Abstract

Uncertainty is an inherent part of a hydrocarbon reservoir modelling and its characterization and assessment is crucial for producing more accurate predictions about the subsurface geology. The present work proposes a new approach, applied to the semi-synthetic Watt field, for uncertainty characterization during reservoir modelling by decoupling different geological descriptions from engineering parameters. The geological descriptions are related to different interpretations of the reservoir’s top structure; fault model; facies definition; and facies’ modelling approach. At each geological level, engineering parameters are optimized through multi-objective history matching processes with adaptive stochastic sampling (Multi-Objective Particle Swarm Optimization) and uncertainty is characterized by Bayesian inference techniques (Neighbourhood Algorithm-Bayes and Bayesian Model Averaging). Results show that top structure presents the best history match results in terms of consistency and misfit minimization, while the modelling approach level produces the narrowest uncertainty interval. Also, new models did not decreased the uncertainty envelope but predict a higher oil production rate at the end of the forecasting period.

Keywords: Multi-level uncertainty characterization, Neighbourhood Algorithm-Bayes, Bayesian Model Averaging, history matching, multi-objective Particle Swarm Optimization.

1. Introduction

Assessing the uncertainty associated with a hydrocarbon reservoir model is essential to properly handle the existing risks associated with field management and developing. Uncertainty results on the lack of knowledge about the internal properties of the reservoir, which may lead to different data interpretations and consequently different reservoir models.

Uncertainty of a reservoir model may be considered at different levels: geological uncertainty - related with the structural geometry of the reservoir, stratigraphy, facies and the spatial distribution of its petrophysical properties; and uncertainty associated with the engineering parameters– related with for example PVT conditions and relative permeability curves. This work aims to assess uncertainty in both domains by tuning the engineering parameters resorting on multi-objective history matching and using a set of Bayesian inference techniques (Neighbourhood Algorithm-Bayes and Bayesian Model Averaging) to characterize uncertainty at different geological levels.

2. History Matching

Integration of production data into reservoir modelling is traditionally referred to as history matching and is done through model calibration to production historical data. The calibration process involves running a fluid flow simulation and changing a set of variables (uncertainty parameters) until the simulated response
matches the observed production historical data.

History matching is mathematically defined as an inverse problem (Christie et al., 2006): we know the response of the system (production history such as flow rates and pressure) to a stimulus (production) but lack the total knowledge of the variables that originate that response (parameters and descriptions of the reservoir model, such as the spatial distribution of permeability and porosity).

2.1. Assisted History Matching
Assisted history matching, also known as automatic history matching, are a set of techniques that use optimization algorithms to automatically calibrate the uncertainty parameters of the reservoir model to match the historical production data. The optimization is driven by a mathematical expression which measures the difference between the observed production and the simulated production is called objective function. The most common objective function used in history matching is the least square norm (Arnold, 2013):

$$ M = \sum_{i=1}^{N} \frac{(obs(t_i) - sim(t_i))^2}{2\sigma_i^2} \quad (1) $$

where $M$ is the misfit between the observed and simulated, $obs$ is the observed data at time $t$, $sim$ is the simulated data at time $t$, $N$ is the number of data points and $\sigma_i^2$ is a representation of the observed data errors (assuming these data errors are independent and have a normal distribution).

Multi-Objective Particle Swarm Optimization (MOPSO) is one of the algorithms which can be used to perform assisted history matching. It is an optimization method developed to simulate the social behaviour observed in bird flocks or fish school while integrating a multi-objective approach, enabling the separation of objectives into different match quality components and balance the trade-offs between the different objectives.

Multi-objective solutions are a set of models resulting on a Pareto Front, i.e. the non-dominated solutions contrary to a unique solution. MOPSO has been shown to be an efficient way to estimate uncertainty due to faster convergence speed and an improve model diversity when comparing to the single objective PSO (Mohamed, 2011, Christie et al. 2013).

3. Uncertainty assessment
Uncertainty in reservoir modelling is widespread across all stages of the geo-modelling workflow. Authors such as Cosentino (2001) refer that uncertainty in reservoir modelling is caused by: (1) quality and data interpretation; (2) interpretation of the structural and stratigraphic concepts; (3) stochastic modelling algorithms and the values of their parameters; (4) multiple realizations.

