Lecture notes on Control Systems applied to Ocean Energy Conversion (Abridged Version)

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Contents

Introduction

These lecture notes introduce Control Theory to the students of the *Wave Energy* course. They have a threefold purpose:

- to provide the vocabulary and the basic notions needed to work together with control engineers;
- to exemplify some of the techniques needed when controlling devices to convert ocean energy;
- to pave the way for an in-depth study which will be based on specialised literature and take more time than that available in this short course.

Another version of these lecture notes includes all the subjects covered until 2021. As the course has been reorganised, this abridged version is also available, covering only the subjects that can now be addressed. The full version is still provided, for the benefit of students who may need to deepen some particular subject.

Chapter 1

Systems

System is the part of the Universe we want to study. *System*

A system made up of physical components may be called a **plant**. A system which is a combination of operations may be called a **process**. *Plant* *Process*

Example 1.1. A Tidal Energy Converter (TEC) is a plant. If we want to study the wave elevation at a certain onshore location as a function of the weather on the middle of the ocean, we will be studying a process.

The variables describing the characteristics of the system that we want to control are its **outputs**. *Outputs*

The variables on which the outputs depend are the system's **inputs**. *Inputs in the general sense*

The inputs of the system that cannot be modified are called **disturbances**. *Disturbances* The inputs of the system we can modify are called **manipulated variables** or **inputs in the strict sense**. *Inputs in the strict sense* From now on, when referring to a variable as input, we mean that it is an input in the strict sense.

Example 1.2. Consider a Wave Energy Converter (WEC) of the Oscillating Water Column (OWC) type, with a valve for pressure relief. To study this plant, we likely want to know, for each time instant, the electric power it is producing, the pressure inside the chamber, the rotation speed of the turbine, and the air mass flow through the turbine. So these will be its outputs. They depend on the incoming wave and on the position of the relief valve, which are the plant's inputs in the general sense. As we cannot modify the incoming wave, this will be a disturbance. As we can open and close the relief valve, its position is an input in the strict sense.

Remark 1.1. Control is by far easier when disturbances are neglectable. Remarkably, when extracting energy from the sea, they never are.

A system with only one input and only one output is a Single-Input, Single-Output (**SISO**) system. *SISO system* A system with more than one input and more than one output is a Multi-Input, Multi-Output (**MIMO**) system. *MIMO system* It is of course possible to have Single-Input, Multiple-Output (**SIMO**) systems, and Multiple-Input, Single-Output (**MISO**) systems. These are usually considered as particular cases of MIMO systems.

Example 1.3. The OWC of Example 1.2 is a MIMO plant. Another one is a car. When we are driving a car, we want to control its velocity and speed (the

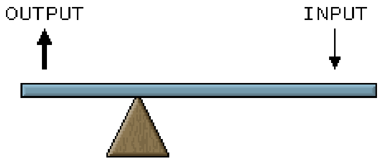

Figure 1.1: A linear system without dynamics (lever; source: Wikimedia Commons).

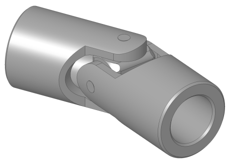

Figure 1.2: A non-linear mechanical system without dynamics (Cardan joint; source: Wikimedia Commons).

outputs). To do this, we can control the angle of the steering wheel, how far the gas pedal, the brake pedal and the clutch are operated, and which gear is engaged (the inputs; an automatic gearbox will mean less inputs). The wind gusts and the road conditions are disturbances. On the other hand, the lever in Figure 1.1 is a SISO system: if the extremities are at heights $x(t)$ and $y(t)$, and the first is actuated, then $y(t)$, the output, depends on position $x(t)$, the input, and nothing more.

Model

A system's **model** is the mathematical relation between its outputs, on the one hand, and its inputs and disturbances, on the other.

Linear system
Non-linear system

A system is **linear** if its exact model is linear, and **non-linear** if its exact model is non-linear. Of course, exact non-linear models can be approximated by linear models, and often are, to simplify calculations.

