Safe routes for hazardous materials transportation

Distribution of Galp liquid fuels in Lisbon

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Abstract

Hazardous materials (hazmats) are essential for the competitiveness of contemporary societies, being its transportation potentially dangerous and expensive. In Portugal, studies enabling the identification of preferable road routes for safe and economic viable distribution of hazmats were not found. This work aims at establishing solid scientific bases that allow, in collaboration with national operators, to balance these two frequently conflicting intrinsic aspects of hazmats transportation. For that purpose, a bi-level linear programming model was applied, defining:

1. In the first level, the safe routes (preferable for the regulator), and
2. In the second level, the most economic routes (preferable for the operator).

Galp Energia provided data concerning the deliveries of white oils (petrol and diesel fuels) to petrol stations and direct clients in Lisbon.

Model implementation used one road network available in a geographical information system (GIS) and the distribution of white oils in Olivais parish was analysed. The model was solved to optimality in a short computation time (2 seconds) for this case study, while the road network has significantly larger size (682 links and 461 nodes) than those found in similar studies in the international literature. The identified routes have been mapped.

The generic formulation of the bi-level linear programming model and the success of the methodology used offer an optimistic prospect for future developments for this study, such as its geographical expansion, indeed already being carried out.

Key words: hazardous materials transportation, bi-level linear programming, road safety in urban areas, geographical information systems (GIS), Galp Energia.

1. Introduction

The economic success of societies requires the transport of considerable amounts of hazardous materials (hazmats). Indeed, in Portugal, 10% of the total materials transported by road are hazmats (ANPC, 2012). Although the transportation of hazmats is associated to a relatively few accidents, their consequences can be severe due to the nature of the cargo (PHMSA, 2011). The transportation of hazmats is, therefore, subject to safety requirements to both their carriers and to the materials themselves. It has been recognized as important to find a balance between the safe requirements for transporting these materials, in order to protect the populations and the environment, and the economic viability of the operation. A common political tool to reduce the risk of hazmat transportation is the interdiction to this transport of certain road sections identified as more vulnerable by the
The carrier is free to choose the routes and manage their risk in the available network. The hazmats industry generally place safety at the centre of their business and the analysis of safe route definitely requires further studies beyond those that have been identified in the literature.

Kara and Verter (2004) developed a bi-level programming model based on a measure that is becoming popular, consisting in the interdiction of hazmat transportation in the links considered to impose a major risk by the regulator, while the carriers choose the shortest path within the available links. In this model, hazmats are grouped into categories according to the risk impact (population exposure) that characterizes them, and a network is attributed to each group. The model is characterized by the decision variables that represent if the link is available or unavailable for the transport of hazmats, by the regulator, and if it is used or not by the carriers. This model was applied to the region of Western Ontario, Canada, and solved as a linear programming model.

Later on, the same authors proposed another linear programming model, Verter and Kara (2008), which was solved through the shortest path formulation. In this formulation, the paths that were considered as economically non feasible were left out of the model, assuring that the carriers wouldn’t be forced to use paths that were their least preferable choices.

Other authors have approached this issue from different forms, such as heuristics (instead of linear programming models). Erkut and Alp (2007) developed an algorithm in two phases: in the first stage a minimum risk network is found, and, in the second, the network is expanded in an iterative procedure, through the addition of paths. This enables the regulator to control the density of the hazmats network and the freedom given to the carriers. Erkut and Gzara (2008) propose a different formulation, in which the authors use a flow problem formulation, in a bi-level network, and compare it to four networks scenarios related to different decision levels: non-regulated model, over-regulated model, two-step model and bi-level model.

2. Optimization model for the routing of hazardous materials transportation

In order to achieve the objectives of this work, namely to identify safe routes for the transportation of hazmats without compromising economic viability, and according to current scientific knowledge, this work will develop a linear programming model based on the model described by Kara and Verter (2004).

2.1 Problem definition

The model presented in Kara and Verter (2004) is a bi-level linear programming model. A bi-level model consists in two optimization problems that are hierarchically related and belong to two distinct decision makers, in which the optimal decision of one of them is constrained by the choices of the other decision maker (Bianco et al., 2009). The regulator assumes the leading role, as its decisions are taken at the first level and the carriers’ path choices will depend of them. Thus, the external problem belongs to the decision maker and allows the determination of which links should be included in the network, according to criteria of total risk minimization resulting from the carriers’ path choices, while the inner problem belongs to the carriers and incorporates the decisions concerning the paths available in the network. Kara and Verter (2004) presents a model that determines the network of the minimum total
risk and assumes the cost minimization of the carriers, achieving significant risk reductions in the transport of hazmats in Western Ontario, Canada.

