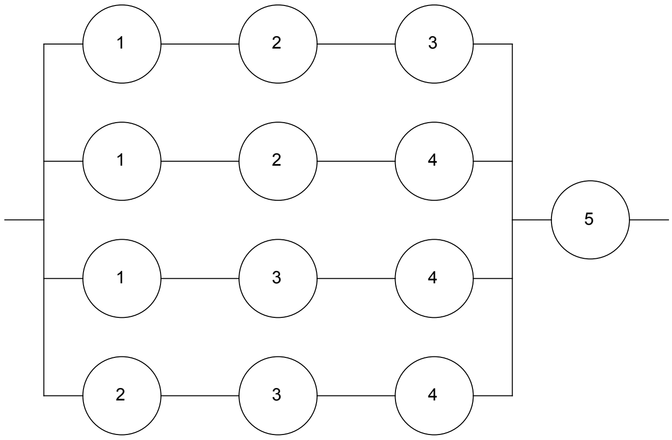
Exam 2

Question 1

Lifetime and Warranty of Solar Panels

```
Show[Graphics[{
   Circle[{-18, 0}, 1],
   Circle[{-14, 0}, 1],
   Circle[{-10, 0}, 1],

   Circle[{-18, -3}, 1],
   Circle[{-14, -3}, 1],
   Circle[{-10, -3}, 1],

   Circle[{-18, -6}, 1],
   Circle[{-14, -6}, 1],
   Circle[{-10, -6}, 1],

   Circle[{-18, -9}, 1],
   Circle[{-14, -9}, 1],
   Circle[{-10, -9}, 1],

   Circle[{-6, -4.5}, 1],

   Text["1", {-18, 0}],
   Text["2", {-14, 0}],
   Text["3", {-10, 0}],

   Text["1", {-18, -3}],
   Text["2", {-14, -3}],
   Text["4", {-10, -3}],

   Text["1", {-18, -6}],
   Text["3", {-14, -6}],
   Text["4", {-10, -6}],

   Text["2", {-18, -9}],
   Text["3", {-14, -9}],
   Text["4", {-10, -9}],

   Text["5", {-6, -4.5}],

   Line[{{-20, 0}, {-19, 0}}],
   Line[{{-17, 0}, {-15, 0}}],
   Line[{{-13, 0}, {-11, 0}}],
   Line[{{-9, 0}, {-8, 0}}],

   Line[{{-20, -3}, {-19, -3}}],
   Line[{{-17, -3}, {-15, -3}}],
   Line[{{-13, -3}, {-11, -3}}],
   Line[{{-9, -3}, {-8, -3}}],

   Line[{{-20, -6}, {-19, -6}}],
   Line[{{-17, -6}, {-15, -6}}],
   Line[{{-13, -6}, {-11, -6}}],
   Line[{{-9, -6}, {-8, -6}}],
```

```
  Line[{{-20, -9}, {-19, -9}}],
  Line[{{-17, -9}, {-15, -9}}],
  Line[{{-13, -9}, {-11, -9}}],
  Line[{{-9, -9}, {-8, -9}}],

  Line[{{-21, -4.5}, {-20, -4.5}}],
  Line[{{-20, 0}, {-20, -9}}],
  Line[{{-8, 0}, {-8, -9}}],
  Line[{{-8, -4.5}, {-7, -4.5}}],
  Line[{{-5, -4.5}, {-4, -4.5}}]
 }]]
```




```
(* https://www.wolfram.com/mathematica/new-in-
  9/reliability/lifetime-and-warranty-of-solar-panels.html *)

ClearAll["Global`*"]
dist = BernoulliDistribution[p];
ℛSolarPanel =
  ReliabilityDistribution[BooleanCountingFunction[{3, 4}, {x, y, z, v}] ∧ w,
   {{x, dist}, {y, dist}, {z, dist}, {v, dist}, {w, dist}}];
r[p_] = FullSimplify[Mean[ℛSolarPanel]]
```

