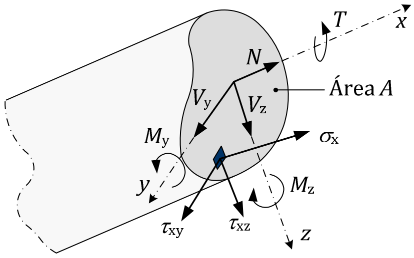
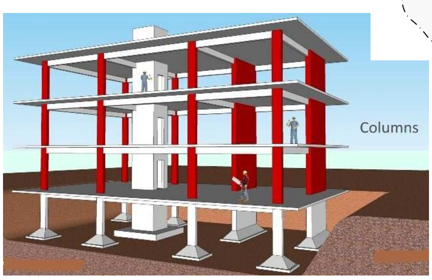
TYPES OF STRUCTURAL ELEMENTS

- **LINEAR ELEMENTS – Beam, Cable, Strut or Column, Arch**
- **PLATE ELEMENTS**
  - **– Flat - Plates and Slabs**
  - **– Curved - Membranes and Shells**
- **MASSIVE ELEMENTS**



Internal forces in the cross-section of a linear element



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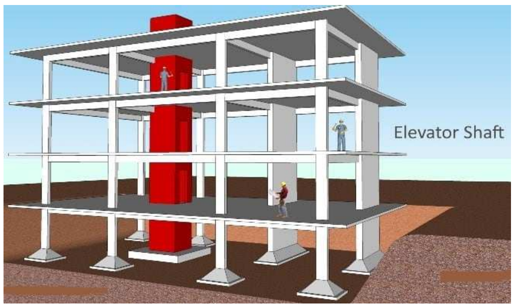

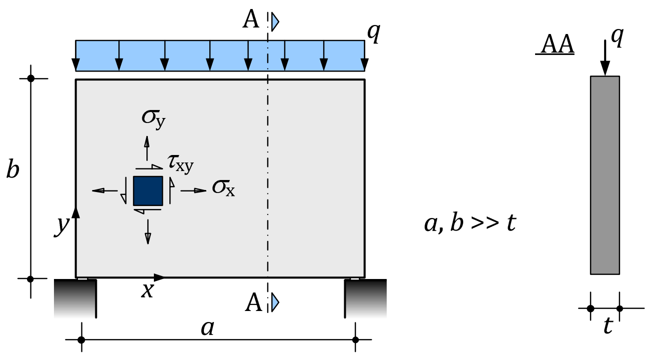

Beam-wall (plate subject mainly to bending)

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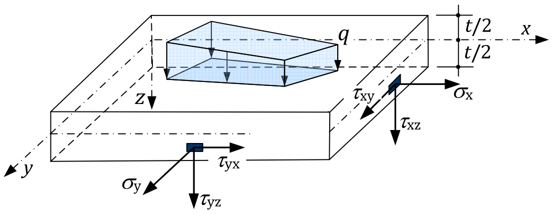

Slab subject to a surface-distributed load

TYPES OF STRUCTURAL ELEMENTS

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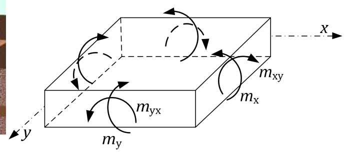

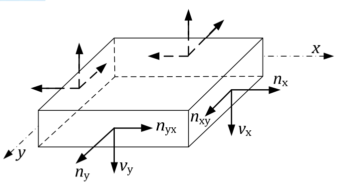

Internal forces on plate elements

TYPES OF STRUCTURAL ELEMENTS

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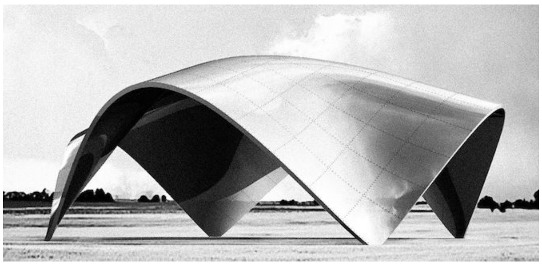
Membrane - tension stresses



