1.

$$K_A = (+1, -1, +1, -1, -1, +1, -1, +1)$$

$$D_A = ("0", "1") = (-1, -1, -1, -1, +1, +1, +1, +1)$$

$$K_B = (+1, +1, -1, -1, -1, -1, +1, +1)$$

$$D_B = ("1", "0") = (+1, +1, +1, +1, -1, -1, -1, -1)$$

$$N = (0, 0, -2, -1, 0, 0, +2, 0)$$

a)

The noise does not affect the BS, so it does not add to the received signal components.

$$R = K_A * D_A + K_B * D_B = (0, +2 - 2,0,0, +2, -2,0)$$

b)

$$R = (0, +2 - 3, -1,0, +1, +2,0)$$

$$R_A^1 = K_A^1 \cdot R^1 = (+1, -1, +1, -1) \cdot (0, +2 - 3, -1) = -4 < -3 \rightarrow "0"$$

$$R_A^2 = K_A^2 \cdot R^2 = (-1, +1, -1, +1) \cdot (0, +1, +2,0) = -1 \in [-3, +3] \rightarrow ?$$

$$R_B^1 = K_B^1 \cdot R^1 = (+1, +1, -1, -1) \cdot (0, +2 - 3, -1) = +6 > +3 \rightarrow "1"$$

$$R_B^2 = K_B^2 \cdot R^2 = (-1, -1, +1, +1) \cdot (0, +1, +2,0) = +1 \in [-3, +3] \rightarrow ?$$

Since there are undefined values, which cannot be doubtlessly decoded, correct answer is “None of the above”.

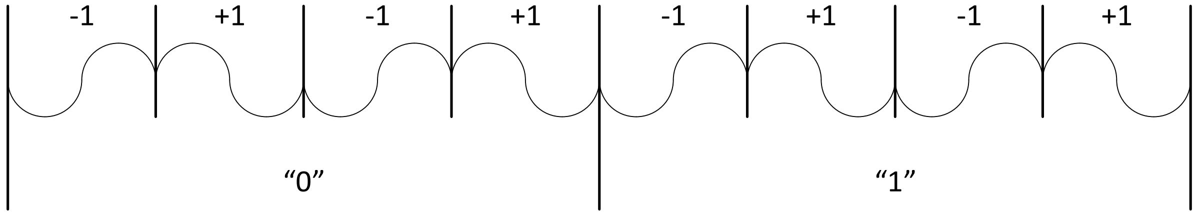
c)

The correct answer is (+1, +1, +1, +1, -1, -1, -1, -1). This is the only key that is completely orthogonal half-by-half with the keys of A and B. This can be easily checked by calculating the internal products.

d)

$$R_b = \frac{R_c}{SF} = \frac{3000000}{4} = 750\ kbit/s$$

e)



2.

a)

One must not forget that although the total effective bandwidth is 40 MHz, the system hops through a sequence of 4 independent frequency channels. From the point of view of the baseband MFSK modulation, only ¼ of the bandwidth is being used.

$$\boldsymbol{T_s = 2 \times T_b = 2 \times \frac{1}{R_b} = 2 \times \frac{1}{B/4}\left(\frac{(1+r)\cdot M}{log_2(M)}\right) = 2 \times \frac{1}{\frac{40 \times 10^6}{4}} \cdot \left(\frac{(1+0)\cdot 4}{2}\right) = 0.4\mu s}$$

$$\boldsymbol{T_c = 2 \cdot T_s = 0.8\mu s}$$

$$\boldsymbol{T_b = \frac{T_s}{2} = 0.2\mu s}$$

b)

The system employs slow frequency hopping, since $T_c \geq T_s$.

c)

Yes. If the respective hopping sequences are orthogonal, transmitting devices will transmit simultaneously using different subcarriers, thus without interfering. This is used in Bluetooth, for example.

d)

The average SNIR of the FHSS system is approximately four times that of the narrowband system. Assuming that the thermal noise power is negligible compared with the interference power:

$$\overline{SNIR_{FHSS-MFS}} \approx \frac{P_r}{\frac{I}{4}} = 4\frac{P_r}{I}$$

$$\overline{SNIR_{NB-MFSK}} \approx \frac{P_r}{I}$$

3.

