Abstract—This study involves the design, construction and experimentation of a novel linear tubular direct-drive triphasic actuator with the stator comprised of a single sided printed flexible circuit for soft robotic applications in exoskeletons and active orthosis. First it’s introduced the actuator characteristics and possible configurations, topologies as well as their constraints being after set the optimization goals. A electromagnetic theoretical model is developed being the predicted force withdrawn of the configuration that brings more advantages. A transient thermal study of the is employed using the electric analog method. The various stages of the prototype’s manufacturing are present going through each step in detail until completion. Electromagnetic and thermal theoretical models are validated at the expense of several experiments. Relevant actuator characteristics as the precision and maximum velocity are investigated.

Keywords: Linear Actuator, Flexible circuit stator, Soft robotics and exoskeletons, Direct-drive three phases actuator, Electromagnetic behavior, Thermal behavior, Topology optimization

I. MOTIVATION

The work developed tries to answer the current lack of solutions and margin for improvement in the market for linear electric actuators comprising features that, not only, are complace with normal human movements but also capable of being applied in devices such as exoskeletons, wearable robotics and active orthosis that can assist individuals, mimicking, helping or rehabilitating their movements. With the intention of filling this empty market slot it’s desired to develop a linear actuator with mandatory characteristics such as:

- Low weight.
- Low energy consumption.
- Forces and velocities within the scope of human normal environment and behavior.
- Working temperature compliant with clothing and human skin.
- Modular distributed volume and weight to facilitate integration into wearable robotic devices.

With that in mind a linear actuator was developed with a few fundamental guidelines such as simplicity provided by few components, pure linear movement obtain without the use of gears or rotational-to-linear conversion and an useful volume maximization withdrawn from a recently new manner of manufacturing the actuator’s stator as described by the patent introduced in [1], for rolled up circuits.

II. STATE OF THE ART

In [2] the authors successfully develop a general framework for the analysis and design of a class of tubular linear permanent magnet machines where analytical expressions for the open circuit and armature reaction fields have been established for radially, axially, and Halbach magnetized machine topologies, and expressions for the force, emf, and self- and mutual-winding inductances have been derived and then validated using finite element calculations and measurements, in [3] the same authors present a modified design of an axially magnetized tubular permanent magnet machine, and it’s magnetic field distribution established, allowing the force capability and force ripple to be expressed as functions of various dimensional ratios, which provide a useful tool for assessing the influence of leading design parameters on the machine performance. In this work it was noted that the force ripple commonly due to irregular magnetic field of the permanent magnets and/or inaccuracy in electronic commutation by the servo amplifier in both machines is less that 0.2% being fairly reasonable to be neglected when compared with the cogging force which results from end effects.

In [4] a multiphysic approach is performed for a model concerning a permanent magnet tubular linear motor, where a coupled electromagnetic and thermal finite element analysis is perform proving the viability of a multiphysics numerical field analysis. Regarding the thermal model, two phenomena were taken into account when study the thermal behavior of the PMTLM prototype:natural convection and conduction. This study was able correctly predict the temperature curves for the copper and iron for the actuator under normal operation rates.

In [5] it’s described the design, analysis, and characterization of a linear permanent-magnet generator and capacitive energy storage system for generating electrical power from a single stroke of a salient-pole armature. An electromagnetic analysis of the generator was described, and a design optimization methodology for the system was presented. Finally,
the performance of a prototype was validated against measurements, having concluded that predicted performance agree well with experimental results for the generator prototype.

An investigation of the influence of the choice of soft magnetic material on the performance of a tubular permanent magnet machine was conducted in [6], and quantified the relative merits of silicon iron laminations and soft magnetic composites (SMCs). The machines here studied are equipped with a modular stator winding and employs a quasi-Halbach magnetized moving-magnet armature. It was shown that, despite its poorer space utilization, a machine whose stator is fabricated from silicon iron laminations has the highest force capability, efficiency and power factor. A machine with a SMC stator, on the other hand, has potential advantages in terms of ease of manufacture and lower cost.

In the pursuit of a bendable motor [7] presents a new two-phase flexible linear actuator. Its flexibility due to elastomers offer the desired actuator flexibility both within the rotor and between the magnets in the mover. Various finite element models were developed, optimizing the dimensional parameters until reaching the desired values. Regarding experimental tests, weights up to 750 g were lifted with a minimal bending radius of 200 mm as well as holding force densities of 49 N/kg in continuous operation.

The authors of [8] present a design and analysis framework for the general class of permanent-magnet electrical machines. In their work they present a surface-mounted linear motors consisting of permanent magnets and iron-less current-carrying coils, they are treated in a uniform way via the magnetic vector potential. This design is accomplished with the purpose of developing novel linear magnetic levitators for driving precision motion control stages.

In [9] a novel modeling design of a three phase tubular PM linear generator is proposed as well as the system definition is presented and analyzed. A parametric evaluation of the machine is done to enforce a finite element model. A parametric approach is adopted to perform a first optimization of TPM-LiG electromagnetic behavior, and the specified features are achieved.