Example 1.4. The lever of Figure 1.1 is a linear plant, since, if its arm lengths are L_x and L_y for the extremities at heights $x(t)$ and $y(t)$ respectively,

$$y(t) = \frac{L_y}{L_x} x(t). \tag{1.1}$$

A Cardan joint (see Figure 1.2) connecting two rotating shafts, with a bent corresponding to angle β, is a non-linear plant, since a rotation of $\theta_1(t)$ in one shaft corresponds to a rotation of the other shaft given by

$$\theta_2(t) = \arctan \frac{\tan \theta_1(t)}{\cos \beta}. \tag{1.2}$$

A car is also an example of a non-linear plant, as any driver knows.

Dynamic system

A system has no dynamics if its outputs in a certain time instant do not depend on past values of the inputs or on past values of the disturbances. Otherwise, it is a **dynamic system**.

Example 1.5. Both mechanical systems in Figures 1.1 and 1.2 have no dynamics, since the output $y(t)$ only depends on the current value of the input $u(t)$. Past values of the input are irrelevant. The same happens with a faucet that delivers a flow rate $Q(t)$ given by

$$Q(t) = k_Q f(t) \tag{1.3}$$

where $f(t) \in [0, 1]$ is a variable that tells is if the faucet is open ($f(t) = 1$) or closed ($f(t) = 0$). But a faucet placed far from the point where the flow exits the pipe will deliver a flow given by

$$Q(t) = k_Q f(t - \tau) \tag{1.4}$$

This is an example of a dynamic plant, since its output at time instant t depends on a past value of $f(t)$. Here, τ is the time the water takes from the faucet to the exit of the pipe. And, again, a car is also an example of a dynamic system, as any driver knows.

Chapter 2

Models

There are basically two ways of modelling a system:

1. A model based upon **first principles** is a theoretical construction, resulting from the application of physical laws to the components of the plant. *First principles model*

2. A model based upon **experimental data** results from applying identification methods to data experimentally obtained with the plant. *Experimental model*

It is also possible to combine both these methods.

Models based upon first principles can be obtained whenever the way the system works is known. They are the only possibility if the plant does not exist yet because it is still being designed, or if no experimental data is available. They may be quite hard to obtain if the system comprises many complicated interacting sub-parts. Simplifications can bring down the model to more manageable configurations, but its theoretical origin may mean that results will differ significantly from reality if parameters are wrongly estimated, if too many simplifications are assumed, or if many phenomena are neglected.

Example 2.1. A WEC consisting in a vertically heaving buoy of mass m can be modelled using Newton's law:

$$m\ddot{x(t)} = \sum F \tag{2.1}$$

Here $x(t)$ is the vertical position of the buoy (likely measured around its position for a calm sea). Finding expressions for all the forces involved — excitation force, radiation force, power take-off (PTO) force... — is not a trivial task, but good approximations can easily be found; in the end an added mass will turn up, as you know; and after some calculations something like this may be obtained:

$$(m + m_\infty)\ddot{x}(t) = F_e(t) + \int_{-\infty}^{t} h(t-\tau)\dot{x}(t)\,\mathrm{d}\tau - \rho g S x(t) + F_{\mathrm{PTO}}(t) \tag{2.2}$$

Dynamic systems can be modelled using **differential equations** if variables involved are continuous. This is the case of (2.1)–(2.2) above. If variables involved are discrete, **difference equations** are used instead. Both models using differential equations and models using difference equations can be, as said in section 1, linear or non-linear, and SISO or MIMO. *Differential equations* *Difference equations*

Models based upon first principles and e
imental data

Experimental data should, whenever available, be used to confirm, and if necessary modify, models based upon first principles. This often means that first principles are used to find a structure for a model (the orders of the derivatives in a differential equation, or the orders of the polynomials in a transfer function, or the size of matrixes in a state-space representation), and then the values of the parameters are found from experimental data: feeding the model the inputs measured, checking the results, and tuning the parameters until they are equal (or at least close) to measured outputs. This can sometimes be done using least squares; sometimes other optimisation methods, such as genetic algorithms, are resorted to.

If the outputs of experimental data cannot be made to agree with those of the model, when the inputs are the same, then another model must be obtained; this often happens just because too many simplifications were assumed when deriving the model from first principles. It may be possible to find, from experimental data itself, what modifications to model structure are needed.

White box model

Grey box model

Models based upon first principles can be called **white box models**, since the reason why the model has a particular structure is known. If experimental data requires changing the structure of the model, a physical interpretation of the new parameters may still be possible. The resulting model is often called a **grey box model**.

