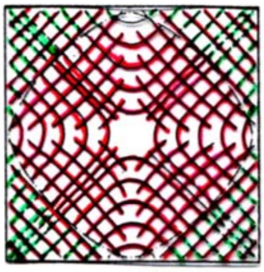
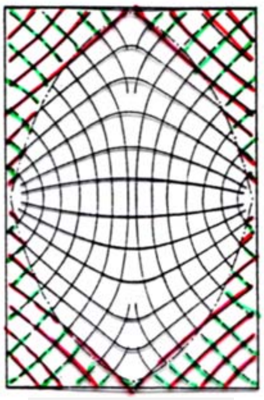
DESIGN OF REINFORCED CONCRETE SLABS

- **Slabs** – Particular cases
  - Panels with simply supported edges - corner reinforcement
  - Panels with free edges
  - Panels with concentrated loads
  - Panels with triangular loads - support walls and rectangular tanks



$b = a$



$b = 1.5\, a$

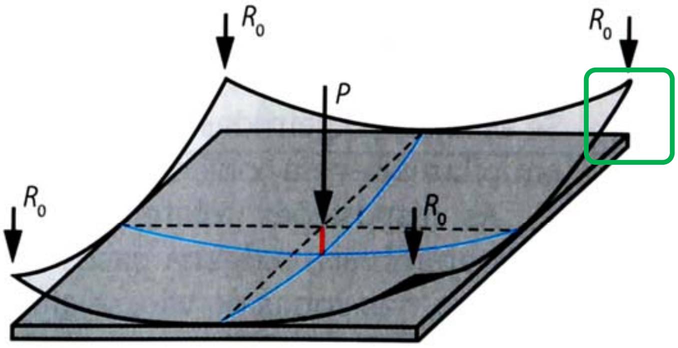
--- Negative bending moments

__ Positive bending moments

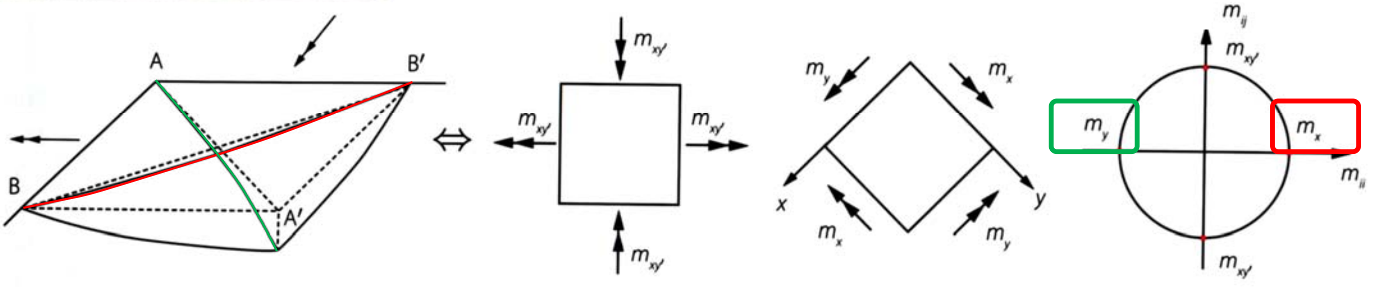
Simply supported panels – main bending moments directions

