Numerical Implementation of a Frequency-Domain Panel Method for Flutter Prediction of a 3D Wing

Eduardo Manuel Pizarro Gomes Pepe
pizarropepe@tecnico.ulisboa.pt

Instituto Superior Técnico, Lisboa, Portugal

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Abstract

Within the field of linear methods for aeroelastic flutter prediction, the Doublet Lattice Method (DLM) and the Strip-Theory are considered standard methodologies. These methods are formulated in the convenient frequency domain, thus allowing computationally efficient solution techniques to extract the aeroelastic system's eigenvalues and infer about its dynamic stability.

Panel methods are based on potential aerodynamic theory and offer greater capability to model complex 3D surfaces, thickness effects, and allow unconstrained surface discretization schemes. However, Panel Methods are seldom applied to flutter analysis since, in general, they are formulated in the time-domain. L. Morino presented a frequency-domain integral solution for the linearized compressible aerodynamic potential equation which constituted the foundations of a unique frequency-domain panel method.

In this thesis, this frequency-domain panel method has been implemented in a MATLAB® environment as part of an in-house aeroelastic design tool specialized in the flutter prediction of 3D cantilever wing structures. A large portion of the work is focused on the implementation aspects and numerical studies concerning convergence and computational efficiency. The modified p-k method is based on the original p-k method and has been developed in order to enforce physical consistency and to increase the accuracy in the process of extracting the system's eigenvalues. The linear flutter analysis of a high-aspect-ratio wing has been performed for validation purposes. While the work completed constitutes a full implementation of the panel method in the frequency domain, the results have shown large discrepancies that require further validation and evaluation of the proposed numerical tool.
Keywords: aeroelastic flutter, potential aerodynamics, panel method, frequency-domain, p-k method

INTRODUCTION

Aeroelastic flutter is a type of dynamic instability caused by a self-feeding mechanism involving the inertial, elastic and aerodynamic forces on a body immersed in a fluid. In the context of airplane wings, the flutter phenomenon is characterized by the onset of wing vibration and subsequent continuous growth of its amplitude, after a certain critical flight speed is reached. Flutter can lead to loss of aircraft control and to the eventual structural failure if the vibration amplitudes are allowed to grow indefinitely. For these reasons, it is of greater importance that the critical flight speed – the flutter speed – is known for a particular aircraft.

Understanding the mechanisms of flutter requires performing flutter tests and the development of suitable analytical models. Due to the hazards of flutter testing, most of today's research involves developing and implementing efficient numerical methods known as the Computational Aeroelasticity (CAE) codes. These numerical techniques couple Computational Fluid Dynamics (CFD) and Computational Solid Dynamics (CSD) codes in order to track or detect the onset of flutter instability. Despite the multidisciplinary nature of flutter, the main effort has been toward the improvement of the unsteady aerodynamic models. These are responsible for the complex self-feeding mechanisms involved and, because of its intrinsic nonlinear character, the aerodynamic models are still an active field of research.

BIBLIOGRAPHICAL REVIEW

Aerodynamic models

Analytical models for flutter calculations have been developed since the early 20s of the XXth century. Among the many contributions in this field, the most prominent were given by Theodorsen [1] in 1934 with the Strip-Theory and Rodden *et al* [2] in 1968 with the DLM. These models rely on unsteady potential aerodynamics and are formulated in the frequency domain by assuming small harmonic deviations of all quantities about a steady, trim state.

While the frequency-domain approach is considered the standard formulation for flutter, there are numerous studies performed in the time-domain. Within the branch of potential aerodynamics, unsteady Panel Methods [3] [4] are possibly the most accurate methodologies for providing time-domain solutions for the pressure field on bodies of arbitrary geometrical complexity. Even though the DLM is preferable for predicting flutter for its speed and accuracy, there have been some successful studies resorting to unsteady panel methods [5]. However, the most common time-domain flutter analysis are associated with more sophisticated aerodynamic theories, including the unsteady Euler [6] and Navier-Stokes [7] equations. The non-linear character of these theories make them more accurate in terms of the quality of the solution, but extremely expensive in the computational point of view.

The main goal of the aerodynamic theories is to obtain the aerodynamic forces on every point representing the structure. This force is either in the form of an Aerodynamic Influence Coefficient's matrix $[AIC]$ or in the form of an external force vector f_A, for frequency or time-domain formulations respectively.

Structural Models

In an aeroelastic analysis, the structural model resembles the whole stiffness of the lifting body and usually is modeled with linear theories such as plate or beam theory. The structural model is independent from the aerodynamic model and different discretization schemes may be used. For this reason, there is often the need of using special methods for transferring the aerodynamic forces from the aerodynamic domain to the structural domain. Examples of these methods are the Rigid Body Attachment [8] and the spline methods such as the Infinite Beam Spline [8] or the Thin Plate Spline [8] [9]. The structural model provides the structural matrices that are key elements for the definition of the full aeroelastic model. These are the mass $[M]$, stiffness $[K]$ and damping $[C]$ matrices, representing the inertial, elastic and damping characteristics of the structure, respectively. The structural matrices allow the calculation of the natural modes and frequencies of vibration that are crucial for performing the flutter analysis, especially in the frequency-domain approaches.