Black box model
NN models
Fuzzy models

There are methods to find a model from experimental data that result in something that has no physical interpretation, neither is it expected to have. Still the resulting mathematical model fits the data available, providing the correct outputs for the inputs used in the experimental plant. Such models are called **black box models**, in the sense that we do not understand how they work. Such models are often **neural network models** (or NN models), or models based upon fuzzy logic (**fuzzy models**). We will not study these modelling techniques, but it is important to know that they exist.

Chapter 3

Basic concepts about control

A system is said to be **stable** if bounded inputs and bounded disturbances always result in bounded outputs. Systems for which such inputs and/or disturbances results in infinite outputs are called unstable. (Of course, in practice, the output never reaches an infinite value, but saturates instead; what is meant is that it would diverge to infinity if there were no limits to its value.) *Stability*

Obviously, one of the objectives of a control system is to maintain stability, or achieve stability if the system is not stable.

Robustness of a control system is its ability of not being affected by undesirable disturbances (robustness to disturbances) or by changing parameters (robustness to parameter uncertainty). *Robustness to disturbances* *Robustness to parameter uncertainty*

3.1 Open-loop control and closed-loop control

The two simplest configurations of control systems are open-loop control and closed-loop control.

In **open-loop control**, a control action is applied, and the result in the output is not verified. If the output is measured, this measurement is not used to correct the control action if there is some deviation from the desired value. In other words, there is no feedback of the output. *Open-loop control*

In **closed-loop control**, the value of the output is compared with the desired reference, and the **error** between the reference and the output is fed to the controller. The control action depends on this error. In other words, there is feedback of the output. *Closed-loop control* *Closed-loop error*

These two control strategies can be represented in **block diagrams** as seen in Figure 3.1, where: *Block diagrams*

- $G(s)$ is the system to be controlled;
- $C(s)$ is the controller;
- $U(s)$ is the control action provided by $C(s)$, and the input of $G(s)$;
- $Y(s)$ is the output of $G(s)$, i.e. the variable we want to control;

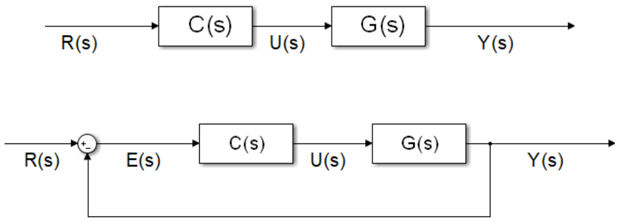

Figure 3.1: Block diagrams of open-loop control (top) and closed-loop control (bottom).

Reference

- $R(s)$ is the reference for $Y(s)$ to follow;
- $E(s)$ is the closed-loop error.

Variable s appears because of a mathematical operator, the Laplace transform, that we will not address. Thanks to this operator, the output of a block can be found using simple rules of block diagram algebra, from which it can be shown that

- when using open-loop control,

$$Y(s) = C(s)G(s)R(s) \Leftrightarrow \frac{Y(s)}{R(s)} = C(s)G(s); \tag{3.1}$$

Closed-loop transfer function

- when using closed-loop control,

$$\begin{aligned} &Y(s) = C(s)G(s)E(s) \Leftrightarrow Y(s) = C(s)G(s)(R(s) - Y(s)) \Leftrightarrow \\ &Y(s) + C(s)G(s)Y(s) = C(s)G(s)R(s) \Leftrightarrow \frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)} \end{aligned} \tag{3.2}$$

Open-loop control only makes sense when the system is very well known, so that the control action needed in each moment can be determined precisely, without need to check the result.

Example 3.1. Open-loop control can be applied in TECs, for instance, because tides are well-known in what concerns both their amplitude and the hours at which they take place; or in a WEC, in a control system that copes with tides. It is very hard to apply open-loop control in the presence of signals hard to predict such as waves; even when there is a very good upstream measurement of the wave it is often better to apply closed-loop control, checking if the control action is achieving or not its purpose.

On-off control

Example 3.2. Open-loop control can be applied to achieve **on-off control**, in which the output can only assume two values. That is for instance the case of a valve that should either be open or closed, actuated by a solenoid: a control action is applied that will bring the valve to one of its two saturation limits, and there may be no need to check that this happened.

