Reliability and Quality Control

LMAC, MMA

2nd. Semester – 2022/2023
2023/06/28 — 8:00
Exam 1

Duration: **120** minutes

- Add your answers to this and the following page.
- Please justify all your answers.
- This test has TWO PAGES. The total of points is 20.0.

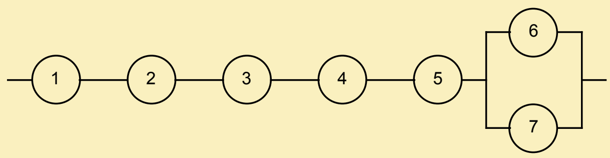
1. A data center needs three servers (components 1, 2, and 3) to work, one power supply (component 4), one cooling system (component 5), and one network connection. This (network) connection consists of two parallel connections (components 6 and 7).

(a) i) Draw a reliability block diagram of the data center system. **(1.5)**

ii) Identify the minimal path sets of this system.

iii) Provide an expression for its structure function in terms of the minimal path sets.

- **Reliability block diagram**



- **Minimal path sets**
  $\mathcal{P}_1 = \{1,2,3,4,5,6\}$
  $\mathcal{P}_2 = \{1,2,3,4,5,7\}$
  $p^* = 2$ minimal path sets

- **Structure function** (in terms of the minimal path sets)

$$\begin{aligned}\phi(\underline{X}) &\overset{Th. 1.30}{=} 1 - \prod_{j=1}^{p^*}\left(1 - \prod_{i\in\mathcal{P}_j} X_i\right)\\ &= 1-(1-X_1X_2X_3X_4X_5X_6)\times(1-X_1X_2X_3X_4X_5X_7)\\ &\overset{X_i^2 \sim X_i}{=} X_1X_2X_3X_4X_5X_6 + X_1X_2X_3X_4X_5X_7 - X_1X_2X_3X_4X_5X_6X_7\end{aligned}$$

(b) Admit that the components of the data center system operate independently, and their reliabilities are equal to $p_i = p$ $(i = 1,\ldots,7)$. **(1.0)**

Determine an upper bound to the reliability of this system.

- **Upper bound**

$$\begin{aligned}r(\underline{p}) = E[\phi(\underline{X})] &\overset{Th., 1.68}{\leq} 1 - \prod_{j=1}^{p^*}\left(1 - \prod_{i\in\mathcal{P}_j} p_i\right)\\ &\overset{p_i=p}{=} 1 - \prod_{j=1}^{p^*}\left(1-p^{\#\mathcal{P}_j}\right)\\ &\overset{\#\mathcal{P}_j=6,\forall j}{=} 1 - \prod_{j=1}^{p^*}\left(1-p^6\right)\\ &\overset{p^*=2}{=} 1-\left(1-p^6\right)^2 \qquad [2p^6 - p^{12} = p^6\left(2-p^6\right)].\end{aligned}$$

(c) Now, admit that: the power supply is a subsystem with component $4a$ and a standby component $4b$; the times to failure of components 1, 2, 3, $4a$, $4b$, 5, 6, and 7 of the data center system are independent and exponentially distributed with parameter λ. **(1.5)**

Write the time to failure of the data center system (T) in terms of the failure times of these eights components. Derive an expression for $R_T(t)$.

- **Time to failure** (components)
  T_i = time to failure of component i
  $T_i \overset{i.i.d.}{\sim} \exp(\lambda)$, $i = 1,2,3,5,6,7$, and $i = 4a, 4b$ [because the power supply has a standby component]
  For $i = 1,2,3,5,6,7$,

$$R_i(t) = P(T_i > t) = R(t) = \begin{cases} e^{-\lambda t}, & t \geq 0 \\ 1, & t < 0. \end{cases} \quad (1)$$

  For $i = 4$, $T_4 = T_{4a} + T_{4b} \sim \text{gamma}(2,\lambda) \sim \text{Erlang}(2,\lambda)$, thus

$$R_4(t) = R_{Erlang(2,\lambda)}(t) \overset{(4.27)}{=} \begin{cases} e^{-\lambda t} + e^{-\lambda t}(\lambda t) = R(t)(1+\lambda t), & t \geq 0 \\ 1, & t < 0. \end{cases} \quad (2)$$