Aeroelastic Solutions

In order to obtain the aeroelastic equilibrium equations it is necessary to couple structural and aerodynamic models in a unified system of equations. The standard procedure is to resort to energy methods and the concept of generalized displacements in order to realize the aeroelastic system of equations:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{f_A(t)\} \qquad (1)$$

Techniques such as the Finite Element Method (FEM) or the Rayleigh-Ritz method are commonly used for space discretization and for obtaining the structural matrices. The above equation is representative of a time-domain formulation. Its method of solution consists in setting an initial displacement field and then using time-marching techniques, for instance the Runge-Kutta scheme [10] or the Newmark algorithm [11], for calculating the displacement field given by q. After each time step, the computed aerodynamic force is transferred to all the structural nodes and the structure is allowed to deform according to the displacements q. The structural matrices are updated for each deformed state and the process is repeated for a prescribed number of time steps or, in the case of flutter analyses [7], until the unstable oscillatory behavior becomes evident in the solution. This type of analysis is referenced as a Fluid-Structure-Interaction analysis (FSI) and is compatible with any kind of aeroelastic problem.

Historically, the flutter analyses have been formulated in the frequency domain by assuming simple harmonic motion of the wing. In conjunction with linear theories, this approach leads to a system of homogeneous equations that can be solved by obtaining the eigenvalues of the system and then the stability of the system is inferred from them:

$$\left[\left(\frac{V_\infty}{c}\right)^2 [M]p^2 + \left(\frac{V_\infty}{c}\right)\left[[C] - [AIC_I(p,M)]\right]p + [K] - [AIC_R(p,M)]\right]\{q\} = 0 \qquad (2)$$

This equation consists in an eigenvalue problem in p and defines the system's stability equation or simply the "flutter equation". Here, $p = k(\gamma + i)$ is the dimensionless complex frequency with k being the reduced frequency and γ the damping coefficient of the oscillatory movement. The aeroelastic system becomes unstable if for a certain flight speed V_∞, the real part of one of the several eigenvalues p becomes positive, i.e., if $\gamma > 0$. However, the flutter equation can seldom be solved as a standard eigenvalue problem because the aerodynamic force is usually a non-linear function of the complex frequency p.

There are three main iterative methods for solving the flutter equation, namely the p-method [12], the k-method [13] [12] and the p-k-method [14]. The p-k-method is the most popular and it is considered superior in efficiency and accuracy.

State-of-the-Art

The modern aeroelastic field of research is focused on studying the effects of both structural and aerodynamic nonlinearities on the structure's dynamic response.

The effect of structural nonlinearities are brought into flutter studies by performing preliminary nonlinear steady state analysis followed by a linear dynamic stability analysis. Unlike the classical flutter analysis, the structural matrices featured in the aeroelastic equations carry the inertial and dynamical proprieties of the deformed wing configuration, leading to different natural and aeroelastic modes.

Patil *et al*. [15] [16] conducted this type of analysis on a High-Altitude-Long-Endurance (HALE) aircraft using nonlinear beam theory and potential aerodynamics for the structural and aerodynamic models, respectively. After comparing the results with linear studies, it became evident that the effect of structural nonlinearities was to decrease the flutter speed.

Xie & Yang [17] conducted a similar study using plate theory for the structure and the DLM for the aerodynamics and arrived at the same conclusion about the effects of the structural nonlinearities on flutter.

Concerning the aerodynamic nonlinearities, a significant amount of flutter studies are focused on the transonic flight regime and rely mostly on time-domain formulations using the Euler or Navier-Stokes (RANS) equations for calculating the pressure forces.

Bendiksen [6] performed a time-domain flutter analysis of a highly swept wing in the high subsonic and transonic Mach number region. The aerodynamics were modeled by the Euler equations and the structure was modeled by the nonlinear Reissner-Mindlin plate theory. The growing Limit-Cycle-Oscillations (LCO) were observed in the transonic region only and were identified as a nonlinear type of flutter. It was concluded that the instable LCO was mainly due to the high nonlinear character of the transonic flight regime.

A complete high-fidelity, high-order study was conducted by Gao [7] for the validation of the AGARD 445 wing. The aerodynamic model consisted in the 3D, finite-volume flow discretization using the RANS equations and the wing was modeled by finite element plate elements. The solution was obtained by a time-marching procedure using the Newmark algorithm, spline methods for force transfer between the aerodynamic and structural domains and structural mesh deformation with each time step. The results covered mostly the subsonic and transonic Mach number regions and in general have shown good agreement with the experimental data available in the literature.

