The Optimization of Production Planning and Scheduling: A Real Case Study in Ice-cream Industry

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Abstract

As other segments of the food industry, the ice-cream industry has its own features that influence the production management of its processes. Amongst these we identify: changeover tasks, products shelf-life and perishability, multiple deliveries during the planning horizon and RMs procurement and inventory control. These aspects have been often left out when studying the production planning and scheduling within the batch food industries. Thus, the problem seeks an optimal solution for the production planning and scheduling of a dairy food company, where a methodology was proposed to address the main features of this industry based on the integration of two supply contracts at the scheduling level. Two mixed integer linear programming (MILP) models are developed where the RMs´ shelf-life aspect is integrated in the operation scheduling, extending them to explore different contractual relationships between suppliers and the company, applying the RTN methodology. Both RTN mathematical approaches are applied to the present case-study in order to evaluate which contract type is more suitable for the present production process, in terms of scheduling, raw-material costs and final products' quality. The obtained results as well as the computational statistics are analysed.

Keywords: scheduling of production, perishable products, inventory control of RMs and supplier contracts.

1. Introduction

The food industry has been growing throughout the years. This growth has been driven by the increase of new market competitors, as well as by an increase on the consumers' demand and requirements. These factors combined have changed the trends in the industry and, production optimization is nowadays a need. In this context the production scheduling became an important activity for companies, allowing them to determine when, where and how a set of products should be produced considering the operational aspects.

Thus, the scheduling activity has an important role for performance improvement while adjusting the resources consumption/production to demand through an accurate allocation. In the artisanal ice-creams production, the major challenge is related to the raw-materials' quality aspects as the final products' qualities/freshness is deeply dependent on these characteristics. Hence, it is important to address this aspect at the scheduling level.

RMs' procurement and inventory control must also be considered as part of the different contractual forms that exist between supplier and companies, since in this type of industry the RMs price and availability suffers a high volatility which has a direct impact in the way that RMs are purchased and their costs.

2. Related Literature

A wide range of methodologies and techniques to deal with the production planning and scheduling is available in the literature. However, the artisanal ice-cream industry is a food industry niche and the literature regarding this subject is very scarce.

Nevertheless, some research has been made considering the problematic of the optimization of the production planning and scheduling in food industry. Entrup et al. (2005) have developed a MILP formulation considering shelf-life restrictions for the final products applied to the yoghurt industry. This aspect was accounted in the objective function, which aims to maximize the margin contribution. Considering the segment of seafood products, Cai et al. (2008) proposed a formulation that takes into account the RMs perishability, considering three type of decisions: i) the type of products to be produced, ii) the processing time of resources to be allocated to each type of product, and finally, iii) the sequence of products production . The authors stated that this model can be applied to any system that has limitations in terms of RM and uncertainty in the delivery dates. Amorim et al. (2011) have also presented a MILP formulation exploring two cases; i) a make-to-order and; ii) a hybrid make-to-order /make-to-stock strategy.

Besides, another research stream has been explored considering the inventory models for perishable products such as the work developed by Goyal (1994) and Soman et al. (2004) based on the application of the Economic Lot Size Problem (ESLP). However, the ESLP has in its base assumptions that are unrealistic for the ice-cream industry namely the constant demand rate assumed in these problems which is not realistic for fresh food industries with seasonal products, as it is the ice-cream industry.

Notwithstanding, the research on production scheduling in the ice-cream industry with perishable goods inventory control is still at its beginning.

Moreover, since in the dairy industry the RMs' procurement plays an important role, the relationship between companies and suppliers becomes a crucial aspect to be integrated in the planning and scheduling production mainly because of the fluctuations on quality and price of RMs delivered. Thus two supply contract types are explored: the Fixed Commitment Contract (FCC) and the Spot Market Contract (SMC). The FCC is a long term contract where two aspects are defined *a priori*: i) quantity supplied and ii) price of RMs. As for i), the quantities of RMs delivered in the period are fixed. Consequently, in industries where demand is uncertain, producers may take a significant risk as regards inventory management as they are unable to forecast the exact amount of RMs they will need. As regards to ii), the price of RMs is negotiated by the parties and remains the same during the contract. Thus, the buyer does not benefit from the fluctuations of RMs' price which may occur in the spot market. However, the purchase price established is lower than the average spot market price. Other benefits can arise from a long term commitment as Minner, S. (2003) points out.

The SMC is characterized both by allowing the buyer to benefit from the fluctuations on RMs' price and by granting the flexibility of moving from one supplier to another without any investment. However, even though one can benefit from the variations on price, in this type of contract the company pays the spot market price.

Having this framework and exploring the existent gap on the literature for addressing all these features at the scheduling of production level, in the present work are developed two mathematical formulations for the artisanal ice-cream batch multipurpose and multi-product process. The aim of the models is to explore the characteristics of this industry, accounting for RMs perishability and their inventory control, changeover tasks and multiple deliveries based on two different supply contracts, simultaneously with the planning and process scheduling.