One of the strategies to solve bi-level linear programming problems consists in the application of the Karush-Kuhn-Tucker conditions (KKT), which were used by Kara and Verter (2004). The application of these conditions transforms the bi-level linear programming model into a one level model, substituting the initial formulation and solving the problem through a commercial solver (Bianco et al., 2009).

The model was solved in a network constituted by a set of points, designated nodes, and lines, designated links, which connect pairs of nodes. The nodes correspond to the intersection of lanes, while the links correspond to segments of road belonging to the network. A path between two nodes is the sequence of distinct links that connect those nodes (Hillier and Lieberman, 2005).

The Kara and Verter (2004) model assumes that the undesired consequences of an eventual incident involving hazmats occur within a determined distance from the place where the incident occurred, varying with the type of hazmat. This means that when a truck crosses a link, only the certain people within that distance are exposed to the materials the truck is carrying. Thus, hazmats are grouped in categories according to the impact of an eventual accident associated to each of them.

In the risk associated to hazmats transport it is assumed an additivity of impacts. It is considered that the risk is known for each link and independent of the direction of each shipment, and that each point of the same link has the same incident probability and level of consequences. The sum of risk of the transport activity in each link results in the linearity of the objective function (Erkut et al., 2007). In this model, the risk is measured by population exposure (by the regulator) and the traveled distance is the criteria for the choice of paths of the carriers, but the methodology can be easily used with other risk and cost measures.

2.2 Mathematical Formulation of the model

As the model is a bi-level model that was transformed in order to be solved, the original mathematical formulation and the formulation that results from the application of KKT conditions are both presented.

Based on the problem description, the following sets, parameters and variables are defined:

**Indices**

- $c$ – shipment
- $p$ – population centre
- $i,j,k$ – node
- $m$ – type of hazmat carried

**Sets**

- $C$ – all shipments across the network, $c \in C$ – Each shipment is characterized by an origin node, a destiny node and a type of hazmat transported
- $P$ – population centre, $p \in P$ – Set of population centres affected by the activity of transport of hazmats
- $N$ – nodes, $i,j \in N$
- $A$ – links, $(i,j) \in A$, where $(i,j)$ designates the link that connects the nodes $i$ and $j$, in the direction $i \rightarrow j$
- $M$ – hazmat types, $m \in M$
Parameters

- \( \rho_{ij}^{p,m} \) – number of people in \( p \) exposed to a truck carrying hazmat \( m \) through link \((i, j)\)
- \( l_{ij} \) – length of link \((i, j)\)
- \( n^c \) – number of trucks used for shipment \( c \)
- \( R \) – an arbitrarily high positive real number

Auxiliary variables

These variables appear with the problem’s transformation with KKT conditions and don’t correspond to decisions, wherefore they are named auxiliary variables.

\( \nu_{ij} \) and \( \lambda_{ij} \) are positive real variables, while \( \omega_{ij} \) is a real variable (positive or negative).

Decision Variables

The model decision variables are binary:

\( Y_{ij}^m = 1 \) if link \((i, j)\) is available for transportation of hazmat type \( m \), \( Y_{ij}^m = 0 \) otherwise.

\( X_{ij}^c = 1 \) if link \((i, j)\) is used for shipment \( c \), \( X_{ij}^c = 0 \) otherwise.