Finally, it is important to mention the Reduced Order Methods (ROMs). These represent a variety of existing and emerging methodologies that either simplify the governing equations or use data from expensive CAE simulations in order to construct a reduced, but compact version of the aeroelastic system. Examples of these methods include the Proper Orthogonal Decomposition method (POD) [18] and system identification processes (SI) such as the Aerodynamics is Aeroelasticity Minus Structure (AAEMS) [19].

PRESENT WORK

Motivation

The idea behind the present work was to develop and implement a numerical method capable of predicting the flutter speed of a 3D airplane wing. The purpose of this study is not to develop a state-of-the-art methodology but rather to implement a distinct aeroelastic tool dedicated to wing flutter. There are essentially five aspects that fully define the numerical method adopted:

1. Object of study
2. Flight regime
3. Theoretical model
4. Methodology
5. Implementation

Object of Study

The object of study is a clean, conventional, slender 3D wing, with low to moderate airfoil camber. The aim is to avoid complicated aerodynamic phenomena such as flow separation which are difficult to predict. Accordingly, the admissible angle of attack should be very small, preferably zero.

Flight Regime

The flight regime is restricted to the subsonic region in order to avoid shockwaves in the flow and the highly nonlinear character of the transonic Mach region.

Theoretical Model

The theoretical model encompasses the aerodynamic and structural models. The aerodynamic model chosen is the linearized potential aerodynamic theory for its compatibility with the geometrical and physical restrictions imposed. On the other hand, 1D beam-rod theory is selected for modeling the wing's structure.

Methodology

The methodology adopted for calculating the pressure field is the subsonic unsteady panel method formulated in the frequency domain, presented by L. Morino [20]. Panel methods are superior in relation to the rival DLM for they allow free surface discretization, include thickness effects and support geometries of greater complexity. The downsides of panel methods are mainly the requirement of wake modeling and discretization and, for this panel method in particular, the lack of exact or efficient numerical formulas for the integral quantities involved. This panel method differs from the conventional panel methods for its extended applicability to unsteady compressible flow and frequency-domain formulation. The method was successfully validated by Yates *et al*. [21] in a study where the flutter boundaries of four cantilever wings with different thickness ratios were obtained and compared with available experimental data. In addition, Cunningham [22] demonstrated the method's feasibility to complex geometries in a study where the unsteady lift and moment coefficients were obtained for an F-5 wing-tip model with and without a tip-mounted missile. The calculations were performed for several Mach numbers and two oscillation frequencies and the results have shown good agreement with previous experiments.

Implementation

The panel method was fully developed in a personal computer using MATLAB® as programming language. Although alternative, faster languages such as C/C++ could have been used, MATLAB® excels in matrix operations while providing plenty of built-in functions that greatly facilitate the realization of the majority of tasks involved.

The structural beam-rod was discretized with the finite element method, using the commercial software ANSYS® for generating the structural matrices. The final aeroelastic system of equations is written in terms of the structural degrees-of-freedom (DoFs) and contains the mass, stiffness and AIC matrices. The AIC matrix expresses the generalized aerodynamic force and is obtained indirectly from the panel method.

FREQUENCY-DOMAIN PANEL METHOD

The panel method is ruled by an integral equation relating the potential $\hat{\varphi}$[1] with the normal perturbation velocity $\widehat{Q}_n$ at any point of the wing's surface S:

$$2\pi\hat{\varphi}(x_0, y_0, z_0) = -\oint\!\!\oint \widehat{Q}_n \frac{e^{-s_0 r_0}}{r_0} dS_{S0} + \oint\!\!\oint \hat{\varphi} \frac{\partial}{\partial n_{01}}\left(\frac{e^{-s_0 r_0}}{r_0}\right) dS_{S0} \quad (3)$$

$$r_0 = \sqrt{(x_0 - x_{01})^2 + (y_0 - y_{01})^2 + (z_0 - z_{01})^2} \quad (4)$$

$$\left(\hat{\varphi}, \widehat{Q}_n, \hat{q}_A\right) = (\tilde{\varphi}, \widetilde{Q}_n, \tilde{q}_A) e^{-s_0 M x_0} \quad (5)$$