DESIGN OF REINFORCED CONCRETE SLABS

Panels with simply supported edges - corner reinforcement



Impossible – edges can't lift from the supports



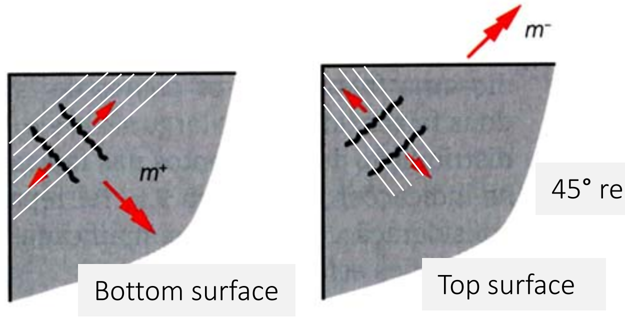
x, y – direcções principais

$|m_{xy'}| = |m_x| = |m_y|$

Ref. - Appleton, J. (2013). Estruturas de betão. Edições Orion

DESIGN OF REINFORCED CONCRETE SLABS

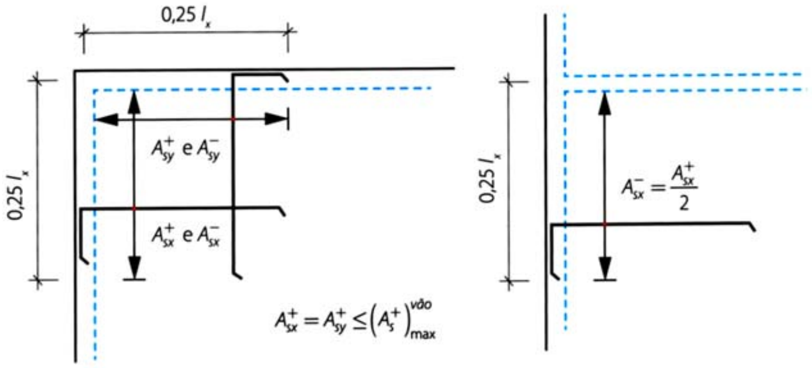
Panels with simply supported edges - corner reinforcement



45° reinforcement is not easy to assemble

It is observed $m_{xy} \leq \{m_x\,;m_y\}_{1/2span}$ thus it is common practice to adopt:

1) Bottom reinforcement – extend the reinforcement from the span to the supports
2) Top reinforcement – put additional corner reinforcement in both directions, equal to the one used at mid-span in $0.25a$ and $0.25b$



Ref. - Appleton, J. (2013). Estruturas de betão. Edições Orion

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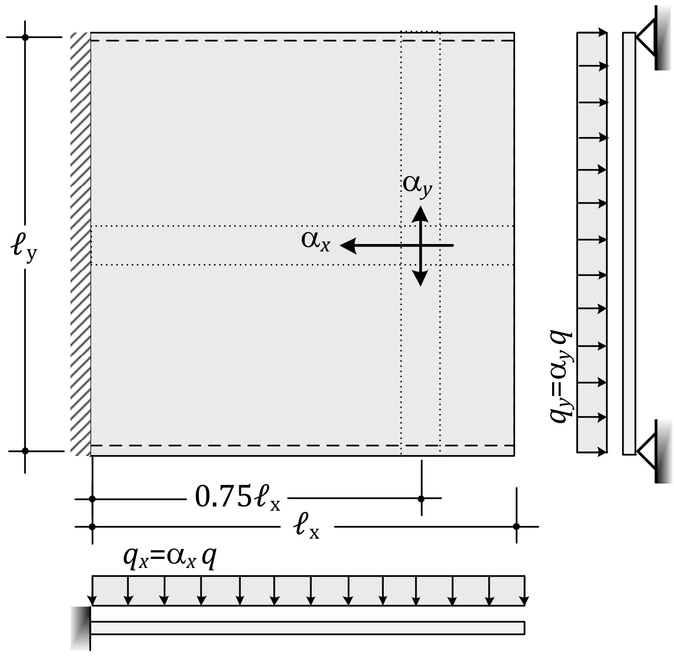
DESIGN OF REINFORCED CONCRETE SLABS

Panels with a free edge



$$\text{if} \quad \ell_x = \ell_y$$

$$\beta = \frac{c_x}{c_y}\left(\frac{\ell_x}{\ell_y}\right)^4 \approx \frac{1/8}{5/384}\left(\frac{0.75\ell_x}{\ell_y}\right)^4 = 3.0$$

$$\therefore \quad \alpha_y = \frac{\beta}{(1+\beta)} \approx 0.75$$

$$m_x = \frac{0.25\,q\cdot\ell_x^2}{2} = -0.125\,q\cdot\ell_x^2$$

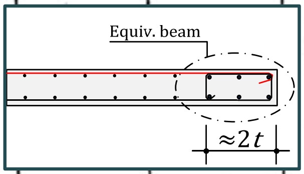
$$m_y = \frac{0.75\,q\cdot\ell_y^2}{8} = 0.094\,q\cdot\ell_y^2$$