- **Time to failure** (system)
  $T = \min\{T_1, T_2, T_3, T_{4a} + T_{4b}, T_5, \max\{T_6, T_7\}\}$

- **Requested reliability**

$$\begin{aligned} R_T(t) &= P(T > t) \\ &= R_1(t) \times R_2(t) \times R_3(t) \times R_4(t) \times R_5(t) \times \{1 - [1 - R_6(t)] \times [1 - R_7(t)]\} \\ &\overset{(1),(2)}{=} [R(t)]^3 \times [R(t)(1+\lambda t)] \times R(t) \times \{1 - [1 - R(t)]^2\} \\ &= [R(t)]^4 \times [R(t)(1+\lambda t)] \times \{[2R(t) - R(t)]^2\} \\ &= [R(t)]^6 \times (1+\lambda t) \times [2 - R(t)] \\ &= e^{-6\lambda t} \times (1+\lambda t) \times \left(2 - e^{-\lambda t}\right), \quad t > 0. \end{aligned}$$

(d) Determine a lower bound (as sharp as reasonably possible) for the expected time to failure of the data center system. **(1.5)**
Note: Assume all failure times are NBUE.

- **[Minimal path sets**
  $\mathcal{P}_1 = \{1,2,3,4,5,6\}$, $\mathcal{P}_2 = \{1,2,3,4,5,7\}$, $p^* = 2$ minimal path sets]

- **Individual expected times to failure**
  $\mu_i = E[\exp(\lambda)] = \lambda^{-1}$, $i = 1,2,3,5,6,7$
  $\mu_4 = E[\text{Erlang}(2,\lambda)] = 2\lambda^{-1}$

- **Upper bound for $\mu = E(T)$**
  Since all times to failure are independent and $T_i \in NBUE$, we can apply Th. 3.65 and provide an lower bound to $\mu = E(T)$:

$$\begin{aligned} \mu &\overset{Th.3.65}{\geq} \max_{j=1,\ldots,p^*} \left\{ \left(\sum_{i \in \mathcal{P}_j} \mu_i^{-1} \right)^{-1} \right\} \\ &= \left(\sum_{i \in \mathcal{P}_1} \mu_i^{-1} \right)^{-1} \quad \text{[two "similar" path sets]} \\ &\overset{\mu_i = \frac{1}{\lambda}, i \neq 4;\ \mu_4 = \frac{2}{\lambda}}{=} \left(\lambda + \lambda + \lambda + \frac{\lambda}{2} + \lambda + \lambda \right)^{-1} \\ &= \frac{2}{11\lambda}. \end{aligned}$$

(e) T has hazard rate function given by $\lambda\left(-\frac{1}{\lambda t+1} + \frac{1}{1-2e^{\lambda t}} + 6\right)$, for $t \geq 0$. **(1.0)**
i) Prove that $T \in IHR$.

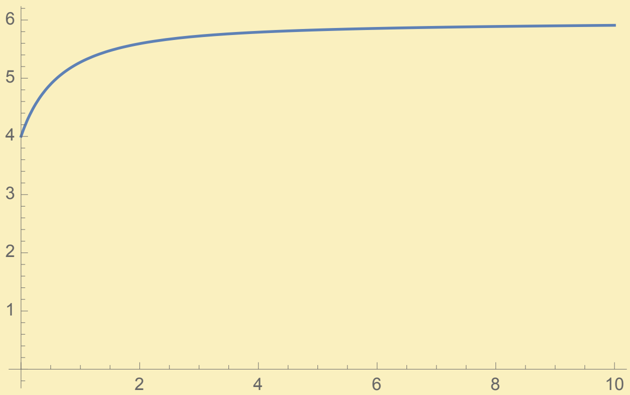
ii) Determine the limit of the hazard rate function when $t \to +\infty$. Comment.