Shell - compression stresses

TYPES OF STRUCTURAL ELEMENTS

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- **MASSIVE ELEMENTS**

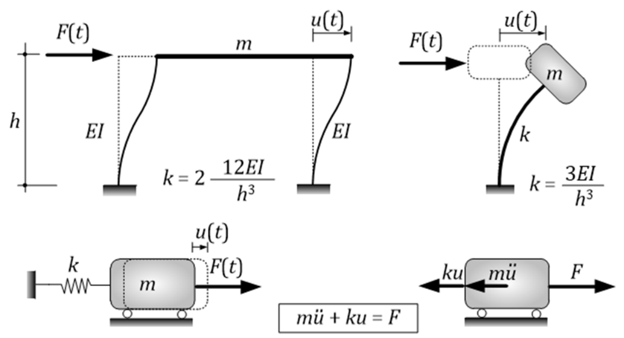
METHODS OF ANALYSIS AND STRUCTURAL DESIGN

- **STATIC ANALYSIS / DYNAMIC ANALYSIS**
- **ELASTIC ANALYSIS / PLASTIC ANALYSIS**
- **LINEAR ANALYSIS / NONLINEAR ANALYSIS**
  - **Geometrically nonlinear analysis**
  - **Physically nonlinear analysis**
  - **Physical and geometrically nonlinear analysis**

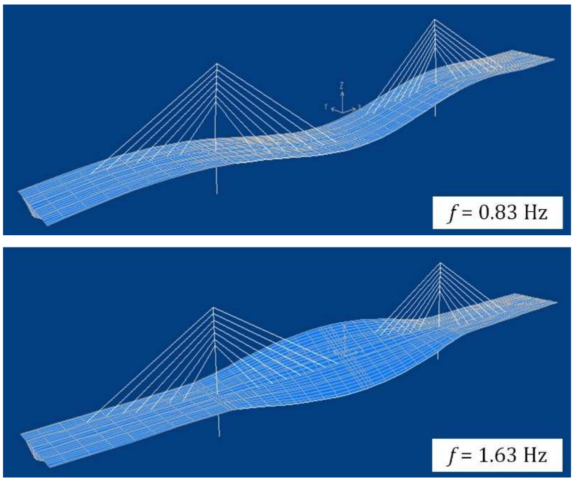
STATIC ANALYSIS / DYNAMIC ANALYSIS

The behaviour of a structure submitted to the applied actions can be achieved through a **static analysis** if the actions do not produce significant accelerations

If the action is rapid (e.g. seismic action) a **dynamic analysis** of the structure should be performed

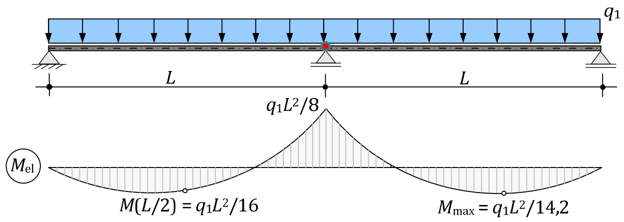


Dynamic equilibrium of a simple 1DL system without internal damping



ELASTIC ANALYSIS / PLASTIC ANALYSIS

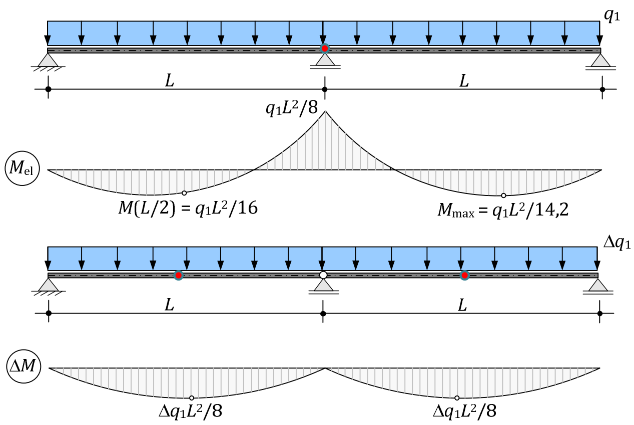
Elastic analysis – a structure behaviour is assumed with a material linear stress-strain laws, regardless of the level of load applied – Serviceability Limit State - SLS