DESIGN OF REINFORCED CONCRETE SLABS

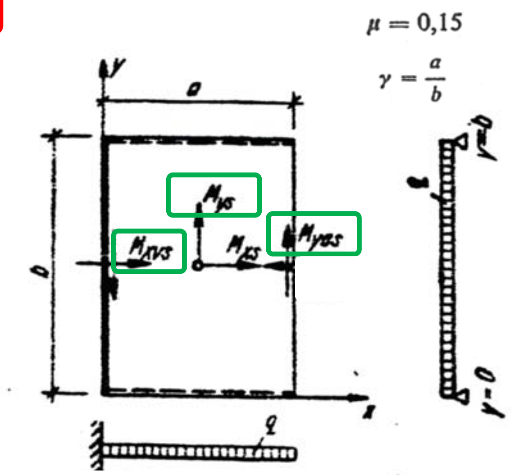
$$m_{xvs} = -0.1182\, q \cdot \ell_x^2$$

$$m_{ys} = 0.0536\, q \cdot \ell_x^2 \quad ; \quad \boxed{m_{yas} = 0.0955\, q \cdot \ell_y^2 \ \ (+78\% m_{ys})}$$

Panels with free edges



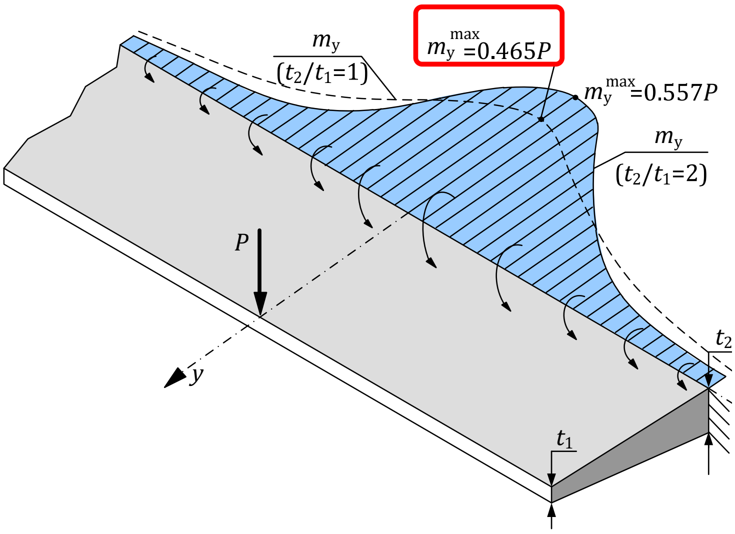
γ	w_s	w_{bs}	M_{xs}	M_{xvs}	M_{ys}	M_{yas}
0.3			−0,0733	−0.4308	0,0007	0.0056
0.4			−0,0300	−0.3687	0.0048	0.0153
0.5			−0.0056	−0.3091	0,0106	0.0288
0.6			0.0106	−0,2513	0.0180	0.0436
0,7			0.0188	−0,2066	0,0269	0.0594
0,8			0.0223	−0.1702	0,0366	0.0736
0,9			0.0228	−0.1416	0,0454	0.0858
1,0	0,0650	0,1176	0,0221	−0,1182	0,0536	0,0955
1,2	0,0392	0,0649	0,0187	−0.0845	0,0680	0.1098
1,5	0,0200	0,0296	0,0113	−0,0548	0,0850	0.1229
2,0	0,0077	0,0100	0,0068	−0,0312	0,1031	0.1308
Fact. de mult.	$\frac{qa^4}{Eh^3}$	$\frac{qa^4}{Eh^3}$	qa^2	qa^2	qb^2	qb^2