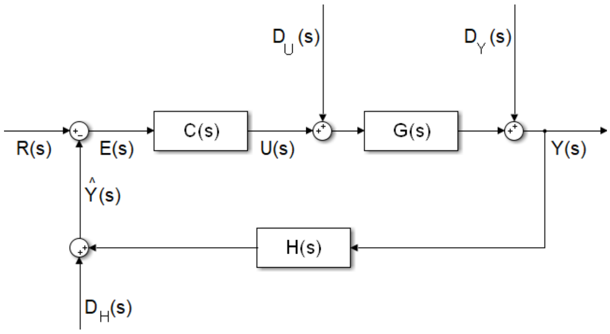

Figure 3.2: Block diagram of a closed-loop with disturbances.

Closed-loop control has a corrective action and does not require a knowledge of the system as exact as that which is necessary to apply open-loop control, though some knowledge is required lest the controller should be badly designed, preventing the closed-loop from achieving the desired performance, or even turning unstable. Notice that closed-loop control only reacts to deviations of the reference, and thus lacks any preventive or predictive action.

A more complete and realistic block diagram for closed-loop control is shown in Figure 3.2, where

- $D_U(s)$ is a disturbance affecting the control action;
- $D_Y(s)$ is a disturbance affecting the output;
- $D_H(s)$ is a disturbance affecting the measured output;
- $H(s)$ is the sensor that measures the output $Y(s)$;
- $\hat{Y}(s)$ is the measured value of the output.

Notice that an ideal sensor verifies $H(s) = 1$ and thus, if there is no sensor disturbance, $\hat{Y}(s) = Y(s)$.

3.2 Closed-loop controllers and how to design them

Closed-loop controllers can have many forms and be designed in many different ways.

The following linear controllers are remarkable for their extended use:

- **Proportional controllers**: *Proportional control*

$$u(t) = K_p e(t). \tag{3.3}$$

  In this way, the larger (smaller) the error, the larger (smaller) the control action.

- **Proportional–integral (PI) controllers**: *PI control*

$$u(t) = K_p e(t) + \int_0^t e(t)\,\mathrm{d}t. \tag{3.4}$$

The control action will have a component proportional to the integral of the error, and so, if the error does not go to zero fast, the control action will increase to try to compensate this non-vanishing error.

PD control

- **Proportional–derivative (PD) controllers**:

$$u(t) = K_p e(t) + K_d \frac{\mathrm{d}e(t)}{\mathrm{d}t}. \tag{3.5}$$

The control action will have a component proportional to the derivative of the error, and so, if the error increases abruptly, the control action will increase to try to compensate this increasing error.

PID control

- **Proportional–integral–derivative (PID) controllers**:

$$u(t) = K_p e(t) + \int_0^t e(t)\,\mathrm{d}t + K_d \frac{\mathrm{d}e(t)}{\mathrm{d}t}. \tag{3.6}$$

This controller has three components, as its name says.

- **Optimal control** consists in defining an **objective function** that should be minimised, usually penalising deviations of the output from the reference, big control actions, and big changes in the control actions (to spare the actuators); controller parameters are then found minimising this objective function. Analytical techniques can be found for LTI systems and quadratic objective functions, and for other well-behaved cases; these control techniques, based upon state-space representations, were among those known in the 1960's as Modern Control. More complicated situations may require other optimisation methods.

The following non-linear controllers deserve to be mentioned:

- **Predictive control** optimises an objective function as well, this time using a non-linear model of the system to predict how different particular control actions will affect performance during a period of time (the **prediction horizon**), finding the best option with a numerical search procedure (often based upon the **branch and bound** algorithm), implementing the best control for a period of time (the **control horizon**) shorter than the prediction horizon (because short-time predictions are more reliable than long-time predictions), and repeating the process (resulting in a **receding horizon** control).

- **Switching control** control consists in having different controllers to be applied in different situations. This has been used to control WECs under different sea states. Switching between controllers has to be done with care, since an abrupt change in controller can easily result in a big change of the control action, which may render the system unstable.

- **On-off control** has already been mentioned; it is in fact a very basic non-linear control technique based upon the saturation of the control variable.