- **Hazard rate function of T**
  $h(t) = \frac{f_T(t)}{R_R(t)} = \lambda\left(-\frac{1}{\lambda t+1}\frac{1}{1-2e^{\lambda t}}+6\right), \quad t > 0.$

- **Devising the stochastic ageing character of T**
  Since T is a non-negative continuous r.v., $T \in IHR$ iff $h(t)$ is an increasing function for $t \geq 0$ (see the second part of Definition 3.14). We have:
  $$\begin{aligned} \frac{d\,h(t)}{dt} &= \frac{\lambda^2}{(\lambda t+1)^2} + \frac{2\lambda^2 e^{\lambda t}}{(1-2e^{\lambda t})^2} \\ &\geq 0, \quad t \geq 0. \end{aligned}$$
  Hence, $h(t) \uparrow_t$ and $T \in IHR$.

- **Requested limit**
  $$\begin{aligned} \lim_{t\to+\infty} \lambda(t) &= \lim_{t\to+\infty} \lambda\left(-\frac{1}{\lambda t+1} + \frac{1}{1-2e^{\lambda t}} + 6\right) \\ &= 6\lambda. \end{aligned}$$

- **Comment**
  The hazard rate function increases and "soon" gets closer to its limiting value 6λ, as shown by the following plot for $\lambda = 1$:



  This means that the time to failure has a "sort of memoryless character" for large values of t.

(f) Capitalize on the stochastic ageing behaviour of T and on the fact that $\xi_{0.5} = F_T^{-1}(0.5) \simeq 0.164423$, when $\lambda = 1$, to provide a lower and an upper limit to $E(T)$. **(1.0)**
Compare the lower limit with the one obtained in (d).

- **Requested lower and upper limit to $E(T)$**
  Since $T \in IHR$, $p = 0.5 \leq 1 - e^{-1} \simeq 0.632121$, and we were given the median of T ($\xi_{0.5} \simeq 0.164423$), we can invoke Theorem 3.52 and write
  $$\begin{aligned} -\frac{\xi_p \times p}{\log(1-p)} &\leq E(T) \leq -\frac{\xi_p}{\log(1-p)} \\ -\frac{0.632121 \times 0.5}{\log(1-0.5)} &\leq E(T) \leq -\frac{0.632121}{\log(1-0.5)} \\ 0.118606 &\leq E(T) \leq 0.237213. \end{aligned}$$

- **Comment**
  The lower limit we just obtained, 0.118606, is less sharp than the one we got in (d), $\frac{2}{11\lambda} = 0.1818(18)$.
  Note that in (d) we used information referring to all the components (NBUE, $E(T_i)$), and the configuration of the system (minimal path sets), whereas in (f) we used *less* information, concerning the system (IHR, median).
  [The true value of $E(T)$ is $\frac{199}{882\lambda} \simeq 0.225624$.]

2. A statistician working in a plant simultaneously activated 20 identical transfer pumps and decided that the test was terminated at the 12th failure. The ordered observed failure times are: 0.8, 3.6, 10.2, 23.3, 28.0, 30.4, 31.6, 41.2, 54.0, 73.2, 89.6, 96.0. **(2.5)**

i) What type of censoring is this and what is the value of the cumulative total time in test?

ii) After stating a reasonable distributional assumption, obtain the ML estimate and a 95% confidence interval for the third quartile of the time to failure of a transfer pump.

- **Censored data**

  Since the test ended when the 12th failure occurred and it seems that none of the 12 (out of 20) transfer pumps that failed were replaced during the test, we are dealing with

  - Type II/item censored testing without replacement
  - $n = 20$
  - $r = 12$
  - $(t_{(1)}, \ldots, t_{(r)}) = (0.8, \ldots, 96.0)$.

- **Cumulative total time in test**

$$\begin{aligned}
\tilde{t} &\overset{Def.\,5.17}{=} \sum_{i=1}^{r} t_{(i)} + (n-r) \times t_{(r)} \\
&= (0.8 + \cdots + 96.0) + (20-12) \times 96.0 \\
&= 1249.9.
\end{aligned}$$

- **Failure times; distribution assumption**

  T_i = time to failure of transfer pump i

  $T_i \overset{i.i.d.}{\sim} T \sim \text{exponential}(\lambda), \quad i = 1, \ldots, n$