a)

We will use the log-distance path loss model, since $\boldsymbol{d_0}$ and $\boldsymbol{PL_0}$ are given.

$$\begin{aligned}
&P_r\ [dBm] = P_t\ [dBm] - PL_0 + G_t\ [dBi] + G_r\ [dBi] - 10 \cdot \alpha \cdot log_{10}\left(\frac{d}{d_0}\right) \Leftrightarrow \\
&-70 = 10 \cdot log_{10}\,(20) - 15 + 0 + 0 - 10 \cdot 3 \cdot log_{10}\left(\frac{d}{1}\right) \Leftrightarrow \\
&d \approx 184{,}9\ m
\end{aligned}$$

b)

Firstly, we have to calculate the bit error ratio (BER), based on E_b/N_0. In order to calculate E_b, we need the time of one bit. We know the modulation, which is 16QAM (i.e., *M=16*), hence:

$$\begin{aligned}
&B = \left(\frac{1+r}{log_2(M)}\right) \cdot R_b \Leftrightarrow 400000 = \left(\frac{1+1}{log_2(16)}\right) \cdot R_b \Leftrightarrow R_b = 800000\ bit/s \\
&T_b = \frac{1}{R} = 1{,}25 \times 10^{-6} s
\end{aligned}$$

We also need the received power:

$$\begin{aligned}
P_r[dBm] &= P_t\ [dBm] - PL_0 + G_t\ [dBi] + G_r\ [dBi] - 10 \cdot \alpha \cdot log_{10}\left(\frac{d}{d_0}\right) \\
&= 10 \cdot log_{10}\,(20) - 15 + 0 + 0 - 10 \cdot 4 \cdot log_{10}\left(\frac{30}{1}\right) \approx -46{,}3\ dBm
\end{aligned}$$

$$\frac{E_b}{N_0} = \frac{T_b \times P_r}{N_0} = \frac{1{,}25 \times 10^{-6} \times 10^{\frac{-46{,}3}{10}}}{10^{\frac{-110}{10}}} \approx 2{,}93\ \mathrm{mJ/mW}/Hz$$

Now, we can calculate the BER:

$$BER_{16QAM} = \frac{4}{log_2(M)} \cdot Q\left(\sqrt{3 \cdot \left(\frac{E_b}{N_0}\right) \cdot \frac{log_2(M)}{M-1}}\right) = Q(1{,}53) \approx 6{,}68 \times 10^{-2}$$

Now, we only need the packet length in order to calculate the FER. Since each data packet has only 30 data symbols and each 16QAM symbol represents 4 bits, the packet length is 120 bits. The FER can now be calculated:

$$FER = 1 - (1 - BER)^{120} \approx 1$$

This means that all packets are lost. The receiver sensitivity must have been measured under different conditions.

c)

The raw throughput of each robot is equal to the number of bits that it transmits per frame, divided by the period of one superframe, which corresponds to its duration. The first thing to do is to calculate the duration of the superframe, knowing that it carries 5 frames (1 downlink and 4 uplink). The duration of the frame corresponds to the duration of 40 symbols.

$$B = \left(\frac{1+r}{log_2(M)}\right) \cdot R_b = (1+\mathrm{r}) \cdot R_s \Leftrightarrow R_s = \frac{B}{1+r} \Leftrightarrow T_s = \frac{1+r}{B} = \frac{2}{400000} = 5 \times 10^{-6}\, s$$

$$T_{frame} = 40 \times T_s = 40 \times 5 \times 10^{-6}\, s = 200 \times 10^{-6}\, s$$

$$T_{superframe} = 5 \times T_{frame} = 1 \times 10^{-3}\, s$$

$$Th_{raw} = \frac{120}{1 \times 10^{-3}} = 120\, kbit/s$$

The effective throughput is equal to the raw throughput multiplied by the probability of successful frame reception:

$$Th_{eff} = Th_{raw} \times (1 - 0{,}1) = 108\, kbit/s$$

d)