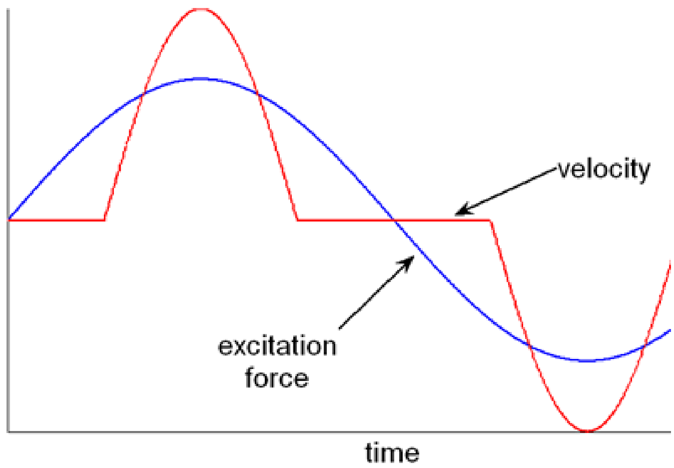

Figure 3.3: Objective of latching control for a heaving point absorber.

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- **Latching** is a non-linear control technique that consists in stopping an oscillating variable when it reaches a maximum or a minimum, to release when it is reckoned that it will be in phase with another variable. This has been extensively used with heaving WECs, which are latched when they stop either at the top or at the bottom of the movement (see Figure 3.3), so that the velocity will be in phase with the excitation force (so as to maximise the extracted power). As sea-waves are irregular, finding the instant to unlatch the WEC is not trivial, and since the mass that has to latched is usually significant, huge forces may be involved. This type of control increases the abrupt variations with time of the power to be injected into the grid; this is an undesirable consequence, that may be mitigated if there is a wave farm with many WECs, with the power variations of the different devices out of phase with each other. Latching control is similar to what happens in an OWC with a relief valve, that may as well be used to try to put the variations of pressure in phase with the excitation force.

3.3 Control of delay systems

Delay systems

Systems with a **delay** do not respond immediately to an input, but only after a while.

Example 3.3. Figure 3.4 shows the unit-step response of two similar systems, one without and another with delay.

Effects of delays in closed-loop

Delays are one of the biggest nuisances in control, as they easily cause closed-loop control systems to become unstable. The reason for this is intuitive: if a control action is applied, but there is no immediate response, the controller may easily increase the control action, to try to elicit some reaction from the system. The control action will then be so big that, when the system finally responds,

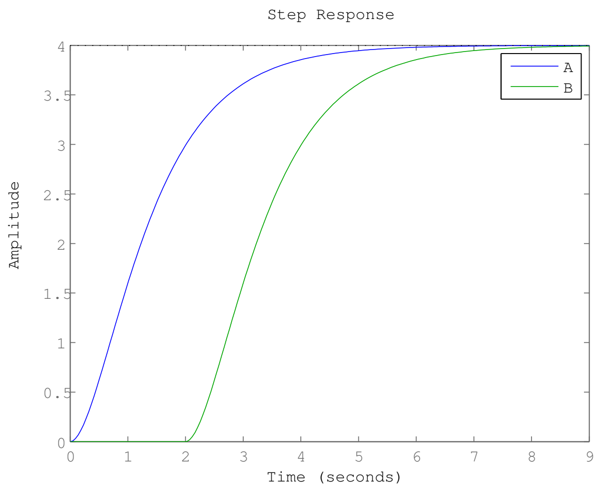

Figure 3.4: Unit-step responses of a system without delay (A) and of a system with delay (B).

it overshoots, and then the controller may easily try to solve this by decreasing too much the control action, resulting in an overshoot in the other direction, and so on. Delays are thus responsible not only for slower responses, but also for oscillations, and eventually unstable closed-loops.

Example 3.4. You likely have had a similar experience when trying to take a shower in a bathroom you do not know, and where the water takes longer to reach the shower than you are used to. You will likely have turned too much the faucet of hot water, and then got burnt; then as reaction you probably turned the faucets so that water went too cold. In such cases it takes some time to get to know the system well enough to obtain a comfortable temperature.