The integrands $\frac{e^{-s_0 r_0}}{r_0}$ and $\frac{\partial}{\partial n_{01}}\left(\frac{e^{-s_0 r_0}}{r_0}\right)$ represent the two punctual source and doublet singularities, respectively. The source integral only covers the wing surface since they would be zero on the wake. The aerodynamic DoFs appear in equation (3) through the normal velocity by applying the zero-flux boundary condition on a harmonically oscillating surface:

$$\widehat{Q}_n = -\frac{\varepsilon}{|\nabla S_S|}\left(\frac{s}{U_\infty} S_U + \frac{\partial S_U}{\partial x}\right) e^{-s_0 M x_0} = -\frac{\beta}{M_\infty} s_0 \mathbf{n} \cdot \hat{\mathbf{d}} - \mathbf{n} \cdot \hat{\mathbf{d}}' \quad (6)$$

$$\hat{\mathbf{d}} = e^{-s_0 M_\infty x_0} \begin{Bmatrix} d_x \\ d_y \\ d_z \end{Bmatrix} \quad \hat{\mathbf{d}}' = e^{-s_0 M_\infty x_0} \begin{Bmatrix} \frac{\partial d_x}{\partial x} \\ \frac{\partial d_y}{\partial x} \\ \frac{\partial d_z}{\partial x} \end{Bmatrix} \quad (7)$$

The vectors (d_x, d_y, d_z) and $\left(\frac{\partial d_x}{\partial x}, \frac{\partial d_y}{\partial x}, \frac{\partial d_z}{\partial x}\right)$ are the aerodynamic translational and rotational DoFs, respectively. The unsteady pressure can be written in terms of the potential $\hat{\varphi}$ by:

$$\tilde{p} = -\frac{1}{2}\rho_\infty U_\infty^2 * \left(\frac{2ik}{c}\left(1 + \left(\frac{M_\infty}{\beta}\right)^2\right)\hat{\varphi} e^{i\frac{M_\infty^2 k}{c\beta} x_0}\right) - \frac{1}{2}\rho_\infty U_\infty^2 * \frac{2}{\beta}\frac{\partial\hat{\varphi}}{\partial x_0} e^{i\frac{M_\infty^2 k}{c\beta} x_0} \quad (8)$$

Discretization Procedure

Equation (3) is discretized by dividing the wing and wake surfaces in several flat panels (with quadrilateral or triangular shapes) and assigning a control point to each panel. The wake surface consists in a flat surface aligned with the incoming flow. The control points are collocated at the centroid of each panel and represent the points where the potential $\hat{\varphi}$ and pressure $\tilde{p}$ are calculated. These quantities are assumed constant inside the respective panel. Accordingly, the integrals are evaluated panel-wise, each one representing the influence of a distribution of

[1] $\hat{\varphi}$ and $\widehat{Q}_n$ are the transformed form of the actual variables $\tilde{\varphi}$ and $\widetilde{Q}_n$ respectively

either source or doublet singularities over a panel, in the control point (x_0, y_0, z_0). Writing the resulting discretized equation for all the control points originates a linear system of equations relating the potential $\hat{\varphi}_i$ with the aerodynamic DoFs $\hat{q}_k$:

$$2\pi\hat{\varphi}_i = \sum_{j=1}^{N_B} B_{ij} D_{ik} \hat{q}_k + \sum_{j=1}^{N_B+N_W} C_{ij} \hat{\varphi}_j \qquad (9)$$

The matrices B_{ij} and C_{ij} contain the integral influence coefficients of the distribution of sources and doublets respectively, over panel "j", on the control point "i". The matrix D_{ik} relates the normal velocity $\hat{Q}_n$ with the aerodynamic DoFs $\hat{q}_k$ by means of equation (6).

Wake Treatment

Since there are $j = 1, \ldots, N_B + N_W$ panels, but only $i = 1, \ldots, N_B$ control points, the system of equations is indeterminate. However, by applying the condition of zero normal force on the wake N_W panels, the influence of wake panels can be fully transferred to the trailing edge panels, thus eliminating the redundancy:

$$\Delta\tilde{p}_{wake} = 0 \qquad (10)$$

$$e^{\frac{s_0 x_{0_{wake}}}{M_\infty}} \Delta\hat{\varphi}_{wake} = e^{\frac{s_0 x_{0_{TE}}}{M_\infty}} \Delta\hat{\varphi}_{TE} \qquad (11)$$

Here, the condition was extended to the trailing edge panels in order to approximately satisfy the physical Kutta condition.

Aerodynamic Force Vector

Using a finite difference scheme, it is possible to write a relation between the pressure $\tilde{p}$ and the potential $\hat{\varphi}$ by means of a coefficient matrix:

$$\{\tilde{p}\} = -\frac{1}{2}\rho_\infty U_\infty^2 * [P]\{\hat{\varphi}\} \qquad (12)$$

Then, by manipulating the terms in equation (9), a relation is obtained between the pressure vector and the aerodynamic DoFs:

$$\{\tilde{p}\} = -\frac{1}{2}\rho_\infty U_\infty^2 * [A]\{\tilde{q}_A\} \qquad (13)$$

In which a transformation was used to convert the modified DoFs $\{\hat{q}_A\}$ into the physical DoFs $\{\tilde{q}_A\}$, using equation (4).

