Railway planning: optimization approaches for the rolling stock rotation problem

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Abstract

Rolling stock is the set of train units and is one of the most expensive assets that a railway company can own. The rolling stock rotation planning aims to schedule the rotation of the rolling stock at minimal costs and is a very complex and time-consuming problem when properly integrated. For that, the rotation must ensure that it covers all trips already defined in the timetable, and respecting the requirements imposed by the railway network. This work integrates a significant number of industrial railway requirements such as vehicle compositions, station capacity, regularity bonification, deadhead trip penalization, shunting, and turn operations. We solved the problem through the hypergraph model methodology, and our contribution is the addition of a new objective function, implying the model to be considered as bi-objective. The new objective function aims to minimize the damaging emissions produced within the operations concerning the rolling stock rotation. To solve the model, we used the augmented e-constraint method. We showed that when considering the minimization of the emissions, the solutions tend to have fewer deadhead trips than the solutions provided by the cost minimization. We also showed a Pareto front that allows the decision-maker to find competitive solutions in which it is possible to decrease the emissions while increasing the costs. Lastly, to validate and test the model, we used the open data of the Chicago railway subway.
Keywords: Rolling stock, Vehicle composition, Shunting, Scheduling, Emissions produced

1. Introduction

Over time, railway companies have witnessed a growing number of passengers, seeking to meet their travel requirements. With the exception of 2020, passenger demand has been steadily increasing over the years (Eurostat, 2022). Due to this continuous increase in railway passenger demand, it is imperative for railway companies to enhance their readiness to satisfy the ongoing emergence of passengers.

Operating railway transportation involves various complexities, but the application of mathematical models and optimization techniques brings advantages for both customers and operators. While customers can experience enhanced service quality, operators might have lower costs. Indeed, one of the earliest applications of mathematical optimization and operations research was in railway planning. (Schrijver, 2002).

The rolling stock rotation planning (RSRP) problem is an essential part of railway planning. It involves scheduling train locomotives, in order to fulfil the timetable trips previously planned, satisfying the passengers' demand, while minimizing the operational costs (Thorlacius, 2015). Having the ability to solve the RSRP problem in the most optimal and efficient way offers railway companies a significant competitive advantage in the railway market (Schlechte et al., 2023).

Furthermore, the continuous increase in the number of passengers using the railway mode of transportation can enable an increase in the pollution associated with the railway network. While often regarded as one of the most environmentally friendly modes of transportation, as noted by Krezo et al. (2018), it is important not to overlook the emissions associated with the rolling stock rotation problem. This is especially pertinent given the growing government encouragement to embrace railway transportation, resulting in an expansion of the railway network.

Thus, our work aims to solve the rolling stock rotation planning problem using the hypergraph model proposed by Borndörfer et al. (2011), i.e., to find the optimal composition of vehicles for each trip, and their rotation, considering the constraints that railway companies have. This will take into consideration the minimization of the costs of this problem as well as the minimization of the emissions emitted by the railway system within the operations required. The duality between the two objective functions will be studied and we aim to find which features have more impact on the costs and which one have more impact on the emissions. It is expected that this project will provide a new vision for the RSRP existing in the literature, through the confrontation of the results obtained from the costs and emissions' optimization.

2. Background

The rolling stock rotation planning problem is a particular case of vehicle scheduling, but for railways. This last scheduling problem is well-known in the literature and has been studied by several authors (see survey Löbel,1997). The RSRP's biggest goal is to cover all the timetabled trips by the existing rolling stock, at the lowest operational costs possible (Thorlacius et al.,2015), respecting all the necessary requirements.

2.1. Rolling stock rotation planning models

In 2011, Borndörfer et al. presented the hypergraph model, whose main benefit is to model changes in the train's compositions at terminal stations, for long-distance railway passenger transport. The authors handled the railway's necessary requirements using hyperarcs. In this way, the representation of the train unit displacements is more detailed, leading to a better granularity level. The authors also proved that the model provides high-quality solutions in reasonable times, using the rapid branching and column generation techniques, for real-world instances. However, the hypergraph model, due to its intricacy is harder to model and leads to higher computational times.

Therefore, an alternative to simplify the hypergraph model is the flow-based approach, which is usually simpler to implement and less time-consuming to compute (Alfieri et al., 2006; Cadarso and Marín, 2011). This type of model focuses on the conservation of the flow, i.e., each vertex ensures that the incoming flow is equal to the outgoing flow. This is a common approach in vehicle routing problems, leading to significantly lower model complexity. Nevertheless, it is not as detailed as the hypergraph model, implying that it cannot capture all the complexities regarding the interactions between the rolling stocks.