3.1. Neighbourhood Algorithm-Bayes
The most common techniques for uncertainty quantification are based on a Bayesian framework, i.e. developed around the Bayes’ Theorem:

$$ p(m|O) = \frac{p(O|m)\, p(m)}{\int_{M} p(O|m)\, p(m)\, dm} \quad (2) $$

where $p(m|O)$ is the posterior probability of the model, which represents the updated uncertainty about the model $m$ based on
observations $O$, $p(O|m)$ is the data likelihood (measure of how good is the fit between model $m$ and observations $O$) and is calculated using equation 3, $p(m)$ is the initial prior probability distribution and $M$ is the space of the model.

$$p(O|m) = e^{-M} \quad (3)$$

where $M$ is the misfit calculated by equation 1.

The posterior probability can be calculated by, for example, using the Neighbourhood Algorithm-Bayes (NAB – Sambridge, 1999b). NAB is a Markov Chain Monte Carlo technique that uses Voronoi cells, at whose centres are the values of the sample points, to divide the parameter space and a Gibbs sampler to estimate the posterior probability density. The posterior probability density at each cell is assumed to be constant and is calculated by the product of the likelihood by the volume of the Voronoi cell. The results of NAB are then used to estimate P10, P50 and P90 curves. P10 and P90 curves define the confidence interval, also known as uncertainty envelope, associated with the reservoir.

**3.2. Bayesian Model Averaging**

Bayesian Model Averaging (BMA, Raftery et al., 2005) is a statistical technique that allows the combination of probability distribution functions (PDF) from different models into a single PDF. The resulting PDF is a weighted average of the individual forecasts based on the uncertainty inferred for each of these models and also the variance between models.

BMA PDF of a quantity $y$ based on $K$ models is given by:

$$p(y|f_1, ..., f_K) = \sum_{k=1}^{K} w_k g_k(y|f_k) \quad (4)$$

where $f_k$ is the forecasting of model $k$, $w_k$ is the posterior weight of model $k$ and $g_k(y|f_k)$ is the PDF of $y$ given the forecast $f_k$.

The posterior weights, $w_k$, can be calculated using:

$$w_k = \frac{B_{kj}}{\sum_{l=1}^{K} B_{lj}} \quad (5)$$

where $B_{kj}$ is the Bayes Factor of model $k$ regarding the assumed best model $j$ (maximum likelihood).

Bayes Factor is a statistical measurement method for different alternative models, allowing to compare their validity according to the data. Bayes Factor of model $k$ ($m_k$) regarding model $j$ ($m_j$), $B_{kj}$, can be calculated recurring to Laplace’s Method (Kass and Raftery, 1995):

$$B_{kj} = \frac{p(\bar{O}|m_k)}{p(\bar{O}|m_j)} \quad (6)$$

where $p(\bar{O}|m_x)$ is the posterior probability of model $x$ around the maximum likelihood model.

**4. Methodology**

The proposed methodology was developed and implemented in a semi-synthetic reservoir - the Watt Field (Arnold et al., 2013).

The available dataset comprises a 41 models created from the combination of different interpretations of the reservoir’s top structure (3 scenarios), fault model (3 scenarios), facies model (3 scenarios) and different modelling approaches for the facies’ spatial distribution (2 scenarios).

The proposed methodology can be divided into three distinct stages (Figure 1):
Stage I: characterization of the uncertainty related to each geological level by history matching the different scenarios tuning the engineering parameters, i.e. fluid and rock fluid properties, and applying NAB and BMA techniques;

Stage II: creation of a new top structure according to the local history matching results of the different Top Structures appraised at Stage I, and subsequently characterize uncertainty;

Stage III: creation of a new modelling scenario according to the local history matching results of the different Modelling Approaches scenarios analysed at Stage I, and subsequently characterize uncertainty.

5. Case Study: Watt Field

5.1. Field Description

Watt Field (Arnold et al., 2013) is a semi-synthetic reservoir where the top structure and wireline data are borrowed from a real hydrocarbon field, while the fluid properties, rock-fluid properties and the field development plan are synthetic. Hydrocarbons present in the field are under-saturated oil and dissolved gas.

Available production history on the Watt Field spans over an 8 year period (2903 days), using a set of 16 production wells, with the help of 7 water injector wells built over a 4 year period (except an injector well which is reconverted as appraisal well).

The relevant dataset within the scope of this thesis consists of 41 models created from the combination of different interpretations of the reservoir’s top structure, fault model, facies model and different modelling approaches (Table 1).