$(4 - 3p)\, p^4$

```
ClearAll["Global`*"]
dist = ExponentialDistribution[λ];
ℛSolarPanel =
  ReliabilityDistribution[BooleanCountingFunction[{3, 4}, {x, y, z, v}] ∧ w,
   {{x, dist}, {y, dist}, {z, dist}, {v, dist}, {w, dist}}];
FullSimplify[SurvivalFunction[ℛSolarPanel, t]]
Mean[ℛSolarPanel]

FullSimplify[HazardFunction[ℛSolarPanel, t]]
h[t_] = FullSimplify[HazardFunction[ℛSolarPanel, t]];
FullSimplify[∂t h[t]]
```

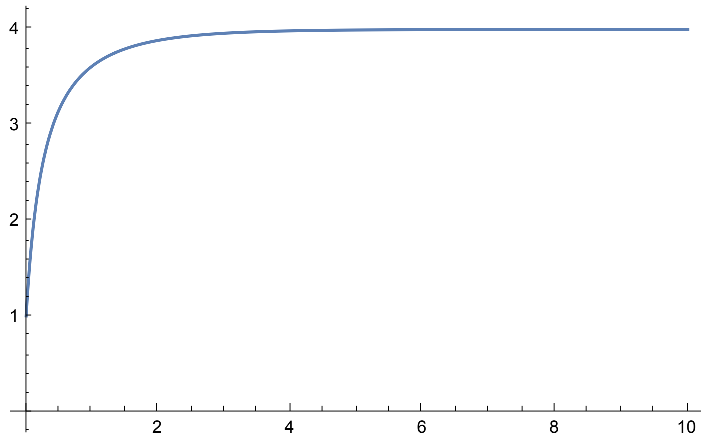
$$\begin{cases} 1 & t < 0 \\ e^{-5 t \lambda} \left(-3 + 4 e^{t \lambda}\right) & \text{True} \end{cases}$$

$$\frac{2}{5 \lambda}$$

$$\begin{cases} \left(4 + \frac{3}{3 - 4 e^{t \lambda}}\right) \lambda & t \geq 0 \\ 0 & \text{True} \end{cases}$$

$$\begin{cases} \text{Indeterminate} & t == 0 \\ \frac{12 e^{t \lambda} \lambda^2}{\left(-3 + 4 e^{t \lambda}\right)^2} & t > 0 \\ 0 & \text{True} \end{cases}$$

```
λ = 1.;
R[t_] = SurvivalFunction[ℛSolarPanel, t];
f[t_] = -∂t R[t];
h[t_] = f[t] / R[t];
Plot[h[t], {t, 0.001, 10.}, AxesOrigin → {0, 0}]
```



```
ClearAll["Global`*"]
dist = ExponentialDistribution[λ];
ℛSolarPanel =
  ReliabilityDistribution[BooleanCountingFunction[{3, 4}, {x, y, z, v}] ∧ w,
   {{x, dist}, {y, dist}, {z, dist}, {v, dist}, {w, dist}}];

Mean[ℛSolarPanel]
Variance[ℛSolarPanel]
Mean[ℛSolarPanel]^2
```