In [10] formulation of the magnetic flux distribution of tubular linear machines based on magnetic vector potential, Laplace’s and Poisson equations is performed. The magnetic field model is validated with FEM results, and employed to analyze the effect of structure parameters on the magnetic field. The results of this paper, as stated by the authors, can be employed to study the force output and design optimization of tubular linear motors in the future.

### III. Prototype Description

The proposed tubular linear actuator is comprised of two distinct parts called mover, the inner part, and stator, the static outer one. The stator by current excitation will generate a magnetic field that will interact with the a permanent field created by the mover’s magnetic rings producing the desired linear movement.

#### A. Prototype Topology and Variables

The mover, inner part, is made of a set of permanent magnet rings sequentially placed in a carbon rod as shown in fig.1, together they make up all mover’s components.

![Fig. 1: Actuator’s configuration and main components.](image)

In turn, the stator is made of a rolled up sheet of flexible circuit, to this approach of build the stator windings, per say, without the usage of actual copper filamented wires and a frame to hold them an keep them in the correct place, is devoted a special attention due to the fact that’s the first time that’s constructed and essayed a linear actuator with this technology.

1) **Mover:** The inner part of the actuator, is comprised of a series of axial magnetized rings sequentially placed along a carbon rod with inverted polarities in such a way that all neighbor rings repel each other. The rings are made from an alloy of neodymium. Dimension-wise (Fig.2), they have an inner and outer diameters $r_{mi}$ and $r_{mo}$ respectively, and an height designed by $h_m$. They fit perfectly into the carbon rod, due to it’s radius being equal to the inner one, $r_{mi}$, of the permanent magnets.

![Fig. 2: Mover’s neodymium rings and carbon rod dimensions.](image)

The rings are axially magnetized, i.e. the magnetization direction is along their geometric axis, being the north and south poles located on their flat faces. Other important magnet rings’ variables are the ones necessary to fully define an electromagnetic model and they are: the Remanence Flux $B_r$, Coercive Force $H_c$, and Magnetic Relative Permeability $(BH)_{max}$.

2) **Stator:** The stator is comprised of only one component, an flexible sheet of circuit. Contrary to common linear actuators that have to rely in sequentially placed coils of rolled up copper wire to induce a magnetic field, the stator’s actuator studied is made of a sheet of flexible circuit that is rolled up forming an hollow cylinder. This sheet has two layers, the bottom layer is made of a polymide composite with thickness $\tau_{p_c}$, and has structural purposes and the top layer is a thin foil of copper of thickness $\tau_{ct}$ (see Fig.4), that is transformed in a circuitry of copper traces after a revelation procedure. The flexible sheet of circuit is then rolled up like in Fig.3 making the same effect as copper windings with the up front advantage of being just a single piece of composite material instead of a large set of components requiring a complex
assembly procedure. The total length of the actuator’s stator is here defined by $L_{act}$.

![Fig. 3: Stator: Rolled up sheet of flexible circuit](image)

After the manufacturing process a cross section of the circuitry looks like fig. 4, where can be seen the copper traces with width $w_{ct}$ separated by gaps of width $g_{ct}$.

![Fig. 4: Section of the stator showing up the copper traces.](image)

A current must be applied to induce the magnetic field and it is convenient to choose the highest current possible respecting both application purposes and physical constrains thus a relation between current, the cross section of the trace in which it is going to flow and the verified increase of temperature needs to be set. To accomplish that it will be used a formula that results from the curve fitting of IPC Temperature Charts [11]. The resultant formula for internal copper traces is given by

$$I = A c k \Delta T^b$$

(1)

where $I$ is the current in Amperes, $\Delta T$ is the increase of temperature in Celsius degrees, $k$, $b$ and $c$ are constants whose values are: $k = 0.024$, $b = 0.44$, $c = 0.725$, and $A$ is the cross section area in $\text{mils}^2$ and is given by $A = \tau_{ct} w_{ct} d$. Where $\tau_{ct}$ and $w_{ct}$ are, respectively, the thickness and width of the copper trace in millimeters and $d$ is a conversion factor between $\text{mm}^2$ and $\text{mils}^2$ equal to $d = (25.4 \times 10^{-3})^{-2}$.

### B. Two Phases and Three Phases Windings Dimensions

To take the maximum advantage of the interaction between the permanent magnetic field of the mover and the induced magnetic field of the stator, it is necessary to analyze whether the best solution is, to implement a two phases or three phases configuration. In fig. 5 a section of the actuator in the two phases arrangement is shown using as representative example the minimum possible functional configuration of the actuator i.e. two magnets (mover) and two traces (stator), here named actuator’s base unit. Notice that, a sinusoidal current wave i.e. two magnets and traces, here named the minimum possible functional configuration of the actuator phases arrangement is shown using as representative example configuration. In fig. 5 a section of the base unit is here defined by $L_{act}$.

![Fig. 5: Planar representation of the traces in two phases configuration with arrows representing the direction in which the current flows through.](image)

The three phases configuration is similar to the one just introduced having now not two phases but three phases shifted of $\frac{2\pi}{3}$ electrical degrees between them, not being placed sequentially in the circuit board, they’re rather placed in the following sequence: $1, -3, 2$. With this architecture it was able to maintain a balanced three phased model but create a $\frac{\pi}{3}$ lag form one trace to the next one. As can be seen in Fig.6, the trigonometric circle will have six current vectors with a shift of $60^\circ$ between them.

