Discrete Symmetries and Proton Decay in the Adjoint $SU(5)$ Model

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Abstract

This extended abstract contains a summary of the Thesis it is associated with. We begin by introducing the Standard Model of Particle Physics because, despite certain exceptions, it provides a reasonable description of strong, weak and electromagnetic interactions at the energy scales available in experiments and because it serves as a prototype for Quantum Field Theories. Afterwards, we briefly discuss some of the Standard Model's shortcomings in order to justify the formulation of Grand Unified Theories. Among these last, the simplest possibility is based on an $SU(5)$ gauge group and it is called minimal $SU(5)$ model. We will see that this new theory still leaves certain Standard Model problems unsolved while bringing along new ones, however, it constitutes a favourable framework on which more realistic models may be obtained. In this context, notable improvements can be achieved by introducing specific scalar or fermion fields and a particular choice of these "fixes" leads us to the Adjoint-$SU(5)$ model. With this model as a starting point we investigate the possibility of using discrete symmetries to simultaneously reduce the number of arbitrary Yukawa parameters and suppress proton decay. Finally, we attempt to build a realistic model using that investigation's results.

Keywords: Grand Unification Theories, Proton Decay, Adjoint SU(5), Discrete Symmetries

I. The Standard Model of Particle Physics

The Standard Model of Particle Physics was proposed in $1973-74$ as a result of adding strong interactions to the Yang-Mills theory unifying weak and electromagnetic interactions introduced by Glashow [1] and improved by Salam [2] and Weinberg [3] through incorporation of the Higgs Mechanism [4–6]. A very important contribution was also given by t'Hooft, who proved the renormalizability of the model [7].

Since we are dealing with a Quantum Field Theory, the interactions between fields and their dependence on space-time coordinates is determined by the Lagrangian function $\mathcal{L}$ upon application of the stationary action principle. Symmetries that are observed in those interactions must, then, be present in the Lagrangian. The Standard Model's gauge group (group of local transformations that leave $\mathcal{L}$ invariant) is

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y, \tag{1}$$

where $SU(3)_C$ is associated with strong interactions, $SU(2)_L$ is associated with weak interactions and $U(1)_Y$ is the hypercharge group.

The local nature of gauge transformation means that invariance can only be achieved by adding vector bosons to the theory. Some symmetries are broken and the bosons associated with the broken generators are massive (something that could not happen if the symmetries were not broken). We could do this breaking explicitly but there is another way that offers more predictivity. If one postulates the existence of a $SU(2)_L$ scalar doublet ϕ, which may be written as

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{2}$$

the most general renormalizable potential involving this field is

$$V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2, \tag{3}$$

where μ and λ are constants. We consider $\lambda > 0$ for otherwise the field oscillations would be unbounded. As for μ, we can have $\mu^2 > 0$ or $\mu^2 < 0$ but, as shown in Figure 1, the second case is more interesting as the potential's minimum becomes $\langle\phi\rangle_0 = -\frac{\mu^2}{\lambda} = v^2$, where v is a real constant. In this situation we say that ϕ has a vacuum expectation value.

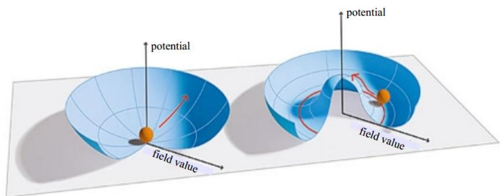

Figure 1: Scalar potential with $\mu^2 > 0$ (left) and $\mu^2 < 0$ (right)

This vacuum expectation value spoils $SU(2)_L \times U(1)_Y$ invariance but the generator defined through

$$Q = \frac{\sigma^3}{2} + Y, \tag{4}$$

where Y is the hypercharge, leaves the system invariant if ϕ has an hypercharge of $\frac{1}{2}$. The $U(1)$ gauge group generated by Q is a symmetry of the system and we identify it with the electromagnetism group (Q corresponds to electric charge).

We see that 3 of the 4 generator of the electroweak gauge group are broken. According to Goldstone's

Theorem, there should be 3 massless Goldstone bosons in the theory but this does not happen. By choosing a specific gauge, the unitary gauge, those new fields vanish, which means that they are not physics fields. In the unitary gauge we have

$$\phi = \begin{pmatrix} 0 \\ (v+H)/\sqrt{2} \end{pmatrix}, \tag{5}$$

where H is a real scalar field. Computing the kinetic terms for the Higgs scalar in this gauge and changing to the mass eigenstate basis we realize that 3 boson fields become massive while 1 (identified as the photon) remains massless. This is called the Higgs Mechanism: the vacuum state of a scalar field breaks a gauge symmetry and the would-be Goldstone bosons associated with the broken generators are absorbed by gauge bosons that get a mass.

As for the fermions, it should be noted that, in the Standard Model, they appear as left- or right-handed fields,

$$\Psi_L = \frac{1-\gamma_5}{2}\Psi, \Psi_R = \frac{1+\gamma_5}{2}\Psi. \tag{6}$$

Their quntum numbers are displayed in Figure 2. Considering this and the quantum numbers (transformation properties) of ϕ, we are allowed to add terms like

$$\mathcal{L}_{Yukawa} = -\bar{Q}^i_L Y^u_{ij}\tilde{\phi}u^j_R - \bar{Q}^i_L Y^d_{ij}\phi d^j_R - \bar{L}^i_L Y^e_{ij}\phi e^j_R + H.c., \tag{7}$$

where $\tilde{\phi} = i\sigma^2\phi^*$ and the $Y^{u,d,e}$ are arbitrary complex 3×3 Yukawa matrices, to the Lagrangian. When ϕ has a vacuum expectation value v, (14) leads to fermion masses:

$$\mathcal{L}_{mass} = -\bar{u}^i_L M^{ij}_u u^j_R - \bar{d}^i_L M^{ij}_d d^j_R - \bar{e}^i_L M^{ij}_e e^j_R + H.c., \tag{8}$$

with

$$M_u = \frac{1}{\sqrt{2}}vY^u, \quad M_d = \frac{1}{\sqrt{2}}vY^d, \quad M_e = \frac{1}{\sqrt{2}}vY^e. \tag{9}$$

The absence of right-handed neutrinos in this model means that they cannot acquire mass like the fermions in (15).

Quark Fields	Quantum Numbers	Lepton Fields	Quantum Numbers
$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	(3,2,1/3)	$L_L = \begin{pmatrix} \nu \\ e^- \end{pmatrix}$	(1,2,-1)
$(u^c)_L$	($\bar{3}$,1,-4/3)	$(e^c)_L$	(1,1,2)
$(d^c)_L$	($\bar{3}$,1,2/3)		

Figure 2: SM fermions and their G_{SM} quantum numbers.