2.1. Description of Ice-cream Production Process

The ice-cream industry has different types of ice-creams production, which are often classified into four categories according to their main ingredient: vanilla, cream, yoghurt and fruits/sorbets. However, when considering the production process of these products they are mainly divided in two categories: sorbets and other types of ice-creams production. Both categories differ in its production process until frozen task is reached. For confidentiality reasons a generic ice-cream production process is characterized in Figure 1.

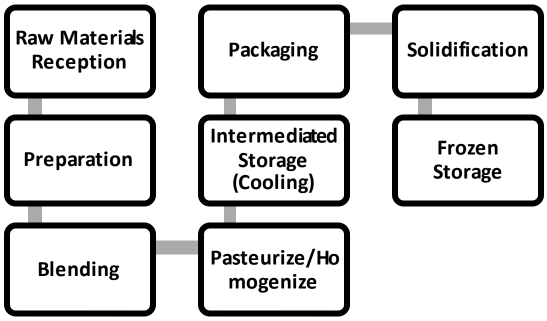


Figure 1 – Generic Ice-cream/Sorbet Production Process.

2.2. Methodology

The artisanal ice-cream industry has special features, such as: changeover tasks between productions, multiple deliveries, shelf-life of products, procurement and inventory control of RMs. All these features must be integrated in the scheduling process in order to make it, as close to reality as possible. Moreover, due the importance of procurement and inventory control methods for this work, it will be considered the integration of the two supply contracts at the scheduling level. To clarify the integration of these aspects in the scheduling, Figure 2, shows the methodology developed to be followed in the next chapters of this work.

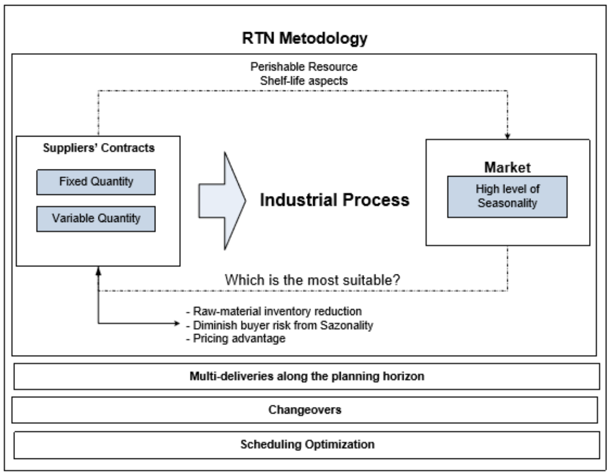
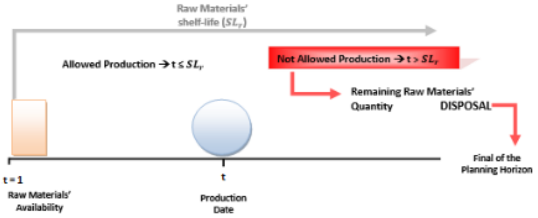


Figure 2 - Framework of the present case study.

The models consider the specific characteristics of the present production process. The multiple deliveries and changeover tasks will be integrated at the same time that it is considered the perishability of RMs. Taking into account the two types of contracts, some features must be adapted to reflect the reality of each contract, which is the case of the perishability of RMs. Thus, in the FCC, it must be integrated the control of RMs' shelf-life in the scheduling of production to account with the reduced shelf-life of these products. In this sense, the RMs' shelf-life control concept will be applied (Figure 2).

Figure 3 - Schematic representation of RMs' shelf-life.



The RMs' shelf-life must be quantified from the beginning of the planning horizon until the production instant. Considering instant t, the moment that a RM is required for production and, SL_r its shelf-life. If t ≤ SL_r its shelf-life is greater than the production date and the RM has the safety and quality properties to be processed. However, in periods that t > SL_r is verified, the the production date t is greater than the RM shelf-life, it will not fulfil the safety and quality patterns, going to the disposal. All the reaming quantity of RMs that have already overcome its shelf-life and consequently, is deteriorated, will be not used in production.

Finally, for the SMC, this feature will not be explicitly considered in the model since in this case the company will work in a JIT production strategy.

3. Problem Statement

In this section with the objective of supporting the production scheduling in an artisanal ice-cream production line of a multiproduct dairy batch plant, it will be developed and characterized both MILP models based on the RTN approach.

In most dairy industries, the quantity of final products to produce is known at the start of the planning horizon. Some assumptions are considered to develop the mathematical formulations, such as:

- All RMs used in the process are received at the begging of the planning horizon.
- The quantity of RMs received will fulfill the weekly production.
- The RMs have different shelf-lives.
- The RMs are available for production until they reach their shelf-life (t ≤ Sl_r).

The optimal scheduling of production can be obtained by solving the following problem:

Given:

- Process description by a RTN representation;
- The maximum amount of each type of resource available;
- Resource characteristics and capacities;
- Time horizon of planning;
- Task and resources operating data;
- RMs' shelf-life;
- RM's spoilage, purchase and storage costs;
- Quantities contracted of raw-materials;
- Selling price of ice-creams and sorbets;
- The production requirements and deliveries dates along the planning horizon.