$$\frac{4}{25 \lambda^2}$$

$$\frac{2}{5 \lambda}$$

$$\frac{1}{10 \lambda^2}$$

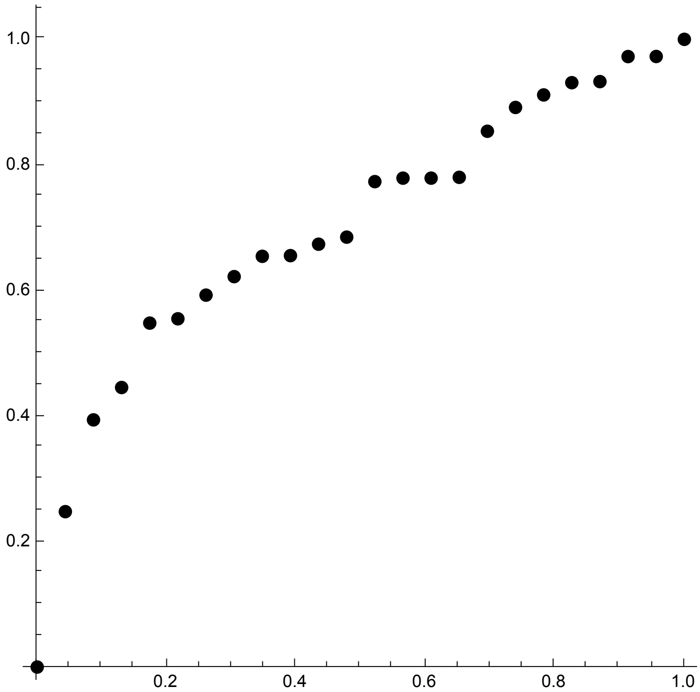
$$\frac{4}{25 \lambda^2}$$

$$\frac{4}{25 \lambda^2}$$

Question 2 — Complete Data

```
ClearAll["Global`*"]

data = {17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.48, 51.84,
51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12,
93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40};
sorteddata = Sort[data]

t = sorteddata;
n = Length[t];
t[[0]] = 0;
ListPlot[Prepend[Table[{i / n, (Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, i}]) / (Sum[(n - j + 1) × (t[[j]] - t[[j - 1]]), {j, 1, n}])}, {i, 1, n}],
  {0, 0}], AspectRatio → 1, PlotStyle → {PointSize[Large], Black}]


𝒟 = EstimatedDistribution[sorteddata, WeibullDistribution[2, 80]]
KolmogorovSmirnovTest[sorteddata, 𝒟, "HypothesisTestData"]

𝒟estim = EstimatedDistribution[sorteddata, WeibullDistribution[α, δ]]
KolmogorovSmirnovTest[sorteddata, 𝒟estim, "HypothesisTestData"]

Quantile[𝒟estim, 0.75]

alphaestim = 2.1;
deltaestim = 81.9;
deltaestim × (-Log[1 - 0.75])^(1/alphaestim)
```

{17.88, 28.92, 33., 41.52, 42.12, 45.6, 48.48, 51.84, 51.96, 54.12, 55.56, 67.8, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.4}




```
WeibullDistribution[2, 80]
```

```
HypothesisTestData[ Type: KolmogorovSmirnovTest
                    p-Value: 0.795 ]
```

```
WeibullDistribution[2.10206, 81.8783]
```

```
HypothesisTestData[ Type: KolmogorovSmirnovTest
                    p-Value: 0.617 ]
```

```
95.643
```

```
95.6829
```

Question 3

```
ClearAll["Global`*"]
data = {{15.8, 16.3, 16.2, 16.1},
{16.3, 15.9, 15.9, 16.2},
{16.1, 16.2, 16.5, 16.4},
{16.3, 16.2, 15.9, 16.4},
{16.1, 16.1, 16.4, 16.5},
{16.1, 15.8, 16.7, 16.6},
{16.1, 16.3, 16.5, 16.1},
{16.2, 16.1, 16.2, 16.1},
{16.3, 16.2, 16.4, 16.3},
{16.6, 16.3, 16.4, 16.1}};
OnesMatrix = Table[1, {i, 1, 4}];
data.OnesMatrix / 4
```

$\mu_0 = 16.2;$

$\sigma_0 = \sqrt{0.01};$

$n = 4;$

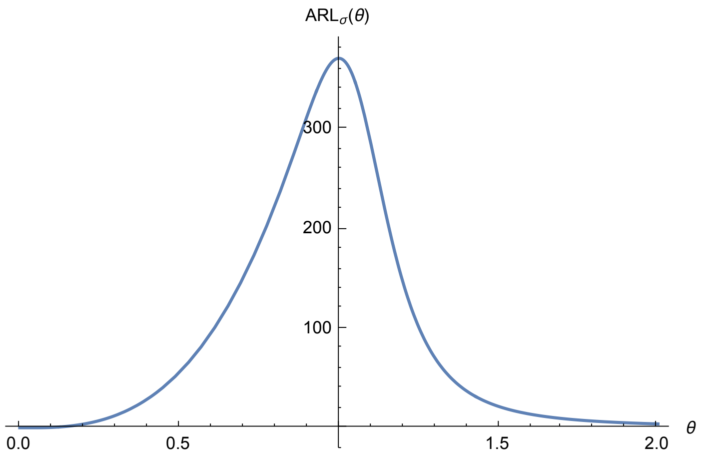
$LCL = \mu_0 - 3 \times \frac{\sigma_0}{\sqrt{n}}$