![Fig. 6: Planar representation of the traces in three phases configuration.](image)

The relational parameters of the stator and mover are play an important role being them the length of flexible circuit sheet necessary to achieve a specific number of turns and the actuator’s base unit relational length. A section of the base unit in the two configurations is represented in Fig.7 being the new variables the inner and the outer stator radius $r_{si}$ and $r_{so}$ and the actuator's base unit length $h_{mon}$.

---

1Following the widely used arrow convention where a circle means that current is flowing out of the paper and a X means that is flowing towards the paper.
2 Phases 3 Phases

Fig. 7: Three phases (on the left) and two phases (on the right) actuator’s base unit configuration.

Since the stator is made out of a sheet of flexible circuit, its thickness is given by \( \tau_s = r_{so} - r_{si} \) and its length by the length of an Archimedes spiral given by the following integral,

\[
L_{fc} = \int_{0}^{2\pi} \left( \left( r + \frac{dr}{d\theta} \right)^2 + \left( \frac{dr}{d\theta} \right)^2 \right)^{1/2} d\theta
\]  

(2)

where \( n \) is the number of turns of the flexible circuit, \( r \) is the initial radius \( r = r_{so} \), and \( \frac{dr}{d\theta} \) is the increase of radius in every loop, i.e. the thickness of the flexible circuit \( \tau_{fc} = \tau_{ct} + \tau_{pc} \).

C. Prototype Constraints

The actuator can be seen as sequentially placed actuator’s base units and since the base units are constituted by two magnets and four or six copper traces depending on which configuration is used, a stator/mover relation be established, i.e. an equation than relates the dimensions of magnets and their configuration is used, a stator/mover relation be established.

Generally in a configuration of \( n \) phases with \( n = 2, 3 \) the following relation holds

\[
h_{mm} = \frac{(h_m + g_m)}{2} = (h_m + g_m) = n(w_{ct} + g_{ct})
\]  

(3)

where, \( h_m \) and \( g_m \) are the magnet’s height and the gap between them and \( w_{ct} \) and \( g_{ct} \) the trace’s width and gap between traces respectively. Regarding the mover, their components are well defined, being the neodymium magnet rings dimensions and magnetic characteristics fixed. One important characteristic is the magnet maximum working temperature since if exceeded the neodymium magnets will lose a lot of their strength. In the N grade type the maximum temperature is 80°C.

Regarding the stator the constrains are related with thickness of both laminated components, the polyimide thickness \( l_{ct} \) and the copper thickness \( t_{pc} \) that are set to the thicknesses of the laminate acquired, respectively, 0.018 mm and 0.025 mm.

D. Optimization Aim

The force generated by the actuator is the product of stator copper traces ampere-turns and magnetic flux density in air gap. Since the air gap magnetic flux is produced by neodymium permanent magnets of the mover, the actual flux value depends upon the demagnetization characteristic of the magnet. One can seen easily that, one way to increase the force is to increase the stator’s ampere-turns which is possible by increasing the amplitude of the exciting 3 phase current and/or number of turns of the flexible laminate, which increases the outer diameters of the actuator. The allowed current density in flat surface coils in printed circuit is very high. However, because of the high joule losses and excessive heating, it is not advisable to increase the current density further for continuous operation. The temperature increase must be monitored and set to limits that protect both neodymium magnets that deteriorate causing a lower gap flux density and lower force. It’s also necessary to control the temperature of the outer surface of the stator due to applications that will be in contact with human skin. Since joule loss of stator traces is proportional with the square of stator’s current, increasing the current to improve the force is not appropriate solution. Therefore for a good design the only way to withdraw the best possible force is:

- Experimentally reduce to the minimum the gap between stator and mover.
- Find which configuration, 2 or 3 phases, created more induced/permanent magnetic field interaction per volume that consequently will increase the actuators force.
- Find out the optimal copper trace width.

IV. ELECTROMAGNETIC ANALYSIS

A. Magnetic Field Theory

Maxwell equation for magnetic field intensity, taking into account that there is no free volume current density due to the movement of free charges, \( J_f \), and no time-dependent effects, equation can be simplified to \( \nabla \times \mathbf{B} = 0 \). Maxwell equation for the magnetic field \( \mathbf{B} \) that yields the impossibility of magnet monopoles and writes as, \( \nabla \cdot \mathbf{B} = 0 \). \( \mathbf{B} \) field can be related to \( \mathbf{H} \) field through the remanent magnetization vector \( \mathbf{M} \) as \( \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \). The magnetic field is confined to two regions, the airspace and windings region in which the permeability is \( \mu_0 \) and the permanent magnet region in which the permeability is \( \mu_0 \mu_r \). Therefore the equations are

\[
\mathbf{B} = \begin{cases} \mu_0 \mathbf{H} & \text{, in the airspace/winding} \\ \mu_0 \mu_r \mathbf{H} + \mu_0 \mathbf{M} & \text{, in the magnets} \end{cases}
\]  