We note that the Yukawa matrices are not, in general, diagonal, so the gauge interaction eigenstates are not mass eigenstates and, as a consequence, there are mixings between fermion flavours. To see how this happens, we consider the charged current associated with $W^+_\mu = \frac{A^1_\mu - A^2_\mu}{\sqrt{2}}$:

$$J^{\mu+}_W = \frac{1}{\sqrt{2}}(\bar{u}_L\gamma^\mu d_L + \bar{\nu}_L\gamma^\mu e_L). \tag{10}$$

Changing to the mass eigenstate basis through

$$u_L = U^u_L\acute{u}_L, \quad u_R = U^u_R\acute{u}_R, \tag{11}$$

$$d_L = U^d_L\acute{d}_L, \quad d_R = U^d_R\acute{d}_R, \tag{12}$$

$$e_L = U^e_L\acute{e}_L, \quad e_R = U^e_R\acute{e}_R, \tag{13}$$

$$\nu_L = U^e_L\acute{\nu}_L, \tag{14}$$

this charged current becomes

$$J^{\mu+}_W = \frac{1}{\sqrt{2}}(\bar{\acute{u}}_L\gamma^\mu V\acute{d}_L + \bar{\acute{\nu}}_L\gamma^\mu\acute{e}_L), \tag{15}$$

where $V = U^{u\dagger}_L U^d_L$ is known as the CKM matrix. It is a 3×3 unitary matrix (when 3 generations of fermions are considered) and, since it is, in general, different from the identity, there will be mixings between different quark flavours. As for the leptons, neutrinos are not massive in the Standard Model so we can put them in their mass eigenstate basis using the same matrix necessary to achieve this for the charged leptons and no mixings arise in this sector.

As stated in the beginning of this extended abstract, it is important for a theory to be renormalizable, otherwise we wold not be able to extract physical quantities form it. In any Quantum field theory one must take account of the possible radiative corrections to every process. Some of these corrections may break the symmetries of the classical Lagrangian, jeopardizing the theory's renormalizability. When this happens we say there is an anomaly.

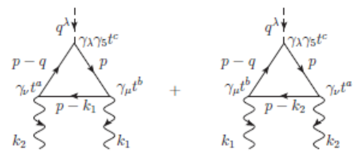

Figure 3: Triangle diagrams with vertices vector-vector-axial for non-Abelian gauges.

This phenomenon was first identified, independently, by Adler [8] and Bell, Jackiw [?]. They studied an Abelian chiral anomaly which implied that some classically forbidden processes may occur. In this work we are more interested in gauge anomalies, the ones that have an effect on renormalizability. Since G_{SM} includes non-Abelian groups, we should study gauge anomalies associated with this kind of groups. We can start by considering the triangle diagrams in Figure 3. The corresponding amplitude is

$$T^{abc}_{\mu\nu\lambda} = -i\int\frac{d^4p}{(2\pi)^4}tr\left[\frac{i}{\not{p}-m}\gamma_\lambda\gamma_5 t^c\frac{i}{\not{p}-\not{q}-m}\gamma_\nu t^a\times \right.$$
$$\left.\times\frac{i}{\not{p}-\not{k}_1-m}\gamma_\mu t^b\right] + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \\ a \leftrightarrow b \end{pmatrix}, \tag{16}$$

where the t's represent non-Abelian group generators. Using algebraic techniques we reach the conclusion that these anomalies are, in general, proportional to

$$tr\left[\left\{t^a,t^b\right\}t^c\right]=0. \tag{17}$$

This is a very important result and combining it with the facts that fermions contribute additively to the anomalies while left-, right-handed fermions contribute with opposite signs we can prove the SM's anomaly freedom.

II. SU(5) based GUTs

In spite of many successes, including the recent discovery of the postulated Higgs scalar [9,10], the Standard Model is plagued by several issues. On of them has to do with failure to quantize gravitational interactions. This is a problem in all Quantum Field Theories and more advances are required in this area.

Moving on to issues that are easier to tackle, we note the absence of neutrino masses, something that is contradicted by the observation of neutrino oscillations [11] (these oscillations are expected to depend on the square of the neutrino's mass differences). There are no right-handed neutrinos in the theory so these fields cannot acquire mass in the same way as the other fermions. Even with only left-handed neutrinos, one could write Majorana mass terms for them if they carried no conserved quantum numbers. The problem is that they have a non-zero lepton number and while instantons can violate lepton number or baryon number individually, the combination remains invariant at quantum level so neutrinos cannot acquire mass through any dynamical process.

Another problem has to do with the large number of arbitrary parameters in the model. This happens both in the scalar and Yukawa sectors, and in this last case it is tied with the absence of explanation for the hierarchy observed between different fermion generation's masses. Also, there is no explanation for the number of fermion generations. New physics is clearly required to shed some light on these issues. The same goes for another very unpleasant feature of the Standard Model, the so-called hierarchy problem. This is a naturality problem since the model's parameters must be fined-tuned, otherwise the radiative corrections to the physical Higgs mass will make it diverge. A possible solution is provided by supersymmetry but we are not interested in exploring this kind of theories in our present work.

Finally, we get to the electric charge quantization issue. In the Standard Model, hypercharges are assigned in order to reproduce observed electric charge values (recall (9)), they are not determined by any theoretical consideration. This is not satisfactory and, although it is possible to constrain the hypercharges imposing anomaly cancellation, we should look at ways of solving this problem. A common one involves postulating a larger gauge group containing G_{SM} as a subgroup. In this scenario, there would be only one gauge coupling when the larger group is effective and, provided we have the same number of fermion fields, there would be a smaller number of arbitrary Yukawa parameters.

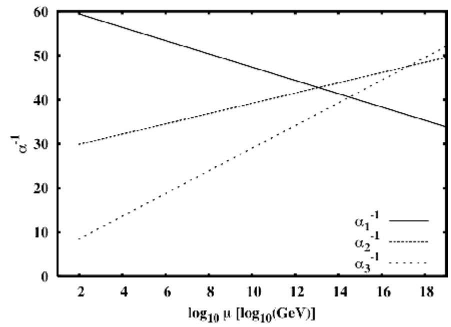

Figure 4: Figure 3. SM gauge couplings running.

Are there grounds to consider this possibility? If we take the Standard Model's gauge couplings and compute their running with the increase in energy scale we obtain the results shown in Figure 4. We can see that unification does not occur, but it can be argued that it "almost" happens. Furthermore, the energy range where this almost happens is consistent with constraints related to proton decay (this subject will be treated later). Since we expect the existence of new physics (and possibly new fields that influence gauge coupling runnings) anyway, it is legitimate to consider that unification might occur.

Theories verifying the conditions listed in the previous paragraphs are called Grand Unified Theories. If we want to have a minimal particle content, the simplest candidate is the minimal $SU(5)$ model. It has the same fermion fields as the Standard Model but the scalar sector is larger. To see why this is so, we start by noting that G_{SM} is a maximal subgroup of $SU(5)$, which means that we can construct this last group's representations using the fundamental representations of $SU(3)$ and $SU(2)$:$5=3\oplus 2$. In the fundamental representation of $SU(5)$ we have, then, an $SU(2)$ doublet analogous to the Standard Model's Higgs but we also have a new scalar colour triplet that we will designate by T^1. Since G_{SM} and $SU(5)$ have the same rank of 4 (number of independent diagonal generators), we need a new scalar in the $SU(5)$ adjoint representation, 24_H, to break this group into the Standard Model's group.