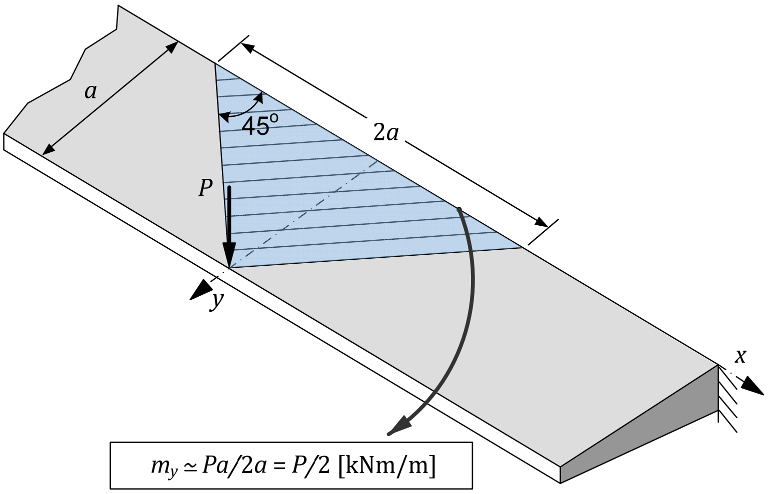


Ref. - Bares, R. (1981) Tablas para el cálculo de placas y vigas pared, Editorial Gustavo Gili.

DESIGN OF REINFORCED CONCRETE SLABS

Panels with concentrated loads

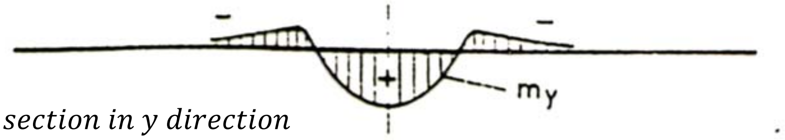




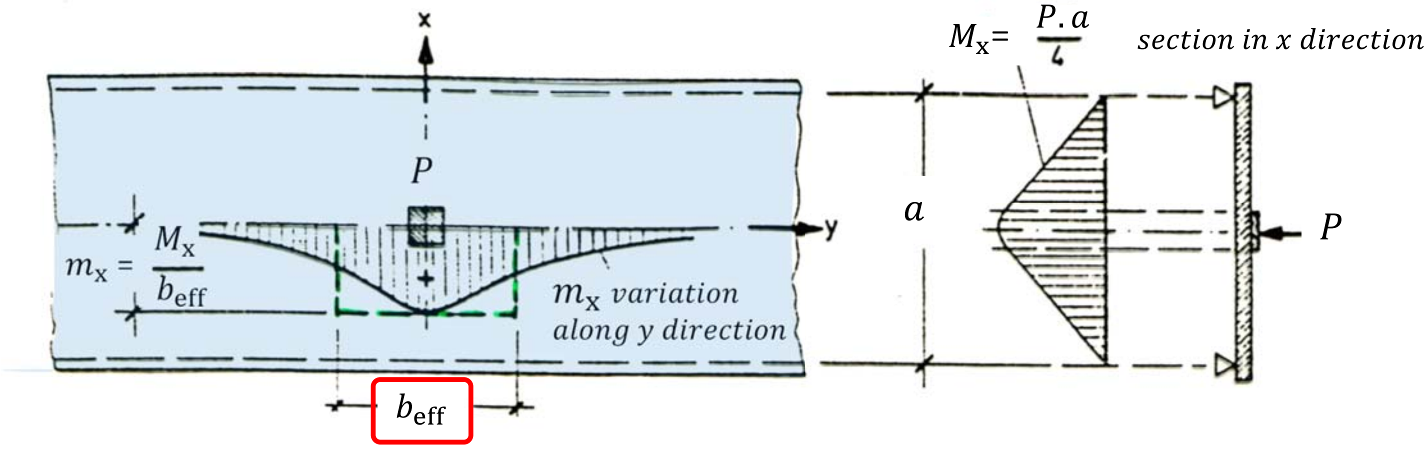
$m_y \simeq Pa/2a = P/2$ [kNm/m]

