Information and Communication Theory

Problem Set 3 - Solutions

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1. Non-singular: C_1, C_3, C_4, C_5
   Uniquely decodable: C_1, C_4,C_5
   Instantaneous: C_1, C_5

   $L(C_1) = 2$, $L(C_2) = \frac{9}{8}$, $L(C_3) = \frac{5}{4}$, $L(C_4) = \frac{15}{8}$,$L(C_5) = \frac{7}{4}$
2. Hint: Construct binary tree
3. It is possible. Ex: 00, 01, 10, 111, 1100. Not optimal since 1100 could be 110.
4. Not possible.
5.
6. $L(C^{SF}) = \frac{8}{3}$, $\rho(C^{SF}) \approx 0.792$
7. $L(C) = \frac{7}{4}$ $\rho(C) = 1$
8. (a) 6. (ternary)

   $L(C^{SF}) = \frac{4}{3}$, $\rho(C^{SF}) = 1$

   (b) 7. (ternary)

   $L(C) = \frac{5}{4}$ $\rho(C) \approx 0.883$
9. *Hint:* Think about reducing the source's entropy.
10. $L(C) = 1$, $\rho(C) \approx 0.544$

    Order-2: $L(C) = 1.36$, $\rho(C) approx 0.80$
11. $L(C) = 1$, $\rho(C) = 1$

    Order-2: $L(C) = 2$, $\rho(C) = 1$
12.
13. 24 optimal codes, 8 Huffman codes.
14. 36 optimal codes, all Huffman.
15. 2 weightings: first with two sets of 3 balls, second with one ball each.
16. Expected number of tastings is 2.39. Use Huffman code to find the mixtures, expected number of tastings lowers to 2.35.