$UCL = \mu_0 + 3 \times \frac{\sigma_0}{\sqrt{n}}$

```
{16.1, 16.075, 16.3, 16.2, 16.275, 16.3, 16.25, 16.15, 16.3, 16.35}

16.05

16.35
```

```
L = σ0^2/(n - 1) * 0.042541;
U = σ0^2/(n - 1) * 18.222384;
ξ[θ_] = 1 - (CDF[ChiSquareDistribution[n - 1], 18.222384/θ^2] -
      CDF[ChiSquareDistribution[n - 1], 0.042541/θ^2]);
ARL[θ_] = 1 / ξ[θ];
ARL[1]
Plot[ARL[θ], {θ, 0, 2}, AxesOrigin → {1, 0}, AxesLabel → {θ, ARLσ[θ]}]
```

370.395




```
Round[18.222384/0.99^2, 0.000001]
Round[CDF[ChiSquareDistribution[n - 1], %], 0.000001]
Round[0.042541/0.99^2, 0.000001]
Round[CDF[ChiSquareDistribution[n - 1], %], 0.000001]
1/(1 - (0.999668 - 0.002374))
ARL[0.99]
```

18.5924

0.999668

0.043405

0.002374

369.549

369.563

```
n = 4;
μ0 = 16.2;
σ0 = 0.1;

distmu = NormalDistribution[0, 1];
γmu = 3;
ξmu[δ_, θ_] = 1 - (CDF[distmu, (γmu - δ)/θ] - CDF[distmu, (-γmu - δ)/θ]);
ARLmu[δ_, θ_] = 1 / ξmu[δ, θ];
ARLmu[0., 1.]
ARLmu[0., .99]

distsigma = ChiSquareDistribution[n - 1];
ξsigma[θ_] = 1 - (CDF[distsigma, 18.222384/θ^2] - CDF[distsigma, 0.042541/θ^2]);
ARLsigma[θ_] = 1 / ξsigma[θ];
ARLsigma[1.]

ξmusigma[δ_, θ_] = ξmu[δ, θ] + ξsigma[θ] - ξmu[δ, θ] × ξsigma[θ];
ARLmusigma[δ_, θ_] = 1 / ξmusigma[δ, θ];
ARLmusigma[0., 1.]
ARLmusigma[δ_, θ_] = (ARLmu[δ, θ] * ARLsigma[θ]) / (ARLmu[δ, θ] + ARLsigma[θ] - 1);
ARLmusigma[0., .99]
```

```
370.398

409.319

370.395

185.449

194.463
```

$$\frac{370.4 * 370.4}{370.4 + 370.4 - 1}$$

$$\frac{1}{1 - \left(\text{CDF}\left[\text{distmu}, \frac{\gamma\text{mu}-0}{0.99}\right] - \text{CDF}\left[\text{distmu}, \frac{-\gamma\text{mu}-0}{0.99}\right]\right)}$$

$$\frac{\% * 369.549}{\% + 369.549 - 1}$$

$$\text{Round}\left[\frac{\gamma\text{mu} - 0}{0.99}, 0.01\right]$$

$$\text{Round}\left[\frac{-\gamma\text{mu} - 0}{0.99}, 0.01\right]$$

$$\frac{1}{1 - (0.998777 - (1 - 0.998777))}$$

$$\frac{\% * 369.549}{\% + 369.549 - 1}$$

```
185.45

409.319

194.459

3.03

-3.03

408.831

194.349
```

Question 4 — Single sampling plan for attributes

```
Clear[Evaluate[Context[] <> "*"]];
p1 = 0.005; (* AQL *)
α = 1 - 0.95; (* producer's risk *)
p2 = 0.035; (* LTPD *)
β = 0.05; (* consumer's risk *)

Q[c_, x_] = Quantile[ChiSquareDistribution[2 × (c + 1)], x];
r[c_] = N[Q[c, 1 - β], 5] / N[Q[c, α], 5];
i = 0;
While[r[i] > p2/p1, Print["Do not use acceptance number c=", i, " because r(c)=",
  N[Q[i, 1 - β], 5], "/", N[Q[i, α], 5] , "=", r[i], ">p2/p1=", p2/p1];
 i++]
Print["Use the acceptance number c=", i, " because r(c)=",
 N[Q[i, 1 - β], 5], "/", N[Q[i, α], 5] , "=", r[i], "≤p2/p1=", p2/p1]
samplesize[c_] = Ceiling[Q[i, 1 - β] / (2 × p2)];
If[Ceiling[Q[i, 1 - β] / (2 × p2)] ≤ Floor[Q[i, α] / (2 × p1)],
 Print["Use the sample size n=", samplesize[i], "."],
 Print["Houston, we have a problem!"]]
```

Do not use acceptance number c=0 because r(c)=5.99146/0.102587=58.404> $\frac{p_2}{p_1}$=7.