(4)

Formulating the magnetic flux density \( \mathbf{B} \) in terms of a magnetic vector potential \( \mathbf{A} \) and setting the Coulomb gauge it’s obtain the magnetic field equations for both regions can be rewritten in terms of the magnetic vector potential \( \mathbf{A} \) and noticing that curl of \( \mathbf{H} \) field is equal to zero the governing equations are simplified to

\[
\nabla^2 \mathbf{A} = \begin{cases} 0 & \text{, in the airspace/winding} \\ -\mu_0 \nabla \times \mathbf{M} & \text{, in the magnets} \end{cases}
\]  

(5)

In cylindrical coordinates taking in consideration the axisymmetry of the actuator’s topology and the fact that the mag-
netization vector $\mathbf{M}$ only has $z$ component, i.e. $\mathbf{M} = M_z \mathbf{e}_z$, the equations can be rewritten, as

$$\begin{cases}
\frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (r A_{1\theta})}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{1}{r} \frac{\partial (r A_{1\theta})}{\partial z} \right) = 0 , & \text{in the airspace/winding} \\
\frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial (r A_{1\theta})}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{1}{r} \frac{\partial (r A_{1\theta})}{\partial z} \right) = 0 , & \text{in the magnets}
\end{cases}$$

(6)

**B. Actuation Force**

The force output delivered by the actuator is proportional to the current applied in their phases. The force between one magnet and one coil as a function of current at many relative displacements was calculated using the Lorentz force equation $\mathbf{F} = \mathbf{j} \times \mathbf{B}$ dV. Since each magnet in the array is assumed to be positioned at the axial center of the coils, the r-component of the force will cancel out. The remaining force is only in the z-direction

$$F_z = \int (-J_\theta \times B_r) \, dV \quad (7)$$

**V. Finite Element Model**

A finite elements study was conducted in order to find the actuator’s configuration and related parameters so that the best performance criteria are meet. This analysis was performed using FEMM - Finite Element Method Magnetics.

**A. 3 Phase vs 2 Phase**

To evaluate the desired configuration, three or two phases, and the copper trace width $w_{ct}$ that ensures the maximum force, a study was conducted. For both, two and three phases circuit configuration it was varied the relative position of the mover in respect to the stator ($Z_m$) from a set position $Z_m = 0\%$ to $Z_m = h_{max}$, i.e. the mover travels a length equivalent to the length of its base unit. Then for each of those positions was simulated the variation of intensity in the electrical phases from 0 to $2\pi$. The result obtained was a surface corresponding to the force produced by the actuator for each position and instantaneous current intensity as seen in Fig. 8. A section of the surface at a constant mover position $Z_m = 10\%$ for the three phase configuration is shown in figure 9 where can be seen the force curve and the correspondent three phases values of the actuator copper traces.

**Fig. 8: Force surface.**

**Fig. 9: Force plot and correspondent three phase currents intensity at $Z_m = 10\%$ of total actuator’s base unit length.**

This force surface study was developed for different types of copper trace widths $w_{ct}$, for both two and three phases configuration, yielding the results present in Figure 10, where can be seen two lines, the red dashed line is the mean of the maximum absolute force’s per unit length values along sections of constant mover’s position for different copper trace’s length. The black continuous line represent for each copper trace width the value with higher probability density based on a normal kernel function.

**Fig. 10: Two (left) and Three (right) Phases Force for different copper traces length**

In both, these two graphics and in a two versus three phases comparative graphic in figure 11, can be seen that
the higher forces per unit length occur in the three phase configuration. Being the maximum force obtained in the three phase configuration with a copper trace width equal to \( w_{ct} = 0.5927\ mm \). Thus this is going to be the configuration chosen to experimentally develop.

![Graph](image)

Fig. 11: Evolution of the maximum force obtained for two and three phases’ configuration.

VI. THERMAL ANALYSIS

The energy dissipation in the Permanent Magnet Linear Actuator happens in the form of temperature increase and is only due to Joule losses in the copper traces since no iron is used in the stator, meaning that hysteresis and eddy-currents will not be taken in consideration in this analysis.

A. Electric-Analog Method Approach

The thermal behavior of the actuator can be, in a simplified 1-D form, modeled appealing to the equivalences of physical quantities between thermal and electrical systems. The thermal behavior of the actuator can be modeled as if it was a electric circuit. A fully transient heat transfer analysis can be conducted using simple circuit components where Kirchhoff’s and Ohm’s law are easily applied transforming a rather complicated problem into a simpler one. In fig. 12 a quarter of a section of the actuator is showed where can be seen mover’s component 1, the neodymium magnets and also the stator’s copper laminate, 3. The air gap between the stator and mover and the surrounding air are defined by numbers 2 and 4.