- **ML estimate of $F_T^{-1}(0.75)$**

  According to tables 5.10 and 5.13 and invoking the invariance property of the ML estimators, the ML estimate of $F_T^{-1}(0.75) = -\frac{1}{\lambda}\ln(1-0.75)$, under Type II/item censored testing without replacement, is equal to

$$\begin{aligned}
\hat{F}_T^{-1}(0.75) &= -\frac{1}{\hat{\lambda}}\ln(1-0.75) = -\frac{1}{r/\tilde{t}}\ln(1-0.75) = -\frac{1249.9}{12} \times \ln(1-0.75) \\
&\simeq 144.394.
\end{aligned}$$

- **Confidence interval for λ**

$$\begin{aligned}
CI_{(1-\alpha)\times 100\%}(\lambda) &\overset{Table\ 5.16}{=} \left[\frac{F^{-1}_{\chi^2_{(2r)}}(\alpha/2)}{2 \times \tilde{t}}; \frac{F^{-1}_{\chi^2_{(2r)}}(1-\alpha/2)}{2 \times \tilde{t}}\right] \\
CI_{95\%}(\lambda) &= \left[\frac{F^{-1}_{\chi^2_{(2\times 12)}}(0.025)}{2 \times \tilde{t}}; \frac{F^{-1}_{\chi^2_{(2\times 12)}}(0.975)}{2 \times \tilde{t}}\right] \\
&\overset{tables}{=} \left[\frac{12.40}{2 \times 1249.9}; \frac{39.36}{2 \times 1249.9}\right] \\
&\simeq [0.0049604; 0.0157453] \\
&= [\lambda_L; \lambda_U].
\end{aligned}$$

- **Confidence interval for $F_T^{-1}(0.75)$**

  $F_T^{-1}(0.75) = -\frac{1}{\lambda}\ln(1-0.75)$ is a decreasing function of $\lambda > 0$, thus

$$\begin{aligned}
CI_{95\%}(F_T^{-1}(0.75)) &= \left[-\frac{1}{\lambda_U}\ln(1-0.75); -\frac{1}{\lambda_L}\ln(1-0.75)\right] \\
&\simeq [88.0452; 279.4725].
\end{aligned}$$

3. A supply chain engineering group monitors shipments of materials. Errors on either the delivered material or the accompanying documentation are tracked on a weekly basis. Fifty randomly selected shipments are examined and the errors recorded. Data for ten weeks are shown below:

Sample	1	2	3	4	5	6	7	8	9	10
Number of errors	2	3	8	1	1	4	1	4	5	1

(a) i) Set up a c-chart with 3–sigma limits and nominal expected number of errors (per sample) equal to $\lambda_0 = 3$. **(1.5)**

ii) Does the production process appear to be in statistical control?

iii) Is this chart able to quickly detect decreases in λ? Justify your answer.

- **Control statistic of the c−chart and its distribution**
  Y_N = number of errors in the N^{th} sample
  $Y_N \overset{indep}{\sim} \text{Poisson}(\lambda), \quad N \in \mathbb{N}$
- **3-sigma control limits**
  [The control statistic only takes values in $\mathbb{N}_0$, therefore the control limits are given by the following ceiling and floor functions of the target expected number of defects (per batch), λ_0:]

$$\begin{aligned} LCL &= \left\lceil \max\{0, \lambda_0 - 3 \times \sqrt{\lambda_0}\} \right\rceil \\ &= \left\lceil \max\left\{0, 3 - 3 \times \sqrt{3}\right\} \right\rceil \\ &= 0 \\ UCL &= \left\lfloor \lambda_0 + 3 \times \sqrt{\lambda_0} \right\rfloor \\ &= \left\lfloor 3 + 3 \times \sqrt{3} \right\rfloor \\ &= 8. \end{aligned}$$

- **Checking whether the process is in statistical control**
  Since $y_N \in [LCL, UCL] = [0, 8]$, for all $N = 1, \ldots, 10$, we deem the production process in-control.
- **Comment**
  No! This chart is unable to quickly detect decreases in λ.
  Since $LCL = 0$, we are dealing with an upper one-sided c-chart with 3-sigma limits, the associated ARL function decreases with λ ($\lambda \in \mathbb{R}^+$), and this c-chart is unable to signal a decrease in λ sooner (in average) than to trigger a false alarm.