Do not use acceptance number c=1 because r(c)=9.48773/0.710723=13.3494> $\frac{p_2}{p_1}$=7.

Do not use acceptance number c=2 because r(c)=12.5916/1.63538=7.69947> $\frac{p_2}{p_1}$=7.

Use the acceptance number c=3 because r(c)=15.5073/2.73264=5.67485≤ $\frac{p_2}{p_1}$=7.

Use the sample size n=222.

```
c = 3;
n = 222;
CDF[BinomialDistribution[n, p1], c]
CDF[BinomialDistribution[n, p2], c]

dist = NormalDistribution[0, 1];
```

CDF[dist, $\frac{c - n\,p_1}{\sqrt{n\,p_1\,(1 - p_1)}}$]

CDF[dist, $\frac{c - n\,p_2}{\sqrt{n\,p_2\,(1 - p_2)}}$]

Round[$\frac{c - n\,p_1}{\sqrt{n\,p_1\,(1 - p_1)}}$, 0.01]

0.9641

Round[$\frac{c - n\,p_2}{\sqrt{n\,p_2\,(1 - p_2)}}$, 0.01]

1 - 0.9591

CDF[dist, $\frac{c - n * 0.03}{\sqrt{n\,0.03\,(1 - 0.03)}}$]

Round[$\frac{c - n * 0.03}{\sqrt{n\,0.03\,(1 - 0.03)}}$, 0.01]

1 - 0.9251

0.973869

0.0467074

0.963944

0.0407556

1.8

0.9641

-1.74

0.0409

0.0749358

-1.44

0.0749

```
(* Unrequested verification *)
ntot = 1000; (* arbitrary lot size *)
exactdist[p_] = HypergeometricDistribution[samplesize[i], Round[ntot × p], ntot];
If[CDF[exactdist[p1], i] ≥ 1 - α && CDF[exactdist[p2], i] ≤ β,
 Print["The single sampling plan for attributes
    complies with the producer's and consumer's risk points."],
 Print["The single sampling plan for attributes does not comply
    with the producer's or the consumer's risk points."]]
```

```
The single sampling plan for attributes
  complies with the producer's and consumer's risk points.
```

Question 5 — Double sampling plan for atributes with rectifying inspection

```
n1 = 20; (* Collect a first sample of size n1 *)
c1 = 0; (* Accept the lot if D1≤c1, reject if D1>c2 *)
n2 = 20; (* Collect a second sample of size n2 if c1<D1≤c2 *)
c2 = 1; (* Accept the lot if D1+D2≤c2, reject otherwise *)
```

```
pI[p_] = CDF[BinomialDistribution[n1, p], c1];
pII[p_] = Sum[PDF[BinomialDistribution[n1, p], k] ×
    CDF[BinomialDistribution[n2, p], c2 - k], {k, c1 + 1, c2}];
```

$$pII[p\_] = \sum_{k=c_1+1}^{c_2} PDF[BinomialDistribution[n_1, p], k] \times CDF[BinomialDistribution[n_2, p], c_2 - k];$$

```
ntot = 500;
```

$$AOQ[p\_] = \frac{p \times ((n_{tot} - n_1) \times pI[p] + (n_{tot} - n_1 - n_2) \times pII[p])}{n_{tot}};$$

```
Round[pI[0.03], 0.0001]

Round[pII[0.03], 0.0001]
(0.8802 - 0.5438) * 0.5438

Round[AOQ[0.03], 0.000001]
0.03 * ((500 - 20) * 0.5438 + (500 - 20 - 20) * 0.182934) / 500
```

```
0.5438

0.1829

0.182934

0.02071

0.0207104
```