Determine:

- The amount of each resource used;
- The task-unit assignment and the batch size;
- The optimal scheduling satisfying not only the multi-deliveries along horizon, but also, the demand at final horizon;
- Raw-materials profile for the time horizon.
- The final quantity of RMs discarded at the end of the planning horizon by not respecting the shelf-life restriction.

3.1 Mathematical formulation for FCC

In this section is presented the FCC model's formulation. The problem can be defined as follows:

3.1.1. Indexes

d	*Deliveries*
k	*Tasks*
r	*Resources*
t	*Time*
θ	*Relative time to define the start of a task*

3.1.2. Sets

C_r	{$r \epsilon R$: set of all material resources, such as: RMs, intermediate products and final products}
D_r	{$r \epsilon R$: set of all equipment and human resources}
D_{chg}	{$r \epsilon D_r$: the equipment resources used for the changeover tasks}
K	{k: Set of all tasks}
K^{proc}	{$k \epsilon K$, $r \epsilon D_r$: set of all processing tasks, k, operating in an equipment resource, r}
K^{chg}	{$k \epsilon K$: set of all changeover tasks, k, to be performed in an equipment resource, r}
R	{r: set of all resources}
R_p	{$r \epsilon C_r$: set of final products}
R_{rm}	{$r \epsilon C_r$: set of RMs}
S_r	{$r \epsilon C_r$: set of material resources with storage}
T_{hr}	{$k \epsilon K, r \epsilon D_r$: set of all tasks, k, that uses operators teams as resources}

3.1.3. Parameters

$C^{spoilage}$	Spoilage cost of deteriorating RMs.
$C^{storage}$	Storage cost of deteriorating RMs.
H	Planning horizon;
$Price_r^{fp}$	Selling price of final products;
$Price_r^{rm}$	Acquisition cost of RMs;
$Q_{rd}^{min} Q_{rd}^{max}$	Minimum /Maximum amount of resource $r \in R_p$ to deliver in instant d.
R_r^{demand}	Demand of final resource r;
$R0_r$	Resource, $r \epsilon R$, available initially;

SL_r	Shelf-life of RMs;
$T^{ed}{}_{rd}\, T^{ld}{}_{rd}$	The earliest/latest time to deliver, $r \in R_p$, in instant d.
$V_{kr}{}^{min}\, V_{kr}{}^{max}$	Minimum/Maximum allowed capacity of resource r to perform task k;
$\tau_k\ \tau_k^{chg}$	Processing time for task k/Processing time for changeover task k;
$\mu^{chg}{}_{kr\theta}$	Resource (processing unit) r consumed at the start/end of the changeover task k^{chg} and at time relative θ;
$\mu_{kr\theta}/\, \nu_{kr\theta}$	Resource produced/consumed of task k at the start/ end of time relative θ;

3.1.4. Variables

In the model it is necessary to considered both binary and continuous variables, as presented below.

Binary Variables

N_{kt} Binary variable that takes the value 1 if task k starts at time t, otherwise is 0;

$N^{chg}{}_{kt}$ Binary variable that takes the value 1 if changeover task k starts at time t, otherwise is 0;

Continuous Variables

Inv_{rm}	Raw-materials inventory level
ξ_{kt}	Quantity of material undergoing task k at the beginning of instant t;
R_{rt}	Quantifies the amount of available resource r at the instant t;
$\prod_{rt}$	Total amount of final product, r, delivered at instant t.
$IDisposal_r$	Quantity of RM r that has to be discarded by not respect the shelf-life restriction
Z	Quantifies the profit

3.1.5. Mathematical formulation for FCC Model

After being defined all the indexes, sets, parameters and variables, the mathematical formulation for the FCC is developed, to be applied to the case study comprising the following constraints:

Objective Function

$$Max\ Z = \sum_{r\in Rp}\sum_{t}\left(\prod_{rt} \times \mathrm{Price}_r^{fp}\right) - \left[\sum_{r\in R_{rm}}\sum_{t}\left(\mathrm{R0}_r \times \mathrm{Price}_r^{rm}\right) + \sum_{r\in R_{rm}}\left(\mathrm{IDisposal}_r \times \mathrm{C}^{Spoilage}\right) + \sum_{r\in R_{rm}}\sum_{t}\left(\mathrm{IDisposal}_r \times \mathrm{C}^{storage}\right)\right] \quad [1]$$