Original bi-level programming model

Using the above definitions, the model is formulated as follows:

Objective function

\[
\min \sum_{p \in P} \sum_{(i, j) \in A} \sum_{l \in C} n^c \rho_{ij}^{p,m(c)} X_{ij}^c
\]

Subject to:

\( Y_{ij}^m \in \{0, 1\} \forall i, j \in A, m \in M \) [2]

Where \( X_{ij}^c \) solves:

\[
\min \sum_{c \in C} \sum_{(i, j) \in A} n^c l_{ij} X_{ij}^c
\]

Subject to:

\[
\sum_{(l, j) \in A} X_{l,j}^c - \sum_{(i, k) \in A} X_{k,i}^c = \begin{cases} +1 & i = o(c) \\ -1 & d(c) \\ 0 & \text{otherwise} \end{cases} \forall i \in N, c \in C
\]

Where:

- \( o(c) \) – origin node of shipment \( c \)
- \( d(c) \) – destiny node of shipment \( c \)

\( X_{ij}^c \leq Y_{ij}^m \) \( \forall (i, j) \in A, c \in C \) [5]

\( m(c) \) – hazmat transported in shipment \( c \)

\( X_{ij}^c \in \{0, 1\} \forall (i, j) \in A, c \in C \) [6]

The external problem (with objective function [1]) refers to the decisions regarding which links should be made available for hazmats transportation, while the interior problem, represented by objective function [3], represents the decisions regarding the actual transportation of hazmats on the available links.
function [3] and constraints [4] - [6], deals with the path choices in the available network. External problem’s binary decision variables (Y^{m}_{ij}) constitute parameters for the interior problem, wherefore given the values of Y^{m}_{ij} the interior problem consists in the determination of the minimum cost flow in the network, minimizing the total distance covered by the trucks (objective function [3]).

Expression [2] assures that binary decision variable Y^{m}_{ij} can only take two values: 1 if the link (i,j) is available to the transport of hazmat m and 0 otherwise.

The requirements of flow balance are verified through equation [4]. This equation assures, in the case of the intermediate nodes, that if one node is neither of origin nor destiny, the hazmat has to follow to another node; in the case of origin nodes, it assures that only one link of the path goes out from this node, while in the case of destiny nodes the equation assures that only one link enters this node.

Constraint [5] assures that only the links made available by the regulator can be used by the carriers. Thus, it indicates that only the links that were previously determined as available links for hazmats transport by the regulator (Y^{m}_{ij} = 1) can be used in the shipment (X^{c}_{ij} = 1).

Constraint [6] indicates that decision variable X^{c}_{ij} is binary, which means that the link is either used for a certain shipment or not used.

Transformed Model with KKT conditions

Objective function

\[
\min \sum_{p \in P} \sum_{(i,j) \in A} \sum_{c \in C} n^{c} p_{ij}^{m(c)} x^{c}_{ij} \tag{1}
\]

Subject to:

\[
\sum_{(i,j) \in A} x^{c}_{ij} - \sum_{(k,j) \in A} x^{c}_{kj} = \begin{cases} +1 & i = o(c) \\ -1 & i = d(c) \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in N, c \in C \tag{2}
\]

\[
x^{c}_{ij} \leq y^{m(c)}_{ij} \quad \forall (i,j) \in A, c \in C \tag{3}
\]

\[
n^{c} l_{ij} - \omega^{c}_{i} + \omega^{c}_{j} - v^{c}_{ij} + \lambda^{c}_{ij} = 0 \quad \forall c \in C, (i,j) \in A \tag{4}
\]

\[
v^{c}_{ij} \leq R(1 - x^{c}_{ij}) \quad \forall c \in C, (i,j) \in A \tag{5}
\]

\[
\lambda^{c}_{ij} \leq R[1 - \left(y^{m(c)}_{ij} - x^{c}_{ij}\right)] \quad \forall c \in C, (i,j) \in A \tag{6}
\]

\[
v^{c}_{ij} \geq 0, \lambda^{c}_{ij} \geq 0 \quad \forall c \in C, (i,j) \in A \tag{7}
\]

\[
\omega^{c}_{i} \in \mathbb{R} \quad \forall c \in C, i \in N \tag{8}
\]

\[
x^{c}_{ij} \in \{0,1\} \quad \forall (i,j) \in A, c \in C \tag{9}
\]

\[
Y^{m}_{ij} \in \{0,1\} \quad \forall (i,j) \in A, m \in M \tag{10}
\]

The model comprising expressions [1] to [10] is a mixed integer linear programming model (MILP), with binary and continuous variables, and consists in the problem of identifying paths for hazmat transportation that minimize the risk of accident without compromising economic viability.

2.3 A small-scale example of model application

A small-scale example of model application was adapted from Verter and Kara (2008). Figure 1 represents the considered network, where the links are represented by lines with the corresponding travel time in minutes, the nodes are represented by numbered circles and the population centres Pop1...
and Pop2 have been added to the original example in order to consider population exposure in the model. Figure 2 shows all the links that comprise the network, with the respective name and direction.