The most general scalar potential is, now, given by

$$V=V(24_H)+V(5_H)+V(24_H,5_H), \tag{18}$$

with

$$\begin{aligned}V(24_H)=&-\frac{\mu_{24}^2}{2}Tr\{24_H^2\}+\frac{\lambda_2}{4}Tr\{24_H^2\}^2+\\&+\frac{\lambda_3}{4}Tr\{24_H^4\}+\frac{a_1}{3}Tr\{24_H^3\},\end{aligned} \tag{19}$$

$$V(5_H)=-\frac{\mu_5^2}{2}5_H^\dagger 5_H+\frac{\lambda_1}{4}(5_H^\dagger 5_H)^2 \tag{20}$$

and

$$\begin{aligned}V(24_H,5_H)=&\lambda_{18}5_H^\dagger 5_H Tr\left\{24_H^2\right\}+\lambda_{19}5_H^\dagger 24_H^2 5_H+\\&+a_3 5_H^\dagger 24_H 5_H,\end{aligned} \tag{21}$$

where μ_{24}, λ_1, λ_2, λ_3, μ_5, a_1, λ_{18}, λ_{19} and a_3 are constants. If the constants in $V(24_H)$ verify certain conditions then this term of the potential has a minimum for $\sigma diag(2,2,2,-3,-3)$ and the $SU(5)$ group is broken to G_{SM}, as desired.

Decomposing the 24_H representation in terms of G_{SM} quantum numbers through

$$24_H = \Sigma_8 \oplus \Sigma_3 \oplus \Sigma_{(3,2)} \oplus \Sigma_{(3^*,2)} \oplus \Sigma_0, \tag{22}$$

where Σ_8 is an SU(3) octet, Σ_3 is an SU(2) triplet and Σ_0 is a singlet (the other 2 field's notation is self-explanatory) and making a parametrization similar to the one in the Standard Model we get

$$\langle 24_H \rangle + 24_H = \begin{pmatrix} \Sigma_8 & \Sigma_{(3^*,2)} \\ \Sigma_{(3,2)} & \Sigma_3 \end{pmatrix} + (\sigma + \Sigma_0)\lambda^{24}. \tag{23}$$

Substituting this in (26) we obtain

$$\begin{aligned} &m^2_{\Sigma_8} = \frac{1}{3}\sigma^2\lambda_3,\ m^2_{\Sigma_3} = \frac{4}{3}\sigma^2\lambda,\ m^2_{\Sigma_0} = 2\mu^2_{24}, \\ &m^2_{\Sigma_{(3,2)}} = m^2_{\Sigma_{(3^*,2)}} = 0. \end{aligned} \tag{24}$$

Given that $SU(5)$ has 24 generators and form considerations made in the last section we conclude that there are 24 gauge bosons in this minimal model, 12 ones that can be identified as the Standard Model's gauge bosons and 12 new ones:

$$\begin{pmatrix} G^1_{1\mu} + \frac{2B_\mu}{\sqrt{30}} & G^1_{2\mu} & G^1_{3\mu} & X^{1c}_\mu & Y^{1c}_\mu \\ G^2_{1\mu} & G^2_{2\mu} + \frac{2B_\mu}{\sqrt{30}} & G^2_{3\mu} & X^{2c}_\mu & Y^{2c}_\mu \\ G^3_{1\mu} & G^3_{2\mu} & G^3_{3\mu} + \frac{2B_\mu}{\sqrt{30}} & X^{3c}_\mu & Y^{3c}_\mu \\ X^1_\mu & X^2_\mu & X^3_\mu & \frac{Z_\mu}{\sqrt{2}} - \sqrt{\frac{3}{10}}B_\mu & W^+_\mu \\ Y^1_\mu & Y^2_\mu & Y^3_\mu & W^-_\mu & -\frac{B_\mu}{\sqrt{2}} - \sqrt{\frac{3}{10}}B_\mu \end{pmatrix} \tag{25}$$

The covariant derivatives for the fundamental representations are, now, given by

$$D_\mu = \partial_\mu + ig_5 \sum_{a=0}^{23} A^a_\mu \frac{\lambda^a}{2} = \partial_\mu + ig_5 \tilde{A}_\mu, \tag{26}$$

where g_5 is the gauge coupling and λ^a are the 24 generalized Gell-Mann matrices that represent the SU(5) generators (in the fundamental representation).

Similarly to what happens in the Standard Model, the 12 new gauge bosons associated with the $SU(5)$ gauge group become massive when this group undergoes spontaneous symmetry breaking to G_{SM}. Through the Higgs mechanism, these bosons related to broken generators absorb the massless would-be Goldstone bosons in (31) to get a mass of

$$M^2_X = M^2_Y = \frac{25}{2} g^2_5 \sigma^2, \tag{27}$$

. An interesting aspect of the new gauge bosons has to do with the fact that they can connect quark and lepton lines and, consequently, mediate proton decays. This is also true for the new scalar colour triplet. The subject of proton decays will be approached in the next section, where it will be seen that experimental results in this area strongly constrains the masses of these mediators.

Regarding fermion fields, we can accommodate them in a 5^* and an anti-symmetric 10 $SU(5)$ representations (for each generation), an assignment which yields the correct $SU(3) \times SU(2)$ quantum numbers and allows us to read the Standard Model's hypercharges from the λ^{24} generator:

$$\lambda^{24} = \frac{1}{\sqrt{15}} \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \tag{28}$$

This gives us charge quantization and explains the fractional charges of quarks.

Just as in the Standard Model, fermion masses arise in the Yukawa sector, which, in this context, is given by

$$\mathcal{L}_Y = \frac{1}{8}\epsilon_5 10^T_F C Y_{10} 10_F 5_H + \bar{5}^T_F C Y_5 10_F 5^*_H + H.c.. \tag{29}$$

When the scalar doublet in 5_H gets a vacuum expectation value of v we have the following relations between masses:

$$Y_e = Y^T_d \tag{30}$$

and

$$Y_u = Y^T_u, \tag{31}$$

where $M_e = vY_e$, $M_u = vY_u$ and $M_d = vY_d$. We have reduced the number of arbitrary Yukawa parameters but the prediction in (30) is wrong, so it is necessary to introduce certain changes. We will return to this issue later.

Beside the problem mentioned in the last paragraph, we note that neutrino masses are still absent in the minimal $SU(5)$ model. In this case, however, we know that baryon-number violating events (proton decay) are not forbidden so, from our previous discussion on this topic, it can be seen that these masses may be obtained through dynamic processes. The Satndard Model's hierarchy problems are not solved in this minimal set-up and new naturality problems arise as the T^1 mass must be very high for this minimal model's predictions to be accurate as far as proton decay is concerned while the scalar doublet in the same 5_H representation is expected to have a mass at the electroweak scale (doublet-triplet splitting problem). This requires a fine-tuning of the model's parameters.