DESIGN OF REINFORCED CONCRETE SLABS

Panels with concentrated loads



section in y direction



DESIGN OF REINFORCED CONCRETE SLABS

Panels with concentrated loads

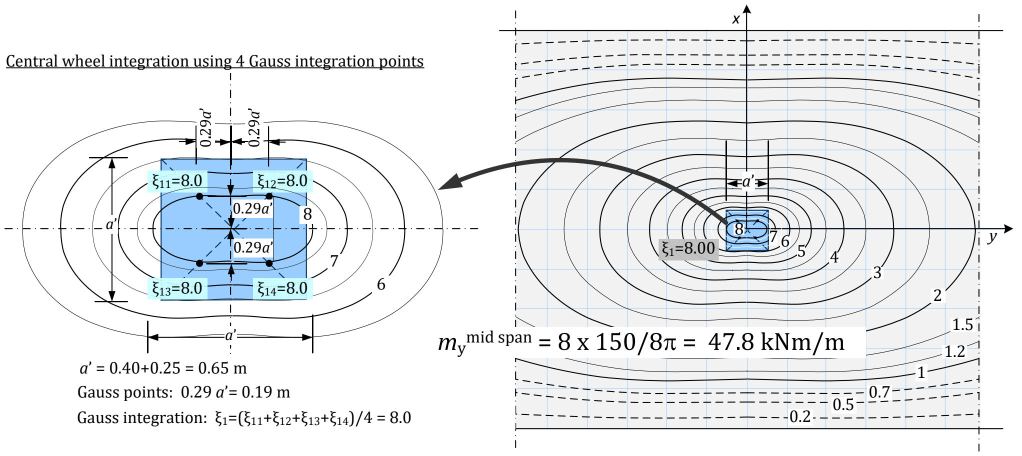
Using the slab influence surface for a mid-span bending moment, and for P = 150 kN distributed in a square area of side 0.40m, it is obtained :

$$m_x = 47.8 \text{ kNm/m}$$

As: $M_x = 0.25 \cdot P \cdot a$

We get: $b_{eff} = \frac{M_x}{m_x} \approx 0.80\,a$

i.e. the specific reinforcement for the concentrated load must be distributed in the width b_{eff} = 0.80 a