(b) Since the leader of the engineering group anticipated both downward and upward shifts, she decided to adopt an ARL-unbiased c−chart. This chart has control limits $L = 0$ and $U = 10$, in-control ARL equal to $ARL(0) \simeq 370.4$, and triggers a signal with: **(2.0)**

- probability one if the sample number of errors is below L or above U;
- probabilities $\gamma_L = 0.038733$ and $\gamma_U = 0.591561$ if the sample number of errors is equal to L and U, respectively.

Compare the in-control ARL with the out-of-control ARL when the expected number of errors (per sample) decreases from its target value $\lambda_0 = 3$ to 2. Comment.

- **Probability of triggering a signal when $\lambda = 3 + \theta = 2$**
  Judging by the description above, when $\lambda = \lambda_0 + \theta$ ($\theta \in (-\lambda_0, +\infty)$), this alternative c−chart triggers a signal with probability

$$\begin{aligned} \xi(\theta) = {} & 1 \times P(Y_N \not\in [L, U] \mid \lambda = \lambda_0 + \theta) \\ & + \gamma_L \times P(Y_N = L \mid \lambda = \lambda_0 + \theta) \\ & + \gamma_{UCL^\star} \times P(Y_N = U \mid \lambda = \lambda_0 + \theta) \end{aligned}$$

$$\begin{aligned}\xi(\theta) &= 1-\left[F_{Poisson(\lambda_0+\theta)}(U)-F_{Poisson(\lambda_0+\theta)}(L-1)\right]\\ &\quad+\gamma_L\times\left[F_{Poisson(\lambda_0+\theta)}(L)-F_{Poisson(\lambda_0+\theta)}(L-1)\right]\\ &\quad\quad+\gamma_U\times\left[F_{Poisson(\lambda_0+\theta)}(U)-F_{Poisson(\lambda_0+\theta)}(U-1)\right].\end{aligned}$$

Thus,

$$\begin{aligned}\xi(-1) &= 1-\left[F_{Poisson(3-1)}(10)-F_{Poisson(3-1)}(0-1)\right]\\ &\quad+0.038733\times\left[F_{Poisson(3-1)}(0)-F_{Poisson(3-1)}(0-1)\right]\\ &\quad\quad+0.591561\times\left[F_{Poisson(3-1)}(10)-F_{Poisson(3-1)}(10-1)\right]\\ \xi(-1) &\overset{tables}{=} 1-(1.0000-0)+0.038733\times(0.1353-0)+0.591561\times(1.0000-0.9998)\\ &\simeq 0.00535889.\end{aligned}$$

- **Requested out-of-control ARL**

  We are still dealing with $RL(\theta)\sim \text{geometric}(\xi(\theta))$ and $ARL(\theta)=\frac{1}{\xi(\theta)}$. Consequently,

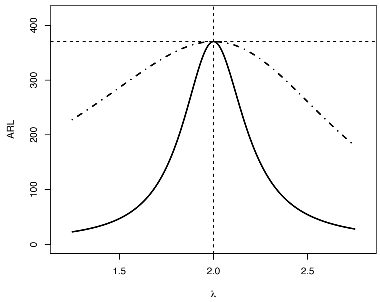
  $$ARL(-1) = \frac{1}{\xi^\star(-1)} \simeq \frac{1}{0.00535889} \simeq 186.606.$$

- **Comment**

  In this case, we have $ARL(-1)\simeq 186.606 < 370.4 = ARL(0)$.

  Unlike the upper one-sided c-chart with 3-sigma limits, the ARL-unbiased c-chart is able to signal a decrease in λ (from the target value 3 to 2) sooner (in average) than to trigger a false alarm — a very desirable property.

(c) A statistician suggested an ARL-unbiased EWMA chart for monitoring this independent Poisson output. **(1.0)**
i) Identify the control statistic of this chart, when the initial value is equal to λ_0.
ii) The ARL profiles of the ARL-unbiased EWMA- and c-charts can be found in the plot on the right. Which profile corresponds to the ARL-unbiased EWMA chart?



