Reliability and Quality Control

LMAC, MMA

2nd. Semester – 2022/2023
2023/07/12 — 8:00
Exam 2

Duration: **120** minutes

- Add your answers to this and the following page.
- Please justify all your answers.
- This test has TWO PAGES. The total of points is 20.0.

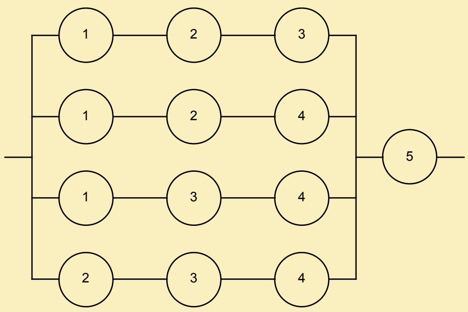
1. A solar panel consists of arrays of photovoltaic cells. The solar panel requires three out of four arrays (components $1,2,3,4$) and an inverter (component 5) to work.

(a) i) Draw a reliability block diagram of the solar panel system. **(2.0)**

ii) Identify the minimal path sets of this system.

iii) Provide an expression for its structure function in terms of the minimal path sets (do not simplify the expression).

- **Reliability block diagram**



- **Minimal path sets**

$\mathcal{P}_1 = \{1,2,3,5\}$, $\mathcal{P}_2 = \{1,2,4,5\}$, $\mathcal{P}_3 = \{1,3,4,5\}$, $\mathcal{P}_4 = \{2,3,4,5\}$,

$p^* = \binom{4}{3}$ minimal path sets

- **Structure function** (in terms of the minimal path sets)

$$\phi(\underline{X}) \overset{Th.1.30}{=} 1 - \prod_{j=1}^{p^*}\left(1 - \prod_{i\in\mathcal{P}_j} X_i\right)$$
$$= 1-(1-X_1X_2X_3X_5)\times(1-X_1X_2X_4X_5)\times(1-X_1X_3X_4X_5)\times(1-X_2X_3X_4X_5)$$

(b) Admit that the components of the solar panel system operate independently, and their reliabilities are equal to $p_i = p$ $(i=1,\ldots,5)$. **(1.5)**

Determine the reliability of this system.

Note: Do not use the structure function previously obtained.

- **Reliability**

$$\begin{aligned}
r(\underline{p}) &= E[\phi(\underline{X})] \\
&= E[\phi_{3-out-4}(X_1,X_2,X_3,X_4)\times X_5] \\
&\overset{X_i\ indep.}{=} P[\phi_{3-out-4}(X_1,X_2,X_3,X_4)=1]\times P(X_5=1) \\
&\overset{(1.6)}{=} P\left[\sum_{i=1}^{4} X_i \geq 4-3\right]\times P(X_5=1) \\
&\overset{X_i\sim_{i.i.d.}Ber(p),\ (1.22)}{=} \left[\binom{4}{3}p^3(1-p)+\binom{4}{3}p^4(1-p)^0\right]\times p \\
&= (4p^3-4p^4+4^4)\times p \\
&= p^4(4-3p).
\end{aligned}$$

(c) Now, admit that: the times to failure of components 1, 2, 3, 4, and 5 of the solar panel system are independent and exponentially distributed with parameter λ. **(2.5)**
Write the time to failure of the data center system (T) in terms of the failure times of these five components. Derive an expression for $R_T(t)$ and $E(T)$.

- **Time to failure** (components)
  T_i = time to failure of component i
  $T_i \stackrel{i.i.d.}{\sim} \exp(\lambda), \quad i = 1,2,3,4,5$
  $R_i(t) = P(T_i > t) = R(t) = \begin{cases} e^{-\lambda t}, & t \geq 0 \\ 1, & t < 0 \end{cases}$
- **Time to failure** (system)
  $T \stackrel{(2.6),(2.2)}{=} \min\{T_{(4-2+1)}, T_5\}$
- **Requested reliability**

$$\begin{aligned} R_T(t) &= P(T > t) \\ &\stackrel{(2.9)}{=} r(e^{-\lambda t}, \ldots, e^{-\lambda t}) \\ &\stackrel{(a),\, p=e^{-\lambda t}}{=} \left(e^{-\lambda t}\right)^4 \times \left(4 - 3e^{-\lambda t}\right) \\ &= e^{-4\lambda t} \times \left(4 - 3e^{-\lambda t}\right), \quad t > 0 \end{aligned}$$