Subject to:

$$\mathrm{R_{rt}} = \mathrm{R_{ro|t=1}} + \mathrm{R_{rt-1|t>2}} + \sum_{k}\sum_{\theta=0}^{\tau_k}\left(\mu_{kr\theta}\mathrm{N}_{kt-\theta}\right) + \sum_{k\in k^{chg}}\sum_{\theta=0}^{\tau_k^{chg}}\left(\mu_{kr\theta}^{chg}\,\mathrm{N}_{kt-\theta}^{chg}\right) \forall r \in \mathrm{D_R}, t \in \mathrm{H} \quad [2]$$

$$\mathrm{R_{rt}} = \mathrm{R0_r} + \sum_{k\in K_r}\sum_{\theta=0}^{\tau_k}\left(\nu_{kr\theta}\xi_{kt-\theta}\right)\ \ \forall\ r \in \mathrm{R_{rm}}, t = 1 \quad [3]$$

$$\mathrm{R_{rt}} = \mathrm{R_{rt-1|t\geq 2}} + \sum_{k\in K_r}\sum_{\theta=0}^{\tau_k}\left(\nu_{kr\theta}\xi_{kt-\theta}\right)\ \ \forall\ r \in \mathrm{R_{rm}}, t \leq SL_r \quad [4]$$

$$\mathrm{R_{rt}} = \mathrm{R_{rt-1}} + \sum_{k\in K_r = Sto1_k}\sum_{\theta=0}^{\tau_k}\left(\nu_{kr\theta}\xi_{kt-\theta}\right) \big|\ t \leq SL_r + \sum_{k\in K_r\setminus Sto_{1k}}\sum_{\theta=0}^{\tau_k}\left(\nu_{kr\theta}\xi_{kt-\theta}\right) - \left(-\prod_{rt}\right)\ \forall\ r\setminus \mathrm{R_{rm}}, t \in \mathrm{H} \quad [5]$$

$$\mathrm{IDisposal_r} = \mathrm{R_{rt}}\ \ \forall\ r \in \mathrm{R_{rm}}\ , t = SL_r - 1 \quad [6]$$

$$V_{kr}^{min} N_{kt} \leq \xi_{kt} \leq V_{kr}^{max} N_{kt} \qquad \forall r \in R, k \in K^{Proc}, t \in H \quad [7]$$

$$\sum_{t=1}^{H}\left(-\prod_{rt}\right) \geq\ R_r^{demand} \qquad \forall r \in R_P, t \in H \quad [8]$$

$$Q_{rd}^{min} \leq \sum_{t=T^{ed}}^{T^{ld}}\ \left(-\prod_{rt}\right) \leq Q_{rd}^{max}\ \forall r \in R_P, d \in D, t \in H \quad [9]$$

$$\sum_{t \leq T^{ed}} \prod_{rt} = 0 \; \forall r \in Rp, d \in D \quad [10]$$

$$\sum_{t \geq T^{ld}} \prod_{rt} = 0 \; \forall r \in Rp, d \in D \quad [11]$$

$$\prod_{rt} = 0 \;\; \forall \; r/Rp, \qquad t \in H \quad [12]$$

$$R_{r|t=tnext} = 0 \; \forall \; r \; \epsilon \; R_P, t \in H \quad [13]$$

Equation (1) defines the objective function which reflects maximization of the relative profit of the company. As aforementioned, the maximization of the profit only accounts with the impact of RMs costs for confidentiality reasons, thus henceforth it is denominated by relative profit. The objective function can be characterized by the revenue, which reflects the quantity of ice-creams/sorbets sold in the first equation term, followed by all costs associated with the RMs. The costs characterization its divided in three terms: i) the purchasing cost of RMs, ii) the RMs disposal costs, and iii) the storage costs of RMs. The first term characterizes the purchase cost of RMs which is expressed by $R0_r \times Price_r^{rm}$. The second term characterizes the RMs disposal costs, which is defined as the quantity of RMs that must be discarded at the end of the planning horizon, representing a lost opportunity cost. In this way we ensure that, only RMs that fulfil the shelf-life restriction are considered for production. Finally, the last term quantifies the RMs storage costs, expressed by $IDisposal_r \times \mathrm{C}^{\mathrm{storage}}$. This cost reflects the storage of RMs that were discarded due to shelf-life limitations along the entire production horizon.

The constraints that must be satisfied are explored in detail as following:

Materials Balance Constraints: For representing the excess resources balance which in general can be expressed by the amount of resource available from one time interval to the next one, several constraints were defined. Typically the material balance constraints are multi-period material balance expressions in which the amount of resource available at time period t is equal to the amount of resource at the previous time period t-1 adjusted by the amount of resource produced/consumed by all tasks starting or ending at t. Thus, the constraint [2] defines the renewable resources balance, for processing and changeover tasks. Where the occurrence of the changeover tasks are defined as follows$\left(\mu_{kr\theta}^{chg} \; N_{kt-\theta}^{chg}\right)$, where $k\epsilon k^{chg}$represents the changeover tasks, the parameter μ^{chg} defines the changeover times between productions and N^{chg} is a binary variable that takes the value 1 if the task k starts at period t, and 0 otherwise. Due to RMs' shelf-life restriction, it is necessary to characterize the non-renewable material balance constraints in three situations. The first one, defined by constraint [3], characterizes the RM balance for the first instant. The second constraint, denominated by [4], considered the RM balance for the RMs having a self-life shorter than t. In this case, only RMs with a shelf-life lower or equal than the period being observed will be considered for production (where the accountability of RMs' shelf-life is considered as explained in section 3.7). The third constraint, constraint [5], characterizes the resource balance for the remaining resources. The second term of [5], expressed by

$$\sum_{k \in K_r = Sto1_k} \sum_{\theta=0}^{\tau_k} (\nu_{kr\theta} \, \xi_{kt-\theta}) \mid t \leq SL_r$$

avoids any production using RMs without enough quality for production, along the production horizon. Based on the assumption that all the RMs are available, this term controls only the storage quality of RMs. Only the RMs that respect the shelf-life restriction will be considered for production. In the production periods for t$\leq SL_r$, RMs will have a good level of quality and are considered for production. However, if the period under analysis is a period $t > SL_r$, the RMs storage will not fulfil the quality standards and consequently, the quantity of RMs required will not be used for production and will be discarded.