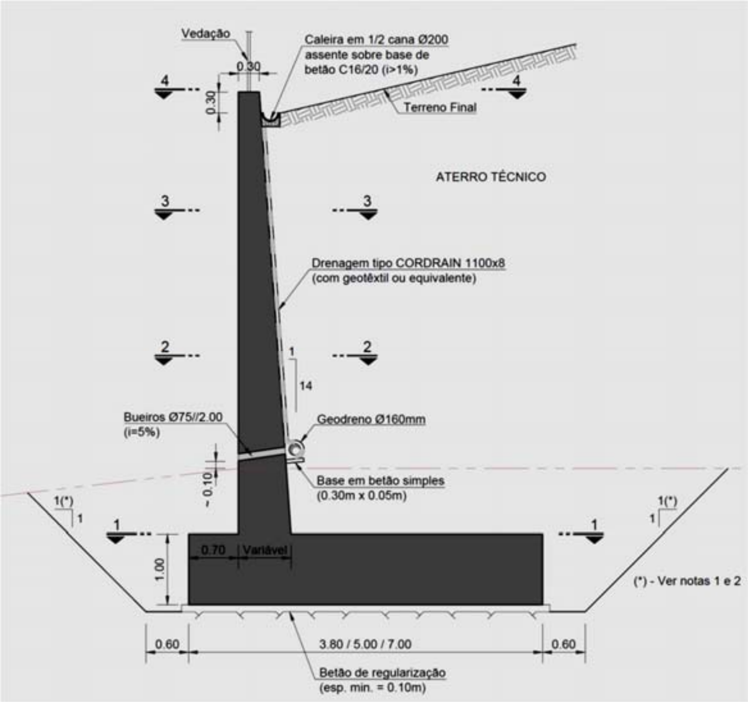
a' = 0.40+0.25 = 0.65 m

Gauss points: 0.29 a'= 0.19 m

Gauss integration: $\xi_1=(\xi_{11}+\xi_{12}+\xi_{13}+\xi_{14})/4 = 8.0$

$m_y^{\text{mid span}}$ = 8 x 150/8π = 47.8 kNm/m

DESIGN OF REINFORCED CONCRETE SLABS



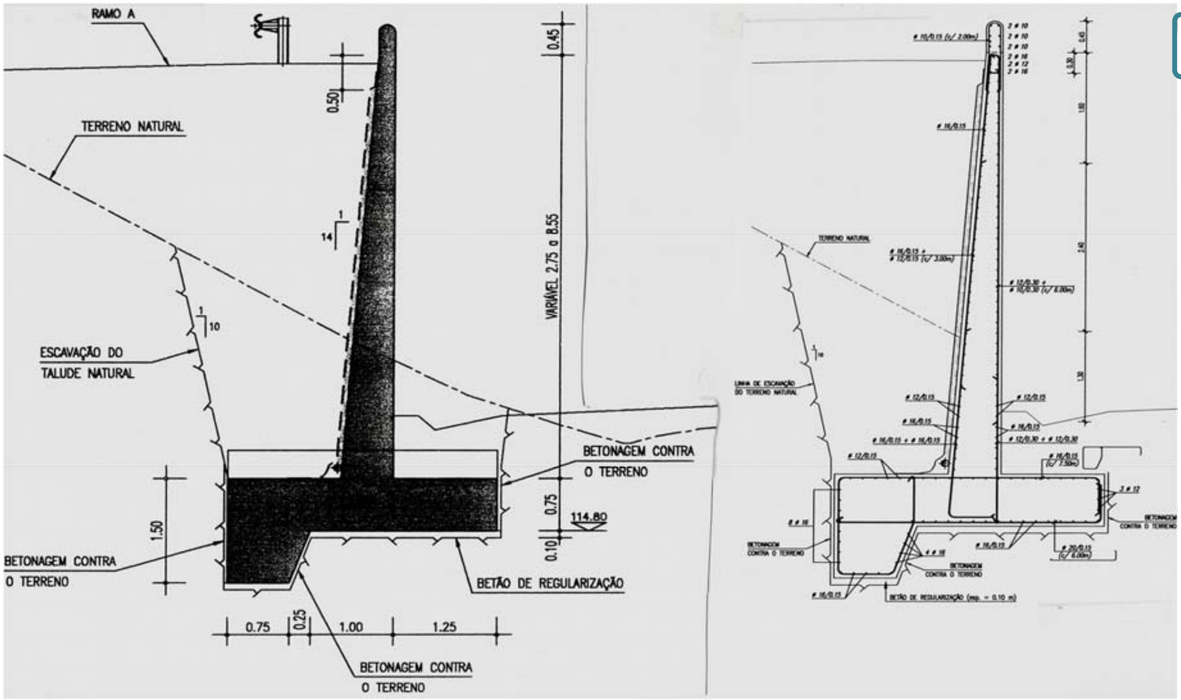
Cantilever wall

$$H < 6\,m$$

$$\frac{H}{2} < B < \frac{2H}{3}$$

$$H_{fund} = \frac{H}{10} > 0.5\text{m}$$

DESIGN OF REINFORCED CONCRETE SLABS



Cantilever wall

$$H < 6\,m$$

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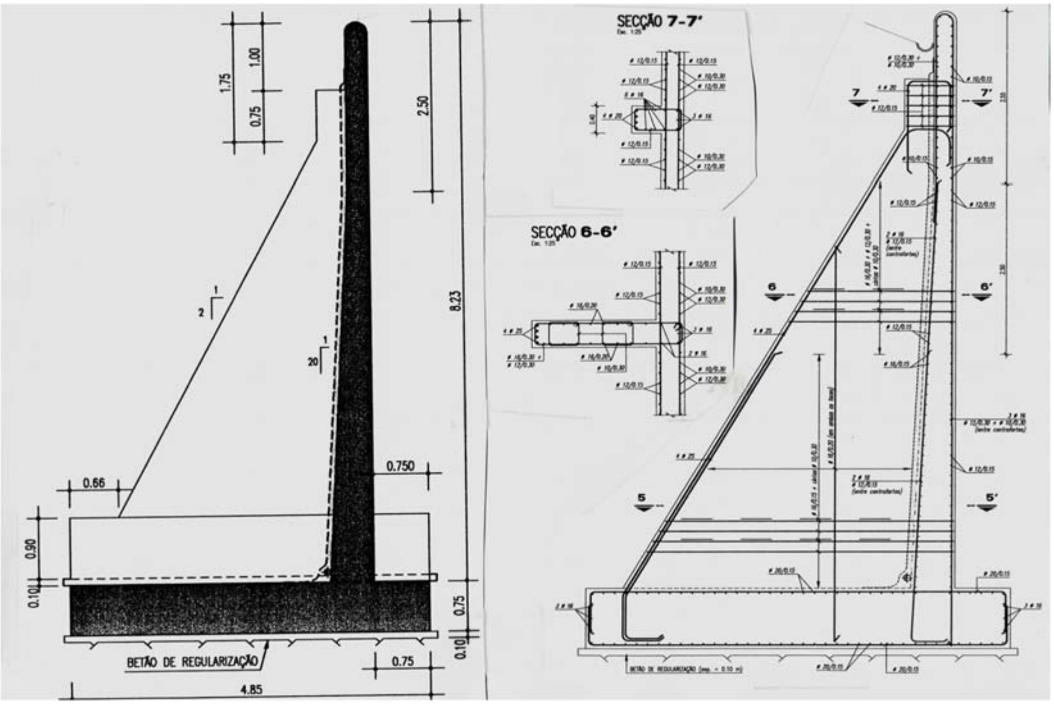
DESIGN OF REINFORCED CONCRETE SLABS