- **Expected value of T**

$$\begin{aligned} E(T) &\stackrel{(2.10)}{=} \int_0^{+\infty} R(t)\, dt \\ &= \int_0^{+\infty} e^{-4\lambda t} \times \left(4 - 3e^{-\lambda t}\right) dt \\ &= \frac{1}{\lambda} \int_0^{+\infty} 4\lambda e^{-4\lambda t}\, dt - \frac{3}{5\lambda} \int_0^{+\infty} 5\lambda\, e^{-\lambda t}\, dt \\ &= \frac{1}{\lambda} \int_0^{+\infty} f_{exp(4\lambda)}(t)\, dt - \frac{3}{5\lambda} \int_0^{+\infty} f_{exp(5\lambda)}(t)\, dt \\ &= \frac{1}{\lambda} \times 1 - \frac{3}{5\lambda} \times 1 \\ &= \frac{2}{5\lambda}. \end{aligned}$$

(d) T has hazard rate function given by $\lambda\left(\frac{3}{3-4e^{\lambda t}} + 4\right)$, for $t \geq 0$. **(1.0)**
i) Prove that $T \in IHR$.
ii) Determine the limit of the hazard rate function when $t \to +\infty$. Comment.

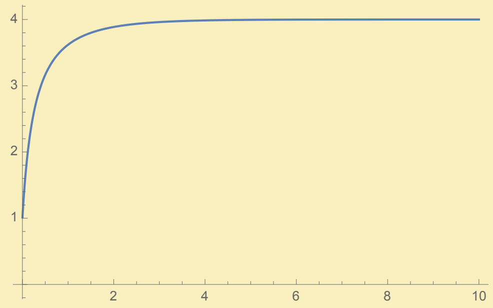
- **Hazard rate function of T**
  $h(t) = \frac{f_T(t)}{R_R(t)} = \lambda\left(\frac{3}{3-4e^{\lambda t}} + 4\right), \quad t > 0.$
- **Devising the stochastic ageing character of T**
  Since T is a non-negative continuous r.v., $T \in IHR$ iff $h(t)$ is an increasing function for $t \geq 0$ (see the second part of Definition 3.14). We have:

$$\frac{d\,h(t)}{dt} = \frac{12\lambda^2\, e^{\lambda t}}{(4 - 3e^{\lambda t})^2} \geq 0, \quad t \geq 0.$$

  Hence, $h(t) \uparrow_t$ and $T \in IHR$.
- **Requested limit**

$$\lim_{t \to +\infty} \lambda(t) = \lim_{t \to +\infty} \lambda\left(\frac{3}{3 - 4e^{\lambda t}} + 4\right) = 4\lambda.$$

- **Comment**
  The hazard rate function increases and "soon" gets closer to its limiting value 4λ, as shown by the following plot for $\lambda = 1$:



This means that the time to failure has a "sort of memoryless character" for large values of t.

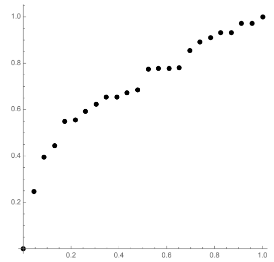
(e) Capitalize on the stochastic ageing behaviour of T to provide an upper limit to $V(T)$. **(1.0)**

- **Requested upper limit to $V(T)$**

  By Proposition 3.36, $T \in IHR \Rightarrow T \in IHRA$, thus we can invoke Corollary 3.54 and write

$$CV(T) = \frac{\sqrt{V(T)}}{E(T)} \leq 1 \Leftrightarrow V(T) \leq E^2(T) = \left(\frac{2}{5\lambda}\right)^2 = \frac{4}{25\lambda}.$$

