Structural analysis of open deck ship hulls subjected to bending, shear and torsional loadings

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“There is nothing stronger than these two old soldiers: patience and time.”

Leo Tolstoy in “War & Peace”
Acknowledgments

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Resumo

O objectivo da presente tese é estudar o comportamento de estruturas de navios de convés aberto sujeitos a flexão, corte e torsão através da utilização de soluções analíticas e de elementos finitos. Dois conjuntos de modelos de elementos finitos simplificados são desenvolvidos como vigas de paredes duplas, compostas de chapas de espessura equivalente acomodando reforços tanto transversais como longitudinais (respeitando as características originais da secção) de forma a poderem realizar-se as análises. O primeiro consiste num modelo com o comprimento equivalente a um tanque de carga, utilizado para estudar o comportamento quando sujeito aos esforços de flexão e corte. Quanto ao segundo, trata-se de um modelo de um casco completo do tipo barcaça, utilizado para estudar a resposta estrutural quando sujeito a um momento de torção. Este segundo conjunto de modelos de elementos finitos é composto por um modelo de convés aberto, um semi-fechado e um fechado, de forma a conseguir quantificar a influência deste factor no comportamento estrutural.

Os valores obtidos através da análise de elementos finitos são comparados com os valores obtidos através dos procedimentos simples da teoria de vigas e de vigas de paredes finas, demonstrando que as tensões globais são previstas, geralmente com sucesso, através destes métodos simples.

A aplicação da teoria das vigas de paredes finas aplicada nesta tese, reside em considerar a secção transversal do navio como uma secção em “U” com áreas aglomeradas, com diferentes condições de fronteira, sujeita a um carregamento de torção, analisada de acordo com o método do bi-momento.

Palavras-Chave: Momento flector, força de corte, momento de torção, estrutura de convés aberto, método dos elementos finitos, teoria da viga, teoria das vigas de paredes finas, método do bi-momento.
Abstract

The objective of the present thesis is to study the structural behavior of open deck ship hull structures subjected to bending, shearing and torsion by using analytic and finite element solutions. Two different sets of simplified finite element models are developed as a double shell box girder composed of plates with equivalent thicknesses that accommodate the stiffeners both in longitudinal and transverse directions (in such way that the original sectional properties are respected) to perform the analyses. The first one, a cargo-hold length model, used to study the behavior under bending and shear design loads, and the second one, a full pontoon-like ship hull model, used to study the structural response under the torsional design load. This second set of FEM models is composed by a open deck model, a partially-closed model and a closed model, in order to quantify the influence of this factor on the structural behavior.

The values obtained by the FEM analysis are compared with the simple procedures from the beam theory and from the thin-walled girder theory, demonstrating that the global stresses are generally successfully predicted by these simple methods.

The thin-walled girder theory application used in this thesis, lies in considering the ship cross-section as a channel section with lumped areas under different boundary conditions, subjected to torsional load analyzed according to the bi-moment method.

Keywords: bending moments, shear loads, torsional moment, open deck structures, finite element method, beam theory, thin-wall girder theory, bi-moment method.
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Chapter 1

Introduction

Ship structure design and analysis has always been a very important and active field of scientific research, in an effort to make those structures more reliable and cost effective. Much of this work was initially aiming to develop methods to determine the hull girder strength, and although these early methods gave adequately safe designs for common ship structures it has been shown by full-scale tests that the mechanisms of failure where frequently different from the predictions of those methods [1]. The major cause for that discrepancy is the non-linear behavior of the individual components and subsequently the entire system. These observations led to an increasing concern with the local phenomena as opposed to the global phenomena. A great amount of research was devoted then to the ultimate strength and behavior of individual ship structural components such as individual plates, stiffened plates and grillages (Fig. 1.1). Based on the knowledge of these individual components, various methods were developed in an attempt to determine the ultimate strength for the entire ship hull [2].

![Diagram of a ship hull girder as a box-like thin-walled stiffened structure.]

Figure 1.1: Ship hull girder as box-like thin-walled stiffened structure.

From a variety of methods, one of the most exhaustive is the one developed by Ostapenko [2][5], where the behavior and ultimate strength of longitudinal stiffened ship hull girder segments of rectangular single-cell cross section, subjected to bending, shear and torque, were analyzed analytically and tested
experimentally. This method produces accurate results for the bending and shear load cases, but not so acceptable results when torsion is considered (up to 40% of overestimation).

Despite of this research work, the torsion problem kept “understudied” since it was not the reported cause of accidents. However, torsion induced buckling damage in deck structures of ships with large deck openings were not uncommon, see for an example Hong et al. [6], where a large ore carrier under rough weather was subjected to deck damages due to excessive warping. Besides, most of the Classification Society criteria for the ship structural design are based on the first yield of hull structures together with buckling checks for structural components (i.e. not for the whole hull structure), which proved themselves to be effective for intact vessels in normal seas and loading conditions, however they fail to do so after accidental situations such as collisions or groundings [7].

Later, Paik [7-9] also worked on the ultimate strength under torsion based on the thin-walled theory and finite element analyses, suggesting a multi-segment model between two neighboring transverse bulkheads or a single-segment model between two neighboring transverse frames as sufficient models to study the warping effects.

Sun and Soares [10] conducted a model experiment, as well as a nonlinear finite element analysis, of a large deck opening ship-type structure to investigate the ultimate strength and collapse mode under a torsional moment.

The elastic behavior of ship structures and its stiffness parameters, under torsion, were a subject of study by Pedersen [11, 12], Pavazza [13, 14] and Senjanović [15-23]. All of these authors developed methods based on the thin-walled girder torsional theory (since ships are a good example of a thin-walled structural application), but while the first two authors modeled the contribution of transverse bulkheads, deck strips and engine rooms as axial elastic foundations, the latter considered them as short closed cross-sections with an increased torsional stiffness and used a finite element analysis and the energy approach to estimate this increment.

Iijima et al. [24] developed a practical method for torsional strength assessment, including a wave load estimation method and a proposal of design loads by a dominant regular wave condition.

The topic of thin-walled girders under torsion is also studied in other engineering fields such as civil engineering, where the methods are not fundamentally different, as for an example, the investigation of Sapountzakis and Mokos [25, 26].

On a more particular level, Villavicencio et al. [27], developed a method, based on finite element analysis, to estimate the normal warping stress fluctuation in the presence of transverse deck strips in large container ships.
1.1 Motivation

Ship hull girders are subjected to shear, bending and torsional loadings. The resulting stresses of these loads depend generally on the distribution of the ship displacement (lightweight plus deadweight) and on the sea effects, namely the wave height and the direction in which the ship is sailing relatively to the encountered waves, and also on the structural configuration.

In the case of shear and bending loads, their effects are well known and have been largely studied, as well as the ultimate ship strength under the effects of such loads. However, that’s not the case of torsional loading effects, for which there are fewer studies performed, from which stand out the studies from Ostapenko [2–5], Pedersen [11, 12] and Senjanović [15–23].

The torsional moments are in general composed by the St. Venant and warping torsional moments, however in most practical cases the effect of one component may be neglected when compared to the other. The warping torsion is negligible in slender members with a closed cross section, while the St. Venant torsion may be negligible in thin-walled open cross sections where the plane sections no longer remain plane and warping deformation takes place [28].

The torsional loading effects become more important in the, so called, open deck ships such as container ships and large bulk carriers, since they have lower torsional strength and rigidity (when compared to tankers for example) due to their wide hatch openings, which allows to consider their cross sections as thin-walled open cross sections.

Normally, the torsional stiffness verified in most ships is more than adequate to prevent undue distortion of the structure [28], however advanced methods for the estimation of a “more closer to the true” hull girder capacity are needed to assess the safety margins as well as a better prediction of fatigue damage, in order to improve the regulations and design requirements [7]. Moreover, since container ships (open deck) are still growing in size, the torsion problem is still a pertinent issue.