What about unification? Defining the B-test [12] as $B \equiv \frac{B_{23}}{B_{12}}$, with $B_{ij} = B_i - B_j$ and [13]

$$B_i \equiv b_i + \sum_I b^I_i r_I, \tag{32}$$

where the b^I_i represent the contribution from particle I and the r_I are ratios that determine how relevant a particle's contribution is as a function of its mass, specifically

$$r_I = \frac{ln(\Lambda/M_I)}{ln(\Lambda/M_Z)}, \tag{33}$$

we see that unification requires $B = 0.718 \pm 0.003$. This cannot be achieved with the minimal $SU(5)$ particle content, which means that this model in not actually a Grand Unified Theory. There is no reason to abandon hope of obtaining unification in the context of $SU(5)$ based gauge theories as possible solutions to neutrino mass absence and wrong mass predictions include the addition of new fields that alter gauge coupling runnings.

Regarding the wrong mass prediction problem, we can look for non-renormalizable or renormalizable solutions. The second case is of more interest in this work, so we focus on it. The mass problem can be fixed by adding a scalar fields that couples to fermions without interfering in the breaking pattern. This new scalar representation should contain a Higgs doublet analogue and acquire a vacuum expectation value alongside the Standard Model's Higgs.

To achieve this, we may add a 45_H representation [14], which decomposes as

$$\begin{aligned} 45_H = &(8,2,1/2) \oplus (\bar{6},1,-1/3) \oplus (\bar{3},2,-7/6)\oplus \\ &\oplus (\bar{3},1,4/3) \oplus (3,3,-1/3) \oplus (3,1,-1/3)\oplus \\ &\oplus (1,2,1/2)) = S_{(8,2)} \oplus S_{(6^*,2)} \oplus S_{(3^*,2)}\oplus \\ &\oplus S_{(3^*,1)} \oplus \Delta \oplus T_2 \oplus H_2. \end{aligned} \tag{34}$$

The penultimate representation can be identified as a new scalar colour triplet, T_2, and the last one can be identified as a new SU(2)doublet scalar, H_2. If the 5_H and 45_H have the appropriate vacuum expectation value structure, given by

$$\langle 5_H \rangle^T = (0,0,0,0,v_5) \tag{35}$$

and

$$\langle 45^{\alpha 5}_{H\beta} \rangle = v_{45}(\delta^\beta_\alpha - 4\delta^\alpha_4\delta^4_\beta),\ \alpha,\beta = 1,...,4, \tag{36}$$

the masses arising from the new Yukawa sector,

$$\begin{aligned} -\mathcal{L}_Y = &\frac{\epsilon_5}{4}((\Gamma^1_u)_{ij}10_i10_j5_H + (\Gamma^2_u)_{ij}10_i10_j45_H)+ \\ &+ \sqrt{2}((\Gamma^1_d)_{ij}10_i5^*_j5_H + (\Gamma^2_d)_{ij}10_i5^*_j45^*_H), \end{aligned} \tag{37}$$

verify

$$M_u = v'\Gamma^1_u + 2v_{45}\Gamma^2_u, \tag{38}$$

$$M_d = v'^*\Gamma^1_d + 2v^*_{45}\Gamma^2_d, \tag{39}$$

$$M^T_e = v'^*\Gamma^1_d - 6v^*_{45}\Gamma^2_d. \tag{40}$$

From this we can see that

$$M_d - M^T_e = 8v^*_{45}\Gamma^2_d, \tag{41}$$

which eliminates the wrong mass predictions. We have corrected one of the model's problems but now we have more arbitrary Yukawa parameters.

Moving on to the neutrino mass absence issue, we could consider adding right-handed neutrinos so these particles would acquire mass in the same way as the other fermions, but in that case it would be difficult to explain the smallness of these masses (another naturality problem). An alternative approach is provided by the seesaw mechanisms. In very broad terms, we introduce heavy fields in the theory and tree-level exchange of those fields generates light neutrino masses through an effective dimension 5 Weinberg operator after the heavy fields are integrated-out [15, 16]:

$$\mathcal{L}_{Weinberg} = -\frac{z^{\alpha\beta}}{\Lambda}(\bar{l}_{L\alpha}\tilde{H})C(\bar{l}_{L\beta}\tilde{H})^T + H.c., \tag{42}$$

where Λ is the high-energy physics cutoff scale and $z^{\alpha\beta}$ are complex constants. Upon SSB this last equation becomes

$$\mathcal{L}_{Weinberg} = -\frac{1}{2}m^{\alpha\beta}_\nu \bar{\nu_{L\alpha}}\nu^c_{L\beta} + H.c. + ..., \tag{43}$$

where $m^{\alpha\beta}_\nu = v^2 z^{\alpha\beta}/\Lambda$ is the 3×3 effective neutrino matrix.

The types of seesaw with more interest in this work are type I and III. In this type I seesaw the new heavy particles are n_R right-handed neutrino fields ν_{Ri}, with quantum numbers given by (1,1,0) [17, 18]. We must add

$$\begin{aligned} \mathcal{L}_I = &\mathcal{L}_{SM} + \frac{i}{2}\bar{\nu}_{Ri}\partial\nu_{Ri} - Y^{\alpha i}_\nu \bar{l}_{L\alpha}\tilde{H}\nu_{Ri} - \\ &- \frac{1}{2}M^{ij}_R \bar{\nu^c_{Ri}}\nu_{Rj} + H.c., \end{aligned} \tag{44}$$

to the Lagrangian, where Y_ν is a $3\times n_R$ complex and arbitrary Yukawa matrix while M_R is a $n_R \times n_R$ symmetric matrix. Changing to the mass eigenstate basis through

$$\nu_{Ri} = R^{ij}_R N_{Rj}, R^T_R M_R R_R = d_R = diag(M_1, ..., M_{nR}). \tag{45}$$

we get

$$\begin{aligned} \mathcal{L}_I = &\mathcal{L}_{SM} + i\bar{N}_{Ri}\partial N_{Ri} - Y^{\alpha i}_R \bar{l}_{L\alpha}\tilde{H}N_{Ri} - \\ &- \frac{1}{2}d^{ij}_R \bar{N}^c_{Ri}N_{Rj} + H.c., \end{aligned} \tag{46}$$

where $Y_R = Y_\nu R_R$.

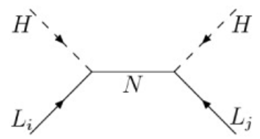

Figure 5: Feynman diagram representing the exchange of heavy particles that generates type I seesaw.