Lastly, the path-based models (Thorlacius et al., 2015; Haahr et al., 2016) have an intermediate quality between the hypergraph and the flow-based models, providing varied results' precision, once that it depends on how well the rolling stock rotation problem is modelled. For this approach, every singular sequence of displacement of the vehicles is modelled, through the association of a path to the individual vehicles.

Hence, depending on the author's expectations, different approaches can be exploited. We choose the hypergraph model, once it allows us to better understand the flows within the rotations, and it is more integrated in the sense that allows us to lead with complex requirements, even if the hypergraph model is the one that leads to higher computational times and is more complex to implement.

2.2. Scope

Cyclic timetables offer passengers the advantage of predictable departure times from their usual stations, along with accommodating numerous passenger transfers. Furthermore, the cyclic timetable can be inefficient, once it can operate trips on days of the week in which the demand is significantly lower than the rest of the days. This is not the case with acyclic timetables, which also offer greater flexibility (Kroon et al., 2008;Lusby et al., 2017).

For models considering cyclic (regular or periodic) timetables, there is an incentive for the train fleet to follow regular trips over the horizon, which can be defined within days, weeks, and sometimes months. These sequences can be established a priori through a set of simple rules (Lusby et al., 2017). Many authors opt for models based on regular timetables.

In contrast, when dealing with acyclic timetables — where trips lack regularity — optimization becomes necessary for each trip sequence, and the trip sequence cannot be provided a priori. This leads to only considering a lower number of train compositions when compared with regular timetables and the train fleet position and order are commonly ignored (Lusby et al., 2017).

2.3. Requirements integration

The rolling stock rotation planning is an intricate problem due to several limitations imposed by the layout of the rail, so the more constraints are integrated into the model, the more complex is the model. As so, it becomes more time-consuming whether on the formulation problem or in the resolution time (Borndörfer et al., 2021).

Considering train composition order significantly boosts model complexity, the integration of this feature is an important decision: some authors prefer to ignore this characteristic such as Maróti and Kroon (2007) and Jha et al. (2008); and others authors implement this feature considering both the composition order and the orientation of the fleet like Haahr et al. (2016) and Borndörfer et al. (2016). In 2021, Borndörfer et al. proposed a three-layered approach using the coarse-to-fine (C2F) method to implement the train composition. This hierarchical column-generation technique initially works at a broad level and then refines calculations based on orientation and position, thus it enables the evaluation at various levels of complexity.

Maintenance requirements are essential to railway fleets. However, due to their complexity, not all models present in the literature include maintenance (Schlechte et al., 2023). It is less complex to add maintenance requirements for the path-based models, once each possible movement sequence of a certain train unit is modeled. However, when it comes to flow-based models, the addition of the maintenance requirements adds more complexity to the model (Thorlacius et al., 2015). For the flow-based models, in which the maintenance increases the complexity of the model, the daily train units are generated to comprise a certain number of arcs related to the maintenance operation, meaning that the fleet can be assigned with sufficient maintenance operations. In this way, Gao et al. (2022), divided the problem into two levels: the first level is to generate the trip sequences while ignoring the maintenance requirements; and the second level is to assign each fleet to its maintenance necessities, according with the optimization of the maintenance operations.

The depot capacity is considered in the majority of the models, and it represents the maximum number of fleets that the depot can take. This feature is easily combined with the depot topology once the topology aims to specify how the shunting and turn operations are performed (Thorlacius et al., 2015; Schlechte et al., 2023).

The regularity constraint comes directly from the cyclic timetable. In 2011, Borndörfer et al. introduced the regularity notion in their hypergraph model. This constraint enhances the model to return rolling stock rotations in which the train compositions are the same, for regular trips. This is achieved through a reduction of the costs for regular trips with the same vehicle composition.

2.4. **Objective Function**

Authors often employ various objective functions for optimization. One widely used approach is cost minimization, which helps companies lower their operational expenses, leading to significant savings. It's important to note that whether the focus is on reducing penalties, train units, or maximizing benefits, all these objectives are intrinsically tied to operational costs. For instance, Jha et al. (2008) included penalties for paths that deviate from due dates in their model, in the form of costs. The penalization is considered in the cost unit, considering the path miles, the deviation, and the due date. Another example is the model presented by Borndörfer et al. in 2016 that penalized the deadhead trips (empty trips) through a time penalization, that was then converted into additional costs. Similarly, in the case of minimizing train units, Cacchiani and Toth (2012) introduced models that achieved this by directly targeting costs associated with train units.