1.2 Aim and scope

The aim of this thesis is to study the overall structural behavior of open deck ship configurations when subjected to bending, shear and torsional loadings. This analysis is performed using the finite element method upon a simplified structural arrangement.

Further more, a particular focus is made on the analysis under torsional loading, where another relevant aspect is discussed and an alternative method of analysis is tested. The mentioned aspect is an attempt of quantifying the differences in the structural response i.e. developed stresses and displacements, between open deck, partially-closed and closed deck structural configurations.

The alternative method of this type of analysis consists on a simple beam theory procedure, aiming to be a fast and practical tool able to be used at an early design stage, in order to predict the expectable stress levels developed on the structure.
1.3 Thesis structure

The present document is divided in six chapters as follows: **Chapter 1** presents a brief introduction to the topic of the study, a review of the previous work developed on the topics of interest and its objectives; **Chapter 2** presents the midship section structural layout to be used throughout the study as well as a description of the element type, boundary conditions and loads applied in the finite element method analysis; **Chapters 3 and 4** deal with the behavior of the structure when subjected to vertical and horizontal bending moments and when subjected to vertical and horizontal shear loads respectively; **Chapter 5** is dedicated to the analysis of the structure when subjected to a torsional load, and presents the alternative simple method of analysis; in **Chapter 6** a review and discussion of the main conclusions of this study is made, as well as an insight to the future work and developments on the subject.
Chapter 2

Case Study

2.1 Midship section

The midship section used for this study is a simplification of the one taken from [28], where the main dimensions are given in Tab. 2.1.

Table 2.1: Ship main dimensions.

<table>
<thead>
<tr>
<th>Main dimensions [m]</th>
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<tr>
<td>Length, $L_{bp}$</td>
<td>152.9</td>
</tr>
<tr>
<td>Beam, $B$</td>
<td>26.0</td>
</tr>
<tr>
<td>Depth, $D$</td>
<td>16.2</td>
</tr>
<tr>
<td>Draft, $T$</td>
<td>10.8</td>
</tr>
</tbody>
</table>

The block coefficient, $C_B$, is not given, and for all computations where is required, it is assumed a typical block coefficient for a container ship, as $C_B = 0.65$.

Figure 2.1: Midship section layout.
Fig. 2.1 presents the original midship section layout in a dashed line, and in a bold line is the simplification that will be used further in the present study. From this midship section layout, a further simplification is made, regarding the bilge. In order to simplify the construction in the finite element model, the curved bilge was replaced by a rectangular bilge.

Also, some modifications have been made, regarding the thicknesses of some segments of the cross-section, in order to fulfill the minimum section modulus, according to the DNV - Det Norske Veritas structural rules [29].

Tab. 2.2 presents the midship cross-sectional properties after all the supra mentioned simplifications.

<table>
<thead>
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<tr>
<td>Area, $A[m^2]$</td>
<td>2.37</td>
</tr>
<tr>
<td>1st moment of area, $S[m^3]$</td>
<td>13.45</td>
</tr>
<tr>
<td>Neutral axis, $Z_{NA}[m]$</td>
<td>5.67</td>
</tr>
<tr>
<td>2nd moment of area, $I_{yy}[m^4]$</td>
<td>154.18</td>
</tr>
<tr>
<td>2nd moment of area, $I_{NA}[m^4]$</td>
<td>77.94</td>
</tr>
<tr>
<td>2nd moment of area, $I_{zz}[m^4]$</td>
<td>249.47</td>
</tr>
<tr>
<td>Section modulus, $Z_{deck}[m^3]$</td>
<td>7.40</td>
</tr>
<tr>
<td>Section modulus, $Z_{bottom}[m^3]$</td>
<td>13.75</td>
</tr>
<tr>
<td>Section modulus, $Z_{side}[m^3]$</td>
<td>19.19</td>
</tr>
<tr>
<td>Required section modulus, $W_{min}[m^3]$</td>
<td>7.36</td>
</tr>
</tbody>
</table>

To define the position of the shear center, a further approximation is used according to [7], as shown in Fig. 2.2 and expressed in Eqn. 2.1.

$$e = \frac{3D_1^2(t_s + t_t)}{B_1(t_b + t_t) + 6D_1(t_s + t_t)}$$  \hspace{1cm} (2.1)
The equivalent thicknesses are defined applying a procedure that consists in redistributing the net sectional areas of the internal girders by their attached plate, and further dividing the new areas by the length of the correspondent segment, obtaining in this way a uniform thickness for each segment. Once this is done, the estimated thicknesses, as well as the shear center position, are given in Tab. 2.3.

Table 2.3: Equivalent thicknesses and shear center location.

<p>| | | |</p>
<table>
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<tr>
<th></th>
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<tbody>
<tr>
<td>$D_t$</td>
<td>15.40</td>
<td></td>
</tr>
<tr>
<td>$B_t$</td>
<td>23.80</td>
<td></td>
</tr>
<tr>
<td>$t_s$</td>
<td>22.16</td>
<td></td>
</tr>
<tr>
<td>$t_l$</td>
<td>16.14</td>
<td></td>
</tr>
<tr>
<td>$t_b$</td>
<td>19.46</td>
<td></td>
</tr>
<tr>
<td>$t_i$</td>
<td>23.00</td>
<td></td>
</tr>
<tr>
<td>$e$</td>
<td>9.07</td>
<td></td>
</tr>
</tbody>
</table>

In respect to the original coordinate system, the shear center is located at the center line of the section at the coordinate $z = -8.27m$.

2.2 Finite element model

To perform the analysis, presented in Chapter 3 and Chapter 4, a three dimensional ship hull segment, equivalent to one cargo hold is modeled (Fig. 2.3) using the commercial software ANSYS.

In order to build the model some assumptions are made, namely a frame spacing, cargo hold length and the thickness of the frame plating. For the frame spacing, the assumed value is estimated as $L_{bp}/20$, which results to 7.645 meters length. The cargo hold length is assumed to be four times the frame spacing, resulting in a 30.58 meter length cargo hold. For the frame plating a thickness of 15 millimeters is assumed. Finally a transverse rigid plate, at one end of the hold, is also modeled as a thicker plate, in order to smoothly transmit the stresses and deformations across the section. This rigid plate is a part of the boundary conditions of the finite element model.

![Figure 2.3: Cargo hold length model.](image-url)
Element type

The type of elements used is the 8-node quadrilateral shell element (Fig. 2.4). It allows six degrees of freedom at each node (translations in the nodal x, y and z directions and rotations about the nodal x, y and z-axes). This type of element has plasticity, stress stiffening, large deflection, and large strain capabilities. This set of characteristics makes this type of element the most suited for this application. In ANSYS this type of element is designated as SHELL93.

![Quadrilateral 8-node shell element.](image)

Figure 2.4: Quadrilateral 8-node shell element.

Boundary conditions

To simulate the interaction between the finite element model of the hull segment with the rest of the ship hull, the nodes along the lines of one of the ends are constrained in every degree of freedom, while all other nodes are kept free, such as in a cantilever beam.

![Boundary conditions applied to the cargo hold length model.](image)

Figure 2.5: Boundary conditions applied to the cargo hold length model.

Applied loads

To model the overall loads to which the finite element model is subjected, it is necessary to transform the loads in equivalent distributed loads or equivalent set of nodal loads and apply them to the specific nodes of the meshed finite element model of the ship hull.
Knowing the bending moment and the second moment of area about the neutral axis of the cross-section, it is possible to determine the normal stresses $\sigma$ at any given point:

$$\sigma_i = \frac{M}{I_{N.A.}} \cdot d_i$$

(2.2)

where $M$ is the bending moment, $I_{N.A.}$ is the second moment of area about the neutral axis and $d_i$ is the distance of any given point to the neutral axis.