Distribution of bending moments obtained in an elastic analysis of the structure

$$M_{max} = \frac{q \cdot L^2}{8} \leq M_{pl} \quad \therefore \quad q_u = 8\,\frac{M_{pl}}{L^2}$$

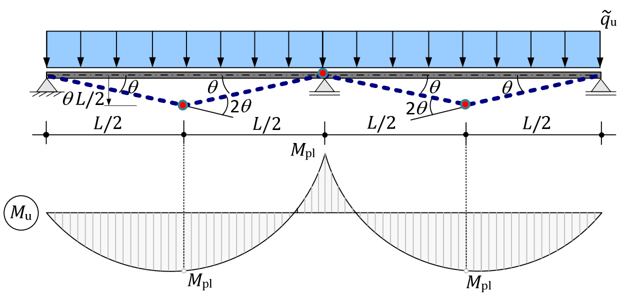
Plastic analysis – considers the plasticity of the materials (e.g. cracking and yielding of certain sections or regions of the structure) in the behaviour of the structure - Ultimate Limit State - ULS



Superposition of bending moments obtained in a plastic analysis of the structure

ANÁLISE ELÁSTICA / ANÁLISE PLÁSTICA

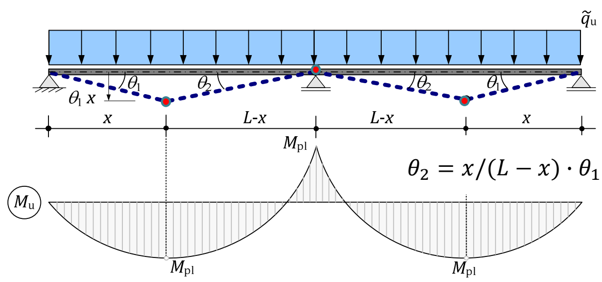
Plastic analysis – assuming the formation of plastic hinges in the mid-spans sections, the ultimate load is obtained by:



$$2\left[\frac{1}{2}\cdot\left(\frac{L}{2}\cdot\theta\right)\cdot L\right]\cdot\tilde{q}_u = 2\cdot[\,2\theta+\theta\,]\cdot M_{pl}$$

$$\therefore \quad \tilde{q}_u = 12\,\frac{M_{pl}}{L^2}$$

Plastic analysis – trying to get the mechanism with the lowest ultimate load:



$$2\left[\frac{1}{2}\cdot(\theta_1\cdot x)\cdot L\right]\cdot\tilde{q}_u(x) = 2[\,(\theta_1+\theta_2)+\theta_2\,]\cdot M_{pl}$$

$$= 2M_{pl}\cdot\theta_1\cdot\left[1+\frac{2x}{L-x}\right] \quad \therefore \quad \tilde{q}_u\,(x) = 2\,\frac{M_{pl}}{L}\cdot\left[\frac{1}{x}+\frac{2}{L-x}\right]$$

$$\left.\frac{\mathrm{d}\tilde{q}_u(x)}{\mathrm{d}x}\right|_{x\,=\,x_s} = 2\,\frac{M_{pl}}{L}\cdot\left[\frac{1}{x_s}+\frac{2}{L-x_s}\right] = 0 \quad \therefore$$

$$x_s = (\sqrt{2}-1)\cdot L = 0{,}414\cdot L \quad \therefore \quad q_u = 11{,}66\,\frac{M_{pl}}{L^2}$$

ELASTIC ANALYSIS / PLASTIC ANALYSIS

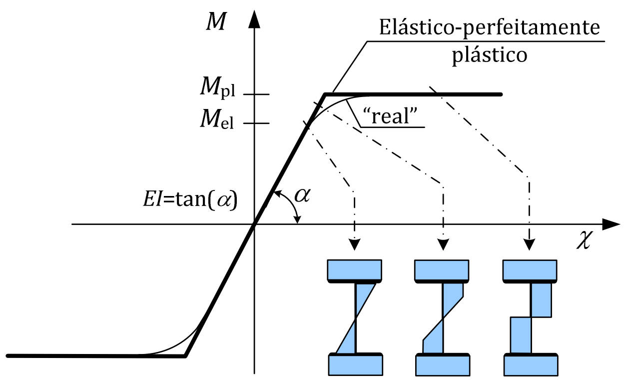
General conditions required to perform a design based on a plastic analysis of the structure :