2.5. **Solution method**

Regardless of the approach chosen to solve the rolling stock rotation planning problem (hypergraph, flow-based, and path-based models), due to the intricacy of the real-world data, the majority of the models implemented a heuristic method to solve the problem. Indeed, all solution methods used in the literature apply commercial solver or heuristics, and, in some cases, the authors use both methods (Thorlacius et al., 2015).

Some authors also applied decomposition techniques, such as column generation and branch-and-price methods, resulting in reduced resolution times for integrating specific model requirements (Borndörfer et al., 2021). This approach also led to faster solution computations (Lusby et al., 2017). Additionally, they demonstrated that applying heuristic methods not only sped up the solving process but also produced effective solutions without significantly increasing costs.

2.6. **The environmental approaches**

RSRP models present in the literature primarily aim to minimize costs, often without a focus on pollution impact. This is because authors typically apply their models to real-world instances in collaboration with railway companies. For instance, Freling et al. (2005) optimized the Netherlands shunting operation for the Zwolle station, Borndörfer et al. (2016) applied the hypergraph model to Deutsche Bahn in Germany, and Hoogervorst et al. (2021) implemented their RSRP's model for the Netherlands Railways.

The International Energy Agency (IEA) considers that the railway is one of the least pollutants modes of transportation, rail transportation as one of the least polluting modes, largely due to the prevalence of electric railway systems, accounting for over 85% of rail operations (Agency, 2023). However, it's important not to overlook potential negative impacts. Emission levels in rail transportation are primarily tied to the type of locomotives used, rather than operations management. In other words, the rolling stock rotation planning problem usually optimizes the costs regarding railway operations, which is not the case for the emissions produced.

Hence, our contribution is to integrate the emissions produced by the rolling stocks, considering a bi-objective model, where it will be possible to compare the solutions obtained from the two perspectives.

3. The rolling stock rotation planning problem

3.1. **Definition**

The rolling stock rotation planning aims to cover all timetable trips, respecting the necessary requirements. A trip $t \in T$ belongs to the timetable trips planned in the standard week and has departure and arrival times, in its respective departure and arrival stations $s \in S$. Let F be the set of the fleet, also referred to as vehicles and C as the set of the compositions that are a group of vehicles f. Take V as the set of nodes. The nodes represent the departure and arrival points of the vehicles that operate the trip t. A standard arc $a = (u, v) \in A$ is an arc that connects two nodes, and operates trip t if $u \in V$ express the departure node of trip t and $v \in V$ stands for the arrival node for that same trip t. A hyperarc $h \in H$ covers trip t if each standard arc $a \in h \subseteq A$, operates trip t. The set of all hyperarcs h that covers trip t is defined as trip hyperarcs $h \in H(t)$, and the set of all hyperarcs connecting two trips is denoted by connection hyperarcs $h \in H(c)$. Additionally, the set of hyperarcs h that enters and exits a node v is denoted as $H(v)^{in} := \{h \in H | \exists a \in h : a = (u, v)\}$, and $H(v)^{out} := \{h \in H | \exists a \in h : a = (v, w)\}$, respectively, and considering nodes $u, v, w \in V$. Finally, the hypergraph model is denoted by $G = (V,\ A,\ H)$.

To solve the rolling stock rotation planning problem we integrated the features that will be presented throughout this section. Moreover, we used the open data from the Chicago Transit Authority - CTA (Authority, 2023). The timetable is cyclic, and the maintenance requirement was left aside from our model. As mentioned previously, the implementation of the maintenance constraints is more complex for flow-based models, therefore, we focused our analysis on other requirements.

3.2. **Constraints**

3.2.1 Vehicle compositions

A vehicle $f \in F$ is a single unit fleet that allows the passengers to travel a trip $t \in T$. A set of grouped vehicles is called a vehicle composition c, that belongs to the set of compositions C. Not all compositions $c \in C$ are feasible for all trips $t \in T$. The set of the feasible compositions is denoted by $C(t) \subseteq C$, and they allow the operation of all the timetable $t \in T$.

The hypergraph model uses flow conservation to solve the RSRP. To ensure that, we need to consider the outgoing and ingoing hyperarcs for each node $v \in V$. Take Figure 1 as an example. For hyperarc h_3, we have two nodes on the left, representing the departure of the compositions represented by h_3, and two nodes representing the arrival. As we can see, all nodes conserve the flow. For example, the let top node has the connection hyperarc h_9 ingoing and the trip hyperarc h_3 outgoing. For the example provided, both these hyperarcs belong to the solution, and thus if hyperarc h_9 is in the solution, hyperarc h_3 will also be, due to the conservation flow.