This is calculated directly from the expression that relates the physical and effective area of the antenna with the gain:

$$\boldsymbol{\eta \times \mathrm{A}_{phy} = \frac{\lambda^2}{4\pi} G \Leftrightarrow 0,6 \times \mathrm{A}_{phy} \approx 0,019 \Leftrightarrow \mathrm{A}_{phy} \approx 0,032\, m^2}$$

4.

a)

If the cyclic prefix is not used, the duration of an OFDM symbol is equal to $1/f_b$, where f_b is the subcarrier separation. When the cyclic prefix is used, it must be added to obtain the total duration. As such, we have:

$$T_{OFDM} = \frac{1}{f_b} + t_{cyclic\_prefix} = \frac{1}{312500} + 0.0000008 = 4\,\mu s$$

The OFDM symbol rate is just the inverse:

$$R_{OFDM} = \frac{1}{T_{OFDM}} = 250000\, symbol/s$$

b)

Each OFDM symbol corresponds to N modulation symbols, each transmitted in a different subcarrier. Consequently, in each subcarrier, the modulation symbol rate is the same as the OFDM symbol rate. The net bitrate is the sum of the net bitrates of all data subcarriers. The net bitrate of a single subcarrier is calculated as follows:

$$R_b^{sc} = R_{OFDM} \times L \times CR,$$

Where L is the number of bits per modulation symbol (equal to $log_2(M)$, where M is the number of different symbols available in the modulation), and CR is the code rate of the FEC (i.e. $\frac{k}{n}$, where k is the number of useful bits transmitted, and n is the total number of bits transmitted).

We have that:

$$R_b^{sc} = \frac{R_b}{N} = \frac{36000000}{48} = 750000.$$

Knowing that $L = 4$ for 16-QAM, we can calculate CR:

$R_b^{sc} = R_{OFDM} \times L \times CR \Leftrightarrow CR = \frac{750000}{4\times 250000}$=3/4

c)

OFDMA is based on OFDM. The difference is that in OFDMA, different subcarriers can be assigned to different users transmitting simultaneously, while in OFDM all subcarriers are assigned to a single user at each time, and the users transmit at different times.

d)

Carrier aggregation consists on joining two or more elementary channels to obtain one channel with more bandwidth. For example, in IEEE 802.11n, two channels of 20 MHz can be aggregated to produce one channel of 40 MHz, allowing a higher transmission speed (this is the purpose).

5.

a)

The duration of one frame is $T_f = 10\ ms = 0.010\ s$. In one hour, the number of transmitted frames is calculated as follows:

$$N_{frames} = \frac{3600}{T_f} = 360000$$

There are only 7 data subframes in one frame, so the number of data subframes in one hour is calculated as follows:

$$N_{sf} = 360000 \times 7 = 2520000$$

Since each device is assumed to transmit 6 subframe messages in one hour, the number of supported devices is calculated as follows:

$$N_{dev} = \frac{N_{sf}}{6} = 420000$$

b)

It is now assumed that $N_{dev} = 500000$.

The first thing to do is to calculate the area of one cell:

$$A_{cell} = 1.5 \times R^2 \times \sqrt{3} = 2\ km^2$$

The number of devices located in each cell is easily calculated based on the density:

$$N_{dev}^{cell} = 30000 \times A_{cell} = 60000$$

The number of cells that can be grouped in a cluster is thus calculated as follows:

$$N_{cell} = \left\lfloor \frac{N_{dev}}{N_{dev}^{cell}} \right\rfloor = 8$$

The number is rounded down in order to guarantee that the number of devices assigned to a cluster is always lower or equal than N_{dev}. However, $N_{cell} = 8$ is not possible, since, in order to be repeatable in space, the cluster size must obey the rule $N_{cell} = I^2 + J^2 + (I \times J),\ st\ I,\ J = 0,\ 1,\ 2,\ etc.$ As such, $N_{cell} = 7$ is right cluster size, corresponding to $I = 2$ and $J = 1$.

c)

This can be done in one of two ways: the mobile operator can assign a GSM channel to serve as NB-IoT PRB, or it can allocate the NB-IoT PRB in a guard band between two LTE bands.