- ✓ Ductility – the structure must have sufficient deformation capacity in areas where plastic behaviour is foreseen including the connections between elements (e.g. bolted connections of steel structures and beam/column connections in concrete structures)
- ✓ Local stability – local buckling of the plastic regions should not occur (e.g. steel structures with class 1 or 2 sections)
- ✓ Global stability – lateral bending in the elements where plastic resistance is attend should be prevented
- ✓ Small deformations – if the effects of the deformation of the structure do not need to be considered in the internal force distributions (i.e. whether the effects $P\text{–}\Delta$ are despicable)

IN ANY CASE, IT REMAINS TO BE NECESSARY TO PERFORM A IN SERVICE DESIGN ASSUMING AN ELASTIC BEHAVIOUR OF THE STRUCTURE (accepting a certain degree of cracking in concrete structures, with the crack openings controlled)

FUNDAMENTAL REQUIREMENT – in any type of analysis used it should be ensured the equilibrium between the applied actions and the corresponding internal force distributions

ELASTIC ANALYSIS / PLASTIC ANALYSIS



Moment-curvature /24 and distribution of normal stresses due to bending (symmetrical, homogeneous, section of elastic-perfectly plastic material)

$$Plastic\ moment - \ M_{Rk} = M_{pl} = W_{pl} \cdot f_y$$

$$Elastic\ moment - \ M_{Rk} = M_{el} = W_{el} \cdot f_y$$

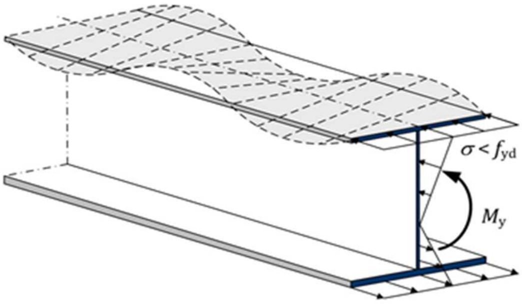
$$Effective\ elast.moment - M_{Rk} = M_{el,eff} = W_{el,eff} \cdot f_y$$

$$plastic\ form\ factor - \alpha = \frac{M_{pl}}{M_{el}} \left(= \frac{W_{pl}}{W_{el}} \right)$$

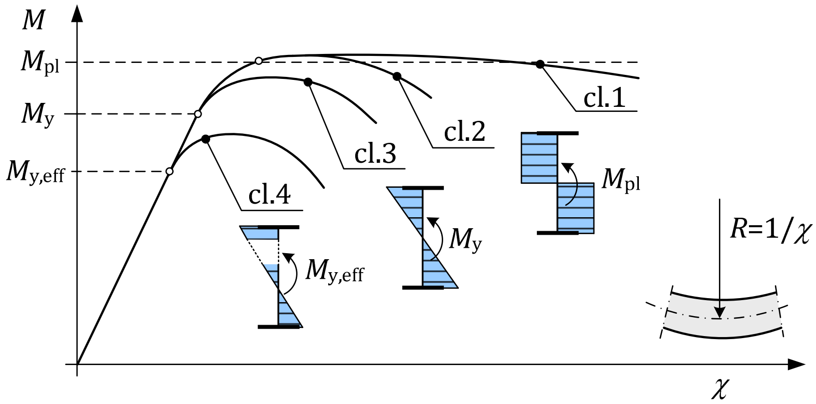
$$Type\ profiles\ IPE\ - \ \alpha \approx 1{,}15$$

Elastic or plastic design of steel sections

ELASTIC ANALYSIS / PLASTIC ANALYSIS



Local plate buckling of a compressive flange of a steel profile with H-section



Classes of steel sections as a function of local plate bending

Elastic or plastic design of steel sections

ELASTIC ANALYSIS / PLASTIC ANALYSIS

Class 1 – cross-sections are those which can form a plastic hinge with the rotation capacity required from plastic analysis without reduction of the resistance;