The resource multiple deliveries along the planning horizon is characterized by the negative variable, $\prod_{rt}$, as shown in constraint [5], which expresses the amount of final product r delivered at time t.

RMs Disposal Constraint: The inventory control of RMs is a critical aspect of this industry because final products' quality is directly related with the quality of the perishable RMs. Hence, it is necessary to track the remaining shelf-life of RMs before they are processed and during the production. As constraint [6] states, the quantity of RMs that is discarded is expressed by the variable $IDisposal_r$ and it is equal to the value of the variable $R_{r,t}$ for the instant,$t = SL_r - 1$, which is immediately the previous instant before RMs had achieved their shelf-life. So the remaining quantity held in $R_{r,t}$ that was not used for production, is disposed at the end of the planning horizon and penalized in the objective function by having an associated cost called disposal cost of RMs ($C^{disposal}$).

Capacity Constraints: The batch size is limited to the equipment resource maximum and minimum capacities, in constraint [7]. For each tasks k, the amount of materials processed must always be limited by upper and lower bounds, V_{kr}^{min} and V_{kr}^{max}, respectively.

Multiple-Deliveries Constraints: The constraints [9] till [12] characterize the multiple delivers. The multiple delivers are defined based on time windows. Each time window is characterized by the lower and upper resource quantity that must be delivered of each product, defined by $Q_{rd}{}^{min}$ and $Q_{rd}{}^{max}$, respectively, as shown in constraint [9].Constraints [10] and [11] set the deliveries variables to zero for periods out of the delivery window, similarly the equation [12] set the delivery variable to zero for products other than final products.

Production Requirements: The production requirements are defined by constraint [8], which quantifies the total amount of each final product to be delivered along the production horizon.

Operational Constraints: The constraint [13] is used to reinforce the model robustness meaning that, it is used to establish that it is not allowed storage of final products in the final of the planning horizon.

3.2. Mathematical formulation for SMC Model

Considering the previous model, its application to the SMC was explored. However some simplification were considered, which are detailed in this section.

The Spot Market Contract formulation was based in the Just-in-Time approach for production and deliveries of RMs. Thus, it assumes that the purchase of the RMs' takes place every time that those are necessary, using it directly for the production process. By doing that, neither initial quantity of RMs nor its storage during the planning horizon are necessary. For these reasons the formulation used in this case is simpler.

Equations [1], [4] and [5] presented in section 4.4.1 will be simplified and replaced by the following, respectively:

$$Max\ Z = \sum\sum(\prod_{rt} \times \mathrm{Price}_r^{fp} - \sum\sum(\mathrm{R0}_r \times \mathrm{Price}_r^{rm}) \qquad [14]$$

$$\mathrm{R}_{rt} = \mathrm{R}_{rt-1} + \sum_{k \in K_r} \sum_{\theta=0}^{\tau_k} (\nu_{kr\theta}\xi_{kt-\theta}) \quad \forall\ r \in \mathrm{R}_{rm}, t \geq 2 \qquad [15]$$

$$\mathrm{R}_{rt} = \mathrm{R}_{rt-1} + \sum_{k \in K_r} \sum_{\theta=0}^{\tau_k} (\nu_{kr\theta}\xi_{kt-\theta}) + \prod_{rt} \quad \forall\ r \backslash \mathrm{R}_{rm}, t \in \mathrm{H} \qquad [16]$$

Equation [14] represents the new objective function, which maximizes the relative profit defined as the difference of revenues and RMs' acquisition cost. Constraint [15] defines the material balance for RMs for instant t>1 and Constraint [16] represents the material balance for intermediaries and final products. To conclude, the model developed for the SMC is characterized by constraints [2], [3], [7] – [13] presented in section 4.1.1 plus the objective function presented on [14] and constraints [15] and [16].

4. Case Study

The mathematical programming models described are applied with data provided by the company XWZ in order to compare the results obtained with both models. The aim is to present the optimal scheduling of production and to evaluate the costs for each contract, giving special emphasis to RMs' procurement and to the level of inventory of deteriorated RMs, to determine which supply contract is the most suitable.

4.1. Data Collection

In this section the relevant data to solve the case study is presented. Note that due to confidentiality requirements, some of the data presented is approximated. Such data is divided into two groups, i) Tasks, ii) Equipment resources and iii) products.

i) Tasks

To develop the planning and scheduling the production process it is important to consider the recipes of the two types of products (sorbets and other ice-creams), as the concrete set of tasks to perform depends according to the desired final product, and its tasks characterization, as shown in Table 1.

