0, F_{\text{th,max}}]$. Thus, by considering only a single thruster, the logical deduction would be to chose the $\tilde{\mathcal{E}} = 1$, leading to the thruster actuation described as

$$u_T = \mathcal{E}u = (\tilde{\mathcal{E}} + W_{\mathcal{E}}\Delta_{\mathcal{E}})u = u + W_{\mathcal{E}}\Delta_{\mathcal{E}}u,$$

where $W_{\mathcal{E}}$ can be interpreted as the percentage of the maximum efficiency loss of the thruster, e.g. a total shutdown is modeled by setting $W_{\mathcal{E}} = -1$. However, in this situation, $\Delta_{\mathcal{E}} \in [0, 1]$. Thus an other set of parameters has to be chosen in order to use the control design methodology described in Section III. Two options are available, as described next. One may set the interval of $\Delta_{\mathcal{E}}$ to $[-1, 1]$, while maintaining the other parameters. However, this would provide to the system the information that it could use a higher actuation than the allowable one. The other option is to modify $W_{\mathcal{E}}$ and $\tilde{\mathcal{E}}$ in order model the allowable actuation, while setting $\Delta_{\mathcal{E}} \in [-1, 1]$, by

$$\tilde{\mathcal{E}} = \frac{u_n + u_f}{2u_n} \quad \text{and} \quad W_{\mathcal{E}} = \frac{u_n - u_f}{2u_n},$$

Figure 10: Fault model using two scalar parameters.
where \( u_n \) is the maximum allowable actuation in a nominal scenario and \( u_f \) is the maximum allowable actuation in a scenario with an efficiency fault. The disadvantage of this approach is that the nominal scenario is no longer the fault-free plant. However, this does not have a high impact on the control design, since the controller is designed to minimize the worst-case scenario and not only the nominal case.

In order to reduce the calculations complexity and without loss of generality, assume that the axes of the chaser reference frame are orthogonal to the surfaces where the thrusters are located. Assuming that there are 24 individual thrusters, it is not feasible to include each individual fault in the model, since the input of mathematical models derived in Section II are total forces and total torques, which are represented in the LVLH and chaser body frame, respectively. Thus, the next step is to derive an expression of total forces and total torques from the input of mathematical models.

The first stage is to connect the thrusters whose actuation is aligned. It is plausible to say that thrusters whose actuation is aligned and yet opposed will never get triggered at the same time, since in regards to total force and total torque they would cancel each other out and lead to a higher fuel consumption.

Thus, when applied to the chaser, it reduces the initial 24 to 12 inputs, which will be located in the corners of 3 virtual orthogonal surfaces to the axes of the chaser reference frame, i.e. there will be 4 actuators per plane in the reference frame. It is possible to verify that the total force per axis is given by

\[
F_{ca} = \sum_{i=1}^{4} F_{ai}, \quad \text{with} \quad F_{ai} \in [-F_{th_{max}}, F_{th_{max}}],
\]

where \( F_{ai} \) represents the force exerted by actuator \( i \) aligned with the \( a \)-axis, with \( a \in \{x, y, z\} \). Thus, it can be said that \( F_{ca} \in [-4F_{th_{max}}, 4F_{th_{max}}] \). Now consider that a total shutdown of one actuator occurs, i.e. the worst-case scenario of an efficiency fault, where one actuator does not provide any output. Thus, the total force in the faulty axis is effectively computed by using only the three healthy actuators. Assume that the faulty actuator is aligned with \( Z_c \). Then, the deteriorated total force in this axis, \( \tilde{F}_{ca} \), will be contained in \([-3F_{th_{max}}, 3F_{th_{max}}] \), which is similar for the remaining axes. Thus, by recalling \( \bar{F}_{ci} \) and \( \Delta_c \), the deteriorated total force per axis, \( \tilde{F}_{ci} \), can be written as

\[
\tilde{F}_{ci} = \left( \frac{1}{2} \frac{4F_{th_{max}} + 3F_{th_{max}}}{4F_{th_{max}}} + \frac{1}{2} \frac{4F_{th_{max}} - 3F_{th_{max}}}{4F_{th_{max}}} \right) F_{ci} = \left( \frac{7}{8} + \frac{1}{8} \Delta_{ci} \right) F_{ci},
\]

with \( \Delta_{ci} \in [-1, 1] \) and \( \forall i = \{x, y, z\} \).

It is highlighted that the use of \( \bar{F}_{ci} \) for all three axes implies that there can be at most 1 actuator fault per axis. Thus, this notation brings some conservatism into the problem, since it also encapsulates the scenarios where there are multiple faults in different axis at the same time.