The feasible compositions $c \in C(t)$ offer the model a large number of degrees of freedom for four main reasons. The first one is due to the shunting operations. Within the RSRP, several shunting operations can be done, and the more vehicles a feasible composition has, the more shunting operations that composition can have. To limit the degrees of freedom for this characteristic, we only allow the model to perform one shunting operation per feasible composition. This example is presented in Figure 1 with hyperarc $h_7 \in H$. The second reason is the orientation of the vehicles, that will be further discussed. The third reason is the position of the vehicles within the composition. Our version of the RSRP considers the position of the vehicle in a composition c, but does not distinguish particular vehicles; it only considers the type of vehicle position. For example, in our model, the composition {Blue_1, Yellow, Blue_2} is the same as {Blue_2, Yellow, Blue_1}, as we only consider the type of vehicles without specifying each particular vehicle. Finally, the fourth reason is the size of a feasible composition $c \in C(t)$. Its size varies based on several factors, such as the demand per trip $t \in T$, the capacity of the vehicles cap_f, and the maximum length, $leng_t$, that a composition c can take for a trip t. If a composition c composed of two vehicles can satisfy the demand and the maximum length for a trip t is six, there are a significant number of possible feasible compositions, considering the order and position of the vehicles.

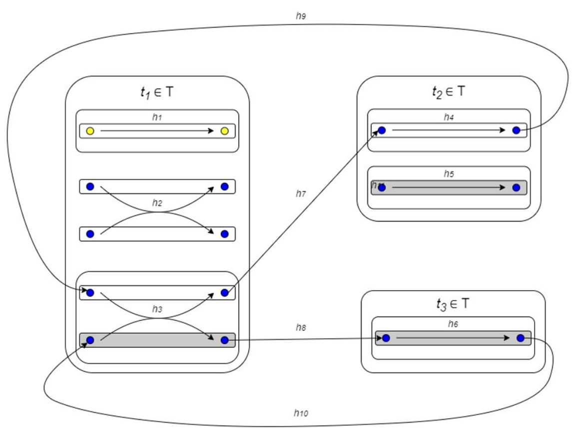


Figure 1: Hypergraph model representation.

There is an associated cost with the vehicle utilization, $util_f$, that is added to the operations if the vehicle f, represented by hyperarcs, is to be selected in the solution. There are two types of costs regarding the rolling stock utilization, that need to be taken into consideration to solve the RSRP: fixed and variable costs. These costs are essential once they are mandatory to operate the timetable in the standard week, at the minimal operational costs possible.

There can be deadhead trips in the solution if the arrival station of trip t_i is not the same as the departure station of trip t_j, shunting operations, or turn operations. These three operations incur additional costs for the model. As a matter of fact, the deadhead trips are not only expensive to the railway companies, but they are an opportunity lost to use the trip in order to satisfy the demand. For this reason, a deadhead trip penalization, $cDhPen$ is included in the model, to ensure that this type of trip is discouraged by the model, optimizing the trips that directly serve the passengers.

3.2.2 Capacity constraint

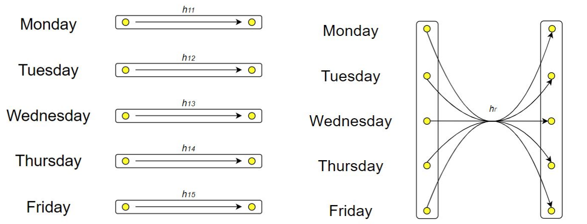
The capacity constraint lies in the fact that each trip $t \in T$ is created knowing the forecasted demand, which is the total number of passengers that will travel on trip t. So, for each trip t, the demand must be satisfied, i.e., the sum of the capacity vehicles, for a composition $c \in C$, operating a certain trip t needs to be able to satisfy the demand.

3.2.3 Regularity constraint

As mentioned, we used a cyclic timetable, which means that the timetable set of trips is defined over a standard week and that the solution is applicable to the following weeks. Figure 2 (a) presents the set of trips having the same departure and arrival stations and times and the same train compositions. Therefore, these trips are referred to as regular, and we can create a regular hyperarc, h_r that has the total operational cost as the sum of all operational costs involving hyperarcs $h_i, i = 11, 15$ minus a reduction, the regularity bonification, $cRegBonif$. The generation of h_r is displayed in Figure 2 (b) and as we can see, this new hyperarc represents the same as the ones in Figure 2 (a), excluding their costs.