Table 1 - Average duration of each task.

Tasks	Equipment Resources	Processing Time (min)
Washing	R1	20
Preparation	R2	20
	R9	20
Extraction	R3 – R4	10
	R5	20
Blending	R6	10
Pasteurization	R10	45
Solidification	R7	15
	R8	10

However some assumptions were considered:

- The processing time of each task is defined as an average value, resulting from the artisanal processes characteristic (e.g. depend from manpower expertise);
- After each processing task requiring cleaning, the changeover task is immediately performed;
- To guarantee a smooth process flow and minimize the storage time, for the intermediate storage it is considered the minimum duration possible (1 time slot).

ii) Equipment Resources

The multipurpose characteristic of the facility allows share equipment and manpower to process different tasks, giving flexibility to the process.

The characterization of the equipment capacity is given in Table 2 followed by the manpower teams are denominated by Hr1 till Hr3 and their characterization is shown in Table 3.

Table 2 – Equipment (maximum) capacity in liters.

Equipment Resources	Resource	Capacity (L)	Total Available
Washing	R1	70	1
Preparation	R2	50	1
	R9	40	1
Extraction	R3 – R4	50	1
	R5	50	1
Blending	R6	30	5
Pasteurization	R10	60	3
Solidification	R7	20	6
	R8	10	4
Storage	R11 -14	-	-

Table 3 – Manpower Allocation.

Allocation of the manpower/Equipment

HR1		HR2		HR3	
Task	**min**	**Task**	**min**	**Task**	**min**
Ext_A	10	-	-	Sol_A	15
Pre_B	10	-	-	Sol_B	10
Ext_C	20	-	-	Sol_C	15
-	-	Prep_D	20	Sol_D	10
-	-	Prep_E	20	Sol_E	10
-	-	Prep_F	20	Sol_F	10

iii) Material Resources

For the final products were given by the company the average of quantity produced per week. The demand for the highest products consumption in high season is defined in Table 4.

Table 4 –Average weekly demand in litters.

Product	Type	Quantity Required
A	Sorbet	1908
B	Sorbet	756
C	Sorbet	768
D	Ice-cream	684
E	Ice-cream	1524
F	Ice-cream	708

For confidentiality reasons, RM quantities are aggregated sets of the specific RMs to produce each flavor (e.g. Amp defines the set of RMs used to produce A). RM self-life in days assumed is: Amp, Emp-3; Bmp-4; Cmp, Dmp, Fmp - < 5.

Another important aspect integrated is the multiple-deliveries. In order to represent this feature 5 time windows were assumed for deliveries of the final products (see Table 5) on a daily basis.

Table 5 – Time Windows for deliveries (in time slots).

Delivery	Earliest Time	Latest Time
D1	10	96
D2	97	192
D3	193	288
D4	289	384
D5	385	480

5. Analysis and Results

For both supply contracts it will be given the optimal scheduling of production for a set of products. It will also be evaluated the costs for each contract, giving special emphasis to the RMs procurement and to the level of inventory of deteriorated RMs.

5.1. Optimal Scheduling of Production

In this section it is analyzed the optimal scheduling of production of a set of final products in order to fulfil the demand during the high season. The analysis is performed for both contracts considered. The demand for final products in high season, to fulfill a weekly plan was characterized in Table 4.

a) Fixed Commitment Contract

The FCC is a long term partnership between the supplier and the buyer where a pre-defined fixed quantity and price of RMs are negotiated by the parties. As mentioned, the buyer will contract higher quantities than required for production, see Table 6.

Table 6 – Quantity of RMs contracted in the FCC.

RMs	Quantity (L)
Amp	2385
Bmp	945
Cmp	845
Dmp	755
Emp	1905
Fmp	780

It is important to note that this increase on quantity can be explained by the fact that in the FCC the buyer will take risk in its inventory management.

b) Spot Market Contract

In the SMC the RMs quantity contracted will be the one strictly necessary to fulfil the demand. However, the buyer will pay the spot market price which is higher than the oone contract in the FCC.

Due to the high complexity associated to the problem and without losing generality, the representations are defined a work day representation is defined, instead the total production horizon. For simplicity, the changeover tasks are omitted from the scheduling representations, despite its processing. The scheduling is characterized based in final product production, instead of equipment usage, because confidentiality requirements which must be guarantee.

As we can observe from Figures 4 and 5, for the FCC, the shelf-life of RMs have a great impact on the sequence of production obtained.

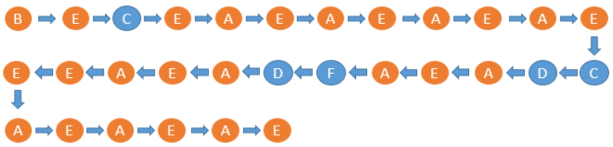


Figure 4 - Sequence of production under FCC.