Finally, the components of the aerodynamic force vector in each control point are obtained by multiplying equation (13) by the panel's area matrix $[S]$ and a rotation matrix $[R]$:

$$\{\tilde{F}\} = \frac{1}{2}\rho_\infty U_\infty^2 * [R][S][A]\{\tilde{q}_A\} \qquad (14)$$

The minus sign vanished as a consequence of the convention used for the direction of the pressure force.

Structural Model

The wing's structure consists in a 1D elastic beam-rod and it is discretized with finite elements using the software ANSYS®. The element used is the BEAM188 that uses a 1st order shear force theory and supports both bending and torsion, with a total of 6 DoFs per node. The structural nodes are evenly spaced in the mid plane of the 3D wing. There are always as many structural nodes as there are panels in the spanwise direction. The beam material proprieties are assumed to be uniform. After ANSYS® extracts the structural matrices and the information about the first set of natural frequencies and modes, the data is transferred to the main program in MATLAB® in order to assemble the aeroelastic model.

AEROELASTIC MODEL

The aeroelastic model is constructed from the coupling of the structural forces with the aerodynamic force and essentially consists in the flutter equation, written in terms of the structural DoFs.

Rigid DoFs Transformation

Since the aerodynamic nodes are located in the region surrounding each structural node, it was required to find a suitable way to relate the aerodynamic DoFs to the structural ones. This was accomplished by a small rigid rotation centered at each structural node. It is assumed that every control point on the wing surface is rigidly connected to the structural nodes by an infinite stiff invisible beam (Figure 1).

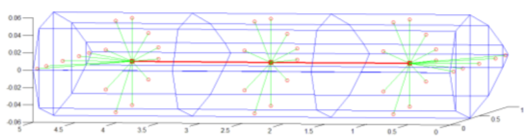

Figure 1: Rigid connections (green lines) between each structural node (red squares) and the aerodynamic ones (red circles). These are virtual beams and are not included in the structural analysis

However, these rigid beams only act on the aerodynamic nodes directly above or below, belonging to the same ring of panels.

$$\{\tilde{q}_A\} = [T]\{\tilde{q}_S\} \qquad (15)$$

The matrix $[T]$ contains the transformation coefficients between the two types of DoFs. The aerodynamic rotations (u_A', v_A', w_A') are solved approximately by finite difference schemes involving the displacements (u_A, v_A, w_A) of different panels.

Equivalent Force System

In order to obtain the AIC matrix, the aerodynamic force vector from the panel method must be transferred to the structural nodes. This was accomplished by adding the components of the pressure forces on each panel surrounding the structural node. In addition, it was required to calculate the moment components of each pressure force at the structural nodes. The procedure followed D'Alembert's Principle for equivalent force systems (Figure 2).

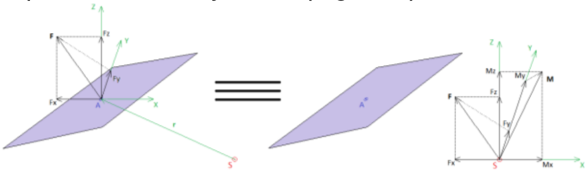

Figure 2: schematics of the pressure force acting on a panel and its equivalent force system on the nearby structural node

AIC Matrix

The AIC matrix is constructed after the aerodynamic forces and moments are written in terms of the structural DoFs and after being transferred to the structural nodes.

$$\{\tilde{F}\} = [AIC]\{\tilde{q}\} \qquad (16)$$

The new vector $\{\tilde{F}\}$ is the generalized force vector and contains the 3 Cartesian components of force and moment, organized per structural node. The columns of $[AIC]$ represent the influence of a certain structural DoF on the generalized aerodynamic force. The AIC matrix is a complex matrix and is a function of the reduced frequency k and the free-stream Mach number M_∞.

The Modified p-k Method

Once the mass, stiffness and AIC matrices are calculated, the aeroelastic stability equation given by equation (2) is fully defined. This equation constitutes a non-linear eigenvalue problem in the reduced frequency k, and a quadratic eigenvalue problem in the complex frequency p, with:

$$p = k(\gamma + i) \qquad (17)$$

The eigenvalues and eigenvectors are extracted using the modified p-k method. The standard p-k-method solves the flutter equation by calculating the AIC matrix for a certain reduced frequency and then by solving the eigenvalue problem for the complex frequency p. If any of the imaginary parts of the extracted eigenvalues is close enough to the reduced frequency, the corresponding complex frequency is stored.

$$\frac{k_{active} - Im\{p\}}{k_{active}} \begin{cases} > \varepsilon \Rightarrow k_{active} = Im\{p\} \\ \leq \varepsilon \Rightarrow p_{converged} = p \end{cases} \qquad (18)$$