As discussed previously, the input for the relative position motion is the force applied in the chaser written in the LVLH reference frame, and the input vector from \( \bar{F}_{ci} \) is described in the chaser reference frame. Thus, a rotation between frames is required, (please refer to [9] for further information), yielding

\[
\tilde{F}_o = R^c_o \bar{F}_c = \frac{7}{8} \bar{R}_c^o \bar{F}_c + \frac{1}{8} \bar{R}_c^o \Delta_c \bar{F}_c = \frac{7}{8} \bar{F}_o + \frac{1}{8} \Delta_c \bar{F}_o,
\]

where \( \Delta_c \) denotes the possible set of actuator faults as described previously and with \( \| \Delta_o \|_\infty \leq 1 \).

The first term of the rotation that yields \( \frac{7}{8} \bar{F}_o \) is simple and straightforward. However the second term, some manipulation has to be made in order to obtain the results shown. This manipulation involves the use of Direct Cosine Matrix (DCM) properties, the inverse rotation matrix of \( R_c^o \) is its transpose, thus \( R_c^o R_c^o \) is equivalent to an identity matrix, yielding

\[
R_c^o \Delta_c \bar{R}_c^o = R_c^o \Delta_c R_c^o \bar{R}_c^o = R_c^o \Delta_c R_c^o \bar{R}_c^o F_o.
\]

It is also important to recall that the output vector of a rotation performed by this type of matrix, i.e. DCM, retains the norm of the original vector, and since by design each possible \( \Delta_c \) is contained in a single axis and \( \| \Delta_o \| \leq 1 \), thus the rotation of the set \( \Delta_c \) to the LVLH reference frame will also retain this characteristics. Due to these described properties, the term \( R_c^o \Delta_c R_c^o \) can be bounded, leading up to

\[
R_c^o \Delta_c R_c^o \leq \Delta_o.
\]

The next step is to introduce the fault model in the relative attitude motion, which can be performed with a similarly approach. Consider that the location of the thrusters is given by the vector \( I_i \) for the actuator \( i \), where \( I_i = [\pm l, \pm l, \pm l] \). Then the total torque per axis will be given by

\[
N_{ca} = \sum_{i=1}^{8} (I \times F_{ci}),
\]

with \( \| F \|_i \in [-F_{th_{max}}, F_{th_{max}}] \) and \( \forall a = \{x, y, z\} \).
where $F_i$ represents the force exerted by actuator $i$ aligned with the axis $a$. Thus, it can be stated that $N_{ca} \in [-8lF_{th_{max}}, 8lF_{th_{max}}]$. Considering now that one actuator failure occurs, similarly to the faulty total force in Equation (29), the total torque per axis including the loss of a possible actuator is given by

$$\tilde{N}_{ca} = \left( \frac{1}{2} \frac{15lF_{th_{max}}}{8lF_{th_{max}}} + 1 \frac{lF_{th_{max}}}{2lF_{th_{max}}} \Delta N_a \right) N_{ca},$$

(36)

with

$$\Delta N_a \in [-1, 1] \quad \text{and} \quad \forall a = \{x, y, z\}$$

(37)

C. Controller Design

By utilizing the parametric uncertainty and fault representations obtained previously one obtains the model described in $\mathbf{38}$ where $w = \Delta z$.

$$\begin{bmatrix} z \\ y \\ u \end{bmatrix} = G \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix},$$

(38)

And by applying simple alterations to the interconnection diagram presented in Section III one obtains the diagram depicted in Figure 12. That will be used in the $\mathcal{H}_\infty$ control theory to design the fault-tolerant controller.

![Figure 12: Signal-based 2 DoF interconnection diagram of an uncertain and faulty system](image)

Since both the parametric uncertainty and actuator faults are formulated as model uncertainty, then

$$\begin{bmatrix} w_\Delta \\ w_f \end{bmatrix} = \begin{bmatrix} \Delta u \\ 0 \\ 0 \\ \Delta \varepsilon \end{bmatrix} \begin{bmatrix} z_\Delta \\ z_f \end{bmatrix},$$

(39)

where $\Delta u$ denotes the normalized parametric uncertainty present in the model, and $\Delta \varepsilon$ denotes the actuator loss of efficiency fault.

Using this approach two controllers were designed: the Fault-Tolerant (FT) controller, that is designed similarly to the Nominal controller designed in Section III but with the model uncertainty and fault information; and the Robust controller, that is designed only with the additional information of the parametric uncertainty.

D. Robust Analysis

Consider that the system is combined back to the 6 DoF model, for analysis purposes. Contrary to Section III the stability and performance evaluation of the controlled system under nominal conditions is not enough to guarantee the same stability and performance levels for uncertainties / faults. Thus, the $\mu$-analysis was used to evaluate the robust stability and performance over the set of models, for further information the reader is referred to $[5, 17]$.