Figure 1 – Illustrative example simplified network

Figure 2 – Illustrative example network with oriented links

Thereafter, Table 1 defines the population of each of the two population centers that are in the influence area of each link of the network. Those values were arbitrarily defined according to the distance that the population centers are of each of the links.

<table>
<thead>
<tr>
<th>Links</th>
<th>Pop 1</th>
<th>Pop 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>arco12</td>
<td>100 000</td>
<td>5 000</td>
</tr>
<tr>
<td>arco21</td>
<td>100 000</td>
<td>5 000</td>
</tr>
<tr>
<td>arco13</td>
<td>70 000</td>
<td>20 000</td>
</tr>
<tr>
<td>arco31</td>
<td>70 000</td>
<td>20 000</td>
</tr>
<tr>
<td>arco41</td>
<td>30 000</td>
<td>35 000</td>
</tr>
<tr>
<td>arco23</td>
<td>70 000</td>
<td>30 000</td>
</tr>
<tr>
<td>arco32</td>
<td>70 000</td>
<td>30 000</td>
</tr>
<tr>
<td>arco25</td>
<td>50 000</td>
<td>35 000</td>
</tr>
<tr>
<td>arco52</td>
<td>50 000</td>
<td>35 000</td>
</tr>
<tr>
<td>arco35</td>
<td>40 000</td>
<td>40 000</td>
</tr>
<tr>
<td>arco53</td>
<td>40 000</td>
<td>40 000</td>
</tr>
<tr>
<td>arco45</td>
<td>10 000</td>
<td>45 000</td>
</tr>
<tr>
<td>arco54</td>
<td>10 000</td>
<td>45 000</td>
</tr>
</tbody>
</table>

The model is solved for a single shipment between node 1 (origin) and node 5 (destination), involving one truck. The model was implemented in GAMS modelling system and solved with CPLEX (version 12.4.0.0) in a computer with Intel® Core™ i3-2350M processor of 2nd generation. Table 2 presents the model characteristics and results: number of variables, binary variables and constraints, CPU time, optimality gap and value of the objective function.

<table>
<thead>
<tr>
<th>No. variables</th>
<th>No. binary variables</th>
<th>No. constraints</th>
<th>No. iterations</th>
<th>CPU Time (s)</th>
<th>Optimality gap (%)</th>
<th>Value of objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>28</td>
<td>62</td>
<td>13</td>
<td>0,047</td>
<td>0</td>
<td>120 000</td>
</tr>
</tbody>
</table>
Table 3 presents the links of the network that are available to the transport of hazmats \((Y_{ij})\) and the links used in its transportation \((X_{ij})\), which result from the model resolution. A schematic representation of the solution is depicted in Figure 3, where the links chosen by the model are shown (arco14 and arco45).

<table>
<thead>
<tr>
<th>Links</th>
<th>(Y_{ij})</th>
<th>(X_{ij})</th>
</tr>
</thead>
<tbody>
<tr>
<td>arco12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco21</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco13</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco31</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco14</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>arco41</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco23</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco32</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco52</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco35</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco53</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>arco45</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>arco54</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The value of the objective function corresponds to the population exposed by the transport (120 000 inhabitants). This value can be imparted through the following expression, through which the same value generated by the objective function is obtained:

\[
\text{Objective function} = Pop_{14} \times N_{trucks} + Pop_{45} \times N_{trucks} = (30 000 + 35 000) \times 1 + (10 000 + 45 000) \times 1 = 120 000
\]

Although the length of the path constituted by links arco14 and arco45 (18 min) is higher than the one of the links arco12 and arco25 (12 min), leading to higher travel costs, the path chosen puts less inhabitants in risk for the fact that it is farthest from the population centres and the population centre that is nearer it has half the population of the population centre that is near arco12.

3. Case study results

In order to apply the developed model to a Portuguese case study, it was considered useful to establish a collaboration with Galp Energia, Portugal’s leading fuel company and predominant in its distribution in the country. It was decided that the model would be applied to the distribution of white oils (petrol and diesel) in the petrol stations and direct clients of the company in Olivais parish, in Lisbon. In the network were included the main routes that, without belonging to Olivais, provide access to it.