3.2.4 Orientation constraint

In 2021, Borndörfer et al. designated the two possible orientations of the vehicles in the following way: if the first class of the vehicle is in the front of the vehicle, considering its direction, the orientation is called *tick*. However, if the first class finds itself



(a) Separate hyperarcs (b) Joined through hyperarc h_r

Figure 2: Regular trips.

on the back of the vehicle, once again considering the trip direction, then the orientation is said to be *tock*. For the Chicago subway, the orientation is not relevant, once that the train locomotives do not possess first and second classes.

3.2.5 Station constraint

Each station $s \in S$ has a length of its platform, and it is noted that if a trip $t \in T$ passes through station s, then station s belongs to $S(t)$. For that reason, some compositions, $c \in C$ cannot be considered, because they are too long to fit in the platform. Moreover, once each trip t is associated with one line, one can calculate the maximum length that a composition c serving a trip t can have, i.e., the feasible compositions $c \in C(t)$.

Furthermore, terminal stations can require other conditions, such as the possibility of turning the train or not, that can lead to a change in the composition and orientation of the train, which needs to be taken into account in the RSRP. Moreover, the shunting operations also occur in terminal stations, although this operation is not allowed to be performed in all terminal stations. Thus, the set of terminal stations allowing shunting operations needs to be defined accordingly with the structure of the terminal stations. Both turn and shunting operations have costs associated, which also need to be taken into account.

Moreover, some terminal stations can have a park in which the trains are allowed to stay from one day to another (or more time), while they wait for their next trip. In these cases, the limitation of the park needs to be taken into consideration. However, for our version of the RSRP, it was assumed that the park has unlimited capacity.

3.3. Emissions of CO_2 equivalent

The literature does not present models with the integration of CO_2 equivalent emissions, hindering the understanding of how emissions can be assessed and allocated within the operations, as we did for the costs, throughout the current chapter.

Once we aim to formulate a model that minimizes the emissions produced by the operations, we evaluate the emissions through the travel distance traveled by the train compositions, $emis_f$. This parameter is correlated to the energetic efficiency of each vehicle, and it is different for each type of vehicle. For instance, electric vehicles have a lower coefficient of emissions of CO_2 equivalent produced than diesel vehicles. As well as the most recent is the fleet, the more energetically efficient it is.

In alignment with the deadhead trip cost penalty, we implemented the deadhead trip emissions penalty, $eDhPen$. This parameter has the same goal as the cost penalty, which is the disadvantage of the model in choosing solutions involving deadhead trips, or at least reducing them. Similarly, as the cost penalty that was created to represent the cost of opportunity for performing an unnecessary trip, the emissions penalty serves to specify to the model that the empty trips do not have the same value as trips that serve passengers. Hence, they should be penalized. Nevertheless, we did not include a regularity bonification through the emissions, once that in terms of emissions, it is not important to compositions of regular trips have the same compositions. Furthermore, a rolling stock rotation solution can be less pollutant by switching compositions for regular trips.

4. Hypergraph model

4.1. Mixed Integer Programming Formulation

In this section, we present the mixed integer programming formulation implemented to solve the rolling stock rotation planning problem. The MIP uses the hypergraph model and thus is focused on the flow conservation for each node $v \in V$, implying that $v^{out} = v^{in}$.

$$\text{minimize} \quad \sum_{h \in H} c_h x_h \tag{1}$$

$$\text{minimize} \quad \sum_{h \in H} e_h x_h \tag{2}$$

$$\text{s.t.} \quad \sum_{h \in H(t)} x_h = 1 \qquad \forall t \in T, \tag{3}$$

$$\sum_{h \in H(v)^{in}} x_h = \sum_{h \in H(v)^{out}} x_h \qquad \forall v \in V, \tag{4}$$

$$x_h \in \{0,1\} \qquad \forall h \in H. \tag{5}$$

A binary decision variable x_h is defined for each hyperarc $h \in H$. For $x_h = 1$, indicates that the hyperarc belongs to the RSRP's solution, and thus it has its cost and emissions produced associated. The solution of current RSRP is solved through a mixed integer programming formulation of the hypergraph model, as it is presented above.