![Diagram](image)

Fig. 12: Cross section of the actuator and the various materials

The two main relevant heat transfer phenomena taken into account in this study are natural convection, here present in the stator-mover air gap and also between the external surface of the stator and the surrounding air represented by the convection resistances \( R_{gap} \) and \( R_{conv} \), respectively, and is caused mainly by the density difference coming from the temperature gradient [12]. The other phenomenon is conduction between solids and represent the transport of energy in a medium due to temperature gradient. Convection is here introduced by the resistances \( R_{neo} \) and \( R_{pec} \) that represent, respectively, the thermal resistances of the neodymium magnets and copper/polyimide laminate fixed with epoxy. Regarding natural convection, the resistances are given by \( R_h = \frac{L}{\kappa A} \), where \( L \) is the specific length of the actuator, \( h \) is the natural convection coefficient in \( W/m^2 \cdot ^\circ C \) and \( A \) is the area in contact with environment in \( m^2 \). The natural convection coefficient \( h \) is based on the empirical non-dimensional numbers of Prandtl \( Pr \) and Rayleigh \( Ra \). Together they define the Nusselt number \( Nu \). Found the non-dimensional numbers the natural convection coefficient \( h \) is given by, \( h = \frac{Nu k}{D} \), being \( k \) the thermal conductivity of the air. According to thermal conduction the resistances of the various components of the actuator are given by, \( R = \frac{L}{k A} \), where \( L \) is the specific length of the actuator, \( k \) is the thermal conductivity of the constitutive materials (\( W/m \cdot ^\circ C \)) and \( A \) is the cross section area of the material. Copper losses can be expressed as \( P_{Cu} = 3I^2 R \), where \( I \) is the current of the stator windings under running condition and \( R \) is the windings’ resistance at a given temperature, rewriting the equation in order to the measured phase resistance \( R_0 \) at a known temperature \( T_0 \) the equation becomes

\[
P_{Cu} = 3I^2 R_0[1 + \alpha_{Cu}(T - T_0)]
\]

where \( \alpha_{Cu} = 0.00386 \) [13] and is defined as the copper temperature coefficient of resistance at \( T_0 = T_a = 20^\circ \) being the measured copper resistance at this temperature equal to \( R_0 = 6.8 \Omega \).

![Table](image)

**TABLE I:** Constituents relevant thermal properties.

<table>
<thead>
<tr>
<th>Material</th>
<th>Mass Density ( \rho ) [kg/m(^3)]</th>
<th>Specific Heat ( c ) [J/(kg K)]</th>
<th>Thermal Conductivity ( k ) @25°C [W/(m K)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neodymium</td>
<td>7600</td>
<td>503</td>
<td>9</td>
</tr>
<tr>
<td>Polyimide</td>
<td>1420</td>
<td>1090</td>
<td>0.46</td>
</tr>
<tr>
<td>Copper</td>
<td>8960</td>
<td>385</td>
<td>401</td>
</tr>
<tr>
<td>Epoxy</td>
<td>1430</td>
<td>1000</td>
<td>1.66</td>
</tr>
<tr>
<td>Air</td>
<td>1.1839</td>
<td>1005</td>
<td>0.0257</td>
</tr>
</tbody>
</table>

Fig.13 illustrates the thermal model where can be seen the resistances and capacitances of the actuators materials and the air resistances. To obtain the transient response it’s necessary to solve iteratively, at each time step the differential equations obtained by applying Kirchoff rules on nodes 1, 2, 3 and 4. The four equations after applying the Laplace transform are,
\[ \begin{align*}
\text{Node 1:} & \quad J_{\text{neo}}(V_{\text{neo}} - V_{\text{gap}}) - sC_{\text{neo}}V_{\text{neo}} = 0 \\
\text{Node 2:} & \quad J_{\text{gap}}(V_{\text{pec}} - V_{\text{gap}}) - J_{\text{neo}}(V_{\text{gap}} - V_{\text{neo}}) = 0 \\
\text{Node 3:} & \quad I - J_{\text{inner}} - J_{\text{gap}}(V_{\text{pec}} - V_{\text{gap}}) - sC_{\text{pec}}V_{\text{pec}} = 0 \\
\text{Node 4:} & \quad J_{\text{pec}}(V_{\text{pec}} - V_{\text{surf}}) - J_{\text{conv}}V_{\text{surf}} = 0
\end{align*} \]

The values for resistances and capacitances of the thermal network are present in Table II.

**TABLE II: Resistances and capacitances of the thermal network.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{neo}}$</td>
<td>92.843 [J/W]</td>
</tr>
<tr>
<td>$R_{\text{neo}}$</td>
<td>0.1766 [K/W]</td>
</tr>
<tr>
<td>$R_{\text{gap}}$</td>
<td>1.213 [K/W]</td>
</tr>
<tr>
<td>$C_{\text{pec}}$</td>
<td>77.157 [J/W]</td>
</tr>
<tr>
<td>$R_{\text{pec}}$</td>
<td>0.083 [K/W]</td>
</tr>
<tr>
<td>$R_{\text{conv}}$</td>
<td>0.713 [K/W]</td>
</tr>
</tbody>
</table>

In figure 14 can be seen the evolution of the temperature rises for the actuator’s different parts during its own operating period. Two curves have a special interest, curves $T_{\text{surf}}$ and $T_{\text{neo}}$. $T_{\text{neo}}$ represents the temperature evolution in the neodymium magnets. Since the temperature in magnets can’t surpass 80°C in order to not lose its own magnetization. The second curve is $T_{\text{surf}}$ and represents the temperature evolution at the stator’s surface, being crucial to monitor since in applications that the actuator is in direct contact with skin, temperature rises can be harmful\(^2\).

\[\text{Fig. 14: Temperature increase of the various components of the actuator.}\]

Thus it’s set as maximum operating time 300 seconds (5 minutes). That ensures that the temperature in the neodymium magnets does not get higher than 337.5 K or 64.35°C and the stator outer surface does not surpasses 318 K or 44.85°C.