Considering the Feynman diagram for ν_{Ri} exchange in Figure 5 and comparing (46) with (42) we obtain

$$\frac{z^{\alpha\beta}}{\Delta} \propto Y^{\alpha i}_R \frac{1}{p - M_i} Y^{\beta i}_R. \tag{47}$$

Since the M_i are much larger than the electroweak scale, we can make the following approximation:

$$\frac{z^{\alpha\beta}}{\Delta} \simeq -Y^{\alpha i}_R \frac{1}{M_i} Y^{\beta i}_R = -Y^{\alpha i}_R \frac{1}{d^{ii}_R} Y^{\beta i}_R. \tag{48}$$

Recalling (52) we can write

$$\begin{aligned} \frac{z^{\alpha\beta}}{\Delta} \simeq &- Y^{\alpha i}_R (d^{ii}_R)^{-1} Y^{\beta i}_R = -(Y_R d^{-1}_R Y^T_R)^{\alpha\beta} = \\ &= -(Y_\nu R_R (R_R)^{-1}(M_R)^{-1}(R^T_R)^{-1}R^T_R Y^T_\nu)^{\alpha\beta} = \\ &= -(Y_\nu M^{-1}_R Y^T_\nu)^{\alpha\beta}. \end{aligned} \tag{49}$$

From this last expression we conclude that:

$$m_\nu = -v^2 Y_\nu M^{-1}_R Y^T_\nu = -m_D M^{-1}_R m^T_D, \tag{50}$$

with $m_D = vY_\nu$. We can easily see that large values of M_R lead to small m_ν, as we wanted in order to reproduce experimental data.

Type III seesaw is obtained by adding n_Σ $SU(2)$ fermion triplets Σ_{Ri}, with quantum numbers given by (1,3,0), to the theory [19]. This process, shown in Figure 6, is analgous to type I seesaw.

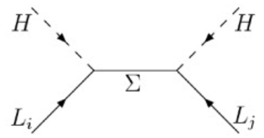

Figure 6: Feynman diagram representing the exchange of heavy particles that generates type III seesaw.

III. Discrete Symmetries and Proton Decay

Adding a 45_H scalar representation and n_ρ adjoint fermion representations to the minimal $SU(5)$ model in order to solve the wrong mass prediction and neutrino mass problems, respectively, we obtain the Adjoint $SU(5)$ model [20]. We choose adjoint representation beause, decomposing it in terms of G_{SM} quantum numbers through

$$\rho(24) = \rho_8 \oplus \rho_3 \oplus \rho_{(3,2)} \oplus \rho_{(3^*,2)} \oplus \rho_0, \tag{51}$$

one sees that it contains a singlet and an SU(2) triplet so both Type I and Type III seesaw are induced. We assume that these new fermion fields acquire unconstrained Majorana masses through some mechanism with which we are not concerned in this work, so the most general Yukawa sector consistent with renormalizability is given by

$$\begin{aligned}-\mathcal{L}_Y =&\frac{\epsilon_5}{4}((\Gamma^1_u)_{ij}10_i10_j5_H+\\ &+(\Gamma^2_u)_{ij}10_i10_j45_H)+\sqrt{2}((\Gamma^1_d)_{ij}10_i5^*_j5_H+\\ &+(\Gamma^2_d)_{ij}10_i5^*_j45^*_H)+MTr(\rho^2)+\\ &+\lambda Tr(\rho^2\Sigma)+(\Gamma^1_\nu)_i5^*_i\rho5_H+(\Gamma^2_\nu)_i5^*_i\rho45_H+H.c.,\end{aligned} \tag{52}$$

where k,l=1,...,n_ρ, M is a symmetric $n_\rho \times n_\rho$ Majorana mass matrix and λ, Γ^1_ν and Γ^2_ν are Yukawa matrices (λ is symmetric).

When the adjoint scalar 24_H gets a vev we have $\rho(24)$ masses given by

$$\begin{aligned}M_0 &= \frac{1}{4}(M-\frac{\sigma}{\sqrt{30}}\lambda),\\ M_3 &= \frac{1}{4}(M-\frac{3\sigma}{\sqrt{30}}\lambda),\\ M_8 &= \frac{1}{4}(M+\frac{2\sigma}{\sqrt{30}}\lambda),\\ M_{(3,2)} &= M_{(3^*,2)} = \frac{1}{4}(M-\frac{\sigma}{2\sqrt{30}}\lambda).\end{aligned} \tag{53}$$

In this context, the Standard Model's scalar vacuum expectation value, v, is a combination of the vacuum expectation values of the $SU(2)$ doublets in 5_H and 45_H, so we perform a rotation by an angle of $\beta \equiv v_{45}/v_5$ in order to have a Higgs field H with a vacuum expectation value of v and another Higgs $\acute{H}$ field without this feature. Now, decomposing the ρ_{24} fields in terms of light fields and following the guidelines for seesaw mechanisms presented in the last section we obtain

$$(m_\nu)_{ij} = -(m^D_0 M_0^{-1} {m^D_0}^T)_{ij} - (m^D_3 M_3^{-1} {m^D_3}^T)_{ij}, \tag{54}$$

with

$$\begin{aligned}m^D_0 &= \frac{\sqrt{15}v}{2}(\frac{\cos\beta}{5}\Gamma^1_\nu+\sin\beta\Gamma^2_\nu),\\ m^D_3 &= \frac{v}{\sqrt{2}}(-\cos\beta\Gamma^1_\nu+3\sin\beta\Gamma^2_\nu).\end{aligned} \tag{55}$$

Since M_0 and M_3 can have very high values, the left-handed neutrinos get very small masses, as we wanted.

We move on to the study of proton decay in this particular model. The sources of proton decay in the Adjoint model are the leptoquarks (the new $SU(5)$ gauge bosons) and coloured scalar triplets. Considering the leptoquarks first,the new interactions involving them are

$$\mathcal{L}_{XY} = -g_5 Tr\{\bar{10}_F\gamma^\mu\tilde{A}^X_\mu 10_F\} + g_5\bar{\bar{5}}_F\gamma^\mu(\tilde{A}^X_\mu)^T\bar{5}_F. \tag{56}$$

Since X and Y fields have the same quantum numbers we regard them as the same field with two indexes. From this, we write (68) as

$$\begin{aligned}\mathcal{L}_{XY} = &\frac{g_5}{\sqrt{2}}\left[((\bar{d_c})^\alpha\gamma^\mu\epsilon_{ab}L^b - \bar{e}^c\epsilon_{ba}\gamma^\mu q^{\alpha b}+\right.\\ &\left.+\bar{q}_{\beta a}\gamma^\mu\epsilon^{\alpha\beta\gamma}u^c_\gamma)(X_\mu)^a_\alpha\right]+H.c..\end{aligned} \tag{57}$$

We now determine the low-energy effective operators leading to proton decay that arise from the previous expression. Applying the equations of motion and integrating-out the heavy leptoquark fields, we arrive at

$$\begin{aligned}\mathcal{L}_{Xeff} = &\frac{g_5^2}{M_X^2}\epsilon_{\alpha\beta\gamma}(\bar{u^c})^\alpha\gamma_\mu q^{a\beta}\left\{\bar{e^c}\epsilon_{ab}\gamma^\mu q^{\gamma b}+\right.\\ &\left.+(\bar{d^c})^\gamma\gamma^\mu\epsilon_{ab}L^b\right\}+H.c.,\end{aligned} \tag{58}$$

which are the dimension 6 effective operators.