This version of the RSRP is presented with two linear objective functions. The first one is presented in Objective Function 1 and it aims to minimize the total cost of the RSRP's operations. On the other hand, the second linear objective function aims to minimize the emissions of CO_2 equivalent produced throughout the operations of the RSRP (Objective Function 2). Equation 3 indicated that for each trip $t \in T$, one hyperarc in H(t) covers the trips, i.e., one hyperarc h is associated with one and only one trip t, ensuring that each trip is considered in the hypergraph model. Additionally, the flow needs to be conserved, and this is achieved through Equation 4. This equation ensures that for each node $v \in V$ that is contained in the solution, it is ensured that the number of outgoing hyperarcs is the same as the ingoing hyperarcs. Finally, the Equation 5 guarantees the binarity of the decision variable x_h that represents the hyperarc h. The binary decision variable will be one of the hyperarc h belongs to the RSRP's solution and zero otherwise.

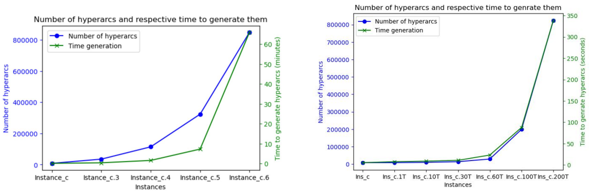
To solve this version of the RSRP, which is a bi-objective problem, we implemented the ε-constraint method, proposed by Mavrotas (2009).

5. Results

5.1. Complexity of the hypergraph model

In this section, we will explore what is most time-consuming for the model. For that, we will analyze the increment of the feasible compositions for one instance against the increment of additional trips on that same instance. The instance is composed of 49 trips and the minimum number of vehicles needed is 2. From this instance, we will create two scenarios: for the first one, we will increase by one at a time, the number of vehicles allowed in the feasible compositions, while keeping constant the number of trips and in the second, we will increase progressively the number of trips, while keeping constant the maximum number of vehicles allowed.

In Figure 3 (a), there is represented the increase of the number of hyperarcs and its respective time to generate them for each instance - scenario 1. Instance_c considers 49 trips and the maximum number for each feasible composition is 2. Instance_c.3 considers 49 trips also, and the maximum number for each feasible composition is 3, until instance_c.6 considers 49 trips and a maximum number for each feasible composition of 6 vehicles. Figure 3 (b) displays the impact of the addition of more trips on the hyperarcs´ number and generation time - scenario 2. The starting instance is the same as the previous case (instance_c), and we progressively increased the number of trips. Firstly, we increased the number of trips by 1 (instance_c.1T), then 10 (instance_c.10T), 30 (instance_c.30T), 60 (instance_c.60T), 100 (instance_c.100T), and finally 200 (instance_c.200T).



(a) Increased of number of vehicles.

(b) Increase of number of trips.

Figure 3: Generation time of the hyperarcs for the several instances scenarios.

Figure 3 (a) allows us to observe that by adding one more vehicle to the set of feasible compositions, the number of hyperarcs and their respective time to generate them also increases, as would be expected. Moreover, the generation time curve increases faster than the one representing the number of hyperarcs, indicating that even if the number of hyperarcs created for the instances does not increase at high rates, its generation time does. This means that, no matter how fast the number of hyperarcs augments, its generation time will tend to exponentially increase. To narrow this issue, we suggest computing limited feasible compositions. For the case of instance c, there is no need to consider feasible compositions with more than 3 vehicles, once the minimum number of vehicles needed is 2. Consequently, by limiting the degrees of freedom for trips, we can significantly decrease the number of feasible cars, and consequently the number of hyperarcs and their generation time.

For scenario 2 (Figure 3 (b)), the speed of increases for both the number of hyperarcs and their generation times, are more similar. Nevertheless, it is also true for scenario 2 that with the progressive addition of trips, the number of hyperarcs generated also increases exponentially.

The number of hyperarcs for instance_c.6 is 848,904 and for instance_c.200T is 823,459. This means that we had to add 200 additional trips all with two vehicles at maximum for the creation of the feasible compositions to achieve roughly the same number of hyperarcs for an instance of 49 trips, but with six as the maximum number of vehicles for the feasible compositions. However, even if the number of hyperarcs is almost the same, the corresponding generation time is very different. For instance c.6 the generation time is 66 minutes, while the generation time for instance_c.200T is almost 6 minutes (eleven times inferior to instance_c.6). This can be explained by the shunting operations. The more sizes of feasible compositions $c \in C(t)$ allowed for a trip $t \in T$, the more shunting operations are allowed, i.e., it is possible to decouple and couple the train in several ways.

