Information and Communication Theory

Problem Set 5 - Solutions

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1. $C = 1 - H(q+p-2pq, 1-(q+p-2pq))$, q and p being the error probabilities of each BSC. Note that the capacity, C, of two channels with capacities C_1 and C_2 satisfies $C \leq min\{C_1, C_2\}$.

2. $C = C_1 + C_2$

3. *Hint:* A monotonic decreasing sequence will converge to its greatest lower bound.

4. $P(X=1) = \frac{3}{5}$, $P(X=1) = \frac{2}{5}$, $C \approx 0.32$

5.

6. *Hint:* $H(Y|X) = H(row)$. Show that for X uniform, Y will be uniform as well, leading to $H(Y) = log(|\mathcal{Y}|)$

7. *Hint:* Note that in the previous demonstration we only used the fact that the sum of the columns were the same and not necessarily permutations of each other.

8. $r = 2$, $n = 3$, $k = 1$, $G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ $H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

9. (a) *Hint:* the codewords form a linear space

   (b) *Hint:* the Hamming distance between two words is the weight (number of 1s) of their sum

10. $1 - P(0 \text{ errors}) - P(1 \text{ error}) = 1 - (1-\alpha)^7 + 7\alpha(1-\alpha)^6$

11. $C = \frac{1}{2} log(1 + \frac{4P}{2\sigma^2(1+\rho)})$

12. For $a > 1$, $C = \frac{1}{a} H(p, 1-p)$, with $p = P(X=1)$.
    For $a < 1$, $C = H(p, 1-p)$.