Wall with counterforts

$$5\text{ m} < H < 10\text{ m}$$

$$\frac{H}{2} < B < \frac{2H}{3}$$

$$H_{fund} = \frac{H}{10} > 0.5\text{m}$$

$$@_{cont.} = 4\text{ a }6\text{m}$$

$$t_{cont.} = 0.4\text{ a }0.5\text{m}$$

DESIGN OF REINFORCED CONCRETE SLABS

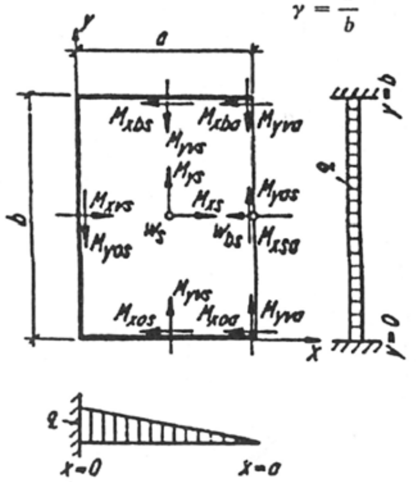
γ	w_s	w_{bs}	M_{xs}	M_{xvs}	M_{ys}	M_{yas}	M_{yvs}	M_{yva}
0,3	0,1158	0,2461	−0,0089	−0,1369	0,0007	0,0024	−0,0048	−0,0083
0,4	0,0733	0,1374	0,0025	−0,1147	0,0021	0,0048	−0,0079	−0,0131
0,5	0,0469	0,0825	0,0080	−0,0916	0,0038	0,0068	−0,0117	−0,0158
0,6	0,0353	0,0516	0,0114	−0,0728	0,0059	0,0083	−0,0160	−0,0166
0,7	0,0264	0,0293	0,0122	−0,0565	0,0081	0,0092	−0,0202	−0,0164
0,8	0,0192	0,0169	0,0122	−0,0453	0,0104	0,0099	−0,0241	−0,0156
0,9	0,0132	0,0102	0,0110	−0,0390	0,0119	0,0099	−0,0272	−0,0138
1,0	0,0095	0,0062	0,0091	−0,0345	0,0129	0,0095	−0,0301	−0,0119
1,2	0,0058	0,0026	0,0060	−0,0260	0,0148	0,0082	−0,0347	−0,0100
1,5	0,0027	0,0008	0,0030	−0,0182	0,0169	0,0063	−0,0382	−0,0074
2,0	0,0009	0,0002	0,0012	−0,0112	0,0191	0,0041	−0,0412	−0,0046
F. m.	$\frac{qa^4}{Eh^3}$	$\frac{qa^4}{Eh^3}$	qa^2	qa^2	qb^2	qb^2	qb^2	qb^2