**VII. PROTOTYPE MANUFACTURING**

**A. Stator Stage - Printing, Developing and Etching**

The first step in the actuator’s manufacturing process is the creation of the stator. The actuator’s stator is made of only one component, a rolled up flexible copper laminate sheet. The manufacturing process starts with a sheet of flexible single-sided cooper clad laminated sheet, this particular sheet is made of a combination of a copper thin foil with a thickness of 18 µm binded to a polyimide composite layer of 25 µm, to this flexible sheet is applied a photosensitive resist that, when exposed to light, loses its resistance or it’s susceptibility to be attacked by an etchant or solvent. In order to print the desired circuit it was used the laser printer to draw the desired traces in the flexible copper laminate.

Finished the printing process the next step is develop the circuit using sodium carbonate, leaving the protective film only covering the copper sheet where the future circuit is going to be. Etching the circuit is the next step in the process, where the copper traces will be reveled, i.e. the surplus of copper is removed leaving a circuit only with the desired electrical traces drawn in it. The result of the etching process can be seen in Fig. 15.

\[\text{Fig. 15: Part of the copper circuit after etching process.}\]

\[\text{B. Stator Stage 4 - Rolling Up}\]

The final step in the stator’s construction is rolling up the circuit as shown in fig. 3. To achieve that some intermediate steps need to be done. Recalling that in the basic trace’s design of the circuit, for the three phase example (fig 6), the three parallel traces carrying currents $I_1$, $I_2$, and $I_3$ induce a magnetic field that is going to interact with the an field created by permanent magnets, need to perform a flip in the their direction in such a way that they positively interact with two magnets of opposite polarities. To manage the change in direction without traces with different phases touching each other after the horizontal connection being folded as shown in figure 16 a soldering mask needs to be applied to function as an insulating layer.

This was the design choose to construct the stator, many other designs were considered in early stages of the construction of the stator and many others would be valid choices to carry a balanced three phase windings. This was chosen over the others because of it’s simplicity and traces soldering. Even though, that was reveled has a difficult task due to the manual soldering process employed showing several positioning and repeatability issues that an automated process could fix. Other valid processes such as the one present in the referred patent [1], where it’s used a two sided copper laminate would with the proper tools and materials be a good but more expensive design solution.

\(^2\)According to the National Institute of Standards and Technology (NIST) in U.S. temperatures above 48°C can inflict first degree burn injuries in the human skin.
After being soldered and the connection closed, as shown in figure 16 the flexible circuit should look like the circuit in figure 17.

Completed the manufacture process of the flexible circuit the last step is to roll up the circuit to make a rigid stator made of only one component as shown in figure 18.

C. Rotor Stage 1 - Assembly

The rotor assembly was performed in such a way that minimum parts and low complexity, following the line of thought of the actuator proposed in this thesis, are an important requirement. Stated that the rotor is assembly just by placing sequentially magnetic rings in a carbon rod. The final prototype can be seen in fig. 19.

Completed the manufacture process of the flexible circuit the last step is to roll up the circuit to make a rigid stator made of only one component as shown in figure 18.

VIII. Practical Results and Analysis

A. Electromagnetic Model Validation

In order to correctly validate the prototype’s theoretical electromagnetic model developed three distinct experiments were conducted. The first one, likewise it was done in Section V-A, consists in the characterization of force experienced by a locked mover for several positions from 0 to 100% of the length of one actuator’s base unit \( h_{\text{mon}} \), experiencing at each discrete position the influence of the three current phases changing their phases from 0 to 360°. Figure 20(left) shows highlighted one actuator’s base unit length to better understand the travel this distance of the mover.

The second experiment consists in performing the same experience, i.e. locking the mover in several positions and for each position vary the current along the three phases in such a way that they perform a whole trigonometric circle \((0 \text{ to } 360°)\), but with small difference. The mover, instead of only travel a distance correspondent to the length of one actuator’s base unit travels 65.024 mm, the length of 8 base units where at each step the force is acquired as can be seen in figure 20.

The last experiment conducted was the acquisition of the variation of force that the mover experienced locked in it’s central position with a imposed variation of current in the windings. Being able to compare the theoretical and experimental forces as function of the phases’ currents.

In the table III(in Appendix section), can be seen all relevant characteristics of the prototype developed. It was selected for construction the three phase configuration with a copper trace width of \( w_{\text{ct}} = 0.5927 \text{ mm} \). In figure 28(in Appendix section) can be seen the force surface obtained by finite elements program, FEMM, when varied the current phases along different movers position within the length of one actuator’s base unit. The Mover is excited by the signals feed to the three phases, having a peak amplitude value of \( I_{\text{peak}} = 0.5814 \text{ A} \), as tabulated in the table above.