- **Control statistic of the EWMA chart for independent Poisson output**

  $$Z_N = \begin{cases}\lambda_0, & N=0\\ (1-\mu)\,Z_{N-1}+\mu\,Y_N, & N\in\mathbb{N},\end{cases}$$

  where $\mu\in(0,1]$.

- **Identifying the requested ARL profile**

  The solid line corresponds to the ARL profile of the EWMA chart because it is well known that the EWMA charts tend to have smaller ARL values than the Shewhart charts, namely in the presence of small and moderate shifts in the parameter being monitored.

4. Admit that we are dealing with samples of size n of independent output and the quality characteristic is normally distributed with nominal mean and variance equal to μ_0 and σ_0^2. When the process mean is off-target (i.e., $\delta=\sqrt{n}(\mu-\mu_0)/\sigma_0=0.1$) and the standard deviation is on-target (i.e., $\theta=\sigma/\sigma_0=1$), the standard $\bar{X}-$chart and the upper one-sided S^2-chart have ARL equal to $ARL_\mu(\delta=0.1,\theta=1)=475.1454$ and $ARL_\sigma(\theta=1)=500$ (resp.). **(2.0)**

Rewrite the formula $PMS_{IV}(\delta)=\frac{[1-\xi_\mu(\delta,1)]\times\xi_\sigma(1)}{\xi_{\mu,\sigma}(\delta,1)}$ in terms of the ARL of the individual charts for μ and σ^2 and obtain $PMS_{IV}(\delta=0.1)$.

- **Quality characteristic**

  $X\sim \text{normal}(\mu,\sigma^2)$, where $\mu=\mu_0+\delta\sigma_0/\sqrt{n}$ and $\sigma^2=\theta^2\sigma_0^2$ represent the process mean and variance, respectively.

- **Probabilities of triggering a signal**

  The STANDARD $\bar{X}-$chart and the UPPER ONE-SIDED S^2-chart trigger a signal with probabilities: $\xi_\mu(\delta,\theta)$, $\delta \in \mathbb{R}$; $\xi_\sigma(\theta)$, $\theta \geq 1$.

  Moreover, according to Exercise 10.38, the joint scheme triggers a signal with probability $\xi_{\mu,\sigma}(\delta,\theta) = \xi_\mu(\delta,\theta) + \xi_\sigma(\theta) - \xi_\mu(\delta,\theta) \times \xi_\sigma(\theta)$, $\delta \in \mathbb{R}$, $\theta \geq 1$.

- **Probability of a misleading signal of Type IV**

  Taking into account the previous result and the fact the RL of the individual charts are geometrically distributed with parameters equal to the probabilities given above, we get:

$$PMS_{IV}(\theta) = \frac{[1-\xi_\mu(\delta,1)] \times \xi_\sigma(1)}{\xi_{\mu,\sigma}(\delta,1)} = \frac{\left[1-\frac{1}{ARL_\mu(\delta,1)}\right] \times \frac{1}{ARL_\sigma(1)}}{\frac{1}{ARL_\mu(\delta,1)} + \frac{1}{ARL_\sigma(1)} - \frac{1}{ARL_\mu(\delta,1)} \times \frac{1}{ARL_\sigma(1)}}. \tag{3}$$

- **Requested PMS of Type IV**

  Capitalizing on (3) and since $ARL_\mu(\delta=0.1,\theta=1) = 475.1454$ and $ARL_\sigma(\theta=1) = 500$,

$$\begin{aligned} PMS_{IV}(\delta=0.1) &= \frac{\left(1-\frac{1}{475.1454}\right) \times \frac{1}{500}}{\frac{1}{475.1454} + \frac{1}{500} - \frac{1}{475.1454} \times \frac{1}{500}} \\ &\simeq 0.486730. \end{aligned}$$

5. A quality control practitioner is using a single sampling plan for attributes with $(n,c) = (20,1)$, which complies with the producer's and consumer's risk points $(p_1 = AQL = 0.5\%, 1-\alpha = 0.95)$ and $(p_2 = LTPD = 17.5\%, \beta = 0.15)$. **(1.5)**

Calculate and comment the ATI when the lot of size $N = 1000$ contains 10% of defective items and rectifying inspection has been adopted.