Class 2 – cross-sections are those which can develop their plastic moment resistance, but have limited rotation capacity because of local buckling;

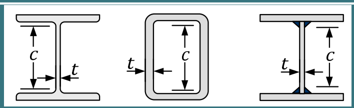
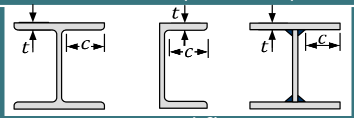
Plastic design

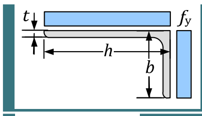
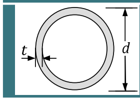
Class 3 – cross-sections are those in which the stress in the extreme compression fibre of the steel member assuming an elastic distribution of stresses can reach the yield strength, but local buckling is liable to prevent development of the plastic moment resistance;

Class 4 – cross-sections are those in which local buckling will occur before the attainment of yield strength in any part of the cross-section.

Elastic design

ELASTIC ANALYSIS / PLASTIC ANALYSIS

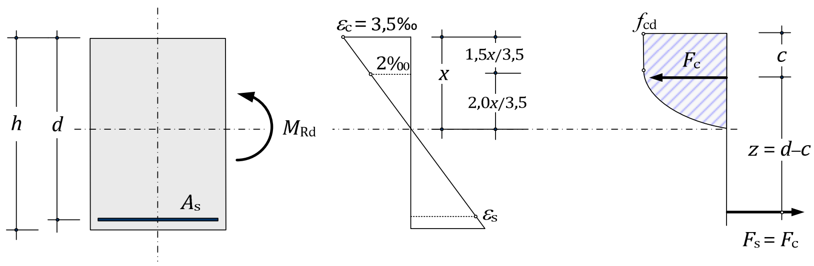
Stress distribution	constant ("C" = simple compression)			linear ("F" = simple bending)		
Class	1(C)	2(C)	3(C)	1(F)	2(F)	3(F)
 Internal compression parts	$33\ \varepsilon$	$38\ \varepsilon$	$42\ \varepsilon$	$72\ \varepsilon$	$83\ \varepsilon$	$124\ \varepsilon$
 Outstand flanges	$9\ \varepsilon$	$10\ \varepsilon$	$14\ \varepsilon$	---	---	---

Class		1	2	3
	Angles with pure compression	---	---	$\frac{h}{t} \leq 15\ \varepsilon$; $\frac{h+b}{2\ t} \leq 11{,}5\ \varepsilon$
	Tubular hollow sections in compression / bending	$d/t \leq 50\ \varepsilon^2$	$d/t \leq 70\ \varepsilon^2$	$d/t \leq 90\ \varepsilon^2$

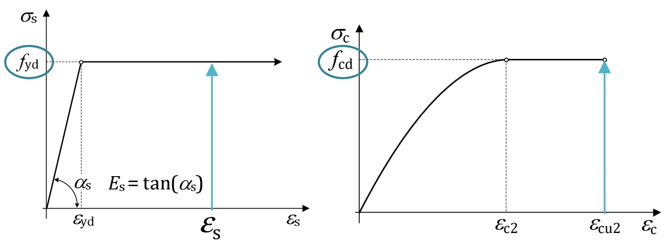
Limits of the "plate width"/thickness ratio (c/t) for internal or outstand compression parts ($\varepsilon = \sqrt{235/f_y}$)

ELASTIC ANALYSIS / PLASTIC ANALYSIS

Determination of M_{Rd} based on the parabola-rectangle diagram



a) cross-section b) stain diagram c) distribution of resulting stresses and forces



$$M_{\mathrm{Ed}} \leq M_{\mathrm{Rd}}$$

$$\mu = \frac{M_{\mathrm{Ed}}}{b \cdot d^2 \cdot f_{\mathrm{cd}}}$$

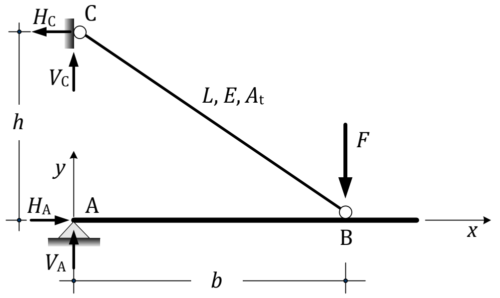
$$\omega(\mu) = 0{,}973 - \sqrt{0{,}946 - \frac{\mu}{0{,}514}}$$