3.1 Data processing

A network was made in the ArcGIS software, adapted from an existing one, in order to include the client’s location in the model. As the network was defined with a number of links superior to those who were predictable that the model could support, it became necessary to reduce its dimension. Thus, the study area was reduced to Olivais and simplifications were made through the junction of links and removal of some sections of minor relevance to this analysis. Originally the Lisbon network comprised 35 981 links; excluding the part of the network that didn’t belong to Lisbon nor was one of the main accesses to it, the network was reduced to 14 195 links; subtracting some path alternatives in the zones...
where don’t exist Galp fuel stations nor direct clients, like Monsanto area, the network stayed with 9759 links; through the reconfiguration of the crossings, a network with 7276 links was obtained; considering only Olivais parish and its main access routes (even those that don’t belong to the parish), the network was reduced to 682 links and it was the network that was used in the case study.

To characterize Galp’s distribution, data concerning the client’s location, the amount of fuel delivered to each client and the seasonality of the sales was collected for Lisbon. As the risk is proportional to the amount of fuel delivered, it was decided the model should consider the number of equivalent trucks. Equivalent trucks consist in the number of trucks that would be necessary to deliver the annual amount of fuel to each client and is obtained by the division of the total fuel distributed per year by the capacity of one truck (30 m\(^3\)).

In order to incorporate the risk in the network, census data (2011) was used to quantify the population living in each geographical information referencing basis (BGRI in Portuguese). Population density was obtained through the division of population by the area of each BGRI. Population exposure is attained through the following expression: \(\text{Pop exp.} = l_{\text{link}} \times (50 + 50) \times \text{Density}\), where \(\text{Pop exp.}\) corresponds to the population exposed, \(l_{\text{link}}\) to the length of each link, \(\text{Density}\) to population density and 50+50 to a buffer of 50m for each side of the link called evacuation distance.

Several strategies were used to deal with the complexity of integrating the data of Galp Energia with data with ArcGIS. Thereby, the following simplifications were assumed:

**Transported materials**

- It was assumed that all white oils belong to the same model category;
- Each truck containing the same type of hazmat imposes the same risk;

**Population density**

- The population density around a road segment is constant;
- In the links that cross two BGRI, it was considered that the population density of that link would correspond to the average of both values;
- Accident probability is constant in each link;

**Network**

- The shipments origin was considered to be the entrance of the A1 highway in Lisbon, because it is through this highway that the shipments enter in Lisbon;
- It was assumed that the trucks circulate at the maximum speed allowed for heavy trucks, reduced through a coefficient of degradation corresponding to 5% of that speed;
- It was assumed that the speed in the curves is the same of the links with straight alignment;
- An additivity of impacts was assumed between two or more connections around a population centre;

**Fuel stations**

- The destiny points (for fuel stations and direct clients) were represented through the projection of their real location in the nearest node of the network;
- Only the six shipments to fuel stations and direct clients of Galp Energia in Olivais parish were included;
Figure 4 presents the graphic representation of the network and the studied shipments. It should be noted that although several fuel stations and direct clients appear in Figure 4, only those from Olivais (those who are numbered) were considered in the model.

![Figure 4 – Location and numbering of the six clients and petrol stations of Galp Energia in Olivais parish](image)

Table 4 shows the name and number of each client, the amount of fuel delivered per year to each client and the number of equivalent trucks that correspond to that amount of fuel.

<table>
<thead>
<tr>
<th>No. of station</th>
<th>Client name</th>
<th>Fuel delivered per year (m³)</th>
<th>No. of equivalent trucks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Delivery location 1</td>
<td>7922</td>
<td>264</td>
</tr>
<tr>
<td>2</td>
<td>Delivery location 2</td>
<td>6492</td>
<td>216</td>
</tr>
<tr>
<td>3</td>
<td>Delivery location 3</td>
<td>2025</td>
<td>68</td>
</tr>
<tr>
<td>4</td>
<td>Delivery location 4</td>
<td>2923</td>
<td>97</td>
</tr>
<tr>
<td>5</td>
<td>Delivery location 5</td>
<td>652</td>
<td>22</td>
</tr>
<tr>
<td>6</td>
<td>Delivery location 6</td>
<td>5585</td>
<td>186</td>
</tr>
</tbody>
</table>

### 3.2 Model results

The model was implemented with the tools mentioned in section 2.3 and was solved to optimality. Table 5 summarises the results and numeric characteristics of the model, comprised by the number of variables, binary variables, constraints and iterations, CPU time, optimality gap and value of the objective function, which corresponds to the population affected by the transport of the six shipments.