All in all, the hypergraph model is a complex and time-consuming methodology to solve the rolling stock rotation planning problem. However, through the control and limitation of the degrees of freedom, it is possible to make it less time-consuming. That is why the Borndörfer et al. (2016) used the column generation method to create the feasible compositions. It allows to progressively generate columns, instead of solving the problem from the beginning.

5.2. Addition of the emissions minimization objective function

The RSRP primarily focuses on minimizing operational costs but, in our version, it also integrates the reduction of emissions. We will explore the impact of this new emissions-related objective function (Equation 2) on the usual cost minimization objective (Equation 1). Table 1 presents results for both objectives, considering only cost minimization (first row of each instance) and emissions minimization (second row of each instance).

To solve larger instances, we reduced the degrees of freedom to manage the number of generated hyperarcs and maintain acceptable resolution times, through the limitation of the maximum number of vehicles per composition. The computational results are based on instances from instance_0 to instance_c.9, which account for several departure times, weekdays, and sets of Chicago's lines.

Instance_c.2 considers shunting operations, while instance_c.3 does not. Apart from that, the instances are identical. The significant difference between the number of hyperarcs generated leads to a massive difference in the hyperarcs' generation time. It is not displayed in this table, but the generation time for instance_c.2 was almost six hours, while for instance_c.3 was only seventeen minutes. For this reason, we decided to ignore this operation to continue with the computational results.

A difference between cost minimization and emissions minimization is the number of deadhead trips traveled. For emissions minimization, the model tends to avoid deadhead trips due to the penalty associated, more than the cost minimization avoids. On the other hand, the regular trips do not seem to be more enhanced for any approach. Due to the regularity bonification, it was expected to observe a significantly higher number of regular trips for cost minimization, which is not the case, as can be observed for instance_2 and instance_9.

To better understand the solutions achieved through our bi-objective function, we have to take a deeper analysis, and for that, we will analyze the Pareto Front of instance_1, which is presented in Figure 4.

Table 1: Computational results.

Instances	$\|T\|$	# Shunting operations	$\|V\|$	# Regularity hyperarcs in the solution	$\|H\|$	#Deadhead trips	Time to solve (hh:mm:ss)	Gap (%)
instance_0	13	0	270	2	2,106	1	00:00:01	0
	13	0	270	0	2,106	1	00:00:01	0
instance_1	34	0	14,384	0	4,178	34	00:00:01	0
	34	0	14,384	0	4,178	32	00:00:01	0
instance_2	222	0	13,114	44	3,024,672	172	00:07:40	0
	222	0	13,114	90	3,024,672	170	00:07:20	3.96E-05
instance_3	222	-	13,114	44	729,384	172	00:05:27	0
	222	-	13,114	90	729,384	170	00:04:44	4,09E-05
instance_4	106	-	1,434	0	18,480	46	00:00:01	0
	106	-	1,434	0	18,480	47	00:00:01	0
instance_5	396	-	5,354	0	294,208	158	00:00:14	0
	396	-	5,354	0	294,208	150	00:00:13	0
instance_6	325	-	4,792	0	457,920	51	00:00:14	0
	325	-	4,792	0	457,920	22	00:01:34	0
instance_7	307	-	3,220	164	944,116	21	00:04:03	0
	307	-	3,220	119	944,116	9	00:06:20	0
instance_8	408	-	4,728	297	2,094,704	26	00:20:24	0
	408	-	4,728	245	2,094,704	16	00:23:40	0
instance_9	273	-	5,110	54	376,298	88	00:01:09	0
	273	-	5,110	78	376,298	85	00:01:54	0

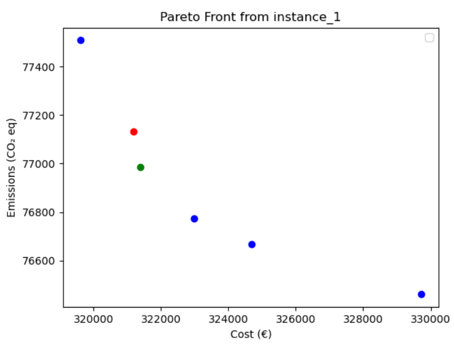


Figure 4: Pareto Front from instance_1.