Once the stresses are known, the sectional forces $F_i$ to apply at each node are determined through the nodal areas, $A_i$, as:

$$\sigma_i = \frac{F_i}{A_i}$$

(2.3)

These nodal forces generate an equivalent bending moment, simulating the vertical or horizontal bending moments as shown in Fig. 2.6.

Figure 2.6: Equivalent vertical (a) and horizontal (b) bending moments.

For the shear load case, when the overall shear load $Q$ is known, the shear stresses $\tau$ are calculated as:

$$\tau = \frac{Q \cdot S^*}{t \cdot I_{N.A.}}$$

(2.4)

where $S^*$ is the static shear moment of area and $t$ is the local thickness.

The local shear stresses, based in the nodal areas, can be used to determine the forces to be applied on each node:

$$\tau_i = \frac{F_i}{A_i}$$

(2.5)
To calculate the shear stress, an approximation is used here. It consists in assuming an average constant value along the element under shear (Fig. 2.7) which in fact is an overestimation of the real shear stress distribution. With this approximation the Eqn. 2.4 can be rewritten as:

$$\tau_{eqv} = \frac{Q}{A_{shearing}}$$  \hspace{1cm} (2.6)

where $A_{shearing}$ is the sectional area of all structural components that are subjected to shear loading.

Figure 2.7: Shear stress approximation for a flat beam case.

These equivalent shear forces generate an equivalent global shear force, both for the vertical and horizontal shearing as shown at Fig. 2.8.

Figure 2.8: Equivalent vertical (a) and horizontal (b) shear forces.
Nodal areas

An important concept at this point of the study is the one of the *nodal area*. This area is the area linked to a specific finite element node (Fig. 2.9). These areas are obtained by multiplying half the distance between two consecutive nodes, \( h \), and the thickness of the respective plates. This procedure is repeated for each node as much times as the number of segments that share the considered node.

\[
A = \sum_i h_i t_i
\]  

(2.7)

The structure analyzed here, only has three different types of nodal area configurations with two, three or four branches as can be seen on Fig. 2.9.

(a)  

(b)  

(c)  

Figure 2.9: Different nodal area configurations: a) node shared by one or two elements; b) node shared by three elements; c) node shared by four elements.
Chapter 3

Ship hull structure subjected to bending load

The bending moment acting upon a ship hull can be divided into a vertical and horizontal component. The sea state along with the longitudinal weight and buoyancy distributions are the principal factors governing these components of the bending moments. Classification society rules provide the design values for these loads. In the present analysis the design values, according to DNV - Det Norske Veritas, are used.

3.1 Vertical bending moment

The design vertical bending moment, according to classification society rules, is divided into two components: the stillwater and the wave induced bending moments:

\[ M_T = M_S + M_W \]  

The stillwater bending moments along the ship length, for sagging and hogging conditions, are given by:

\[ M_S = k_{sm} M_{SO} \]  

\[ M_{SO} = -0.065 C_W L^2 B (C_B + 0.7) \ \text{(kN.m) in sagging} \]  
\[ = C_W L^2 B (0.1225 - 0.015 C_B) \ \text{(kN.m) in hogging} \]

where \( L \) is the length between perpendiculars of the ship, \( k_{sm} \) is a distribution factor, as shown at Fig. 3.1 and \( C_W \) is the wave coefficient given as:

\[ C_W = 10.75 - \left( \frac{300 - L}{100} \right)^{1.5} \]
The wave induced bending moments along the length of the ship are given by:

\[ M_W = k_{wm} M_{WO} \]  \hspace{1cm} (3.6)

\[ M_{WO} = -0.11\alpha C_W L^2 B(C_B + 0.7) \text{ (kN.m) in sagging} \]  \hspace{1cm} (3.7)

\[ = 0.19\alpha C_W L^2 B C_B \text{ (kN.m) in hogging} \]  \hspace{1cm} (3.8)

where \( \alpha = 1 \) is for seagoing conditions and \( \alpha = 0.5 \) for sheltered water conditions, and \( k_{wm} \) is a distribution factor as shown at Fig. 3.1. For this analysis \( \alpha \) has its unitary value.

Figure 3.1: Stillwater design bending moment distribution (left), wave induced design bending moment distribution (right).

The estimated values for both stillwater and wave-induced design bending moments are presented in Tab. 3.1.

<table>
<thead>
<tr>
<th>Vertical bending moments</th>
<th>( M_s ) sagging [kN.m]</th>
<th>( M_s ) hogging [kN.m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stillwater bending moments</td>
<td>-478222</td>
<td>614467</td>
</tr>
<tr>
<td>Wave-induced bending moments</td>
<td>-809298</td>
<td>673053</td>
</tr>
</tbody>
</table>

Summing the bending moment components, both for sagging and hogging cases, according to Eqn. 3.1 it can be observed that the total design bending moment, in absolute value, is approximately the same for both conditions. Due to that fact, the hogging bending load case is chosen, \( M_{T_{hog}} = 1287520 \text{ kN.m} \), to be used in the analysis.
### 3.2 Horizontal bending moment

According to the classification society rules, the designing wave-induced bending moment for the horizontal bending load is given by:

\[
M_{WH} = 0.22L^{9/4}(T + 0.3B)C_B\left[1 - \cos\left(360 \frac{x}{L}\right)\right] \quad (kN.m) \tag{3.9}
\]

where \( T \) represents the considered draft.

By plotting the trigonometric parcel of Eqn. 3.9, it is possible to obtain the horizontal design bending moment distribution.

![Figure 3.2: Horizontal design bending moment distribution.](image)

Considering the distribution shape represented in Fig. 3.2 is clear that the maximum bending moment occurs when \( x/L = 0.5 \), which in fact is the midship. Computing its value, according to Eqn. 3.9 the estimated value is \( M_{WH} = 437316 \text{ kN.m} \).
3.3 Finite element analysis

Prior to the presentation and discussion of the results according to the procedures described on Chapter 2, it is important to mention that the results obtained by the finite element method are affected by the particularities of the finite element model. It is expected that stress concentrations may appear, namely in the proximity of the fully-constrained end of the FE model. Since the boundary conditions applied to the FE model approximately represent the real case, these stress concentrations are not to be taken into consideration, for in the real case they would not exist. Hence the readings of the results will be taken at mid-length of the FE model, at a position where they are as far as possible away from the constrained end and from the free-end where the nodal forces are applied.

Another important factor, visible throughout the axial stress analysis, is the shear lag effect. This happens when there are interconnected concurrent plates so that relative displacements between them cannot occur. This effect leads to a non-uniform axial stress distribution due to a considerable increase on the stress values on the region when these intersections occur.

Concerning the shear stress results, it is known that its values are considerably affected by the thickness variations, and hence, notorious discontinuities are expected at the regions where the structure elements intersect each other.

3.3.1 Vertical bending moment

The results obtained through finite element method analysis, regarding the axial stresses $\sigma_x$ as a function of the vertical bending moment, give a maximum tension stress (on deck) in the range of 146 to 190 MPa, and a maximum compression stress (on bottom) in the range of 70 to 113 MPa, as shown in Fig. 3.3. These results are consistent with the values obtained from the beam theory (Fig. 3.4), which are 170 MPa for tensile stress at the deck and 93 MPa for compressive stress at the bottom.

![Figure 3.3: Axial stresses under vertical bending moment.](image)
The normal stresses taken along the depth of the hull at the side shell at mid-length of the FE model, are presented in Fig. 3.4, demonstrate tensile stresses on deck of 159 MPa and compressive stresses on the bottom of 86 MPa, which leads to a difference of about 6% for tensile and 8% for compressive stresses with respect to the results estimated by the beam theory.

![Figure 3.4: Axial normal stress distribution along the side shell, under vertical bending moment, obtained by beam theory (solid line) and FEM (dashed line).](image)

Figure 3.4: Axial normal stress distribution along the side shell, under vertical bending moment, obtained by beam theory (solid line) and FEM (dashed line).