In Figure 13 it is seen that the upper bounds of the structured singular value for all three controllers are well under 1. Thus, it is possible to conclude that all the controllers are quite robust, since it covers a wide range of uncertainties with a very safe margin. One interesting phenomena that can be observed is that the peak of the Nominal controlled system has been flatten out, i.e. during the control design of the robust and fault-tolerant controllers the algorithm tried to find a controller that minimized this unwanted effect of the uncertainty. Also to remark that the curves of both robust and fault-tolerant controller are quite similar, this is due to the actuator fault model having a similar representation to the model uncertainties that impact the plant input.

In Figure 14 it is clear that the system controlled by the nominal controller cannot maintain the required performance for the set of uncertainties, since it has structured singular values larger than 1, as it will further evidenced during the simulations in Section IV. The robust and fault-tolerant controllers on the other hand show their maximum $\mu$-value around the operating frequency, and it has a good margin. The robust design can accommodate about 166% uncertainty at that frequency, while the fault-tolerant controller can accommodate 185%.
V. CONTROLLER COMPARISON

Due to the nature of the $H_{\infty}$-control design, if it is proved
that the controller has the desired stability and performance for
the worst-case parameter characterization, then the remaining
set of models will also have at least that level of stability and
performance. Thus, the simulations will be performed under
worst-case conditions. In order to evaluate the full capacities
of the controllers two different fault scenarios were considered:
total loss-of-efficiency of one thruster that operates in the
orbital plane during the CRR stage; and total loss-of-efficiency
of one thruster that operates in the orbital plane during a
collision avoidance maneuver.

Another interesting result obtained from Figure 15 is that
the Robust controller demonstrates a slightly better perfor-
manence than the FT controller in both fault and fault-free
circumstances. This can be explained by the fact that, in the
design of the Robust controller, the chaser mass is a parametric
uncertainty, which influences the plant input, as seen in (2),
leading to an impact similar to the one due of modeling of the
loss of efficiency faults, since both can be seen as an uncertain
parameter that multiplies the plant input. However, the FT
controller can guarantee stability of the controlled system over
a larger set of uncertainties.

As expected, a fault in a thruster whose actuation direction
is along the $X_c$-axis causes momentum around the remaining
axes. Thus, the controller has to counteract this torque and
restore the constant angular velocity. Figure 16 shows the
Euler angle errors around the time of the fault occurrence. It is
clear that the Nominal controller has a degraded performance,
since it does not compensate for the fault sufficiently fast
enough. This is evidenced by the error signal of the Euler
angles $\theta$ and $\psi$ reaching a maximum of 25 degrees. After
the steady state is reached, this controller presents an error
4 times larger compared with the fault-free counterpart. Both
the Robust and the FT controllers restore the attitude in steady
state within 10 to 15 s, and present a maximum error of 3 and
2 deg., respectively.

The loss of efficiency faults of one thruster during the CRR
do not have a meaningful impact in the motion perpendicular
to the orbital plane, thus the passive FTC designed to control
this motion does not to demonstrate the full extent of its
capacities. Hence, a second scenario is considered, where
the chaser is performing a collision avoidance maneuver. In
this scenario, the system is controlled by the Robust or the
FT controller, and the fault impacts the system from the
beginning of the simulation. In order to be able to compare
these results, fault-free simulations were also performed and
using the corresponding controller. The results of the Nominal
Controller are not presented, because when a faults occurs the
Nominal controller requires that the plant input to be higher
than the allowable actuation.
Figure 17 indicates that although the robust controller presents the best performance without the presence of any fault, regarding the rise and settling time, when the chaser has one less functional thruster, due to the fault, its performance is highly deteriorated evidenced by the large overshoot and increase of the rise and settling times. The Fault Tolerant controller provides a similar step response with and without the fault in the thruster, and although the rise and settling times of these controlled systems are substantially higher when compared to the times obtained by the Robust controlled system under fault-free circumstances, they fulfill the performance and safety requirements.

VI. CONCLUSION

When a control designer is considering the use of a fault-tolerant control methodology, he/she has to take into consideration that there is a significant trade-off between the robustness of the controlled system and the performance levels in fault-free circumstances. A passive Fault Tolerant controller, such as the one designed in this paper, has the advantage of retaining similar performance levels with and without an actuator fault, by sacrificing the simplicity of the designing methodology. I.e., in order to design a controller that is robust to model uncertainty, faults, or and external disturbances, the augmented plant used for the controller synthesis grows in complexity and consequently the controller will be of higher order. Additionally, the use of such controlling techniques allows for stability and performance robustness analysis to be done systematically over the set of plant models, by the means of the so called $\mu$ analysis. However, the success of the control design process revolves around the proper selection of weights, which is a known limitation of this methodology.

REFERENCES