2. An engineer analyzed data on the endurance of deep groove ball bearings. **(2.0)**



She registered the number of million revolutions before failure for 23 such ball bearings. The distinct ordered failure times led her to the TTT plot on the right. She also performed a Kolmogorov-Smirnov test with null hypothesis $H_0 : T \sim \text{Weibull}(\delta = 80, \alpha = 2)$, and obtained the ML estimates $\hat{\delta} = 81.9$ and $\hat{\alpha} = 2.1$.

i) What conclusion can the engineer draw from the TTT plot?

ii) Comment on the p-value of the goodness-of-fit test ($p - value = 0.795$), namely in light of the TTT plot above.

iii) Determine the ML estimate of the third quartile of the number of million revolutions before failure of a single ball bearing.

- **Comment on the TTT plot**

  The TTT plot suggests a concave curve above the 45^o line, thus the data can be fitted by an IHR distribution,[1] according to Note 5.5 of the lecture notes.

- **Comment on the p-value of the Kolmogorov-Smirnov test**

  [Recall that the $p-$value is the largest significance level leading to the non rejection of the null hypothesis. Thus,] for these particular data set and null hypothesis $H_0 : T \sim \text{Weibull}(\delta = 1, \alpha = 2)$: should not reject H_0 for any significance level $\alpha_0 \leq p-\text{value} = 0.795$, specifically the usual significance levels $(1\%, 5\%, 10\%)$. This decision is consistent with the TTT plot because the conjectured Weibull distribution in H_0 is IHR after all it has a shape parameter $\alpha = 2$ larger than 1.

- **Failure time**

  $T \sim \text{Weibull}(\delta, \alpha), \quad \delta, \alpha > 0$ (UNKNOWN)

- **ML estimate of the third quartile of the failure time**

  Since

$$\xi_{0.75} \quad : \quad P(T \leq \xi_{0.75}) = 0.75 \Leftrightarrow R_T(\xi_{0.75}) = 1 - 0.75 \Leftrightarrow \exp\left[-\left(\frac{\xi_{0.75}}{\delta}\right)^{\alpha}\right] = 1 - 0.75$$
$$\xi_{0.75} = \delta\,[-\ln(1-0.75)]^{1/\alpha} = h(\delta, \alpha),$$

  we can invoke the invariance property of the ML estimators and get the requested ML estimate:

$$\widehat{me} = \hat{h}(\delta, \alpha) = h(\hat{\delta}, \hat{\alpha}) = \hat{\delta}\,[-\ln(1-0.75)]^{1/\hat{\alpha}} = 81.9 \times [-\ln(1-0.75)]^{1/2.1} \simeq 95.682854.$$

3. The net weight (in oz) of a dry bleach product is to be monitored by a $\bar{X}$-chart with 3-sigma limits, using a sample size of $n = 4$. Data for ten days are shown below:

Sample (N)	1	2	3	4	5	6	7	8	9	10
Mean net weight $(\bar{x}_N)$	16.1	16.075	16.3	16.2	16.275	16.3	16.25	16.15	16.3	16.35

Admit that the data refer to independent output and a normally distributed quality characteristic with nominal mean and variance equal to $\mu_0 = 16.2$ and $\sigma_0^2 = 0.01$.

(a) i) Obtain the control limits, LCL_μ and UCL_μ, of this control chart. **(1.5)**

ii) Does the production process appear to be in statistical control?

iii) Is this chart able to detect shifts in the process standard deviation σ? Justify your answer.

- **Quality characteristic**
  $X \sim \text{normal}(\mu, \sigma^2)$, where $\mu = \mu_0 + \delta\sigma_0/\sqrt{n}$ and $\sigma^2 = \theta^2\sigma_0^2$ represent the process mean and variance (respectively).
- **Control statistic of the $\bar{X}$-chart**
  $\bar{X}_N$ = mean of random sample N, $\quad N \in \mathbb{N}$
- **3-sigma control limits**

$$LCL_\mu = \mu_0 - 3 \times \frac{\sigma_0}{\sqrt{n}} = 16.2 - 3 \times \frac{0.1}{\sqrt{4}} = 16.05$$

$$UCL_\mu = \mu_0 + 3 \times \frac{\sigma_0}{\sqrt{n}} = 16.2 + 3 \times \frac{0.1}{\sqrt{4}} = 16.35$$