<table>
<thead>
<tr>
<th>Hierarchical level</th>
<th>Abbreviation</th>
<th>Differences between scenarios</th>
<th>Scenario</th>
<th>Characteristic of the scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Structure</td>
<td>TS</td>
<td>Depth, size and shape of the top structure</td>
<td>TS1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TS2</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TS3</td>
<td>-</td>
</tr>
<tr>
<td>Fault Model</td>
<td>FM</td>
<td>Number and extension of the faults</td>
<td>FM1</td>
<td>10 faults</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>FM2</td>
<td>9 faults</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>FM3</td>
<td>26 faults</td>
</tr>
<tr>
<td>Facies Model</td>
<td>CO</td>
<td>Cut-off values based on Relative Porosity Difference</td>
<td>CO1</td>
<td>Cut-off: 0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>CO2</td>
<td>Cut-off: 0.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>CO3</td>
<td>Cut-off: 0.8</td>
</tr>
<tr>
<td>Modelling Approach</td>
<td>MA</td>
<td>Technique for modelling facies distribution</td>
<td>Object</td>
<td>Object based algorithm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Pixel</td>
<td>Pixel based algorithm</td>
</tr>
</tbody>
</table>

Figure 1 - Schematic representation of the methodology adopted within the scope of this thesis
5.2. Stage I results

5.2.1. Model Screening

The pre-existing 41 models were used as input for a fluid flow simulator and the obtained FOPR and FWPR curves used to calculate the total misfit (FOPR+FWPR), as defined in equation 1.

Fluid flow simulation occurred for a period of 8 years and was constrained, primarily, to a maximum liquid rate of 8000 STB/day and, secondarily, to a maximum borehole pressure (BHP) of 1000 psi.

The lowest total misfit obtained from the pre-existing scenarios is composed of the combination of Top Structure 2, Fault Model 2, Facies Model 3 and Pixel as Modelling Approach (TS2-FM2-CO3-PIXEL: the base case model). This model configuration was then used for the uncertainty characterization associated with each hierarchical level (Table 2).

Table 2 - Models used in history matching and forecasting and uncertainty quantification for each level with the scenarios highlighted

<table>
<thead>
<tr>
<th>Hierarchical level</th>
<th>Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Structure</td>
<td>TS1-FM2-CO3-PIXEL</td>
</tr>
<tr>
<td></td>
<td>TS2-FM2-CO3-PIXEL</td>
</tr>
<tr>
<td></td>
<td>TS3-FM2-CO3-PIXEL</td>
</tr>
<tr>
<td>Fault Model</td>
<td>TS2-FM1-CO3-PIXEL</td>
</tr>
<tr>
<td></td>
<td>TS2-FM2-CO3-PIXEL</td>
</tr>
<tr>
<td></td>
<td>TS2-FM3-CO3-PIXEL</td>
</tr>
<tr>
<td>Facies Model</td>
<td>TS2-FM2-CO1-PIXEL</td>
</tr>
<tr>
<td></td>
<td>TS2-FM2-CO2-PIXEL</td>
</tr>
<tr>
<td></td>
<td>TS2-FM2-CO3-PIXEL</td>
</tr>
<tr>
<td>Modelling Approach</td>
<td>TS2-FM2-CO3-Object</td>
</tr>
<tr>
<td></td>
<td>TS2-FM2-CO3-PIXEL</td>
</tr>
</tbody>
</table>

5.2.2. History Matching

From the original dataset, the available parameters related to fluids and rock-fluids properties are: relative permeability, fault transmissibility multipliers, water-oil capillary pressure and PVT (Pressure-Volume-Temperature) data. Arnold et al. (2013), observed that capillary pressure has no significant value in the Watt Field, while PVT data does not show any variation across wells, therefore these parameters are assumed as constants on the field and hence with no uncertainty associated with them.

Relative permeability is part of Watt Field uncertainty levels and the existing scenarios follow the Brooks-Corey correlation model:

\[ k_{ro} = k_{r_{omax}} \left[ \frac{S_o - S_{or}}{1 - S_{or} - S_{wc}} \right]^{c_o} \]  
\[ k_{rw} = k_{r_{wmax}} \left[ \frac{S_w - S_{wc}}{1 - S_{or} - S_{wc}} \right]^{c_w} \]

where \( k_{ro} \) is the relative permeability for oil, \( k_{r_{omax}} \) is the maximum relative permeability for oil, \( S_o \) is the saturation of oil, \( S_{or} \) is the Residual saturation of oil, \( S_{wc} \) is the critical saturation of water, \( c_o \) is the oil exponent for Brooks-Corey correlation, \( k_{rw} \) is the Relative permeability for water, \( k_{r_{wmax}} \) is the maximum relative permeability for water and \( c_w \) is the water exponent for Brooks-Corey correlation.