Wall with counterforts

Tabla 1.92

$\mu = 0,15$

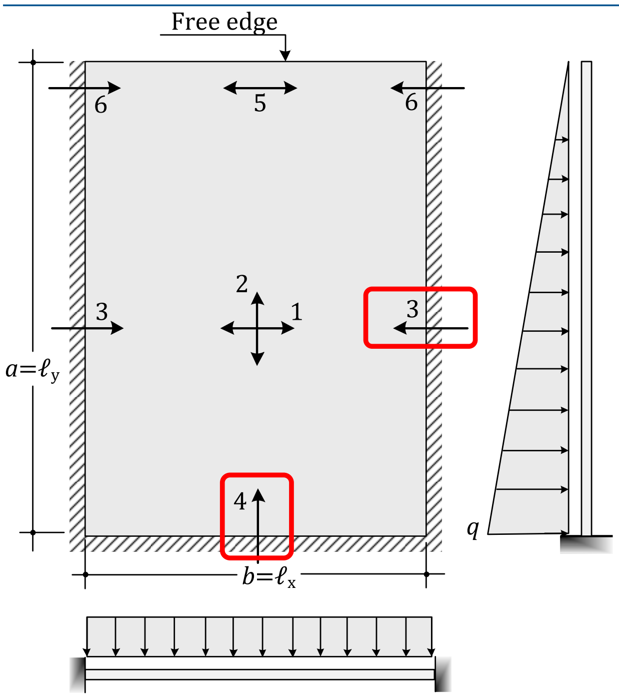
$\gamma = \frac{a}{b}$



Ref. - Bares, R. (1981) Tablas para el cálculo de placas y vigas pared, Editorial Gustavo Gili.

DESIGN OF REINFORCED CONCRETE SLABS

Wall with counterforts



if

$$\gamma = \frac{\ell_y = a = 7.5}{\ell_x = b = 5.0} = 1.5$$

and

$$q = 7.5 \cdot 10 = 75\ \text{kN/m}^2$$

we get

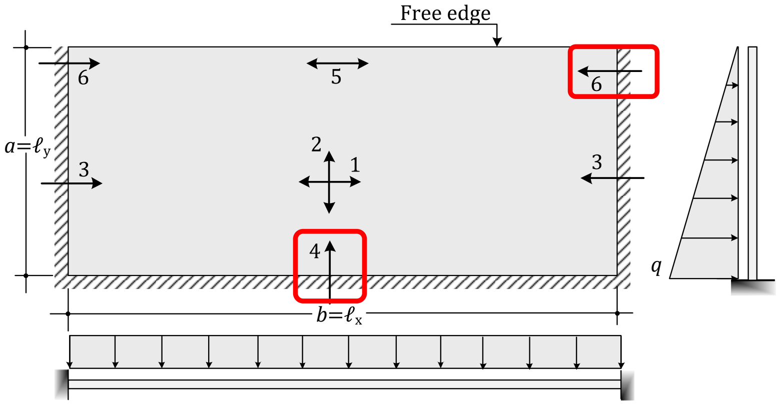
$$m_1 = +0.0169\, q \cdot \ell_x^2 = +31.7\ \text{kNm/m}$$
$$m_2 = +0.0030\, q \cdot \ell_y^2 = +12.7\ \text{kNm/m}$$
$$m_3 = -0.0382\, q \cdot \ell_x^2 = \boxed{-71.6\ \text{kNm/m}}$$
$$m_4 = -0.0182\, q \cdot \ell_y^2 = \boxed{-76.8\ \text{kNm/m}}$$
$$m_5 = +0.0063\, q \cdot \ell_x^2 = +11.8\ \text{kNm/m}$$
$$m_6 = -0.0074\, q \cdot \ell_x^2 = -13.9\ \text{kNm/m}$$

notice that if the it was a cantilever wall

$m_4 = -1/6\, q \cdot \ell_y^2 = -703.1\ \text{kNm/m}$ (9 x bigger)

DESIGN OF REINFORCED CONCRETE SLABS

Tank with 10x10x3m^3



we get

$$m_1 = +0.0007\, q \cdot \ell_x^2 = +2.10\ \text{kNm/m}$$
$$m_2 = -0.0089\, q \cdot \ell_y^2 = -2.40\ \text{kNm/m}$$
$$m_3 = -0.0048\, q \cdot \ell_x^2 = -14.4\ \text{kNm/m}$$
$$m_4 = -0.1369\, q \cdot \ell_y^2 = \boxed{-40.0\ \text{kNm/m}}$$
$$m_5 = +0.0024\, q \cdot \ell_x^2 = +7.20\ \text{kNm/m}$$
$$m_6 = -0.0083\, q \cdot \ell_x^2 = \boxed{-24.9\ \text{kNm/m}}$$

if

$$\gamma = \frac{\ell_y = a = 3.0}{\ell_x = b = 10.0} = 0.3$$

and

$$q = 3.0 \cdot 10 = 30\ \text{kN/m}^2$$

notice that if the slab was in cantilever

$m_4 = -1/6\, q \cdot \ell_y^2 = -45.0\ \text{kNm/m}$ (almost the same value!)