To get an estimate of the leptoquark masses required for consistency with proton decay experimental data we consider the process

$$p \to \pi^0 e^+. \tag{59}$$

The amplitudes contributing to this process are

$$\begin{aligned}M_1 &= \frac{g_5^2}{M_V^2}\epsilon_{\alpha\beta\gamma}(\bar{u^c})^\alpha u^\beta\bar{e^c}\gamma_\mu\gamma^\mu d^\gamma,\\ M_2 &= \frac{g_5^2}{M_V^2}\epsilon_{\alpha\beta\gamma}(\bar{u^c})^\alpha\gamma_\mu u^\beta(\bar{d^c})^\gamma\gamma^\mu e,\end{aligned} \tag{60}$$

so one can state that [21]

$$\Gamma_{pd} \sim \alpha_5^2\frac{m_p^5}{M_X^4}. \tag{61}$$

Using the most restrictive constraints on the proton lifetime available at this time [22], which are

$$\tau(p \to \pi^0 e^+) > 1.4 \times 10^{34} years, \tag{62}$$

and the fact that unification requires $26 \leq \alpha_5^{-1} \leq 35$, we obtain

$$M_V > (4.9 - 5.7) \times 10^{15} GeV. \tag{63}$$

This result is not troublesome as we expect, from naturality arguments, that leptoquark masses should be around unification scale values.

As for scalar-mediated proton decay, the 5_H scalar multiplet includes a coloured scalar triplet, T_1. Its interactions with fermions are given by:

$$-\mathcal{L}_Y^{5_H-fermion} = \frac{\epsilon_{\alpha\beta\gamma\delta\eta}}{4}(\Gamma_u^1)_{ij} 10_i^{\alpha\beta} 10_j^{\gamma\delta}(5_H)^\eta + \\ + \sqrt{2}(\Gamma_d^1)_{ij} 10_i^{\alpha\beta} 5_{j\alpha}^*(5_H^*)_\beta, \tag{64}$$

where the i,j are generation indexes and α, β, γ, δ, η are SU(5) indexes. Keeping only the terms with T_1 we get

$$(\Gamma_u^1)_{ij}(\frac{1}{2}Q^i Q^j + u^{iC} e^{jC})T + (\Gamma_d^1)(Q^i L^j + u^{iC} d^{jC})T^*, \tag{65}$$

from which it can be seen that these scalars can mediate proton decay. After adding the triplet's mass term,

$$-m_{T_1}^2 T_1^* T_1, \tag{66}$$

to (77), we can repeat the procedure used to obtain the effective operators for leptoquark mediated proton decay to get

$$\frac{(\Gamma_u^1)_{ij}(\Gamma_d^1)_{kl}}{M_{T_1}^2}\left[\frac{1}{2}(Q_i Q_j)(Q_k L_l) + (u_i^c e_j^c)(u_k^c d_l^c)\right], \tag{67}$$

the dimension 6 effective operators contributing to the decays by means of T_1 bosons exchange. There is another coloured scalar triplet in the Adjoint model, T_2, which belongs to the 45_H scalar representation. Starting form the Yukawa Lagrangian and retaking the steps already described for the leptoquarks and T_1, one arrives at the following effective operators:

$$\frac{4(\Gamma_u^2)_{ij}(\Gamma_d^2)_{kl}}{M_{T_2}^2}(u_i^c e_j^c)(u_k^c d_l^c). \tag{68}$$

Also, the 45_H multiplet contains other coloured representations that may lead to proton decay, specifically, the $(3^*, 1, 4/5)$ triplet and the three triplets $\Delta^{-1/3}$, $\Delta^{2/3}$ and $\Delta^{-4/3}$, which belong to the $(3, 3, -1/5)$ triplet-triplet. By expanding the Lagrangian terms involving these fields one can see that $(3^*, 1, 4/5)$ and $\Delta^{-4/3}$ make no contribution for proton decay as they only couple to pairs of up-quarks while (Γ_u^2) is antisymmetric (therefore it cannot couple two equal particles). For the $\Delta^{-1/3}$ and $\Delta^{2/3}$ fields the effective operators are

$$\frac{(\Gamma_u^2)_{ij}(\Gamma_d^2)_{kl}}{2M_{\Delta^{-1/3}}^2}\left[(u_i d_j)(u_k e_l) + (u_i d_j)(d_k \nu_l)\right] \tag{69}$$

and

$$-\frac{(\Gamma_u^2)_{ij}(\Gamma_d^2)_{kl}}{2M_{\Delta^{2/3}}^2}(d_i d_j)(u_k \nu_l) \tag{70}$$

respectively.

We can use the same procedure as in the leptoquark case to estimate constraints on these scalar's masses. The fact that the amplitudes in this situation depend on the Yukawa parameters instead of the gauge couplings means that the constraints are milder, with

$$m_{T_{1,2}}^2 \geq 10^{12} GeV. \tag{71}$$

Taking advantage of the aforementioned dependence on the Yukawas, we will try to supress these decays, at tree-level, using discrete symmetries.

The use of these symmetries allows not only this suppression but it also diminishes the number of arbitrary parameters in the model. We begin by collecting the conditions that must be obeyed for scalar-mediated proton decay to be forbidden:

$$\rightarrow (\Gamma_u^1)_{11} = 0,\ (\Gamma_d^1)_{11} = 0,\ (\Gamma_d^1)_{12} = 0,\ (\Gamma_d^1)_{13} = 0; \tag{72}$$

$$\rightarrow (\Gamma_u^1)_{11} = 0,\ (\Gamma_u^1)_{12} = 0; \tag{73}$$

$$\begin{aligned} \rightarrow &(\Gamma_d^1)_{11} = 0,\ (\Gamma_d^1)_{12} = 0,\ (\Gamma_d^1)_{21} = 0, \\ &(\Gamma_d^1)_{22} = 0, (\Gamma_d^1)_{13} = 0,\ (\Gamma_d^1)_{23} = 0. \end{aligned} \tag{74}$$

If any of this sets of conditions is verified, the $T_{1,2}$ do not contribute to these processes at tree-level. For this to happen with the $\Delta^{-1/3}$ and $\Delta^{2/3}$, any of the following sets of conditions should be observed:

$$\rightarrow (\Gamma_u^2)_{11} = 0,\ (\Gamma_d^2)_{11} = 0,\ (\Gamma_d^2)_{12} = 0,\ (\Gamma_d^2)_{13} = 0; \tag{75}$$

$$\rightarrow (\Gamma_u^2)_{11} = 0,\ (\Gamma_u^2)_{12} = 0; \tag{76}$$

$$\begin{aligned} \rightarrow &(\Gamma_d^2)_{11} = 0,\ (\Gamma_d^2)_{12} = 0,\ (\Gamma_d^2)_{21} = 0, \\ &(\Gamma_d^2)_{22} = 0,\ (\Gamma_d^2)_{13} = 0,\ (\Gamma_d^2)_{23} = 0. \end{aligned} \tag{77}$$

Regarding the possible ways of satisfying these conditions through the use of discrete symmetries, we label the different solutions according to the number of zeros obtained in the up-quark (rows) and down-quark (columns) mass matrices. We show here two distinct examples:

	0	1	2	3	4	5	6
0	4	0	0	0	0	0	0
1	4	6	12	0	3	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	42	138	54	0
5	0	0	0	30	105	78	0
6	0	0	0	0	0	0	6

Figure 7: The first column indicates the number of M_u zeros and the first row indicates the number of M_d zeros. The Z_4 symmetry was used.