- **Single sampling plan for attributes with rectifying inspection**

  $N = 1000$ (lot size), $n = 20$ (sample size), $c = 1$ (acceptance number)

- **Auxiliary r.v. and its approximate distribution**

  $D =$ number of defective items in the sample $\stackrel{a}{\sim}$ binomial$(20,p)$

- **Probability of lot acceptance**

  $P_a(p) = P(D \leq c) \simeq F_{binomial(n,p)}(c)$

- **Requested average total inspection**

$$\begin{aligned} ATI(p) &\stackrel{(13.15)}{=} n\,P_a(p) + N\,[1-P_a(p)] \\ ATI(p=0.1) &\simeq 20 \times 0.3917 + 1000 \times (1-0.3917) \\ &\simeq 616.134. \end{aligned}$$

- **Comment**

  Since $AQL = 0.5\% < p = 10\% < 17.5\% = LTPD$, the probability of lot acceptance is between $\beta = 0.05$ and $1-\alpha = 0.95$. Consequently, we reject the lot rather frequently, around 39% of the time, and we have to inspect the remaining $N - n = 1000 - 20$ items of the rejected lot more often [than we wished for]. Unsurprisingly, the average number of items we have to inspected is larger than half the lot size, $N = 1000$.

6. Consider a sampling plan by variables with unknown standard deviation, $n_s = 59$, and $k_s = 1.937713$. **(2.0)**

i) Verify that it meets the risk points $(p_1, 1-\alpha) = (1\%, 0.95)$ and $(p_2, \beta) = (5\%, 0.1)$.

ii) Make the necessary calculations to determine whether the lot should be accepted when the upper specification limit and the sample mean and standard deviation are equal to $U = 3$, $\bar{x} = 3.207$, and $s =$

0.120.

- **Sampling plan by variables with** UNKNOWN STANDARD DEVIATION
  $n_s = 59$ (sample size),
  $k_s = 1.937713$ (acceptance constant)

- **Producer's and consumer's risk points**
  $(p_1, 1-\alpha) = (1\%, 0.95)$
  $(p_2, \beta) = (10\%, 0.1)$

- **Requested verification**
  If we consider $n_s = 59$ and $k_s = 1.937713$ then

$$
\begin{aligned}
P_a(p_1) &\overset{(13.39)}{\simeq} \Phi(\theta_{p_1}) \\
&\overset{(13.41)}{\simeq} \Phi\left[\frac{\Phi^{-1}(1-p_1) - k_s\sqrt{\frac{3n_s-4}{3n_s-3}}}{\sqrt{\frac{1+\frac{3n_s k_s^2}{6n_s-8}}{n_s}}}\right] \\
&= \Phi\left[\frac{2.3263 - 1.937713\sqrt{\frac{3\times 59-4}{3\times 59-3}}}{\sqrt{\frac{1+\frac{3\times 59\times 1.937713^2}{6\times 59-8}}{59}}}\right] \\
&\simeq \Phi(1.77) \\
&\overset{table}{=} 0.9616 \\
&\geq 1-\alpha = 0.95 \\
P_a(p_2) &= \Phi\left[\frac{1.6449 - 1.937713\sqrt{\frac{3\times 59-4}{3\times 59-3}}}{\sqrt{\frac{1+\frac{3\times 59\times 1.937713^2}{6\times 59-8}}{59}}}\right] \\
&\simeq \Phi(-1.29) \\
&\overset{table}{=} 1 - 0.9015 \\
&= 0.0985 \\
&\leq \beta = 0.1.
\end{aligned}
$$

  Hence, the sampling plan complies with the two given risk points.

- **Checking whether or not the lot should be accepted**
  The lot should be accepted iff

$$
Q = \frac{U - \bar{X}}{S} \geq k_s,
$$

  [where Q is the quality index, U is the upper specification limit, $\bar{X}$ and S represent the mean and standard deviation of a random sample with size n_s, and k_s the acceptance constant]. For this sample, we have

$$
\begin{aligned}
q &= \frac{U - \bar{x}}{s} \\
&= \frac{3 - 3.207}{0.120} \\
&= -1.725 \\
&\ngeq 1.937713,
\end{aligned}
$$

  therefore we should reject the lot.