- **Checking whether the process is in statistical control**
  Since $\bar{x}_N \in [LCL_\mu, UCL_\mu] = [16.05, 16.35]$, for all $N = 1, \ldots, 10$, we deem the production process (mean) in-control.
- **Comment**
  Yes! The $\bar{X}$-chart is sensitive to shifts in σ^2 because the distribution of its control statistic depends of the magnitude of shift in the standard deviation, $\theta = \sigma/\sigma_0$. Indeed, $\bar{X}_N \overset{indep}{\sim} \text{normal}(\mu = \mu_0 + \delta\sigma_0/\sqrt{n}, \sigma^2 = (\theta\sigma_0)^2)$, for $N \in \mathbb{N}$[, and the probability that this chart triggers a signal is equal to $\xi_\mu(\delta,\theta) = 1 - \{\Phi[(3-\delta)/\theta] - \Phi[(3-\delta)/\theta]\}$, according to the formula (9.7) of the lecture notes.]

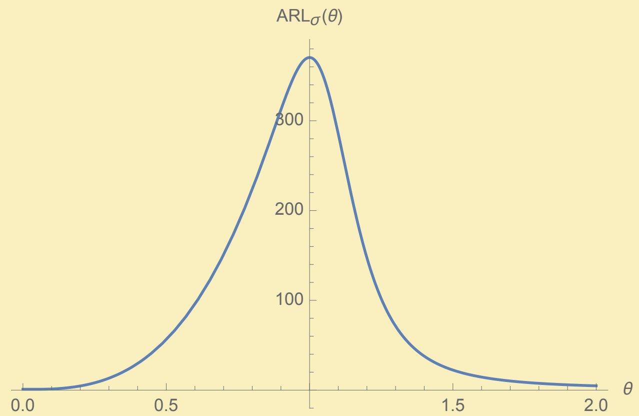
(b) Since the leader of the quality-improvement team anticipated both downward and upward shifts in the process variance, she decided to adopt an ARL-unbiased S^2−chart with in-control ARL equal to $ARL_\sigma(1) \simeq 370.4$. This chart has control limits $LCL_\sigma = \frac{\sigma_0^2}{n-1} \times 0.042541$ and $UCL_\sigma = \frac{\sigma_0^2}{n-1} \times 18.222384$. **(2.0)**

i) Sketch and comment the ARL profile of this chart.

ii) Obtain the ARL value of this chart in the presence of a 1% decrease in the process standard deviation (i.e., $\theta = \sigma/\sigma_0 = 0.99$).

Note: $F_{\chi_{(3)}}(0.043405) \simeq 0.002374$ and $F_{\chi_{(3)}}(18.592372) \simeq 0.999668$.

- **(Sketch of the)** $ARL_\sigma(\theta)$ **curve**



- **Comment**
  The ARL curve achieves a maximum at $\theta = 1$ (in-control situation), thus this chart takes longer in average to trigger a false alarm than to detect any increase or decrease in the process standard deviation.

- **Control statistic**
  S_N^2 = variance of the random sample N, $\quad N \in \mathbb{N}$

- **Relevant distribution**
  $\frac{(n-1)S_N^2}{(\theta\sigma_0)^2} \sim \chi^2_{(n-1)}$, where $\theta = \sigma/\sigma_0$ $(\theta > 0)$ represents a shift in the standard deviation σ

- **Control limits of the ARL-unbiased S^2-chart**
  $LCL_\sigma = \frac{\sigma_0^2}{n-1} \times 0.042541$
  $UCL_\sigma = \frac{\sigma_0^2}{n-1} \times 18.222384$