![Figure 3.5: Evolution of the axial stress distribution on the bottom, across the length of the model, measured on the free end (solid line), at half length of the model (dashed line) and at about two thirds of the model length (dashdotted line).](image)

Figure 3.5: Evolution of the axial stress distribution on the bottom, across the length of the model, measured on the free end (solid line), at half length of the model (dashed line) and at about two thirds of the model length (dashdotted line).

Analyzing Figs. 3.3 and 3.5, it is possible to observe the effects of the stress concentrations near the fixed end of the FE model, and a very irregular stress distribution at the free end of the model, where the loads are applied.

Other important factor to check is the von Mises yield stress criterion. According to this criterion, a material is said to start yielding when its von Mises stress \( \sigma_v \) reaches the material yield strength \( \sigma_y \). The von Mises stresses are computed accordingly to:

\[
\sigma_v = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}}
\]  

(3.10)

where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the principal stresses measured at the considered point.
As it can be seen in Fig. 3.6, the von Mises stresses, estimated by FEM are similar to the absolute values of the axial normal stresses, since the axial normal stresses are the dominant stresses in this loading scenario. Also, it can be observed that the maximum von Mises stresses are below the yield strength of the common shipbuilding steels (around 250 MPa), which means that the structure does not reach the yielding point and is capable to resist against the subjected load.

![Figure 3.6: von Mises stress distribution along the side shell, measured at the model mid-length.](image)

### 3.3.2 Horizontal bending moment

The maximum axial stresses $\sigma_x$, for the horizontal bending load case, estimated by FEM, are in the range of 21 to 30 MPa, both for tension and compression (Fig. 3.7), which is consistent with the fact that the cross section is symmetrical in relation to its central line.

![Figure 3.7: Axial stresses under horizontal bending moment.](image)
Figure 3.8: Axial normal stress distribution along the bottom, under horizontal bending moment, obtained by beam theory (solid line) and by FEM (dashed line).

The stresses estimated by the beam theory (Fig. 3.8) give a value of 23 MPa both for tension and compression, which lays in the interval of global values estimated by FEM.

The axial stress values across the beam of the ship at the bottom (Fig. 3.8) are 21 MPa as the maximum stress, both for tension and compression, at each side of the ship. These values represent a difference of 9% in relation with those from the beam theory.

The structural response under horizontal bending moment is one order of magnitude lower, when compared to the results under vertical bending moment. This observation is in accordance to the fact that the horizontal bending moment is also one order of magnitude lower than the vertical bending moment, while the side, bottom and deck moduli are all of the same order of magnitude.

Figure 3.9: Axial stress distribution along the side shell.

Fig. 3.9 denotes the structural asymmetry between the bottom and the deck, where slightly higher stresses are attained, due to the fact that there is much less net sectional area on the deck than on the bottom.

Finally, and since the bending moments, both in vertical and horizontal cases, are generated by a set of linearly varying axial forces, distributed around the neutral axis, there are no significant shear stresses produced (Fig. 3.10).
Figure 3.10: Shear stress components under vertical (top) and horizontal (bottom) bending moments. From left to right: YZ, ZX, XY.
Chapter 4

Ship hull structure subjected to shear load

4.1 Vertical shear forces

The vertical shear forces acting on a ship while she’s sailing are divided in stillwater shear forces, caused by the weight and buoyancy distributions; and in wave induced shear forces, caused by the encountered sea effects.

The classification society rule values for the stillwater shear forces along the length of the ship, both in sagging and hogging are determined as:

\[ Q_S = k_{sq}Q_{SO} \]  \hspace{1cm} (4.1)

\[ Q_{SO} = 5 \frac{M_{SO}}{L} \text{ (kN)} \]  \hspace{1cm} (4.2)

where \( M_{SO} \) are the stillwater design bending moment as determined in Chapter 3 and \( k_{sq} \) is a distribution factor as illustrated at Fig. 4.1.

The wave-induced shear force rule values can be positive or negative, depending on the sign of the stillwater shear force.

\[ Q_{WP} = 0.3\beta k_{wqp}C_WLB(C_B + 0.7) \text{ (kN)} \]  \hspace{1cm} (4.3)

\[ Q_{WN} = -0.3\beta k_{wqn}C_WLB(C_B + 0.7) \text{ (kN)} \]  \hspace{1cm} (4.4)

where \( \beta \) is a factor with unitary a value for seagoing conditions (in use) and 0.5 for sheltered waters and \( k_{wqp} \) and \( k_{wqn} \) are the distribution factors varying as shown on Fig. 4.1.
Testing the different possible scenarios resulting from Eqns. 4.1 to 4.4 and considering the distributions from Fig. 4.1, the conclusion is that the maximum vertical design shear force case takes place when $x/L \in [0.7; 0.85]$. The structural analysis is performed at $x/L = 0.7$, since it coincides with a transverse bulkhead and the division between the cargo holds. The results are presented at Tab. 4.1.

Table 4.1: Vertical design shear forces.

<table>
<thead>
<tr>
<th></th>
<th>$Q_S - [kN]$</th>
<th>$Q_S + [kN]$</th>
<th>$Q_{WN}[kN]$</th>
<th>$Q_{WP}[kN]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stillwater shear forces</td>
<td>-15638</td>
<td>20094</td>
<td>-12025</td>
<td>14435</td>
</tr>
<tr>
<td>Wave-induced shear forces</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Summing the stillwater and wave-induced shear loads, results in an higher load for the positive shear force scenario. The obtained final vertical shear force is $Q_T = 34529$ kN.
4.2 Wave-induced horizontal shear forces

The horizontal wave-induced design shear force along the ship length is given by the classification society rules as:

\[ Q_h = 0.6 Q_{\text{max}} \sin \left( \frac{360 x}{L} \right) \text{(kN)} \]  \hspace{1cm} (4.5)

\[ Q_{\text{max}} = (Q_S + Q_W)_{\text{max}} \text{(kN)} \]  \hspace{1cm} (4.6)

where \( Q_S \) and \( Q_W \) are the values of the extreme scenario obtained in Tab. 4.1.

From the Eqn. 4.5 the maximum horizontal shear force will take place when \( x/L = 0.25 \) and \( x/L = 0.75 \), however, to keep the consistency of the analysis, the point of interest is \( x/L = 0.7 \) considering that the discrepancy in the results can be neglected.

Thus, the obtained horizontal shear force, according to Eqn. 4.5, is \( Q_h = -19327 \text{ kN} \).

![Horizontal design shear force distribution.](image)

4.3 Finite element analysis

4.3.1 Vertical shear forces

The vertical shear force \( Q_z \) is modeled as forces applied on the nodal areas of the longitudinal vertical elements \( A_{zx} \) (shear area) on the free end of the structure, so the \( \tau_{zx} \) is the main shear stress component affecting the structure (Fig. 4.3), since:

\[ \tau_{zx} = \frac{Q_z}{A_{zx}} \]  \hspace{1cm} (4.7)

with its maximum values around 97 MPa. Also, there is some visible shear stress (\( \tau_{yx} \)) at the bottom plates (Fig. 4.4). However, its values are less relevant (on the order of 10 MPa).
Analyzing the shear stress $\tau_{zx}$ distribution along the side (Figs. 4.3 and 4.5), it is visible that its maximum values occur on the cross section neutral axis, which is in accordance with Eqn. 2.4, where the term corresponding to the static moment of area $S^*$ guarantees that the maximum shear stress value occurs at the neutral axis location. Analyzing its distribution across the double bottom plating (Fig. 4.5), it is observed that there is shear stress only in the regions where there are vertical plating.
Figure 4.5: ZX shear stress component measured at the model mid-length along the side (left) and along
the double bottom (right).