Smith predictor

What is needed is a controller that waits for the delay and takes it into account when determining the control action. This is possible with a variation of IMC called **Smith predictor**, shown in Figure 3.5, where

- $e^{-\theta s}$ represents a delay of θ seconds (this exponential is used for this purpose again because of the Laplace transform, that we will not enter into);
- $G(s)e^{-\theta s}$ is a system with a delay;
- $G^*(s)$ is a model of the system without delay;
- $\hat{\theta}$ is an estimate of the system's delay.

If $G^*(s)$ and $\hat{\theta}$ are exact (i.e. if the model is perfect), then $Y(s) = \hat{Y}(s)$, and thus what is being fed back is

$$G^*(s)U(s) = G^*(s)C(s)E(s) = G(s)C(s)E(s). \tag{3.7}$$

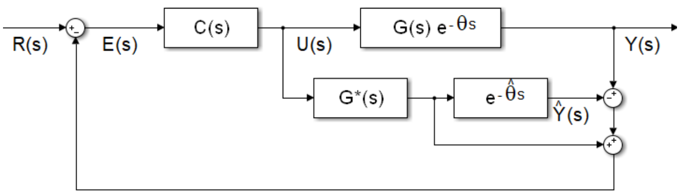

Figure 3.5: Smith predictor.

Consequently, the error is given by

$$E(s) = R(s) - G(s)C(s)E(s) \Leftrightarrow E(s)\,(1 + G(s)C(s)) = R(s) \Leftrightarrow E(s) = \frac{R(s)}{1 + G(s)C(s)}, \tag{3.8}$$

and since $Y(s) = G(s)e^{-\theta s}C(s)E(s)$ then

$$Y(s) = \frac{G(s)e^{-\theta s}C(s)R(s)}{1 + G(s)C(s)} \Leftrightarrow \frac{Y(s)}{R(s)} = \frac{G(s)e^{-\theta s}C(s)}{1 + G(s)C(s)} \tag{3.9}$$

Notice the difference between this and the result of a usual closed-loop:

$$\frac{Y(s)}{R(s)} = \frac{G(s)e^{-\theta s}C(s)}{1 + G(s)e^{-\theta s}C(s)} \tag{3.10}$$

Remark 3.1. It should be stressed that the Smith predictor does *not* eliminate the effect of the delay. The response of the controlled system will always be delayed. What the Smith predictor does is to eliminate the effect of the delay in the stability of the closed-loop — and this is already much.

Chapter 4

Signals

A **signal** is a function of time or space that conveys information about a system. *Signal*

Example 4.1. One of the outputs of a WEC is the electric power it injects into the grid. This is a signal that depends on time. One of its disturbances is the wave it extracts energy from. Wave elevation is a signal that depends on both time and space.

Some signals can only take values in a discrete set; they are called **quantised signals**. Others can take values in a continuous set; they are called **analogical signals**. *Quantised signal* *Analogical signal*

Example 4.2. The rotation speed of a turbine is real valued; it takes values in a continuous set. So does the position of the break pedal of a car. The number of blades of the turbine is an integer number; it takes values in a discrete set. So does the shift engaged by the gearbox of a car.

Remark 4.1. In engineering, most signals (if not all) are **bounded**. For instance, the wave elevation at a certain point cannot be less than the depth of the sea, and cannot be arbitrarily large (a sea wave 1 km high, for instance, is a physical impossibility). Again, the rotation speed of a turbine, or the linear velocity of a shaft, or a voltage in a circuit, are always limited by physical constraints. Still, as these signals can assume values in a continuous set, there are infinite values they can assume. On the other hand, discrete signals, being limited, can only assume a limited number of values: a wave farm with a very large number of WECs can be conceived, but there is a physical limit for this too (probably well below the number of devices that would entirely cover all the oceans on Earth!).

Most signals are nowadays measured using digital equipment. Irrespective of the number of bits employed, there is a finite resolution implied, and so signals take only discrete values in practice. In other words, most signals are nowadays quantised.

The **resolution** of a quantised signal is the difference between the consecutive discrete values that it may assume. In practice, this depends on how the signal is measured. The **precision** of a measurement is the range of values where the real value may be; in other words, it is the maximum error that can occur. Precision and resolution should not be confused. *Resolution* *Precision*

Example 4.3. Figure 4.1 shows an input-output USB device. It can be used as an analog-to-digital (AD) converter. It reads signals in the -10 V to $+10$ V range using 12 bits. Consequently, it has an input resolution of $\frac{20}{2^{12}} = 4.88 \times 10^{-3}$ V. It can also be used as a digital-to-analog (DA) converter. It outputs signals in the 0 V to $+5$ V range using 12 bits. Consequently, it has an output resolution of $\frac{5}{2^{12}} = 1.22 \times 10^{-3}$ V.