- **Probability of triggering a signal**
  Adapting formula (9.8) of the lecture notes, we can add that the ARL-unbiased S^2-chart triggers a signal with probability

$$\begin{aligned}
\xi_\sigma(\theta) &= P\left(S_N^2 \not\in [LCL_\sigma, UCL_\sigma] \mid \theta\right) \\
&= 1 - \left\{F_{\chi^2_{(n-1)}}\left[\frac{(n-1)\,UCL_\sigma}{\sigma^2}\right] - F_{\chi^2_{(n-1)}}\left[\frac{(n-1)\,LCL_\sigma}{\sigma^2}\right]\right\} \\
&= \ldots \\
&= 1 - \left[F_{\chi^2_{(n-1)}}\left(\frac{18.222384}{\theta^2}\right) - F_{\chi^2_{(n-1)}}\left(\frac{0.042541}{\theta^2}\right)\right], \quad \theta > 0.
\end{aligned}$$

- **Run length**
  $RL_\sigma(\theta) \sim \text{geometric}\,(\xi_\sigma(\theta))$

- **Requested ARL**

$$\begin{aligned}
ARL_\sigma(0.99) &= \frac{1}{\xi_\sigma(0.99)} \\
&= \frac{1}{1 - \left[F_{\chi^2_{(3)}}\left(\frac{18.222384}{0.99^2}\right) - F_{\chi^2_{(3)}}\left(\frac{0.042541}{0.99^2}\right)\right]} \\
&\simeq \frac{1}{1 - \left[F_{\chi^2_{(3)}}(18.592372) - F_{\chi^2_{(3)}}(0.043405)\right]} \\
&\simeq \frac{1}{1-(0.999668 - 0.002374)} \\
&\simeq 369.549.
\end{aligned}$$

  [Expectedly, this out-of-control ARL of the ARL-unbiased S^2-chart is smaller than than its in-control ARL, $ARL(1) \simeq 370.4$.]

(c) Find and compare the ARL of the joint scheme whose constituent charts are described in (a) and (b), when $(\delta,\theta) = (\frac{\mu-\mu_0}{\sigma_0/\sqrt{n}}, \frac{\sigma}{\sigma_0}) = (0,1)$ and $(\delta,\theta) = (0, 0.99)$. **(1.5)**

- **Probability of a signal**
  [Following Exercise 10.38 of the lecture notes, the probability of a false alarm being triggered by the joint scheme is given by]

$$\xi_{\mu,\sigma}(\delta,\theta) = \xi_\mu(\delta,\theta) + \xi_\sigma(\theta) - \xi_\mu(\delta,\theta) \times \xi_\sigma(\theta),$$

  where $\xi_\mu(\delta,\theta) = 1 - \{\Phi[(3-\delta)/\theta] - \Phi[(3-\delta)/\theta]\}$.

- **ARL of the joint scheme**
  The in-control RL of this Shewhart-type joint scheme is such that

$$\begin{aligned}
RL_{\mu,\sigma}(\delta,\theta) &\stackrel{st}{=} \min\{RL_\mu(\delta,\theta), RL_\sigma(\theta)\} \sim \text{geometric}\left(\xi_{\mu,\sigma}(\delta,\theta)\right) \\
ARL_{\mu,\sigma}(\delta,\theta) &= \frac{1}{\xi_{\mu,\sigma}(\delta,\theta)}
\end{aligned}$$

$$
\begin{aligned}
ARL_{\mu,\sigma}(\delta,\theta) &= \frac{1}{\xi_\mu(\delta,\theta)+\xi_\sigma(\theta)-\xi_\mu(\delta,\theta)\times\xi_\sigma(\theta)} \\
&= \frac{1}{\frac{1}{ARL_\mu(\delta,\theta)}+\frac{1}{ARL_\sigma(\theta)}-\frac{1}{ARL_\mu(\delta,\theta)}\times\frac{1}{ARL_\sigma(\theta)}} \\
&= \frac{ARL_\mu(\delta,\theta)\times ARL_\sigma(\theta)}{ARL_\mu(\delta,\theta)+ARL_\sigma(\theta)-1}.
\end{aligned}
$$