Otherwise, the aerodynamic and eigenvalue calculations are repeated for the closest reduced frequency until convergence is achieved. The "Modified p-k Method" does not rely exclusively on the relative error between the new and the previous reduced frequencies for eigenvalue convergence. Instead of directly comparing the values of reduced frequencies, the method selects the eigenvalue that best suits the physical nature of the eigenvalue being converged. Doing so requires accessing the eigenvectors extracted from the flutter equation. This process involves three steps:

1. Mode amplitude inspection: the mode amplitudes of the active DoF are stored
2. Mode shape detection: the mode shapes of the active DoF are characterized by the modes' order numbers. These numbers are stored
3. Mode candidate selection: the modes whose orders' numbers match those of the active mode are stored, and the one whose amplitude is the greatest is selected as best candidate

After these three steps are performed, the eigenvalue corresponding to the selected mode is compared to the one being converged, in agreement with the standard p-k method iterative criterion (see eq. (18)).

EFFICIENT AIC MATRIX CALCULATION

The AIC matrix is the most demanding variable in terms of computational resources usage. A single calculation corresponding to one flight speed and one frequency may take from a few minutes to several hours, depending on the number of nodes constituting the aerodynamic mesh. The number of aerodynamic calculations involved in the full p-k method's cycle can make the aeroelastic program unfeasible, even for coarse meshes, considering the total CPU time required. In order to reduce the computation time, interpolation functions were used in the calculation of the AIC matrix. In the parametric study performed, it was shown that the AIC matrix varies smoothly with both reduced frequency k and free-stream Mach number M_∞, providing excellent conditions for the use of interpolation functions.

k-Interpolation

For a constant Mach number, interpolation over the reduced frequency is feasible and highly advisable. In general, the variations are slow and the region of reduced frequencies of interest is narrow, which allows higher quality interpolation functions at the cost of fewer interpolation points.

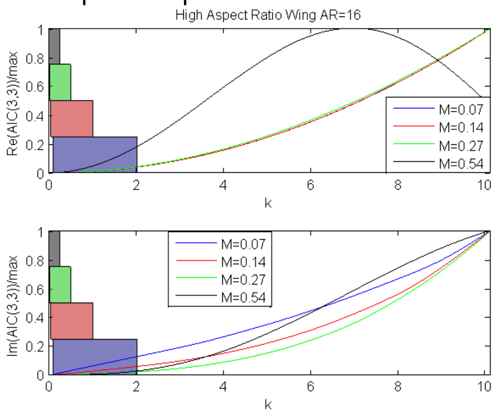

Figure 3: typical curve behavior of an entry (3,3) of the complex matrix AIC versus the reduced frequency for several Mach numbers. The horizontal bars represent the reduced frequency bands corresponding to the first four wing natural modes. Each color corresponds to a different Mach number

The frequency interpolation is extremely advantageous when the system is being solved for several frequencies that are packed together in a narrow band – a characteristic of a high aspect ratio wing.

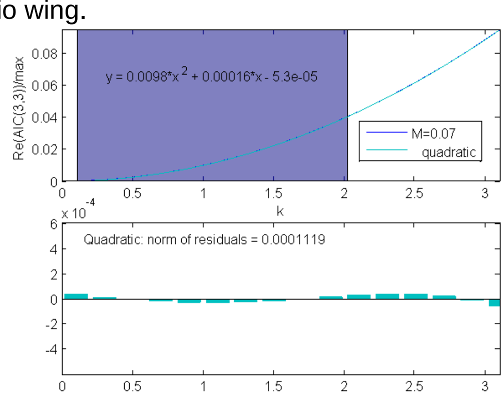

Figure 4: quadratic curve fitting for the real part of AIC(3,3). The norm of the residuals is exposed below

Resorting to curve-fitting techniques (Figure 4), it became clear that 2nd degree polynomials were excellent approximants to both real and imaginary parts of most entries of the AIC matrix.

Implementation

In the code, the interpolation procedure involves storing the AIC matrix for three equally spaced reduced frequencies, followed by the storage of the coefficients of a 2nd degree polynomial that intersects those three points.

For the cases where abrupt variations are detected, a two-parameter rectangular hyperbola is used instead. In such cases, the hyperbolas are fit to the 3 points by solving a nonlinear least squares problem.

The success of this type of interpolation allows the enforcement of small errors in the p-k method, saving substantial amounts of CPU time, at a small cost of accuracy in the solution.

M-Interpolation

After performing some frequency interpolations at different speeds, it becomes possible to resort to the second type of interpolation – the velocity interpolation. This interpolation attempts to find the aerodynamic coefficients for a certain reduced frequency as a function of the Mach number. Although attractive in terms of CPU time saving, the velocity interpolations are considerably dangerous because they rely on the accuracy of several frequency interpolations. Thus, if high accuracy is intended, such procedure should be avoided.