\( k_{r_{omax}}, S_{or} \) and \( k_{r_{wmax}} \) are kept constant in the field, therefore they are not considered as variables in the history matching process of the current study. According to Arnold et al. (2013), critical saturation of water (\( S_{wc} \)) in the reservoir is between 0.1 and 0.5. Exponents for Brooks-Corey correlation can have values between 1 and 6.

Fault transmissibility multipliers are a simple representation of the sealing capacity of a fault, and can possess values between 0, for sealing faults, and 1, for leaking faults.
The variables matched during History Matching are: (1) Field Oil Production Rate (FOPR); (2) Field Water Production Rate (FWPR).

For consistency purposes and statistical significance, the screened models were ran five times each, each run composed of 260 iterations. The Pareto Front results of these runs, i.e. the models that may be solution to the history matching process of each run, are shown in Figure 2.

It is possible to interpret that TS3 is the TS scenario originating the most consistent Pareto Front models across runs, i.e. it has the less spread of models, while minimizing the FWPR Misfit.

In the FM, CO and MA levels, the base case model, not only gets more consistent runs in terms of Pareto Front, but it is also better minimizing the misfit.

Globally, TS scenarios produce more similar Pareto Fronts than any other scenario, with FM1 also achieving good results. From all the models, TS3 is the scenario with less spread while also minimizing the FWPR misfit.

5.2.3. Uncertainty Characterization

NAB algorithm was used to perform forecasting based on history matching results of the first run of each scenario and predict the uncertainty envelope of all scenarios (Figure 3). The forecasting period used in this work is 1440 days, giving a total 4373 simulated days.

Within the TS level, TS2 (the base case scenario) is the scenario that produces the narrowest forecasting envelope in both FOPR (1087.66 STB/day) and FWPR (1204.42 STB/day), while TS1 originates the widest envelope in both production rates (5194.23 STB/day for oil and 5056.72 STB/day for water). TS3 achieves envelopes of 3172.86 STB/day and 2436.05 for FOPR and FWPR, respectively.

FM3 achieves the narrowest uncertainty envelope of the FM level (127.86 STB/day for oil and 144.29 STB/day for water), which is not very realistic because this scenario produced very inconsistent runs at the previous appraisal step. FM1 has the widest uncertainty envelope of the FM level with an uncertainty of 1620.65 STB/day for oil production and 1750.48 STB/day for water production.

At the CO level, CO1’s first run produces the narrowest FOPR and FWPR uncertainty envelopes, 131.64 STB/day for oil production, while water has no uncertainty at the end of the forecasting period (P10 and P90 curves have the same value). CO1 produces a thin interval due to a high values of total misfit and to the isolation of the models with low misfit. CO2 achieves an uncertainty of 2722.94 STB/day and 687.98 STB/day for oil and water production, respectively, originating the widest uncertainty in oil production, while in the water production the widest is created by the CO3 – the base case scenario. Due to the inconsistency history matching results across runs of CO1 and CO2, these scenarios should be
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considered not fully characterized and should be subject of a detailed study. In the MA level, Pixel scenario achieves the widest envelopes for oil and water (1087.66 STB/day and 1204.41 STB/day, respectively).

Interpreting all levels together, it is possible to observe that CO level originates the greatest range of the uncertainty for FOPR, 9414.77 STB/day, while Top Structure level (TS) has the widest range of uncertainty envelopes for FWPR (7007.78 STB/day). Modelling Approach (MA) level achieves the lowest range of the uncertainty envelopes for both FOPR (3449.40 STB/day) and FWPR (3388.08 STB/day).

BMA may be used to combine scenarios generating global uncertainty envelopes for FOPR and FWPR based on the highest maximum likelihood model (achieved in this case by TS3 scenario, Figure 4).

While combined FOPR uncertainty envelope is a combination of most of the scenarios, FWPR envelope is mainly based on the maximum likelihood scenario (TS3), ignoring the remaining scenarios mainly because TS3’s maximum likelihood for FWPR is much higher than the maximum likelihoods of the remaining scenarios.