- **Requested ARL values**

  Since

$$
\begin{aligned}
ARL_\mu(0,1) &\simeq ARL_\sigma(1)\simeq 370.4 \\
ARL_\mu(0,0.99) &= \frac{1}{1-\{\Phi[(3-0)/0.99]\times\Phi[(-3-0)/0.99]\}} \\
&\simeq \frac{1}{1-[\Phi(3.03)\times\Phi(-3.03)]} \\
&\simeq \frac{1}{1-[0.998777-(1-0.998777)]} \\
&\simeq 408.831 \\
ARL_\sigma(0.99) &\stackrel{(b)}{=} 369.549,
\end{aligned}
$$

  we get

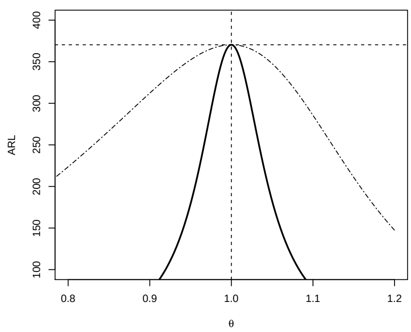
$$
\begin{aligned}
ARL_{\mu,\sigma}(0,1) &= \frac{ARL_\mu(0,1)\times ARL_\sigma(1)}{ARL_\mu(0,1)+ARL_\sigma(1)-1} \\
&\simeq \frac{370.4\times 370.4}{370.4+370.4-1} \\
&\simeq 185.450 \\
ARL_{\mu,\sigma}(0,0.99) &\stackrel{(b)}{=} \frac{408.831\times 369.549}{408.831+369.549-1} \\
&\simeq 194.349.
\end{aligned}
$$

  [In this case, we have $ARL(0,1) = 185.450 < ARL(0,0.99) = 194.349$. Consequently, the joint scheme is ARL-biased in the presence of sustained shifts in σ.
  Note that the $\bar{X}$-chart is ARL-unbiased when it comes to shifts in μ; however, it is ARL-biased in the presence of shifts in σ because $ARL_\mu(0,1) \simeq 370.4 < 408.831$. Hence, the undesirable behaviour of the joint scheme for μ and σ.]

(d) A statistician suggested an ARL-unbiased EWMA-S^2 chart for monitoring the variance of this independent normal output. **(1.0)**

i) Identify the control statistic of this chart, when the initial value of the latter is equal to σ_0^2.

ii) The ARL profiles of the ARL-unbiased EWMA-S^2 and S^2-charts can be found in the plot on the right. Which profile corresponds to the ARL-unbiased EWMA-S^2 chart? Comment.



- **Control statistic of the EWMA-S^2 chart for the variance of independent normal output**

  [On page 9 of the report entitled *On the pivotal role of ARL-unbiased charts in SPC*, we find]

$$
Z_N = \begin{cases} \sigma_0^0, & N=0 \\ (1-\lambda)\,Z_{N-1}+\lambda\,S_N^2, & N\in\mathbb{N}, \end{cases}
$$

  where $\lambda \in (0,1]$.

- **Identifying the requested ARL profile**

  The solid line corresponds to the ARL profile of the EWMA-S^2 chart because it is well known that EWMA charts tend to have smaller ARL values than the Shewhart charts, namely in the presence of small and moderate shifts in the parameter being monitored.

4. A single sampling plan for attributes is to be developed based on a producer's and consumer's risk points $(p_1 = AQL = 0.5\%, 1-\alpha = 0.95)$ and $(p_2 = LTPD = 3.5\%, \beta = 0.05)$. **(2.0)**

i) Verify that the pair $(n, c) = (222, 3)$ meets these risk points.

ii) Compute the approximate probability of acceptance if incoming lots contain 3% nonconforming units.

Note: Use the normal approximation without continuity correction to solve i) and ii).