Or $\omega(\mu) \approx \mu \cdot (1 + \mu)$

$$A_s \geq \frac{b \cdot d \cdot f_{\mathrm{cd}}}{f_{\mathrm{yd}}}\ \omega$$

Resistance of reinforced concrete sections subjected to simple bending

LINEAR ANALYSIS / NONLINEAR ANALYSIS



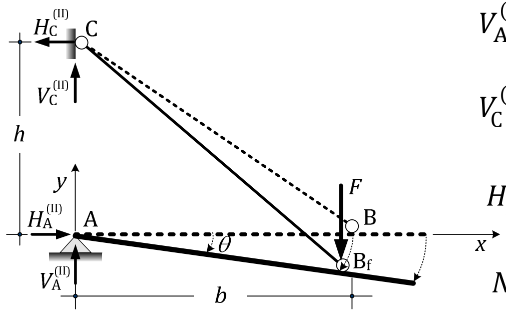
$$V_A^{(I)} = 0$$

$$V_C^{(I)} = F$$

$$H_A^{(I)} = F \cdot \frac{b}{h} = H_C^{(I)}$$

$$N_t^{(I)} = F \cdot \frac{L}{h}$$

Geometrically linear analysis – equilibrium performed in the initial undeformed position



$$V_A^{(II)} = -F \cdot \frac{b}{h} \cdot \text{sen}(\theta)$$

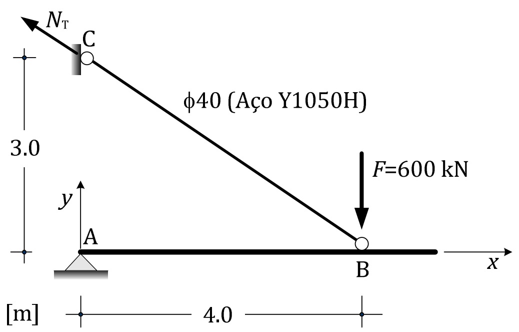
$$V_C^{(II)} = F \cdot \left[1 + \frac{b}{h} \cdot \text{sen}(\theta)\right]$$

$$H_A^{(II)} = F \cdot \frac{b}{h} \cdot \cos(\theta) = H_C^{(II)}$$

$$N_t^{(II)} = \sqrt{V_C^{(II)2} + H_C^{(II)2}}$$

Geometrically nonlinear analysis – equilibrium performed in the current deformed position

LINEAR ANALYSIS / NONLINEAR ANALYSIS



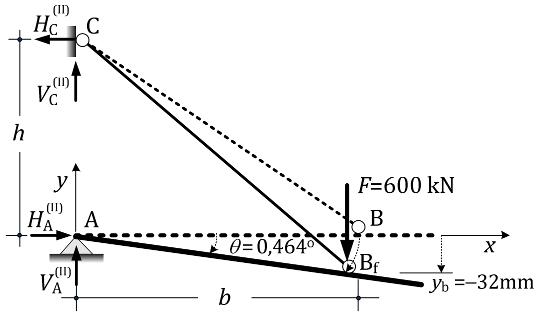
$$V_A^{(I)} = 0 \text{ kN}$$

$$V_C^{(I)} = 600 \text{ kN}$$

$$H_A^{(I)} = 800 \text{ kN} = H_C^{(I)}$$

$$N_t^{(I)} = 1000 \text{ kN}$$

Geometrically linear (1st order) analysis – equilibrium performed in the initial undeformed position



$$V_A^{(II)} = -6{,}5 \text{ kN}$$

$$V_C^{(II)} = 606{,}5 \text{ kN}$$

$$H_A^{(II)} = 799{,}97 \text{ kN} = H_C^{(II)}$$

$$N_t^{(II)} = 1003{,}9 \text{ kN}$$

Geometrically nonlinear (2nd order) analysis – equilibrium performed in the current deformed position resulting from the applied loads