![Figure 3](image)

**Figure 3** – Detail of the Uncertainty Envelope at the end of the forecasting period of all scenarios: a) for FOPR; b) for FWPR;

![Figure 4](image)

**Figure 4** – Detail of the combined Uncertainty Envelope at the end of the forecasting period for all scenarios for: a) FOPR; b) FWPR
5.3. Stage II results

Based on the well’s history matching results of the Pareto Front models for the Top Structure scenarios, the reservoir was divided into different regions (Figure 5) according to the scenario which minimizes the average of the total misfit per well. Faults and water streamlines were used to define the boundaries of these regions, because water production is responsible for the majority of the misfit, as interpreted from the previous stage.

Figure 5 - Top view of the Top Structure regionalization according to local history match and the water production streamlines

To perfectly integrate the different regions within a single reservoir’s top structure, Collocated Co-Krigging was used to smooth the differences of depth near the boundaries between each region.

This new scenario, named TSx, was combined with the base case scenarios of the remaining levels and submitted to a history matching and forecasting and uncertainty characterization as introduced in Stage I.

5.3.1. History Matching

The Pareto Front results obtained for the five runs of the new scenario (Figure 6) proved to achieve a similar consistency to TS3 and TS2 runs and also increasing the wide of the fronts thus improving the diversity of the possible solutions. Increasing the diversity allows better uncertainty characterization.

Figure 6 - Pareto Front models for TS level (including TSx)

5.3.2. Uncertainty Characterization

Figure 7 shows the uncertainty envelopes for FOPR and FWPR, at the end of the forecasting period, obtained for TSx together with the envelopes of the pre-existing Top Structures and BMA’s combined envelope for the four scenarios.

It can be interpreted that the new model does not decreases the thickness of the uncertainty envelopes (1843.97 STB/day for oil and 2094.02 STB/day for water), when comparing to the narrowest of the pre-existing scenarios (TS2: 1087.66 STB/day and 1204.42 STB/day), but the new Top Structure’s envelope indicates a larger production of oil at the end of the forecasting period than TS1, TS2 and TS3, while also minimizing the water production.

BMA’s envelope for FOPR is a combination of the individual envelopes due to the similarity between the maximum likelihoods for oil of the scenarios. FWPR’s combined envelope follows almost exactly the envelope for the new model, disregarding the pre-existing scenarios. This happens due to the fact that TSx achieves a much higher maximum likelihood for FWPR than the pre-existing scenarios.
5.4. Stage III results

Similar to Stage I, the results of the local match of the models belonging to the Pareto Front models for the Modelling Approach scenarios were used to regionalize the reservoir (Figure 8) according to the scenario which minimizes the average total misfit at the wells. Faults and water streamlines were used to define the boundaries of these regions.

This new scenario, named MAx, was combined with the base case scenarios of the remaining levels and submitted to a history matching and forecasting & uncertainty characterization processes similar to Stage I and Stage II.

5.4.1. History Matching

Like the base case scenario the Pareto Front models of the new scenario (Figure 9) show that all runs produce consistent results, slightly increasing the FOPR Misfit range when comparing with Pixel.

5.4.2. Uncertainty Characterization

Figure 10 shows that the resulting envelopes of MAx scenario (2172.16 STB/day for oil and 1734.81 STB/day for water production) have increased the uncertainty envelope thickness comparing with the pre-existing scenarios. Despite this, the new model predicts a more optimistic scenario for oil production and potentially less production of water.

The combined uncertainty envelope for FOPR of the three scenarios encompasses most of the models since all models have similar weights, but, for FWPR, the combined envelope mainly follows the MAx’s envelope since this model possesses a maximum likelihood value much higher than the pre-existing scenarios.
6. Summary and Conclusions

History Matching results showed that TS3 achieves the best results for TS level and when all levels are considered, with more consistent runs and minimizing the water misfit, while for the remaining levels the base case scenarios (FM2, CO3 and Pixel) got the best results.

Uncertainty characterization on the Top Structure and Modelling Approach levels predicts the narrowest uncertainty envelopes for TS2 and Object scenarios, respectively. The scenarios that produced the narrowest envelopes for the Fault Model (FM3) and Facies Model (CO1) should not be considered as a real uncertainty assessment on these scenarios, given the inconsistency of their history matching runs.

BMA results at every level and when combining all scenarios show an uncertainty envelope for FOPR based on the respective scenarios’ envelopes, while for FWPR, the differences between likelihoods results in an envelope that resembles the maximum likelihood scenario.

Creating new scenarios for Top Structure and Modelling Approach based on the local match outcome of the history matching process, did not improved the uncertainty envelopes but showed a slightly improvement on the history matching results, predicting higher values for oil production and lower values for water production than the pre-existing scenarios.

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