

Probability Theory — LMAC/MMA, 1st Sem. 2021/22

General info

- **Faculty**

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- **Contacts**

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Floor 5 – Room 5.49 – Mathematics Building

- **General objectives**

To introduce students to probability theory at an intermediate-level, covering: probability spaces, random variables and distributions, transforms of random variables and vectors; independence; integration and expectation; convergence of sequences of random variables and classical limit theorems; conditional expectation and martingales.

- **Website**

- **Faculty website**

Detailed program

1. Probability spaces

- 1.1 Random experiments

- 1.2 Events and classes of sets

- 1.3 Probabilities and probability functions

- 1.4 Distribution functions; discrete, absolutely continuous and mixed probabilities

- 1.5 Conditional probability

2. Random variables

- 2.1 Fundamentals

- 2.2 Combining random variables

- 2.3 Distributions and distribution functions

- 2.4 Key r.v. and random vectors and distributions

- 2.4.1 Discrete continuous r.v. and random vectors

- 2.4.2 Absolutely continuous r.v. and random vectors

- 2.5 Transformation theory

- 2.5.1 Transformations of r.v., general case

- 2.5.2 Transformations of discrete r.v.

- 2.5.3 Transformations of absolutely continuous r.v.

- 2.5.4 Transformations of random vectors, general case

- 2.5.5 Transformations of discrete random vectors

- 2.5.6 Transformations of absolutely continuous random vectors

- 2.5.7 Random variables with prescribed distributions

3. Independence

- 3.1 Fundamentals

- 3.2 Independent r.v.

- 3.3 Functions of independent r.v.

- 3.4 Order statistics
- 3.5 Constructing independent r.v.
- 3.6 Bernoulli process
- 3.7 Poisson process
- 3.8 Generalizations of the Poisson process
- 4. Expectation
 - 4.1 Definition and fundamental properties
 - 4.1.1 Simple r.v.
 - 4.1.2 Non negative r.v.
 - 4.1.3 Integrable r.v.
 - 4.1.4 Complex r.v.
 - 4.2 Integrals with respect to distribution functions
 - 4.2.1 On integration
 - 4.2.2 Generalities
 - 4.2.3 Discrete distribution functions
 - 4.2.4 Absolutely continuous distribution functions
 - 4.2.5 Mixed distribution functions
 - 4.3 Computation of expectations
 - 4.3.1 Non negative r.v.
 - 4.3.2 Integrable r.v.
 - 4.3.3 Mixed r.v.
 - 4.3.4 Functions of r.v.
 - 4.3.5 Functions of random vectors
 - 4.3.6 Functions of independent r.v.
 - 4.3.7 Sum of independent r.v.
 - 4.4 L^p spaces
 - 4.5 Key inequalities
 - 4.5.1 Young's inequality
 - 4.5.2 Hölder's moment inequality
 - 4.5.3 Cauchy-Schwarz's moment inequality
 - 4.5.4 Lyapunov's moment inequality
 - 4.5.5 Minkowski's moment inequality
 - 4.5.6 Jensen's moment inequality
 - 4.5.7 Chebyshev's inequality
- 5. Convergence of random sequences and classical limit theorems
 - 5.1 Modes of convergence
 - 5.1.1 Convergence of r.v. as functions on Ω
 - 5.1.2 Convergence in distribution
 - 5.1.3 Alternative criteria
 - 5.2 Relationships among the modes of convergence
 - 5.2.1 Implications always valid
 - 5.2.2 Counterexamples
 - 5.2.3 Implications of restricted validity
 - 5.3 Convergence under transformations
 - 5.3.1 Continuous mappings