Figure 4.1: National Instruments USB-6008, `http://www.ni.com/en-gb/support/model.usb-6008.html`.

Example 4.4. Analog measurements have resolution and precision too. Consider the weighing scale in Figure 4.2. Mass can be measured in the 5 kg to 100 kg range, with a resolution of 0.1 kg, and a precision of 0.1 kg. There is no finer resolution because the graduation has ten marks per kilogram. The precision is a result of the characteristics of the device.

Example 4.5. In Example 4.4 the resolution and the precision have the same value, but this is often not the case. Figure 4.3 shows luggage scales with a resolution of 1 g, but with a precision of 5 g or 10 g depending on the range where the measurement falls. (Varying precisions are found for some types of sensors, especially because of non-linearities.)

Remark 4.2. It makes sense to have a resolution equal to the precision, as in Example 4.4, in which case all figures of the measurement are certain; and it makes sense to have a resolution higher than the precision, as in Example 4.5, in which case the last figure of the measurement is uncertain. It would make no sense to have a resolution more than 10 times larger than the precision, since in that case at least the last figure of the measurement would have no significance. It would also make no sense to have a resolution coarser than the precision, since the capacities of the sensor would be wasted.

Remark 4.3. The precision of a signal depends on all the elements of the measuring chain. The value shown at the display of the device in Figure 4.3 has a precision resulting from both the sensor used, and its particular precision, and the AD converter that the sensor's signal goes through, with its precision.

Remark 4.4. A naturally quantised signal can be measured with a coarser resolution. For instance, the population of a country has a resolution of one, but very often statistics give values rounded to thousands. This is because the

Figure 4.2: Weighing scales once used in the Bełchatów coal mine, Poland.

Figure 4.3: Luggage scales.

uncertainty of the measured signal does not allow for finer resolutions, the last figures of which would have no significance.

Some signals take values for all time instants: they are said to be **continuous**. Others take values only at some time instants: they are said to be discrete in time, or, in short, **discrete**. The time interval between two consecutive values of a discrete signal is the **sampling time**. The sampling time may be variable (if it changes between different samples), or constant. In the later case, which makes mathematical treatment far more simple, the inverse of the sampling time is the **sampling frequency**.

Continuous signal
Discrete signal
Sampling time
Sampling frequency

Example 4.6. The air pressure inside the chamber of an OWC is a continuous signal: it takes a value for every time instant. The number of students attending the several classes of a course along the semester is a discrete signal: there is a value for each class, and the sampling time is the time between consecutive classes. The sampling time may be constant (if there is e.g. one lecture every Monday) or variable (if there are e.g. two lectures per week on Mondays and Tuesdays).

Most sensors nowadays measure the signal they are intended for only at some time instants, that is to say, with a given sampling time. In other words, nearly all measurements are discretised in time, just as they are quantised. If we want to store our data, and since data is normally recorded digitally, this makes all sense, as it would of course be impossible to record digitally a signal for *all* time instants. (An analogical record may sometimes be possible.)

Discretising a signal has to be done with care. If the sampling time is too big, we will miss many intermediate values that may be important. If it is too small, we will end up with many consecutive measurements that are either equal (because the signal does not change that fast) or where changes are irrelevant (because the measured value changes only due to some noise source).

Example 4.7. It is intuitive that, to study tides, we need not measure the sea level with a 1 ms sampling time (i.e. a 1000 Hz sampling frequency), as the period of tides is in the range of hours. It is also intuitive that, to study sea waves, we cannot measure the sea level once every minute, as we would miss most of the wave crests and troughs.