<table>
<thead>
<tr>
<th>No. variables</th>
<th>No. binary variables</th>
<th>No. constraints</th>
<th>No. iterations</th>
<th>CPU Time (s)</th>
<th>Optimality gap (%)</th>
<th>Value of objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 763</td>
<td>4 788</td>
<td>33 007</td>
<td>3 222</td>
<td>1.92</td>
<td>0</td>
<td>838 335</td>
</tr>
</tbody>
</table>

During the GAMS pre-processing, the model was reduced to 1 470 constraints (lines), 3 718 variables (columns) and 4 072 binary variables, which are lower values than the original ones. The model was
solved in a reduced CPU time, for a quite superior model dimension than those encountered in similar problems described in the literature.

Table 6 presents the number of equivalent trucks and some characteristics of the results for each shipment: number of links used and the total path travel time. The solution obtained was validated with the verification that the links for each shipment are sequential, which means that where one link ends the other one starts, from the origin to the destiny of the shipment.

Table 6 – Characteristics of the result obtained for each shipment

<table>
<thead>
<tr>
<th>Shipment</th>
<th>No. trucks</th>
<th>No. links</th>
<th>Travel time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shipment 1</td>
<td>264</td>
<td>58</td>
<td>21.1</td>
</tr>
<tr>
<td>Shipment 2</td>
<td>216</td>
<td>38</td>
<td>18.5</td>
</tr>
<tr>
<td>Shipment 3</td>
<td>68</td>
<td>43</td>
<td>19.5</td>
</tr>
<tr>
<td>Shipment 4</td>
<td>97</td>
<td>73</td>
<td>27.9</td>
</tr>
<tr>
<td>Shipment 5</td>
<td>22</td>
<td>71</td>
<td>27.1</td>
</tr>
<tr>
<td>Shipment 6</td>
<td>186</td>
<td>72</td>
<td>27.8</td>
</tr>
</tbody>
</table>

In order to obtain the graphic representation of the path recommended for each of the studied shipments (Figure 4), the results were exported to Excel and then integrated with the network in ArcGIS, with the aim of visualising the result.

3.3 Sensitivity analysis

A sensitivity analysis was made in order to verify the consistency of the model. Different realities were considered within the same network, to verify in which way the paths defined by the model are susceptible to variations of population density and the viability of the model (and corresponding value of objective function) to the variation of R.

Variation of R (Big-M)

An analysis was made to parameter R, often mentioned in the literature as Big-M, varying it by multiples of 10 between 10 and 10^{12}. For each value of R, it was analysed if the solution of the model was possible or impossible and which were the respective values of the objective function and CPU time. With the aim of obtaining the values presented in the table below, the model was run three times for each value of R and the average of CPU time computed.
Table 7 – Results obtained with the model for different values of R (Big-M)

<table>
<thead>
<tr>
<th>R</th>
<th>Result</th>
<th>Objective function</th>
<th>CPU time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Impossible solution</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>100</td>
<td>Impossible solution</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1000</td>
<td>Impossible solution</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$10^4$</td>
<td>Possible solution</td>
<td>838 335</td>
<td>1.644</td>
</tr>
<tr>
<td>$10^5$</td>
<td>Possible solution</td>
<td>838 335</td>
<td>2.335</td>
</tr>
<tr>
<td>$10^6$</td>
<td>Possible solution</td>
<td>838 335</td>
<td>2.636</td>
</tr>
<tr>
<td>$10^7$</td>
<td>Possible solution</td>
<td>838 335</td>
<td>3.671</td>
</tr>
<tr>
<td>$10^8$</td>
<td>Possible solution</td>
<td>838 335</td>
<td>9.703</td>
</tr>
<tr>
<td>$10^9$</td>
<td>Possible solution</td>
<td>838 335</td>
<td>8.991</td>
</tr>
<tr>
<td>$10^{10}$</td>
<td>Possible solution</td>
<td>838 335</td>
<td>2.184</td>
</tr>
<tr>
<td>$10^{11}$</td>
<td>Possible solution</td>
<td>838 335</td>
<td>2.007</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>Possible solution</td>
<td>838 335</td>
<td>1.721</td>
</tr>
</tbody>
</table>

Thus it is verified that the model is sensitive to the value of R, as if it is too much low it is impossible to solve the model (CPLEX ends with the message "Problem is integer infeasible"). It is also to be noted that the value of the objective function keeps constant for the values of R for which the solution is possible. CPU time varied between 1,6s and 9,7s and no relation was found between this value and the value of Big-M.