- 5.3.2 Algebraic operations
- 5.4 Convergence of random vectors
- 5.5 Limit theorems for Bernoulli summands
 - 5.5.1 Laws of large numbers for Bernoulli summands
 - 5.5.2 Central limit theorems for Bernoulli summands
 - 5.5.3 The Poisson limit theorem
- 5.6 Weak law of large numbers
- 5.7 Strong law of large numbers
- 5.8 Characteristic functions
- 5.9 The Central Limit Theorem
- 5.10 The law of the iterated logarithm
- 5.11 Applications of the limit theorems
- 6. Conditional expectation†
 - 6.1 Conditional expectation given a finite set of random variables
 - 6.2 Conditional expectation for $X \in L^2$
 - 6.3 Positive and integrable r.v.
 - 6.4 Conditional distributions
 - 6.5 Computational techniques
 - 6.6 Complements
- 7. Martingales†
 - 7.1 Fundamentals
 - 7.2 Stopping times
 - 7.3 Optional sampling theorems
 - 7.4 Martingale convergence theorems
 - 7.5 Applications of convergence theorems
 - 7.6 Complements

Bibliography

- **Recommended**

Karr, A.F. (1993). *Probability*. Springer-Verlag.

This book *is not encyclopaedic and quite deliberately incomplete, globally and locally* (Karr, 1993, p. v). The author also recommends that *students should be made aware of the role of probability within the broader context of measure theory* and also adds that *to dismiss probability as the special case of a ‘space with total measure one’ is neither honest nor helpful*.

- **Optional**

Resnick, S.I. (1999). *A Probability Path*. Birkhäuser. (QA273.4-.67.RES.49925)

Rohatgi, V.K. (1976). *An Introduction to Probability Theory and Mathematical Statistics*. John Wiley & Sons.

Teaching material

- **Lectures notes** (sent by e-mail, chapter by chapter)
- **Tables**
- **Detailed solutions** of the tests and exams from 2009/10, 2010/11, 2013/14, 2014/15.

Classes and schedule

- **Classes**

In English, in case there are any students who do not speak Portuguese, and with the following structure: motivation, result, example, exercise. The exercises are chosen and assigned to students in advance, and are solved by the students on the blackboard for *Mathematics is better learned actively than passively* (Karr, 1993, p. ix).

- **Schedule (classrooms)**

Mon., 13:30–15:30 (V1.11, Civil Engineering Building)

Tue., 9:00–11:00 (V1.27, Civil Engineering Building)

Tue., 19:00–20:00 (P8, Mathematics Building)

- **Office hours**

Proposed: Mon., 11:00–13:30, zoom (<https://videoconf-colibri.zoom.us/j/86203397812>)

The professor will eventually leave the zoom room if the students are more than 30 minutes late.

- **Extra office hours**

By appointment preferably by sms (927941249), tel. (218417047 or Ext. 1047) or e-mail (maj@math.ist.utl.pt), at least 24 hours in advance.

The professor will eventually leave the zoom room if the students are more than 30 minutes late.

Assessment method

- The assessment method comprises two compulsory components:
 - a 2 hour exam (50%);
 - at most five 30 minute tests, one per chapter (50%).
- The exam will cover all chapters/sections taught. The date of Exam 1 (resp. Exam 2) is February 7 (resp. February 18), 2022.
- To pass the course:
 - the mark of the exam has to be exceed 7.4 (out of 20) points;
 - and the average of the mark of the exam and of the mean of the scores of the 30 minute tests has to exceed 9.4 points (out of 20) points.

Whenever a student takes Exam 2, the mark obtained in it is considered for the final mark if and only if it is higher than that obtained in Exam 1.

- **Permitted material**

The only material allowed during the 2 hour exams and the 30 minute tests is:

- test/exam sheet and scrap paper (WITHOUT any written COMMENTS or ADDITIONS);
- calculator (GRAPHING calculators are NOT ALLOWED);
- tables and formulae available.

The use of any other material is punishable and can lead to the annulment of the exam.

The use of mobile phones is also forbidden during the 2 hour exams and the 30 minute tests.

- **Personal identification**

The student (or identity card) should be taken to the exam and the 30 minute tests.