IV. Adjoint SU(5) with Discrete Symmetry Results

In order to check the realism of an Adjoint $SU(5)$ with a discrete symmetry we investigate its compatibility with low-energy experimental data. In the quark sector, we can see if we have enough Yukawa parameters to reproduce the measured quark masses and mixings (using the CKM matrix). As for the leptonic sector, the fact that we now have neutrino masses and

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	4	6	12	0	3	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	42	111	54	0
5	0	0	0	0	27	84	42
6	0	0	0	0	24	96	78

Figure 8: The first column indicates the number of M_u zeros and the first row indicates the number of M_d zeros. The continuous symmetry was used.

flavour oscillations means that we need to take a new look at this part of low-energy phenomenology.

Our considerations so far (neutrino oscillations) imply a distinction between the interaction and mass basis, specifically,

$$\acute{\nu}_\alpha = \sum_{j=1}^{3} U_{\alpha j}\nu_j, \tag{78}$$

where U is an unitary matrix, $\acute{\nu}$ are interaction eigenstates and the ν are mass eigenstates. After spontaneous symmetry breaking of the GUT gauge group we may write the lepton masses in the interaction basis as

$$\mathcal{L}_{leptonmasses} = -M_l\bar{\acute{e}}_L\acute{e}_R - \frac{1}{2}m_\mu\bar{\acute{\nu}}_L\acute{\nu}_L^c + H.c.. \tag{79}$$

The charged lepton mass matrix can be diagonalized by means of a bi-unitary transformation, while the neutrino Majorana mass matrix,being symmetric and multiplied form both sides by the same vector (in family space), ν_L, can be diagonalized using a single unitary matrix. Interaction and mass eigenstates are, then, related through

$$\acute{l}_R = U_{Rl}l_R, \acute{l}_L = U_{Ll}l_L, \acute{\nu}_L = U_n u\nu_L. \tag{80}$$

Applying these transformations to the charged current in (10) we obtain

$$\mathcal{L}_{char.current} = \frac{g_W}{\sqrt{2}}(\bar{\acute{u}}_L\gamma^\mu\acute{d}_L + \bar{\acute{\nu}}_L\gamma^\mu\acute{e}_L)W_\mu^+ + H.c. =$$
$$= \frac{g_W}{\sqrt{2}}(\bar{u_L}V_{CKM}\gamma^\mu d_L + \bar{\nu_L}U_{PMNS}^\dagger e_L)W_\mu^+ + H.c., \tag{81}$$

where

$$U_{PMNS} = U_{Ll}^\dagger U_\nu = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}. \tag{82}$$

The experimental results related to the physical quantities in U_{PMNS} may, then, be used to test our model.

From what we have seen before, it is clear that anomalies are highly undesirable. The Adjoint $SU(5)$ model is anomaly-free, but what about the $SU(5)\times Z_N$ case? Quantum gravity effects may destroy the discrete symmetry and, consequently, the benefits that come from them, unless the $U(1)$ symmetry from which it originates verifies certain "discrete anomaly cancellation" conditions [23].

If we have a discrete Z_N gauge symmetry under which the fermion fields transform as $\Psi_i \rightarrow exp(i2\pi q_i/N)\Psi_1$, the $U(1)$ charges before spontaneous symmetry breaking were necessarily of the form

$$Q_i = q_i + m_i N, \tag{83}$$

where q_i and m_i are integers (q_i are the Z_N charges). We may use (83) to redefine the $U(1)$ charges so that possible anomalies are annulled, but this is not always possible. We can also consider the existence of heavy fermions that become massive upon spontaneous symmetry breaking and contribute to the anomaly-freedom conditions. There are two types of masses fermions may acquire at this stage [24]:

-Pairs of different Weyl fermions with charges that verify

$$Q_{W1}^j + Q_{W2}^j = p_j N, \tag{84}$$

where p_j is an integer, combine to get a mass.

-One fermion that is an $SU(5)$ singlet gets a Majorana mass and it must verify

$$Q_\chi^j = \frac{1}{2}\acute{p}_j N \tag{85}$$

when N is even (there is no contribution to the anomalies when N is odd), $\acute{p}_j$ are integers.

Admitting the possibility of having several of these new fields and considering the charge redefinitions we are free to make, the anomaly-freedom constraints become

$$\sum_i T_i(q_i) = rN, \tag{86}$$

$$\sum_i (q_i)^3 = mN + \eta n\frac{N^3}{8}, \tag{87}$$

and

$$\sum_i q_i = pN + \eta q\frac{N}{2}, \tag{88}$$

where η is 1, 0 for N even, odd, respectively, and the integers r, m, n, p, q are related to the new heavy fields and the possible charge redefinitions in (83). If the conditions in (86), (87) and (88) are satisfied we still have no guarantee of anomaly-freedom, since we are dealing with diophantine equations that do not always admit solutions. In this context, the obtained conditions are useful in excluding theories that cannot be made anomaly-free.

Finally, we investigate realizations of an Adjoint $SU(5)$ model with a Z_N discrete symmetries. The fact that there are no continuous ($U(1)$) cases for which one can obtain tree-level scalar-mediated proton decay suppression and an up-, down-quark mass matrices with 5, 3 texture zeros, respectively, coupled with "discrete gauge anomaly" considerations, lead us to choose a Z_8 symmetry. Making the following charge assignments,

$$Q(10_1) = 2, \quad Q(10_2) = 0, \quad Q(10_3) = 4, \quad Q(5_1^*) = 6,$$
$$Q(5_2^*) = 0, \quad Q(5_3^*) = 6, Q(5_H) = 0, Q(45_H) = 2, \tag{89}$$

we get

$$M_u = \begin{pmatrix} 0 & 0 & * \\ 0 & 0 & 0 \\ * & 0 & * \end{pmatrix} \tag{90}$$

and

$$M_d = \begin{pmatrix} * & * & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}. \tag{91}$$

From these mass matrices we calculate the V_{CKM} matrix and it can be seen that we have enough free Yukawa parameters to obtain the quark masses and this matrix within one standard deviation. Attempts at finding a possible new correlation between certain V_{CKM} values are presented in Figures 9 and 10:

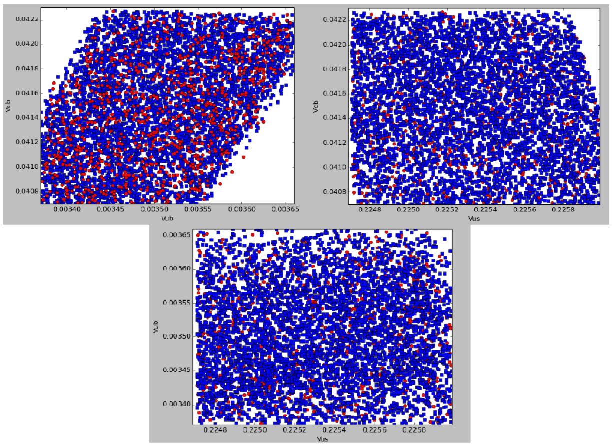

Figure 9: Correlation plots involving V_{CKM} moduli. The red dots represent this work's results, the blue ones represent all the possibilities.