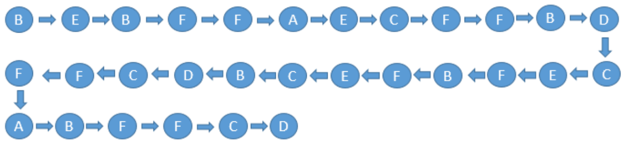


Figure 5 - Sequence of production under SMC

The production starts with the products which require RMs with a shorter shelf-life, notably B and E. Furthermore, if we consider the three products (A, B and E) with shelf-life restrictions (highlighted in orange), by looking at the sequence of production (Figure 4) it is possible to observe that, despite the fact that product B has a shorter RM shelf-life than products A and E, the quantity needed of product B is significantly lower than the necessary quantities of products A and E. For this reason, the scheduling obtained will firstly consider the production for A and E. When compared to the SCC (see Figure 5) the scheduling in this case, will not be restricted by the shelf-life of RMs. In fact, the company is producing through a JIT production policy, where the RMs will be purchased as they are required for the production process. Thus, the SMC model gives an optimal scheduling of production where all the products have the same priority on being produced.

Taking into account the results on Table 7, it is verified that the plan of 6348L is accomplished by both contracts. By comparing the results one can conclude that in the FCC the total quantity produced for the planning horizon is 8.7% higher when compared to the required quantity of demand. In the SMC only the quantity demanded is the one produced.

Table 7 - Comparison of total quantities produced.

Contracts	FCC	SMC
Demand to be fulfilled	6348 L	6348 L
Total quantity Produced	6900 L	6348 L
Variation (%)	8.7 %	0%

This increase obtained under the FCC can be explained by the fact that in this type of contract the quantity of RMs available is higher than the one required. Thus, the model will maximize the quantity of final products produced, considering not only the maximization of the utilization of the equipment capacity but also the shelf-life restriction for RMs. However, we must recognize that we are considering a perfect operation, where process variability is not considered. In Table 8 is presented the percentages of the increase in production of each final product under the FCC.

Table 8 – Percentage increase of final products produced.

Final Products	A	B	C	D	E	F
Quantity produced (L)	1950	760	800	755	1905	780
Quantity required to fulfil the demand (L)	1908	756	768	684	1524	708
Δ (%)	2%	1%	4%	10%	25%	10%

As we can observe, the increase of 8.7% is divided by an increase of production in all final products, being the must representative the increase on production of E with a percentage of 25%. For the SMC is not verified an increase on production since in this case, the quantity available of RMs is the one strictly needed for production.

On the other hand, when comparing the percentage of utilization of RMs, the SMC has 100% of usage of RMs (see Table 9), meaning that all the quantity available for production is entirely used, in contrast, the FCC has a utilization of only 89.6%.

Table 9 – Utilization of RMs (litters)

Contracts	FCC	SMC
Quantity of RM contracted	7615	6348
Quantity used of RMs	6900	6348
% of RMs Utilization	89,60%	100%

Regarding the percentage of usage of RMs in the FCC, it includes both the quantity of spoilage of RMs and the quantity of the excess of RMs that can be utilized in the next production (see Table 10).

Table 10- Structure of quantities obtained for FCC.

Final Products	A	B	C	D	E	F
RMs Contracted (L)	2385	945	845	755	1905	780
Total quantity produced (L)	1950	760	800	705	1905	780
Disposal Inventory (L)	**435**	**185**	0	0	0	0
Excess of RMs (L)	**0**	**0**	**45**	**50**	**0**	**0**

Despite the fact that during the production the proportion of final products which are made from RM with shorter shelf-life were maximized, at the end of the horizon some RMs were spoiled, 435 L and 185 L, for A and B final products, respectively.

For products C and D were left over 45 l and 50 l of RMs, respectively. This quantity remaining of these RMs is not deteriorated since their shelf-lives are greater than the planning horizon. Thus, this quantity is an excess of RMs that could not be used in the production process due to production capacity restrictions. Thus the percentage of 89.6% of usage of RMs under the FCC, results from the combination of the abovementoned aspects. Thus, if this quantity was not penalized the percentage of utilization of RMs would increase for 91.1%. However, that value is still lower than the obtained with the SMC.

In the SMC an excessive quantity of RMs is not verified due to the JIT production policy applied, which enables the company to contract only the quantity that is strictly required. For that reason there is neither spoilage nor an excess of quantitity of RMs.

The six final products deliveries were characterized through 5 time windows during the planning horizon, as is shown in Figure 6.

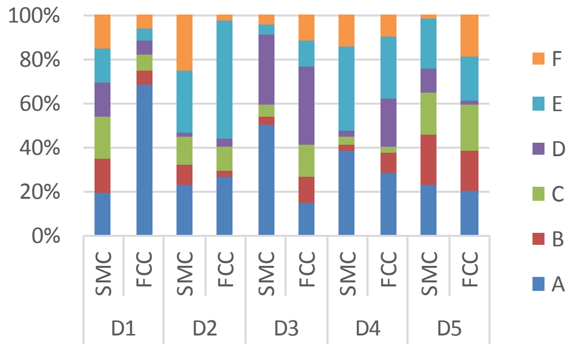


Figure 6 – Deliveries for both contracts.