CODE VALIDATION

The validation of the code was performed by simulating the linear flutter analysis of a high aspect ratio wing conducted by Patil *et al.* [15].

Case Study

Table 1: wing model input data required for performing the aeroelastic calculations. The values refer to the structural, geometrical and flight conditions of the linear flutter analysis present in [15]

Geometry	
Half-span	16 m
Chord	1 m
Given Material Proprieties	
Mass per unit length	0.75 kg/m
Moment of Inertia (50% chord)	0.1 kg m
Spanwise elastic axis	50% chord
Center of Gravity	50% chord
Bending Rigidity	2E4 N m^2
Torsional Rigidity	1E4 N m^2
Bending Rigidity (chordwise)	4E6 N m^2
Calculated Structural Proprieties	
Equivalent beam width	1.262 m
Equivalent beam height	8.922E-2 m
Elastic Modulus	267.823 MPa

Torsional Constant	2.854E-4 m^4
Shear Modulus	35.039 MPa
Flight Conditions	
Altitude	20 km
Air Density	0.0889 kg/m^3
Speed of Sound	295.1 m/s

Preliminary Modal Analysis

Patil *et al*. used a similar 1D beam-rod model to represent the wing's structure. In order to validate the structural model, a modal analysis was performed using ANSYS® and the first five natural frequencies were extracted and compared with the results obtained in [15]:

Table 2: wing's first five natural frequencies calculated using ANSYS® and comparison with the same ones obtained in [15]

	Present Analysis	**Patil *et al* [15]**	**Relative Error (%)**
1st spanwise bending	2.243	2.247	-0.18
2nd spanwise bending	14.064	14.606	-3.71
1st chordwise bending	31.341	31.739	-1.25
1st torsion	31.523	31.146	1.21
3rd spanwise bending	39.445	44.012	-10.38

Despite the 3rd spanwise bending, the frequencies are very similar with relative errors below 5%. Also, each frequency corresponds to the same mode on either analysis. An exception occurs in the 3rd and 4th frequencies that seem to be swapped in position. However, since they are numerically very close, such may not be an issue concerning the following aeroelastic analysis.

Aeroelastic Analysis

This case study was simulated in MATLAB® with the mesh parameters based on the conclusions of the convergence study. The complete list of MATLAB® input parameters is shown in the following table:

Table 3: input computational parameters used in MATLAB® concerning the mesh, number of frequencies and speeds

Aeroelastic Analysis	
Angle of Attack	0
Airfoil	NACA 0012
Critical Mach[2]	0.818
"nseg_X"	12
"nseg_Y"	128
"nseg_wake"	36
"n_panels"	3132
k-interpolation	Yes
M-interpolation	No
"n_freqs"	5
Mach step (Speed step)	0.0339 10 m/s
Mach range (Speed range)	[0.0339; 0.9149] (10; 270) m/s
Mach Loops	27
CPU time (equivalent time)	311631 seconds (3d 14h 33min 51s)

[2] Critical Mach estimated using NACA 0012 in JavaFoil [24]

Results

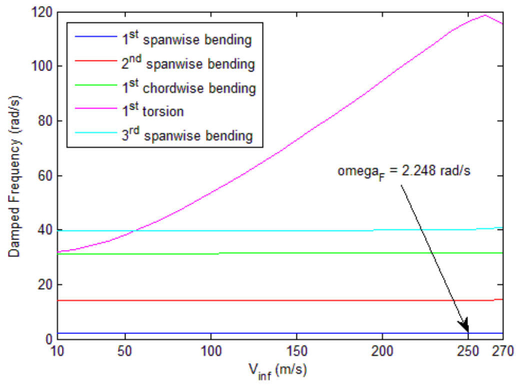

Figure 5: imaginary part of the dimensional part of "p" corresponding to the aeroelastic damped frequencies versus the flight speed

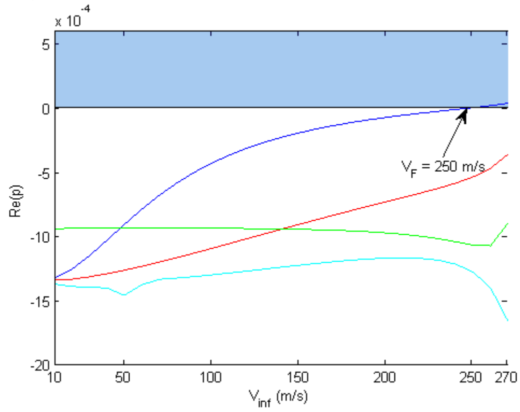

Figure 6: real part of "p" corresponding to the aeroelastic damping versus the flight speed

Table 4: aeroelastic results for both studies and comparison

	Present Study	**Patil *et al* [15]**	**Relative Error (%)**
Flutter speed (Flutter Mach)	250 m/s (0.8472)	32.21 m/s	676.16
Flutter Frequency	2.243 rad/s	22.61 rad/s	-90.08

