MEMEG - Chapter 4 - Principal Components

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Exercise 2

Given the covariance matrix

$$\mathbf{\Sigma} = \begin{pmatrix} 8 & 0 & 1 \\ 0 & 8 & 3 \\ 1 & 3 & 5 \end{pmatrix}.$$

Use the R to:

1. Compute the eigenvalues λ_i and the eigenvectors $\boldsymbol{\gamma}_i$, for $i = 1, \ldots, 3$, of $\mathbf{\Sigma}$.

```
Sig=matrix(c(8,0,1,0,8,3,1,3,5),nrow = 3,ncol=3)
eigen(Sig)
```

2. Verify that $\lambda_1 + \lambda_2 + \lambda_3 = \text{tr}(\mathbf{\Sigma})$, where the trace of a matrix equals the sum of its diagonal elements, i.e, total variance of $\mathbf{X}$ equals the sum of the eigenvalues of $\mathbf{\Sigma}$.

```
val=eigen(Sig)$values
sum(val)
library(psych)
tr(Sig)
```

3. Verify that $\lambda_1\lambda_2\lambda_3 = |\mathbf{\Sigma}|$, where $|\mathbf{\Sigma}|$ is the determinant of $\mathbf{\Sigma}$, ie, generalized variance of $\mathbf{X}$ equals the product of the eigenvalues of $\mathbf{\Sigma}$.

```
prod(val)
det(Sig)
```

4. Verify the spectral decomposition $\mathbf{\Sigma} = \mathbf{\Gamma}\mathbf{\Lambda}\mathbf{\Gamma}^{\text{T}}$, where $\mathbf{\Gamma}$ is the matrix with the eigenvectors and $\mathbf{\Lambda} = diag(\lambda_1, \lambda_2, \lambda_3)$

```
vec=eigen(Sig)$vector
round(vec%*%diag(val)%*%t(vec),2)
```

5. Compute $\mathbf{\Sigma}^{-1}$. Use the spectral decomposition of $\mathbf{\Sigma}$ to obtain $\mathbf{\Sigma}^{-1}$.

```
round(solve(Sig),2)
round(vec%*%solve(diag(val))%*%t(vec),2)
```