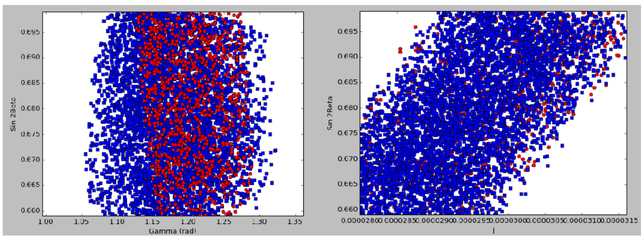

Figure 10: Correlation plots involving V_{CKM} angles. The red dots represent this work's results, the blue ones represent all the possibilities.

. We note that no correlation at all exists except in the J vs. $\sin 2\beta$ plot.

Considering the lepton sector now, we assign $Q(\rho_1) = 5$, $Q(\rho_2) = 6$, $Q(\rho_3) = 2$ and add two scalar singlets with Z_8 charges of 5 and 6, obtaining

$$M_e = \begin{pmatrix} * & 0 & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}, \tag{92}$$

$$M_{Total} = \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & 0 \end{pmatrix}, \tag{93}$$

and, from our discussion of seesaw mechanisms,

$$(m_\nu)_{ij} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix}. \tag{94}$$

Taking into account the studies of neutrino textures in ref. [25], we conclude that this case is compatible with experimental results, although it provides little to no predictivity. To end the study of this case, we note that the conditions for "discrete gauge anomaly" cancellation are satisfied.

The other case we study is a Z_7 symmetry that puts the quark mass matrics in the NNI (Nearest Neighbour Interaction) form. We make assign the discrete symmetry charges according to

$$Q(10_1) = 1, \quad Q(10_2) = 3, \quad Q(10_3) = 2, \quad Q(5_1^*) = 6, \\ Q(5_2^*) = 1, \quad Q(5_3^*) = 0, Q(5_H) = 3, Q(45_H) = 2, \tag{95}$$

so

$$M_u = \begin{pmatrix} 0 & 0 & * \\ 0 & 0 & 0 \\ * & 0 & * \end{pmatrix} \tag{96}$$

and

$$M_d = \begin{pmatrix} * & * & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}, \tag{97}$$

as expected. Instead of investigating compatibility with low-energy quark phenomenology, we focus on the lepton sector. Choosing $Q(\rho_1) = 4$, $Q(\rho_2) = 5$, $Q(\rho_3) = 6$ and adding a scalar singlet with a charge of 4 we get

$$M_{Total} = \begin{pmatrix} 0 & 0 & * \\ 0 & * & 0 \\ * & 0 & 0 \end{pmatrix} \tag{98}$$

and

$$(m_\nu)_{ij} = \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}. \tag{99}$$

Plugging this result and the form of the charged lepton mass matrix into a program developed with the *minuit2* minimization package for C and C++ we tested this model's predictions. The Chi-square test performed returns a minimum value of 7.65627×10^{-6}, so we conclude that the free Yukawa parameters in the theory are sufficient to satisfactorily reproduce experimental data.

	$S_{(8,2)}$	$S_{(6^*,1)}$	$S_{(3^*,2)}$	$S_{(3^*,1)}$	Δ	$T_{1,2}$	$H_{1,2}$
B_{23}	$-\frac{2}{3}r_{S_{(8,2)}}$	$-\frac{5}{6}r_{S_{(6^*,1)}}$	$\frac{1}{6}r_{S_{(3^*,2)}}$	$-\frac{1}{6}r_{S_{(3^*,1)}}$	$\frac{2}{3}r_\Delta$	$-\frac{1}{6}r_{T_{1,2}}$	$\frac{1}{6}r_{H_{1,2}}$
B_{12}	$-\frac{8}{15}r_{S_{(8,2)}}$	$\frac{2}{15}r_{S_{(6^*,1)}}$	$\frac{17}{15}r_{S_{(3^*,2)}}$	$\frac{16}{15}r_{S_{(3^*,1)}}$	$-\frac{9}{5}r_\Delta$	$\frac{1}{15}r_{T_2}$	$-\frac{1}{15}r_{H_2}$

	Σ_8	Σ_3	ρ_8	$\rho_{(3,2)}$	ρ_3
B_{23}	$-\frac{1}{2}r_{\Sigma_8}$	$\frac{1}{3}r_{\Sigma_3}$	$-2r_{\rho_8}$	$\frac{1}{3}r_{\rho(3,2)}$	$\frac{4}{3}r_{\rho_3}$
B_{12}	0	$-\frac{1}{3}r_{\Sigma_3}$	0	$\frac{2}{3}r_{\rho(3,2)}$	$-\frac{4}{3}r_{\rho_3}$

Figure 11: Possibly relevant contributions for gauge coupling runnings.

To conclude this section, we check weather or not unification can be achieved in this scenario. The relevant contributions for the aforementioned B-test are shown in Figure 11. It can be seen that the only fields contributing towards unification (increasing the

B value in relation to the SM one) are the Higgs doublets, ρ_3, Δ and Σ_3. These Higgs fields are expected to have masses around the electroweak scale, so their r_I is taken to be 1. As for the Δ and the coloured triplets $T_{1,2}$, their masses are constrained to be large due to proton decay. These constraints are not as strict as the ones for the leptoquarks (which are not even considered here because they should have an r_I very close to zero) but the T's are not favourable for unification so we assume they have masses around unification scale and do not contribute to the running of gauge couplings. Computing these runnings with the other masses left as free parameters we obtain unification for a wide variety of possible energy scales and intermediate masses, as expressed in Figure 12. The fact that unification can be achieved is possible but the intermediate masses are not very constrained among each other so it is difficult to obtain predictions in this context.

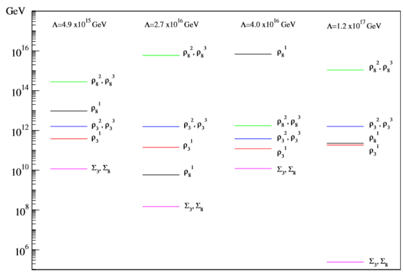

Figure 12: Mass spectrum of the adjoint fermionic fields, Σ_8 and Σ_3 for different unification scales.

V. Conclusions

We were able to obtain a realistic theory based on the Adjoint $SU(5)$ and a discrete symmetry. There are enough arbitrary parameters to reproduce low-energy experimental results but the predictivity is limited. The same applies for unification and the intermediate masses involved. It woulds be interesting to study other more restrictive cases.

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