Comments

Both flutter speed and frequency are an order of magnitude above the one obtained for the exact same wing by Patil *et al*. In addition, the calculated flutter speed is clearly in the transonic regime which is beyond the linear scope of the theory in use. Due to the high discrepancies between both results, it was not possible to successfully validate the developed tool for flutter calculation. The numerical error might be associated with the following:

- Lack of accuracy in the numerical integrations corresponding to the influence coefficients
- Lack of accuracy in the finite differences' schemes for the unsteady pressure formulas and for the aerodynamic rotational DoFs
- Oversimplified DoFs transformation method from one mesh to another
- General implementation errors, either due to the lack of information available or poor interpretation

Spanwise Lift Plots

Despite the unacceptable numerical error, there is a possibility of validating the program from a qualitative point of view. In another publication, Patil & Hodges [16] provided the unsteady pressure coefficient curve along the span for the same high aspect ratio wing.

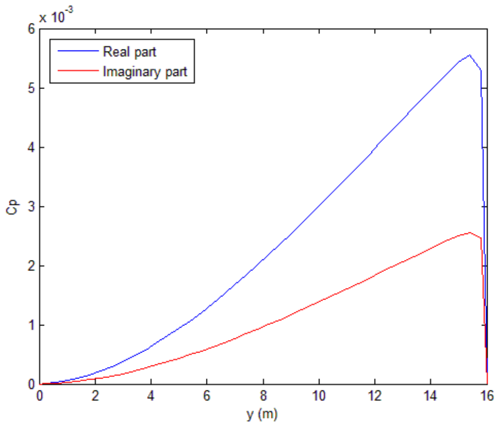

Figure 7: spanwise unsteady lift coefficient corresponding to the 1st spanwise bending and k=0,4 obtained by the developed program

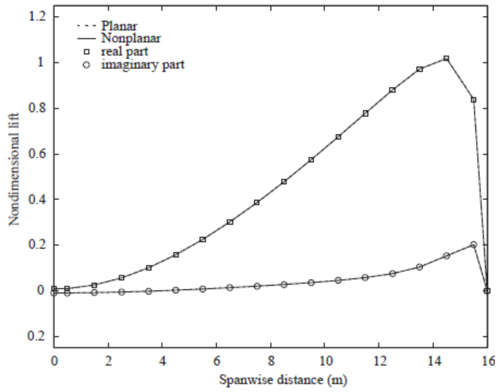

Figure 8: spanwise unsteady lift coefficient corresponding to the 1st spanwise bending and k=0,4 extracted from reference [16]

It is important to notice that despite the difference by several orders of magnitude, the real and imaginary parts behave similarly in shape in both analysis (Figure 7 and Figure 8). The pressure increases slowly from zero at the wing root and drops quickly after the peak just before the wing tip, as would be expected from a 3D aerodynamic theory. This similarity does support the qualitative value of the present work.

CONCLUSION

A computational tool was developed with the purpose of determining the aeroelastic flutter speed of a 3D aircraft wing. The aeroelastic model was created by coupling a 1D beam-rod structural model with a 3D compressible, frequency-domain panel method aerodynamic model. The equations of motion were discretized by a Finite Element Method approach, centered at the structural nodes. The stability of the aeroelastic system were obtained by eigenvalue extraction, using the modified p-k method. Unlike the original p-k method, this method prevents tracking the incorrect mode during the frequency matching process, by inspecting the several eigenvectors extracted.

In the attempt of validating the program, the linear flutter calculation of a high aspect ratio wing performed by Patil *et al* in [15] was simulated, and the frequency and flutter speed were calculated for comparison. However, even though flutter was evident after inspecting the results, the corresponding values of frequency and speed were far above the ones obtained in [15], with relative deviations around 700% and 90% respectively. In the preliminary modal analysis performed in ANSYS®, the values and types of natural frequencies obtained were similar to the ones provided in [15], suggesting that the error was originated in the aerodynamic calculations.

By comparing the non-dimensional spanwise lift plot (Figure 7) with the one provided in [16] (Figure 8) corresponding to the same wing, it was possible to notice strong similarities between the curves, indicating that the obtained results may have some qualitative significance. However, the numerical values still revealed great deviation from the reference.

From a practical point of view, the developed tool did not bring any advantage over the common methods in use today for linear flutter calculation, namely the DLM or even the classical Strip Theory.

From the academic point of view, considering the distinct and broader domain of applicability of the present method over others within the framework of potential aerodynamics, there might be interest in revisiting the methodology implemented.

In a hypothetical future approach, the aspects concerning the theoretical formulation and the implementation procedures of the panel method should be reviewed in order to gain some additional insight about the nature of the errors and, if possible, to increase the overall efficiency of the code.

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