4.3.2 Horizontal shear forces

In this load case, the shear force $Q_y$ is modeled as forces applied on the nodal areas of longitudinal
horizontal elements. Thus, the main shear stress component affecting the structure is $\tau_{yx}$, as shown in
Fig. 4.6 with its values around 74 MPa.

Besides the dominant $\tau_{yx}$ shear stress component, as in the vertical shear force load case, there is a
residual shear stress propagation through the plating in the normal orientation to the load, in the present
case, the $\tau_{zx}$ component. Once again, its values are considerably lower, around 30 MPa (Fig. 4.7).
The shear stress $\tau_{yx}$ along the bottom at mid-length of the FE model (Fig. 4.8) gives a clear notion of the symmetry of the structure, since the maximum shear stress occurs at the neutral axis. Also, the effects of the presence of the vertical elements of the structure are clearly noted (shear lag effect). Analyzing its distribution along the side shell, it is clear that the shear stress only affects the structure on the proximity of the horizontal structural elements and that its values are almost irrelevant.

Figure 4.8: YX shear stress component measured at the model mid-length along the bottom (left) and along the side (right).
Chapter 5

Ship hull structure subjected to torsional loading

The distribution of the torsional moment along the ship length depends on the cargo load distribution, both longitudinally and transversely, as well as on the ship advance direction relatively to the encountered waves, which can produce asymmetric loads on the hull. The torsional moment \( T \) is divided in two components, namely the St. Venant torsional moment \( T_S \) and warping torsional moment \( T_\omega \) [28], hence:

\[
T = T_S + T_\omega
\] (5.1)

The St. Venant component accounts for the torsion effects assuming that there are no in-plane deformations, i.e. a plane cross section remains plane during the twist. This kind of torsion is only verified in circular closed cross sections, while in the remaining cases the warping component must be considered, since the cross sections no longer remain in plane during the twist. These warping deformations vary with the rate of twist and as a function of the position across the cross section.

5.1 Bi-moment method analysis

In the preliminary design stages, when the torsional loading is not yet known, the application of a simplified procedure for structural analysis is very important. This approximate method gives a general procedure for calculating the shear and flexural warping stresses as well as the torsional deformations induced in open ships by torsional loading. It is based on the sectorial properties of the ship section, which are obtained using a simplified idealization of the ship section configuration [28].

The shear and flexural (normal) warping stresses are calculated as follows:

\[
\tau(\omega) = \frac{T(\omega) \cdot S(\omega)}{J(\omega) t} \] (5.2)

\[
\sigma(\omega) = \frac{\omega \cdot M(\omega)}{J(\omega)} \] (5.3)
where \( T(\omega) \) is the warping torsional moment, \( S(\omega) \) is the sectorial static moment, \( M(\omega) \) is the bi-moment, \( J(\omega) \) is the sectorial moment of inertia (or warping constant of the section), \( \omega \) is the sectorial coordinate and \( t \) is the thickness.

\[
T(\omega) = -EJ(\omega) \frac{d^3\phi}{dx^3}
\]

\[
M(\omega) = -EJ(\omega) \frac{d^2\phi}{dx^2}
\]

In order to proceed with the determination of the warping stresses, the sectorial properties of the ship section (i.e. \( \omega \), \( S(\omega) \) and \( J(\omega) \)) and the solution of the torsion equation (i.e. \( \phi(x) \)) must be determined.

The differential equation for non-uniform torsion, equivalent to Eqn. 5.1 is given by:

\[
GJ_t \frac{d\phi}{dx} - EJ(\omega) \frac{d^3\phi}{dx^3} = T
\]

where \( E \) is the Young modulus, \( G \) is the shear modulus, \( J_t \) is the torsional modulus and \( T \) is the sectional torque applied to the structure.

Equation 5.6 can be rewritten as:

\[
\frac{d\phi}{dx} - k^2 \frac{d^3\phi}{dx^3} = \frac{T}{GJ_t}
\]

where:

\[
k^2 = \frac{EJ(\omega)}{GJ_t}
\]

The solution of the differential equation (Eqn. 5.7) depends on the boundary conditions and on the torque characteristics expressed below as:

**A) Fixed end with free warping**

This boundary condition, satisfy:

\[
\phi = \frac{d^2\phi}{dx^2} = 0
\]

**B) Fixed end with constrained warping**

This boundary condition, satisfy:

\[
\phi = \frac{d\phi}{dx} = 0
\]

**Applied torque**

The magnitude and distribution of the torsional loading, acting upon a ship hull, depend on a multitude of factors, namely the hull form, the sea state, the cargo distribution, the shear center position and etc.
Different formulations are given by different classification societies. However, for this simplified method, a concentrated sectional torque amidship will be used (Fig 5.1).

![Figure 5.1: Torsion loading applied on the cargo hold length.](image)

This concentrated sectional torque is to be divided by the two halves of the beam, as follows:

\[
T = \begin{cases} 
  T_0/2 & \text{for the left half}, \\
  -T_0/2 & \text{for the right half}.
\end{cases} \tag{5.11a//b}
\]

**Solution of the differential torsion equation**

For the present study, two different scenarios are considered. In the first scenario warping deformations are allowed at both ends (boundary conditions expressed by Eqn. 5.9), on the second scenario warping deformations are not allowed at both ends (boundary conditions expressed in Eqn. 5.10). In both cases torsion deformations are not allowed at both ends.

Combining Eqns. 5.7, 5.11a and 5.11b and applying the above mentioned boundary conditions the following sets of solutions are obtained (consistent to the American Institute of Steel Construction solutions [30]):

**Free warping at both ends**

\[
\phi(x) = \begin{cases} 
  e^{-\frac{x}{L}T_0A} & x \in [0, L], \\
  e^{-\frac{x+2L}{L}T_0B} & x \in [L, L].
\end{cases} \tag{5.12a//b}
\]

where:

\[
A = \left(-ke^\frac{x}{L}+ke^\frac{2L}{L}x-e^\frac{2L}{L}x-e^\frac{x}{L}x\right) \tag{5.13}
\]

\[
B = \left(k(e^\frac{L}{L}-e^\frac{x}{L})+e^\frac{2L}{L}(L-x)+e^\frac{2L}{L}(L-x)\right) \tag{5.14}
\]
Constrained warping at both ends

\[
\phi(x) = \begin{cases} 
-\frac{e^{-\frac{x}{T_0}}TC}{2(1 + e^{\frac{x}{T_0}})GJ_t}, & x \in [0, \frac{L}{2}], \\
\frac{e^{-\frac{x+2x}{T_0}}TD}{2(1 + e^{\frac{x}{T_0}})GJ_t}, & x \in [\frac{L}{2}, L].
\end{cases}
\]  

(5.15a)

(5.15b)

where:

\[
C = \left( -ke^{\frac{x}{T_0}} - e^{\frac{x}{T_0}}(k + x) + ke^{\frac{2x}{T_0}} + e^{\frac{k+2x}{T_0}}(k - x) \right)
\]  

(5.16)

\[
D = \left( k(e^{\frac{2x}{T_0}} - e^{\frac{3x}{T_0}}) + e^{\frac{k+2x}{T_0}}(k + L - x) + e^{\frac{k+x}{T_0}}(-k + L - x) \right)
\]  

(5.17)

5.1.1 Simplified structural response

In order to simplify the computations, the cross sectional configuration of the double-skin structure of an open ship is simplified as a thin-walled open section [28]. This procedure, consists in transforming a section, like the one shown on Fig. 5.2a, on a simplified one, Fig. 5.2b, considering its effective bottom and side structures thicknesses. The sectional areas of the horizontal and vertical girders, on the side shell and double bottom respectively, are idealized as lumped areas.

For the present case study, the original section given in Fig. 2.1, is simplified into the idealization presented at Fig. 5.3.