While there is a theorem (the Nyquist theorem) about the lowest possible sampling time that can be used to sample a periodic signal, in practice a higher sampling frequency should be employed. The following rule of thumb is a fair indication of how the sampling time should be chosen: let ω_b be the highest frequency, in rad/s, we may be interested in studying. Then the sampling time T_s should verify

Nyquist theorem
Rule of thumb for T_s

$$\frac{2\pi}{20\omega_b} \leq T_s \leq \frac{2\pi}{10\omega_b} \tag{4.1}$$

(Frequency ω_b should more precisely be the system's bandwidth; we will mention this later on.) If $t_b = \frac{2\pi}{\omega_b}$ is the smallest time interval we are interested in studying, (4.1) is the same as

Bandwidth

$$\frac{t_b}{10} \leq T_s \leq \frac{t_b}{20} \tag{4.2}$$

Example 4.8. If a tide has a period of 12 hours, then it should suffice to measure the sea level every 1.2 hours (70 minutes) at the least, or every 0.6 hours (36 minutes) at the most. As (4.1)–(4.2) are a rule of thumb, there would likely be no problem in measuring the sea level every half hour, or even every quarter of an hour.

Example 4.9. If we are interested in studying the waves at a shore where we know that during the entire year the wave spectra have neglectable content above 2 Hz = 12.57 rad/s, then a sampling time between 25 ms and 50 ms will be in order.

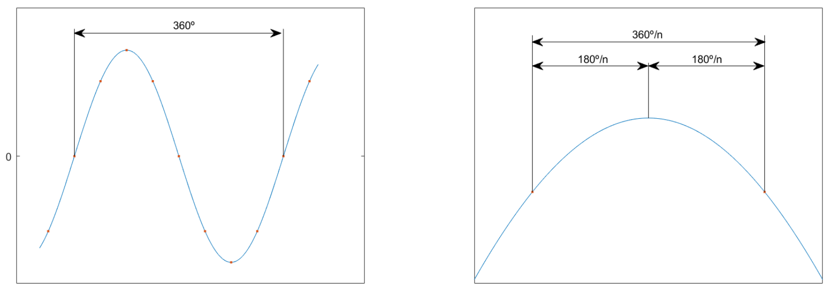

Figure 4.4: Sampled sinusoid. Left: in a very luck situation, sampling instants fall on zero crossings, crests and troughs. Right: in an equally unlucky situation, a crest is missed by as much as possible.

The lower value for the sampling time in rule 4.1 can be justified in the following way. When sampling a sinusoid, with some luck, sampling instants may fall on zero crossings and extreme values (crests and troughs), as seen in Figure 4.4. With an equal lack of luck, zero crossings and extreme values will be missed by as much as possible, as also shown in the Figure 4.4, for the case in which there are n sampling instants per period. If $n = 10$, this corresponds to an error of 18°. Since

$$\cos 18° = 0.95 \tag{4.3}$$

we see that, using 10 points per period, the amplitude of the sinusoid can be found from sampled data with an error of, at most, 5%. Decreasing the sampling time, lower errors in the measured amplitude will be obtained.

Chapter 5

Applications

In what follows, three papers are introduced that concern the control of two WECs, and cover most of the subjects addressed so far.

5.1 The Archimedes Wave Swing (AWS)

The Archimedes Wave Swing was an off-shore submerged heaving point absorber, using a linear electric generator as PTO, and deployed in the north of Portugal (Figure 5.1).

[4] addresses the following issues:

- identification of a transfer function model;
- reactive control;
- phase and amplitude control (including proportional control);
- latching;
- feedback linearisation.

[5] addresses the following issues:

- identification of a NN model;
- IMC;
- switching control.

5.2 The Inertial Sea Wave Energy Converter (ISWEC)

The Inertial Sea Wave Energy Converter is an off-shore floating pitching point absorber, using gyroscopes and an electric generator as PTO, and deployed off Pantelleria island, Italy (Figure 5.2)

[6] addresses the following issues:

- PID control;
- switching control;

Figure 5.1: The Archimedes Wave Swing.

Figure 5.2: The Inertial Sea Wave Energy Converter.

- optimal control;
- nonlinear control.

Bibliography

In case you ever need to study these subjects in greater depth, you may wish to consult one or more of the following textbooks. [3] is a classical book, covering the control of continuous systems. [2], by the same author, covers discrete systems. [1] covers NNs and fuzzy modelling and control.

* * *

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[5] Duarte Valério, Mário J. G. C. Mendes, Pedro Beirão, and José Sá da Costa. Identification and control of the AWS using Neural Network models. *Applied Ocean Research*, 30(3):178–188, 2008.

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