The deliveries have an impact on the sequence of production as all final products are delivered on a daily basis. In fact, even though in some cases the quantity delivered per product is the minimum batch size, all products have a quantity available for delivery every day. From the results, one concludes that the profiles for both supply contracts are quite different. In the FCC more quantity of products A and E is delivered in the first days. In fact, A is the product with a greater impact on the quantity delivered in the D1. In D2, product E comprises 53% of the quantity delivered, that is for two reasons: i) the short shelf-life of these two products' RMs, which forces the model to produce firstly; ii) these two products are the ones with the highest demand and, hence, must be produced in higher quantity.

In contrast, with the SMC where the quantites delivered of all products are more distributed throughout each time window, as exception of products A and E which have more quantities to be delivered due the higher quantities required to produce for fulfil the demand.

5.2. Cost Analysis

For the present cost analysis it is important to firstly describe all costs associated with each supply contract type.

For the SMC the following costs are considered:

- Selling Price of Final Products of 22.7€/L
- Acquisition costs of 11.26 €/L

In the FCC the following costs are considered:

- Selling Price of Final Products of 22.7€/L
- Acquisition costs of 10.7 €/L
- Disposal Cost of RM of 3.2 €/L
- Storage cost of RM of 4.5 €/L

The acquisition cost of RMs under the FCC will be 5% lower than the spot market price (11.2 €/L), and hence the acquisition cost is 10.7€/L. Due to the restrictions on RMs' shelf-life and the higher quantities contracted with the supplier in the FCC, it will be necessary to account with the disposal and storage costs of RMs. In this analysis the disposal cost considered is 30% of RMs' acquisition cost. Likewise, storage costs are 20% of the final products' selling price. Note that both costs vary according to the quantities of RMs discarded and stored during the horizon. For confidentiality requirements these costs were estimated.

From a comparison of the two contracts types, in terms of profits and costs, it can be said the in the SMC the profit is 1.56% higher than the profit obtained through the FCC (see Table 11).

Table 11 – Comparison of Costs results.

Contracts	FCC	SMC	Δ
Relative Profit (€)	71, 485.7 €	72 621,12 €	1.56%
Spoilage Cost(€)	1,984 €	-	-1,984 €
Storage Cost(€)	2814.8 €	-	-2814.8 €

Although the acquisition price of RMs is lower in the FCC, the costs associated with the spoilage RMs and its storage penalize the relative profit of the company. As a consequence, with the latter contractual type the company will achieve a worst result in terms of profit. As regards the FCC, in Table 12 is presented a summary of the quantities and costs associated with spoilage of raw materials.

Table 12 – Deteriorated RM quantification for the FCC.

Raw materials	Quantity of spoilage (L)	Disposal Cost (€)	Storage Cost (€)
Amp	435	1392	1974.9
Bmp	185	592	839.9
Emp	0	0	0

In order to avoid spoilage of RMs, it would be necessary for the model to be able to process all the RMs received during their shelf-life. In Table 12, can be observed that despite of Emp's 3days of shelf-life, the quantity necessary to fulfill the demand for the respective final product is produced. Moreover, the remaining quantity of Emp available is also totally used for production within its shelf-life period. In this case, it will not be considered neither the disposal nor the storage costs.

However, after analyzing Amp and Bmp one concludes that there is a remaining quantity of both that was not used during the process. As it was explained in section 3.7, in this case the RMs could not be produced as they did not comply with the shelf-life restriction at the moment of production. Thus, these quantities were penalized in the objective function with the costs presented above.

6. Conclusions and Future Research

In this paper, two MILP models based on two supply contracts (FCC and SMC), with the integration of the main features of this industry such as: different RMs' shelf-life, its inventory control, multiple-deliveries during the planning horizon, changeover tasks and the integration of different procurement policies of RMs, were developed. Thus, these models enable to provide to the company an optimal sequence of production according to the supply contract followed, as the same time that they maximize the utilization of resources. Furthermore, due to the integration of the inventory control of RMs it could be also assed the impact of the RMs' shelf-life in the production process.

For both supply contracts the models can solve the problem under analysis, in fact for the SMC was obtained a CPU Time of 372.375s, such time is faster than the one obtained with the FCC. In terms of the relative gap, the FCC achieved the optimal solution with a relative gap of 0.39%, with the SMC was achieved a relative gap was 0%.

The main contribution of this work is to fill in the gap existent in the literature regarding the integration of inventory models of perishable products, the consideration of different procurement policies at the scheduling level, as the same time that the shelf-life of RMs as identified as main feature of the present production process is considered at the production. With the models results it was possible to identify the SMC as the most suitable supply contract type to be applied in the present study. Not only presents better economic results presented in the cost analysis performed, but also allows better final products quality due to the characteristics of the procurement policy of RMs in this contract.

In future research it can be explored: i)The inclusion in the model of the shelf-life control of final products as it would enable to maximize the freshness of final products; and ii) the lead times of RMs delivered as it may have an impact both in terms of costs and scheduling.

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