The general relation between the main geometrical properties (e.g. beam and draft) of the original structure is expressed by the following equation:

\[
D_{\text{idealization}} = \frac{D_{\text{external}} + D_{\text{internal}}}{2}
\]  

(5.18)

where \( D \) is a generic main dimension.
Figure 5.3: Idealized ship section.

**Torsion constant, }_{t}**

The torsion constant is determined from the original section [28]:

\[
J_{t} = \sum_{i=1}^{r} \frac{4A_{i}^2}{I_{Y}} \tag{5.19}
\]

where \( r \) is the number of closed cells and \( A_{i} \) is the enclosed area of cell \( i \).

**Shear center position, }_{Y}**

Since the simplified structure is symmetric in respect to the Y-axis, its shear center is located along that axis, at a distance \( }_{Y} \) from an assumed pole \( P \), and \( I_{Y} \) is the second moment of area of the simplified structure with respect to its Y-axis. The distance \( }_{Y} \) is determined according to the following formula:

\[
}_{Y} = \frac{1}{I_{Y}} \int_{A} zw'dA \tag{5.20}
\]

where \( z' \) is the sectorial coordinate based on an assumed pole \( P \), and represents the double area swept by the radius vector from \( s = 0 \) to the required point on the contour (Fig. 5.4a).

The sectorial coordinate is determined as follows:

\[
z' = \int r ds \tag{5.21}
\]

where \( r \) is the perpendicular distance from the assumed pole \( P \) to the tangent at the point under consideration, as shown in Fig. 5.4a. The sectorial area has a positive value when the radius vector rotates in a clockwise direction, and a negative value for the opposite case.
The warping constant, also known as sectorial moment of inertia, is obtained by means of the principal sectorial coordinate $\omega$, accordingly to the following formula:

$$J(\omega) = \int_{A} \omega^2 dA \tag{5.22}$$

The principal sectorial coordinate is determined in the same way as the assumed sectorial coordinate (Eqn. 5.21), but with the pole coincident with the shear center of the section, as shown in Fig. 5.5, instead of the earlier assumed position $P$.

The sectorial static moment is a result of the integration of the principal sectorial coordinate given as:

$$S(\omega) = \int_{A} \omega dA \tag{5.23}$$
5.1.2 Results and analysis

For the procedures described in the previous section, two main classes of inputs are required. The material and geometric properties of the simplified structure (Tab. 5.1) and the applied torsional load.

Table 5.1: Structural input parameters.

<table>
<thead>
<tr>
<th>Steel properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young modulus, $E$ [GPa]</td>
<td>200.0</td>
</tr>
<tr>
<td>Shear modulus, $G$ [GPa]</td>
<td>79.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Section properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam, $b$ [m]</td>
<td>23.80</td>
</tr>
<tr>
<td>Depth, $d$ [m]</td>
<td>15.40</td>
</tr>
<tr>
<td>Sheer strake height, $a$ [m]</td>
<td>2.40</td>
</tr>
<tr>
<td>Bottom thickness, $t_{b}$ [mm]</td>
<td>37.00</td>
</tr>
<tr>
<td>Side thickness, $t_{w1}$ [mm]</td>
<td>30.00</td>
</tr>
<tr>
<td>Sheer strake thickness, $t_{w2}$ [mm]</td>
<td>42.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lumped areas, $A_i$</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{1,2}$, [m$^2$]</td>
<td>2.16E-2</td>
</tr>
<tr>
<td>$A_{3,4}$, [m$^2$]</td>
<td>2.20E-2</td>
</tr>
<tr>
<td>$A_5$, [m$^2$]</td>
<td>2.64E-2</td>
</tr>
<tr>
<td>$A_6$, [m$^2$]</td>
<td>8.28E-2</td>
</tr>
</tbody>
</table>

As discussed in section 5.1, the torsional load applied to the structure is a concentrated sectional moment amidship. This load is determined according to classification society rules wave-induced torsional moment equation (Eqn. 5.24) computed at midship. The estimated value is 144 MN.m.

$$M_{WT} = K_{T1}L^{5/4}(T + 0.3B)C_Bz_e \pm K_{T2}L^{4/3}B^2C_{SWP} \quad (kN.m) \quad (5.24)$$

where

$$K_{T1} = 1.4 \sin \left( 360 \frac{x}{L} \right) \quad (5.25)$$

$$K_{T2} = 0.13(1 - \cos \left( 360 \frac{x}{L} \right)) \quad (5.26)$$

$$C_{SWP} = \frac{A_{WP}}{LB} \quad (5.27)$$

and $z_e$ is the height of the neutral axis. Since the waterplane area $A_{WP}$ is not known, the waterplane coefficient $C_{SWP}$ (Eqn. 5.27) has to be assumed. The same logic of “worst case scenario” is applied, so the assumption is that the waterplane coefficient has an unitary value ($C_{SWP} = 1$).
Once the input parameters are all defined, the results of the procedures described in the previous section (i.e. geometric properties, the solutions of the differential torsion equation and the bi-moments and warping torsional moments) can be determined. The principal results are summarized in Tab. 5.2.

Table 5.2: Calculation results.

<table>
<thead>
<tr>
<th>Geometric properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Torsion constant, $J_t [m^4]$</td>
<td>4.06</td>
</tr>
<tr>
<td>$\int \omega' dA [m^5]$</td>
<td>1657.84</td>
</tr>
<tr>
<td>$2^{nd}$ moment of area, $I_Y [m^4]$</td>
<td>225.97</td>
</tr>
<tr>
<td>Shear center position, $e_Y [m]$</td>
<td>7.34</td>
</tr>
<tr>
<td>Warping constant, $J(\omega)[m^6]$</td>
<td>5241.17</td>
</tr>
<tr>
<td>Sectorial static moment, $S(\omega)[m^4]$</td>
<td>22.02</td>
</tr>
</tbody>
</table>

Free warping case

| Twisting angle, $\phi_{max} [rad]$ | 3.59E-03 |
| Bi-moment, $M(\omega)_{x=0}[N.m^2]$ | 0 |
| Bi-moment, $M(\omega)_{max}[N.m^2]$ | 3.24E09 |
| Warping torsional moment, $T(\omega)_{x=0}[N.m]$ | 4.40E07 |
| Warping torsional moment, $T(\omega)_{x=0.5L}[N.m]$ | 7.19E07 |

Constrained warping case

| Twisting angle, $\phi_{max} [rad]$ | 1.17E-03 |
| Bi-moment, $M(\omega)_{x=0}[N.m^2]$ | -2.01E09 |
| Bi-moment, $M(\omega)_{max}[N.m^2]$ | 2.01E09 |
| Warping torsional moment, $T(\omega)_{x=0}[N.m]$ | 7.19E07 |
| Warping torsional moment, $T(\omega)_{x=0.5L}[N.m]$ | 7.19E07 |

Figure 5.6: Distribution of the twist angle considering the two warping constraining scenarios: free warping (solid line) and constrained warping (dashed line).

Tab. 5.3 presents the twist angles at the middle of the ship open length (maximum twist angles). Analyzing this data is possible to observe that, for the analyzed structure, the warping constraining can affect the twist angle up to 67%.

Once the twist angle distribution is determined, is then possible to determine the shear $\tau(\omega)$ and normal $\sigma(\omega)$ stresses induced by warping, and its maximum values.
Table 5.3: Twist angle at the middle of the ship open length.

<table>
<thead>
<tr>
<th>Warping</th>
<th>$\phi$ [rad]</th>
<th>$\phi$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td>3.59E-03</td>
<td>0.206</td>
</tr>
<tr>
<td>Constrained</td>
<td>1.17E-03</td>
<td>0.067</td>
</tr>
</tbody>
</table>

Fig. 5.7 represents the distributions of the shear and normal stresses induced by warping on a simple beam. In the shear stress case, the maximum value $\tau_{\omega 1}$ is attained in the flange at a position symmetrical to the shear center position in relation with the base line, being zero at the flange top. However in the present case study the shear stress distribution (Fig. 5.8) must take into consideration the thickness variation in the side structure and also the contribution of the girders for the sectorial static moment, as stated in Eqn. 5.2. These factors produce a significant change, since the shear stress at the top of the side structure is not zero anymore.