- **Single sampling plan for attributes**

  $n = 222$ (sample size), $c = 3$ (acceptance number)

- **Producer's and consumer's risk points**

  $(p_1 = AQL = 0.5\%,\ 1-\alpha = 0.95)$, $(p_2 = LTPD = 3.5\%,\ \beta = 0.05)$

- **Auxiliary r.v. and its approximate distribution**

  D = number of defective items in the sample

  $D \stackrel{a}{\sim} \text{binomial}(n,p) \stackrel{a}{\sim} \text{normal}(np, np(1-p))$

- **Probability of lot acceptance**

  $P_a(p) \simeq P(D \leq c) \simeq F_{binomial(n,p)}(c) \simeq \Phi\left[\frac{c-np}{\sqrt{np(1-p)}}\right]$

  Requested verification

  If we consider $n = 222$ and $c = 3$ then

$$\begin{aligned}
P_a(p_1) &\simeq \Phi\left[\frac{3 - 222 \times 0.005}{\sqrt{222 \times 0.005 \times (1-0.005)}}\right] \\
&\simeq \Phi(1.8) \\
&\stackrel{table}{=} 0.9641 \\
&\geq 1-\alpha = 0.95 \\
P_a(p_2) &= \Phi\left[\frac{3 - 222 \times 0.035}{\sqrt{222 \times 0.035 \times (1-0.035)}}\right] \\
&\simeq \Phi(-1.74) \\
&\stackrel{table}{=} 1 - 0.9591 \\
&= 0.0409 \\
&\leq \beta = 0.05.
\end{aligned}$$

  Hence, the sampling plan complies with the two given risk points.

- **Requested acceptance probability**

$$P_a(0.03) = \Phi\left[\frac{3 - 222 \times 0.03}{\sqrt{222 \times 0.03 \times (1-0.03)}}\right] \simeq \Phi(-1.44) = 1 - \Phi(1.44) \stackrel{table}{=} 1 - 0.9251 = 0.0749.$$

5. A producer of electronic components for the automobile industry uses the following double sampling plan to inspect batches of $N = 500$ incoming integrated circuits with $n_1 = n_2 = 20$, $c_1 = 0$, and $c_2 = 1$. Moreover, rejected lots are screened and all defective items reworked and returned to the lot. **(2.0)**

Calculate the average outgoing quality (AOQ) when incoming lots contain 3% nonconforming units.

- **Double sampling plan for attributes with rectifying inspection**

  $n_1 = n_2 = 20$ (sample sizes)

  $c_1 = 0$, $c_2 = 1$ (acceptance numbers)

- **Auxiliary r.v. and their approximate distributions**

  D_i = number of defective units in the i^{th} sample

$D_i \overset{a}{\sim} \text{binomial}(n_i, p),\ i = 1, 2$

- **Probability of accepting the lot in the first stage of the sampling plan**

$$\begin{aligned} P_a^I(p) &\overset{(13.16)}{=} P(D_1 \leq c_1) \\ &\simeq F_{binomial(n_1,p)}(c_1) \\ &\overset{p=0.03}{=} F_{binomial(20,0.03)}(0) \\ &\overset{table}{=} 0.5438 \end{aligned}$$

- **Probability of accepting the lot in the second stage of the sampling plan**

$$\begin{aligned} P_a^{II}(p) &\overset{(13.17)}{=} P(c_1 < D_1 \leq c_2,\ D_1 + D_2 \leq c_2) \\ &= \sum_{k=c_1+1}^{c_2} P(D_1 = k) \times P(D_2 \leq c_2 - k) \\ &\simeq \sum_{k=c_1+1}^{c_2} P_{binomial(n_1,p)}(k) \times F_{binomial(n_2,p)}(c_2 - k) \\ &\overset{p=0.03}{=} \sum_{k=1}^{1} [F_{binomial(20,0.03)}(k) - F_{binomial(20,0.03)}(k-1)] \times F_{binomial(20,0.03)}(2-k) \\ &= (0.8802 - 0.5438) \times 0.5438 \\ &\simeq 0.182934 \end{aligned}$$

- **Average outgoing quality of a double sampling plan with rectifying inspection**

$$\begin{aligned} AOQ(p) &\overset{(13.26)}{=} \frac{p[(N-n_1)\,P_a^I(p) + (N-n_1-n_2)\,P_a^{II}(p)]}{N} \\ &\overset{(a)}{=} \frac{0.03 \times [(500-20) \times 0.5438 + (500-20-20) \times 0.182934]}{500} \\ &\simeq 0.0207104. \end{aligned}$$