In figure 29(in Appendix section) it’s present the experimental curve acquired using a scale with a precision of 0.01 grams. Being the surface comprised of 6624 measurements. In figure 21 it’s plotted the difference between the predicted values and the values obtained in the experimental measurements. The errors between the two surfaces are more prominent, as expected due to the sine wave nature, the places where maximum and minimum forces occur. Being the maximum error relative to the experimental value of 16.08%. The lower forces experienced by the actuator in the experimental can be related to main three factors: The friction present between the mover and the stator since it was necessary to add kapton film in several places to ensure a perfectly concentric mover and stator. Even though the areas of contact added between the stator and mover where ensured to be the minimum ones.
necessary to a normal behavior in the actuator’s operational movement the static and kinetic friction coefficients between two kapton films are, respectively, 0.63 and 0.48 at 23°C as state in document of the manufacturer Dupont [14].

The second factor is the back-emf due to the relative movement of the mover’s magnets with the copper traces of the stator. To understand the magnitude of the back-emf force, experiments were conducted where it was used the actuator as a generation. The potential difference experienced to a mover velocity equal to the one used to acquire the force surface of figure 29 can be seen in figure 22 (on the left). With peaks of potential difference not higher than 50 mV and considering the theoretical relation between force produced by the actuator and the corresponding current present in figure 24 the back-emf force does not exceed 1.5 g accounting for 10% of the difference in theoretical and experimental results.

Since the stroke length is the maximum length that the mover can travel without exposing the interior of the stator, being equal to \( L_{cr} - L_{act} = 65.024 \text{ mm} \), all peaks exhibit the same heights, fact that predictably does not occur when the mover surpasses these length since less copper traces interact with the magnet rings.

In the third experiment conducted to validate the theoretical model the mover was, as previously stated, looked in a central position with respect to the stator. Measurements were taken and the maximum forces were acquired for each increase of current, from 5% up to 100% of the maximum current able to be provided to the phases, \( I_{max} = 1.1628 \text{ A} \).

Increasing the velocity of the actuator to critical values when controlling the actuator in open-loop in order to show the capabilities of generation potential difference, the voltage across two phases can be seen in figure 22 (on the right), where the actuator generates almost 1.3 V between two phases.

The third important loss that explains the lower experimental force is the joule losses in the copper traces due the flow of current in a conductor.

The generalization of the previous analysis to the stroke length of the actuator is present in figures 30 (in Appendix section) and 31 (in Appendix section) where in the right figure is present the forces calculated using finite elements and in the left figure it’s shown the acquired experimental force surface.

The surface correspondent to the difference between the signal acquired experimentally and the theoretical one can be seen in figure 23.

In figure 24 can be seen the comparison between the experimental and the predicted results of force, showing an appreciable agreement. This validates the theoretical model used in the electromagnetic analysis.

**B. Precision and Maximum velocity**

Precision is an important measure in linear actuators since measures the repeatability or reproducibility of the movements performed by the actuator. To quantify the actuator’s precision
and the impact of the velocity in said measurement, the actuator was tested performing ten consecutive forward and backwards movements, each equal to $3h_{mon} = 24.384 \text{ mm}$. Figure 25 illustrate the initial and final positions of the experiments. The acquired measurements have a precision of 0.0703 mm since the captured image has a width of 45 mm corresponding to resolution of 640 pixels.

![Fig. 25: Initial and Final positions of the mover.](image)

The experiment was developed exciting the phases with the signals represented in appendix figure 32 (on the left). The velocity of the mover was varied for each acquisition by changing the interval of time between the signals sent to the three phases. The equation for the actuator velocity is given by

$$v_z = \frac{h_{mon}}{s} t$$

(9)

Figure 26 shows the evolution of maximum error demonstrated in the actuator’s final position when velocity is increased up to 225 mm/s. For velocities higher than this the mover because of it’s own dynamic characteristics isn’t able to follow the moving wave, not being able to be reasonably controlled in open loop.

From the graphic can be observed an generalized increase in the difference between the initial and final positions. For the maximum velocity without instability the error between positions is 0.7031 mm which is very large since the pace of the actuator is 0.0226. Meaning that the actuator is off more than 31 discrete positions from the initial one.

![Fig. 26: Maximum error as function of the actuator’s velocity.](image)

In further experiments, for all velocities, the error was able to be removed by exciting, after the sinusoidal signal, the three phases for about 100 milliseconds with the last value of the signal as seen in figure 32 (on the right) locking the mover in the correct position.

Being set the maximum admissible velocity that the actuator’s mover can travel without position error, the force associated with the said velocity must be determined since plays an important roll in the actuator’s characteristics. An experiment with the same molds of the ones done to acquire the forces of figure 24 was conducted and the maximum force acquired. For the actuator comprised of For the actuator with the characteristics of table III and a velocity of $v_z = 225 \text{ mm/s}$ and a current value of $I = I_{max} = 1.1628 A$ the force done by the actuator is equal to 0.672 N.