![Figure 5.7: Channel section cross-sectional distribution of shear (a) and normal (b) stresses due to warping.](image)

Figure 5.7: Channel section cross-sectional distribution of shear (a) and normal (b) stresses due to warping.

Regarding the normal stress distribution, there is no difference between the example shown in Fig. 5.7 and the studied structure, with its maximum value being on the flange tip and attaining zero value at a position symmetrical to the shear center position in relation to the base line ($z = 7.4m$).

At this point it is possible to determine the warping induced stresses at any point of the simplified structure. It is observed that the higher values of the shear stress are found (Tab. 5.4 and Fig. 5.9) at the mid-length of the beam model, and are equal for both warping-free and constrained warping cases. Also in the constrained warping case the shear stresses have the same values at the mid-length and extremities.
Figure 5.8: Distribution of the dimensionless warping induced shear stress from the top of the side structure to its connection with the bottom structure.

For the normal stress case, its maximum values are observed at the beam models mid-length (Fig. 5.10) and with a value about 38% higher for the warping free model.

Table 5.4: Maximum stress values (values in Pa).

<table>
<thead>
<tr>
<th></th>
<th>free warping</th>
<th>const. warping</th>
<th>diff. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau(\omega)_{\text{max}} )</td>
<td>9.80E06</td>
<td>9.80E06</td>
<td>0</td>
</tr>
<tr>
<td>( \tau(\omega)_{\text{sheer strake}} )</td>
<td>2.57E06</td>
<td>2.57E06</td>
<td>0</td>
</tr>
<tr>
<td>( \tau(\omega)_{\text{bilge}} )</td>
<td>4.05E06</td>
<td>4.05E06</td>
<td>0</td>
</tr>
<tr>
<td>( \tau(\omega)_{\text{keel}} )</td>
<td>-3.50E06</td>
<td>-3.50E06</td>
<td>0</td>
</tr>
<tr>
<td>( \sigma(\omega)_{\text{sheer strake}} )</td>
<td>5.93E07</td>
<td>3.68E07</td>
<td>38</td>
</tr>
<tr>
<td>( \sigma(\omega)_{\text{bilge}} )</td>
<td>-5.40E07</td>
<td>-3.35E07</td>
<td>38</td>
</tr>
</tbody>
</table>

Figure 5.9: Warping induced maximum shear stress distribution along the length considering the two warping constraining scenarios: free warping (solid line) and constrained warping (dashed line).
Another stress component that was not yet considered (because its effects are neglectable) is the pure torsional shear stress. This component is always present on a cross-section of a member subject to torsion, and its maximum values are determined according to:

\[ \tau_t = G t \frac{d\phi}{dx} \]  

(5.28)

In a contrary to the warping induced shear stress, the pure torsional shear stress is not constant through the thickness of the element, varying linearly between the two edges of the element (Fig. 5.11). Consequently, the maximum value of this shear stress component is found at the thickest element of the cross-section, in the present case the sheer strake.

Figure 5.11: Channel cross-sectional pure torsional shear stress distribution.
In order to determine the total shear stress \( f_v \) at any determined point of the cross-section, the different shear stress components are combined using the superposition principle according to:

\[
f_v = \tau(\omega) \pm \tau_t
\]

(5.29)

Since the maximum values for the pure torsional shear stress are of the order of 0.3 MPa (Fig. 5.12) and the ones from the warping induced shear stress are of the order of 10 MPa (Fig. 5.9), the former component might be considered neglectable when compared to the latter.

At a scenario where the pure torsional shear stress is higher, its contribution represents only about 4.7% of the total shear stress (Fig. 5.13).

---

Figure 5.12: Pure torsional shear stress distribution at sheer strake along the length considering the two warping constraining scenarios: free warping (solid line) and constrained warping (dashed line).

Figure 5.13: Distribution of the total shear stress and its components along the side structure at a cross-section at x=0 in an unrestricted warping scenario.
5.2 Finite element analysis

Another approach applied here is the finite element analysis of the open ship structure subjected to torsional loading. This option represents a much more exhaustive and time consuming task, since it requires a somewhat complex FE structural model. For this reason, this approach can only be used at a later stage of the design process when more details are known.

At this point of the study another aspect is to be considered. The differences of the structural responses in the cases where the ship structure is open, partially closed or fully closed, are investigated through the analysis of three different three dimensional FE models.

5.2.1 Model generation

For this analysis, the whole simplified ship hull structure is modeled. The model consists on the replication of the model used in the previous sections, in such way that it now forms a cylindrical body with 122.32 meters length, divided in four cargo holds separated by transversal bulkheads. Also simplified symmetric bow and stern closings with closed cross-sections are modeled, creating a 154.81 meters length overall barge-like model.

Three different models are created with the objective of comparing the different structural responses. The first model (open model, Fig. 5.14a) has four open cargo holds, the second model (partially-closed model, Fig. 5.14b) has its forward and after holds closed and the third model (closed model, Fig. 5.14c) has all its cargo holds closed, in a “tanker-like” configuration.

The element type and meshing parameters used, are the same as already described in Chapter 2.
Boundary conditions and applied loads

The boundary conditions applied to the FE model are such that all the nodes on both end planes, since the models have both bow and stern vertical transoms, are constrained in every degree of freedom (Fig. 5.15).
The loads applied to the FE model are linearly varying loads distributed around the four exterior edges of the midship section, as shown at Fig. 5.16. These loads are determined in a way that they form an equivalent moment around the shear center with a magnitude $M_{WT}$ as determined by Eqn. 5.24.

The loads are computed according to the following formula:

$$M_{WT} = \sum \frac{1}{2}P_i a_i L_i$$

(5.30)

where $M_{WT}$ is the torsional load according to Eqn. 5.24, $P_i$ is the maximum value of the load, $a_i$ is the span of the load (the length of the edge) and $L_i$ is the distance from the shear center to the equivalent force application point. Additionally, a restriction is applied in order to ensure the balance between the four loads. This restriction assures that the four loads produce an equal moment around the shear center:

$$\left| \frac{1}{2}P_1 a_1 L_1 \right| = \left| \frac{1}{2}P_2 a_2 L_2 \right| = ... = \left| \frac{1}{2}P_4 a_4 L_4 \right|$$

(5.31)

The applied loads and their respective parameters are presented at the following table:

<table>
<thead>
<tr>
<th>$i$</th>
<th>$P$ [N/m]</th>
<th>$a$ [m]</th>
<th>$L$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.28E6</td>
<td>26.00</td>
<td>4.33</td>
</tr>
<tr>
<td>2</td>
<td>4.66E5</td>
<td>16.20</td>
<td>19.02</td>
</tr>
<tr>
<td>3</td>
<td>1.28E6</td>
<td>26.00</td>
<td>4.33</td>
</tr>
<tr>
<td>4</td>
<td>6.51E5</td>
<td>16.20</td>
<td>13.62</td>
</tr>
</tbody>
</table>

### 5.2.2 Results and analysis

Performing the finite element analysis, and plotting the normal stresses of the open ship structure (Fig. 5.17), it is possible to observe the development of the global normal stresses from the ends of the open length towards the mid-length. It is also possible to observe some high stress concentrations, namely on the connections between the open length and the bow and stern closings and at the mid-length on the
connection between the inner side and the transversal bulkhead. However these local stress concentrations will not be regarded.

![Figure 5.17: Normal stress under torsional loading.](image)

As can be seen in Fig. 5.17, the normal stresses tend to be higher on the sheer strake region. According to this, the normal stress distribution along the sheer strake are plotted (Fig. 5.18) for the three different structural configurations presented before. These plots confirm that the maximum normal stress levels are much higher for an open structure (45.4 MPa) than for a closed structure configuration (9.2 MPa).