Variation of population density

As mentioned, one of the criteria used by the model to define the paths of less risk is population density. Therefore, it was considered interesting to verify how the solution varies if every link presents the same population density. It is intended, through this analysis, to verify that a population variation effectively leads to different path choices.

Table 8 discriminates the travel times (Time), the population exposed (Pop. Exposed), the amount of trucks delivered per year (No. trucks) for each shipment and the number of links that constitute the path chosen for each shipment (No. links), for two distinct scenarios: the base case study (with the real population density) and the alternative scenario described above, in which the population density is altered to the same value for all the links of the network. This corresponds to the weighted average of the population density in the network, where the population density of each link is multiplied by the corresponding travel time.

Population exposure increased considerably in the model with the same population density in comparison with the real population density, as in this scenario the model lacks links with less population density for which to opt for in order to minimize the risk. Therefore, it is emphasised that the interpretation of the number of links that constitute the chosen path should be made carefully, as there are links with very different lengths. It is hence justified that in the model with the same population density the shipments present higher travel times although they are constituted by a smaller number of links.
### Table 8 – Comparison of results between the model with the real population density and the same population density in all links

<table>
<thead>
<tr>
<th></th>
<th>Real population density</th>
<th>Same population density</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time</td>
<td>Pop.</td>
</tr>
<tr>
<td>Shipment 1</td>
<td>21,1</td>
<td>966</td>
</tr>
<tr>
<td>Shipment 2</td>
<td>18,5</td>
<td>706</td>
</tr>
<tr>
<td>Shipment 3</td>
<td>19,5</td>
<td>710</td>
</tr>
<tr>
<td>Shipment 4</td>
<td>27,9</td>
<td>1117</td>
</tr>
<tr>
<td>Shipment 5</td>
<td>27,1</td>
<td>1117</td>
</tr>
<tr>
<td>Shipment 6</td>
<td>27,8</td>
<td>1342</td>
</tr>
<tr>
<td>Sum 6 shipments</td>
<td>142</td>
<td>5,958</td>
</tr>
<tr>
<td>Total/year</td>
<td>19,366</td>
<td>838,335</td>
</tr>
<tr>
<td>Total/truck/year</td>
<td>22,7</td>
<td>983</td>
</tr>
</tbody>
</table>

4. **Conclusions**

The developed model aims to find a balance between the exposed population and the economic viability of the hazmats transport, namely white oils (petrol and diesel fuels). The application of linear programming models to hazmats transport problems with this dimension of links and nodes was not found in the literature (because of its application to an urban environment) nor applied to the Portuguese geographic area, wherefore it is considered to be an innovative work that filled a gap. It’s particularly relevant the fact that the model was applied to a real case study, which brings complexity to its development, and that it was developed with a partnership with *Galp Energia*, the Portuguese company of this sector.

The optimal solution was obtained in a quite short time (less than 2 s) and indicated which links should be used for hazmat transportation and, within them, which should be used for the transport of each of the six studied shipments of *Olivais* district. Through results analysis it is possible to verify the population exposed in each shipment and the travel time for the paths defined by the model.

In the future, it would be interesting to apply the model to a larger geographic area, using another network or doing more simplifications in the one used in this work; For simplification reasons, the population exposed in the model was considered to be the resident population, but it would be more accurate if the resident population (population overnight) was distinguished from the population that is effectively present in that zone during the day, for which would have to considered the generator poles of each zone, such as services and jobs/schools. It is also recommended for future analysis the comparison between the paths currently used by the carriers trucks with those resulting from the model, which would allow the calculation of the population exposed to hazmats in both scenarios.

**References**

Autoridade Nacional de Protecção Civil (ANPC),