From Figure 4, it can be seen six solutions found using the augmented ϵ-constraint method. Once we do not have a decision-maker, we cannot discuss with them in order to properly find a solution that would fit their expectations and objectives. However, we can better understand the solutions found, using a bi-objective function. First of all, the extreme solutions are the ones that only minimize the costs and emissions. One of the many advantages of being able to see the Pareto front is the ability to compare two solutions. For instance, take a decision-maker who would like to consider a decrease in their emissions produced, but still keep the costs reduced, and let z_1 be the solution for the costs and z_2 for the emissions. Let´s say that he is willing to increase its costs by 0.5% (compared with the minimum costs) if it implies a reduction of the emissions produced. If we didn't have the whole picture that the Pareto front allows us, a solution that fits this description is the one with $z_1 = 321{,}188$€ and $z_2 = 77{,}133$ CO_2 eq (red point). This solution increases the costs by 0.496% while reducing the emissions by 0.484%, which complies with the decision-maker's expectations. However, this solution increases more the costs, than it decreases the emissions. By taking a deeper look, we can see that there is another interesting solution, the one represented by the green point. Even if this solution increases the costs by 0.556%, which is more than the decision-maker is willing to accept, it decreases the emissions by 0.673%, which is significantly more than the previous solution, and consequently competitive with the red solution. Hence, the decision-maker could be tempted to choose this rotation solution, even if the costs are slightly superior than initially expected.

Considering the DM chooses the green point solution (consider solution_2), Table 2 displays the differences in the solutions for the cost minimization (solution_1) and the green point solution. The only difference between both solutions is the number of deadhead trips present in the solution. Indeed, the more importance we give to the minimization of emissions, the less we provide to the minimization of the costs, and the implications stated above, can already be seen between these two solutions. Hence, a reduction in the number of deadhead trips is expected once the emissions penalization is quite significant when compared with the cost penalization of the deadhead trips.

Table 2: Comparison between solution_1 and solution _2.

	Shunting	Turn	Deadhead	Regular
solution_1	0	4	34	0
solution_2	0	4	31	0

In short, the addition of the minimization of the emissions' objective function brings new possible rotations for the RSRP. The objective function related to the emissions tends to benefit rotations by reducing the number of deadhead trips.

6. Conclusions

To solve the RSRP, we used the hypergraph model, proposed by Borndörfer et al. (2011), which allows us to have a deeper understanding of the rolling stocks flow, once the model enhances the modeling of the several changes in train compositions. Our version of the RSRP integrated the emissions of CO_2 equivalent produced throughout the operations related to this problem, as a second objective function, making the problem bi-objective. This feature was not implemented in any other work that we found in the literature and thus, brings a new perspective in the RSRP.

We found that is possible to reduce the computational time, by restricting the maximum number of vehicles per train composition. The more vehicles can a train composition consider, the more complex and time-consuming the model. By using the hypergraph model, we considered every possible and feasible composition of each trip, however, it was revealed that this was unnecessary and even adverse to the computational time. Once demand is a priori known, by reducing the number of possible vehicles for the train composition, the degrees of freedom are reduced, thus the computational time is also decreased. Moreover, by reducing the maximum number of vehicles per composition, we reduced the number of shunting operations. The more possibilities of feasible compositions, with different sizes, the more hyperarcs to perform the several possible shunting operations are generated. Even with our initial limitation of one shunting operation per feasible composition, we found that hyperarcs related to the shunting operations still represented the majority of the hyperarcs, and were not present in our solution. Thus, by eliminating their possibilities, we were able to reduce even more the degrees of freedom. Nevertheless, it is to be noted that the shunting operations are relevant in the rolling stock rotation planning. However, one way to reduce the complexity would be ignoring them or only considering them for certain stations, for example.

Furthermore, we were able to properly integrate the objective function regarding the emissions produced, and understand the implications and changes of having this new objective function. First of all, when the emissions are minimized, the model tends to avoid more deadhead trips than when only minimizing the costs. Secondly, through a bi-objective analysis, we are prepared to show a potential decision-maker the results and find unexpected solutions that can lead to low increases in the operational costs, but higher decreases in the emissions produced by the operations, which is an interesting point of view.

Even if our version of the rolling stock rotation planning problem, using the hypergraph model works, it can be improved. Shunting operations and larger numbers of maximum vehicles allowed per composition lead to consecutive increases in computational times. Besides the limitation of this characteristic, to decrease the degrees of freedom, we suggest the integration of heuristics into the model.

Another limitation that we had to deal with was regarding the data. Even if the Chicago subway open data is revealed to be satisfactory and appropriate for the RSRP, it was not enough to really understand and model all the potential requirements that the Chicago subway must have. Another limitation was the lack of data and information regarding the allocation and description of the emissions produced for rolling stock rotation operations. It is strongly recommended to collaborate with a railway company to better explore the impact of the emissions produced when facing the operations' costs, and thus, developing further this new version of the rolling stock rotation planning.

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