![Figure 5.18: Normal stress distributions at sheer strake along the length for open-deck (solid line), closed-deck (dashed line) and partially closed-deck (dashdotted line) cases.](image)
Another important aspect, visible in Fig. 5.18, is the effect of the transversal bulkheads on the normal stress. These bulkheads are positioned at 30.58 m, 61.16 m and 91.74 m, respectively 1/4, 1/2 and 3/4 of the open length. Its effects are more visible on the open structure configuration and gradually less visible, until they are almost not noticeable at the closed structure configuration.

Concerning the normal stress distributions along the depth and beam of the hull, they are to be checked at a section where the stress levels are maximum at the sheer strake. This section is expected to be the midship section, however as can be seen in Fig. 5.18, the presence of the transversal bulkhead causes a sudden reduction of the normal stress level at this section, therefore, the chosen section as at 66.5 m. Figs. 5.19 and 5.20 show the normal stress distribution along the depth and beam, respectively.

Figure 5.19: Normal stress distribution along the depth at a section located at $x = 66.5$ m for open-deck (solid line), closed-deck (dashed line) and partially closed-deck (dashdotted line) cases.

Figure 5.20: Normal stress distribution along the bottom at a section located at $x = 66.5$ m for open-deck (solid line), closed-deck (dashed line) and partially closed-deck (dashdotted line) cases.

Fig. 5.19 present normal stress distributions that tend to be linear. The plot of the open deck scenario
shows that the zero value is attained at the coordinate $z = 9.65\,m$. This, represents a deviation of around 16.7% when compared to the extrapolation from the beam model, where this value should occur at a distance from the base line equal to the shear center distance to the same base line ($z = 8.27\,m$). This observation also means that the normal stress levels are higher at the bottom region than they are on the deck.

Observing Figs. 5.21 and 5.22 where the warping effect on a cross-section at a transverse bulkhead is shown causing an anti-symmetrical distortion, the understanding that the warping has a considerably higher influence on an open cross-section profile is reinforced. The difference, at a point where the distortion is greater, is about 146%.

![Figure 5.21: Distortion of the open cross-section: longitudinal displacements on deck (solid line), bottom (dashed line) and double bottom (dashdotted line) cases.](image)

![Figure 5.22: Distortion of the closed cross-section: longitudinal displacements on deck (solid line), bottom (dashed line) and double bottom (dashdotted line) cases.](image)
Regarding the shear stresses, its higher values are attained in the side shell, which means that the $\tau_{zx}$ component is the most relevant (Fig. 5.23). The maximum values occur at $z = 8.6m$ for the open and partially-closed deck cases and at $z = 5.6m$ for the closed deck case (Fig. 5.24).

Figure 5.23: Shear stress $\tau_{zx}$ component under torsional loading.

The maximum shear stress $\tau_{zx}$ component values are 28.6MPa, 24.4MPa and 12.9MPa for open-deck, partially closed-deck and closed-deck configurations respectively (Fig. 5.24), and are obtained at a length of $x = 20m$ for open-deck and $x = 49m$ for partially closed and closed deck configurations (Fig. 5.25).

Figure 5.24: Shear stress $\tau_{zx}$ component distribution along the side for open-deck (solid line), closed-deck (dashed line) and partially closed-deck (dashdotted line) cases. (Non-average results)
As could be foreseen, given the normal and shear stress values, the von Mises yield stress criterion is complied. As can be seen at Fig. 5.26, the maximum von Mises stress attained at the open structure (only the open structure is analyzed since it is the one with higher stress levels) is around $230\,\text{MPa}$, while the yield strength of the structural steal is $250\,\text{MPa}$. Also relevant is the fact that this maximum stress level is attained at a stress concentration point and that in the remaining structure the stresses are much lower (one order of magnitude lower).
5.3 **Comparison and discussion**

Comparing the axial stress longitudinal distribution given by the two approximate methods (Fig. 5.27) along the sheer strake, an almost perfect fit of the FEM results between the two limit cases of the bi-moment method (free and constrained warping) is observed. This fit is only broke due to the warping stress fluctuation at the presence of the transversal bulkheads and respective deck strips as explained by Villavicencio et al. 2015 [27].

![Figure 5.27: Comparison between open-deck finite element method and free and constrained warping by the bi-moment method for axial stress results along the sheer strake.](image)

As for the situation of the axial stress distribution across the bottom, at a section where the stresses levels are higher (theoretically the midship section, however due to warping stress fluctuations in the FEM model this section is slightly forward) it is possible to see (Fig. 5.28) a quite good fit between the results and particularly good at the extreme values.
Figure 5.28: Comparison between open-deck finite element method and free and constrained warping by the bi-moment method for normal stress distribution along the bottom at a section located at \( x = 66.5m \) for the FEM model at midship (\( x = 61.16m \)) for the bi-moment method model.

Figure 5.29: Comparison between open-deck finite element method and free and constrained warping by the bi-moment method for normal stress distribution along the side at a section located at \( x = 66.5m \) for the FEM model at midship (\( x = 61.16m \)) for the bi-moment method model.

Relatively to the normal stress distribution on the side structure (Fig. 5.29), analyzed in the same conditions and at the same cross-section as before, the fit of the FEM results to the bi-moment method estimates is also satisfactorily met.

Considering the shear stress results (Fig. 5.30), at their maximum values (\( z_{\text{baseline}} = 7.34m \) for the bi-moment method model and \( z_{\text{baseline}} = 8.60m \) for the FEM model), a considerably large discrepancy (approximately 194\%) between the methods is observed. This discrepancy, determines this simple bi-moment method as a not so efficient method to predict the overall shear stresses as verified by the FEM model. However, since both methods consist in a series of simplifications, it is reasonable to consider it as a good reference. Also considering that the bi-moment method warping induced shear stress values are
estimated for a thicker plate (equivalent to both plates from the double shell structure) and that these stresses are inversely proportional to the plate thickness, it is safe to assume that the bi-moment method predictions may be significantly lower than the real case.

Figure 5.30: Comparison between open-deck finite element method and free and constrained warping by the bi-moment method for shear stress $\tau_{zx}$ component (warping induced plus pure shear stresses) along the length.
Chapter 6

Final remarks and further work

The work presented in this dissertation deals with the structural response of an open deck pontoon-like ship in its elastic domain of material, under vertical and horizontal bending moments, shear forces and torsional moment, analyzed using both finite element method and simplified approaches. One of its main focus was to understand if a very simplified thin-wall girder theory application may predict, within an acceptable range, the expected stress levels under torsion in open deck ships. These expected stress levels were estimated by finite element method.

The thin-walled girder application uses an idealization of the original structure through a channel-section with lumped areas, analyzed under the bi-moment method. To create a range of valid stress values, and since there is no simple methodology to determine the degree of warping constrain, the thin-walled girder ends were subjected to two different sets of boundary conditions: torsion constrained and free warping and both torsion and warping constrained.

The conclusion of this study is that the simplified thin-walled girder application, within the predefined boundary conditions, provides an almost perfect “envelope” for the axial warping stresses verified in the FE analysis, but its prediction differs in about 194% for the shear warping stresses comparing to the FE results.

The discrepancy between the warping induced shear stress results of the two different methods, and in the absence of experimental observations, do not allow to rule any method as either better or worse since both are built upon many simplifications and constrains. In any way, since the analytical methods are usually more conservative, the results of the bi-moment method can be regarded as a valid estimation in an initial design stage, fulfilling thus the purpose of being a reliable prediction method. However, this is a point to develop and improve in future research, adding some complexity to this otherwise simple method, by considering the effect of transverse bulkheads as already proposed by some authors.
Bibliography