C. Thermal Model Validation

In order to validate the temperature rise experienced by the outer surface of the stator that’s in direct contact with the surrounding air at $25^\circ C$, an experiment was conducted. The actuator was excited by a moving wave with low velocity and amplitude of 1.2 A. The experimental curve obtain can be seen in blue in figure 27 and is compared with the theoretical one, seen in red with dashed lines.

![Fig. 27: Theoretical versus experimental temperature data.](image)

One can observe that the theoretical model developed using the electric analog of the thermal behavior shows to be quite proper when applied to this actuator since the theoretical curve seems to replicate the experimental data with a great deal of accuracy except for the final value that shows a disparity of 0.65°C when compared with the stationary value in the theoretical case.

CONCLUSIONS

In this thesis, the design, construction, testing and theoretical model validation of a novel tubular direct-drive triphasic linear motor were discussed. A theoretical part was present where the tubular linear actuator possible topologies were introduced, after the model parameterization was conducted and an optimization aim was set. The theoretical force was withdraw. A thermal transient behavior was introduced predicting the increase of temperature due to joule losses in the laminate copper traces.

After was explained the manufacturing procedure that was developed. Various designs for the printed circuit were tested as well as several methods for connecting the traces being the design present here the result of several essays.

Finally experimental results were taken from various tests being analyzed the validated of both electromagnetic and thermal models. Being both accepted as valid theoretical models presenting small errors. furthermore a couple of tests were conducted to investigate the actuator characteristics.
APPENDIX

Fig. 28: Theoretical force surface obtained by finite elements.

Fig. 29: Experimental force surface.

Fig. 30: Theoretical force surface obtained by finite elements for the actuator’s stroke.

Fig. 31: Practical force surface obtained by finite elements for the actuator’s stroke.

Fig. 32: Exciting three phases current with sudden break (on the left), and with hold in the last current value (on the right).

TABLE III: Actuator’s Prototype properties and dimensions.

<table>
<thead>
<tr>
<th>Actuator’s Part</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mover</td>
<td>( r_{mi} )</td>
<td>Magnet’s ring interior radius</td>
<td>1 mm</td>
</tr>
<tr>
<td></td>
<td>( r_{mo} )</td>
<td>Magnet’s ring exterior radius</td>
<td>3 mm</td>
</tr>
<tr>
<td></td>
<td>( h_m )</td>
<td>Magnet’s high</td>
<td>4 mm</td>
</tr>
<tr>
<td></td>
<td>( n_m )</td>
<td>Number of magnet rings</td>
<td>104</td>
</tr>
<tr>
<td></td>
<td>( h_{mon} )</td>
<td>Monomer’s length</td>
<td>8.128 mm</td>
</tr>
<tr>
<td></td>
<td>( g_m )</td>
<td>Gap between magnets</td>
<td>0.0640 mm</td>
</tr>
<tr>
<td></td>
<td>( m_m )</td>
<td>Mover Mass</td>
<td>39.40 g</td>
</tr>
<tr>
<td></td>
<td>( L_{cr} )</td>
<td>Length of the mover</td>
<td>211.328 mm</td>
</tr>
<tr>
<td>Stator</td>
<td>( L_{act} )</td>
<td>Length of the stator</td>
<td>139.9 mm</td>
</tr>
<tr>
<td></td>
<td>( g_{ct} )</td>
<td>Gap between copper traces</td>
<td>0.0640 mm</td>
</tr>
<tr>
<td></td>
<td>( w_{ct} )</td>
<td>Copper trace width</td>
<td>1.1853 mm</td>
</tr>
<tr>
<td></td>
<td>( \tau_{ct} )</td>
<td>Copper trace thickness</td>
<td>0.018 mm</td>
</tr>
<tr>
<td></td>
<td>( \tau_{pc} )</td>
<td>Polymide thickness</td>
<td>0.025 mm</td>
</tr>
<tr>
<td></td>
<td>( n_{ph} )</td>
<td>Number of phases</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>( I_p )</td>
<td>Rated Current Amplitude</td>
<td>0.5814 A</td>
</tr>
<tr>
<td></td>
<td>( R_p )</td>
<td>Resistance Per Phase</td>
<td>8.6 Ω</td>
</tr>
<tr>
<td></td>
<td>( r_{si} )</td>
<td>Stator inner radius</td>
<td>3.5 mm</td>
</tr>
<tr>
<td></td>
<td>( r_{so} )</td>
<td>Stator outer radius</td>
<td>4.75 mm</td>
</tr>
<tr>
<td></td>
<td>( s_m )</td>
<td>Stator Mass</td>
<td>10.88 g</td>
</tr>
<tr>
<td></td>
<td>( n_w )</td>
<td>Number of winding turns</td>
<td>8</td>
</tr>
</tbody>
</table>
REFERENCES


[32] Saso Ježernik, Gery Colombo, and Manfred Morari. Au-
tomactic gait-pattern adaptation algorithms for rehabilitation with a 4-dof robotic orthosis. IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION, June 2004.