LINEAR ANALYSIS / NONLINEAR ANALYSIS

The following three situations should be identified according to the relevance of the second-order effects:

- the second-order effects are despicable => the design can be based on the results of first-order analyses and it is not necessary to amplify these results to take into account, indirectly, the second-order effects;
- the second-order effects are moderately significant => the design can be based on the results of first-order analyses, amplified to take into account, indirectly, the second-order effects;
- the second-order effects are very significant and should be considered directly in the analysis.

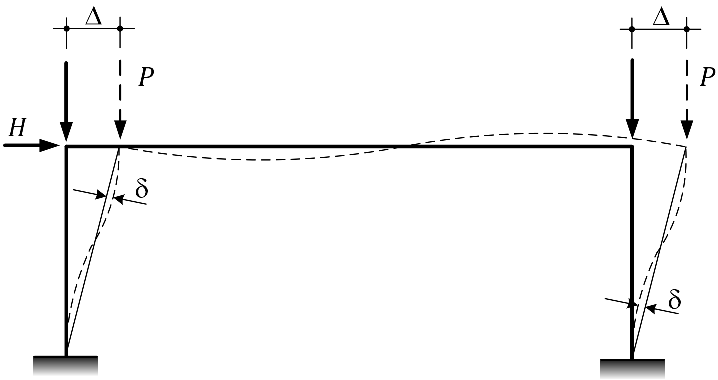
Geometrically linear (1st order) analysis – possibly with an amplification factor of the effects

Geometrically nonlinear (2nd order) analysis – cases of very deformable structures

LINEAR ANALYSIS / NONLINEAR ANALYSIS

In reticulated structures, it is usual to subdivide second-order effects into two following categories:

- **the effects P–Δ** (designated by **global second-order effects**), associated with relative displacements of the ends of compressed elements, and
- **the effects P–δ** (designated by **local second-order effects**), transverse displacements along each compressed element, measured in relation to the respective chord.



In situations where it is necessary to explicitly consider the effects of 2nd order The ***P*-Δ** effects are usually incorporated into the global analysis, while the ***P*-δ** effects are considered in the resistance formulas used to assess structural safety (the same procedure that is used to take into account the initial geometric imperfections).

LINEAR ANALYSIS / NONLINEAR ANALYSIS

For **reinforced concrete building structures**, EC2-1-1 states that second-order global effects can be disregarded if they correspond to less than 10% of the respective first-order effects.

For **steel building structures**, in EC3-1-1 it is indicated that ***P*–Δ** effects may be disregarded for a given combination of actions if the following condition is met:

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 10$$

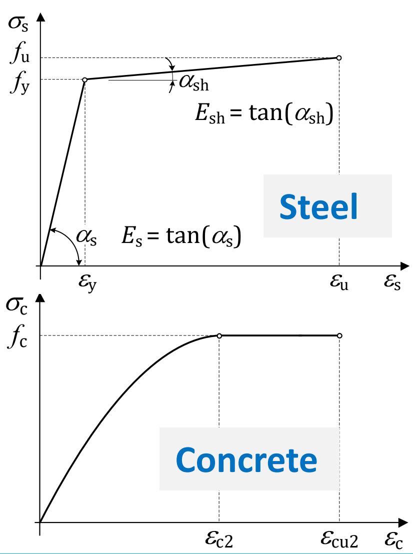
where α_{cr} is the factor by which the design load would have to be multiplied to "cause" elastic instability in a global in-plane sway mode.

If $3 \leq \alpha_{cr} < 10$ the 2nd order effects can be considered by increasing the effects of the 1st order analysis by the factor:

$$\beta = \frac{\alpha_{cr}}{\alpha_{cr} - 1} \qquad (1{,}0 < \beta \leq 1{,}5 \quad \text{with} \quad \alpha_{cr} \geq 3{,}0)$$

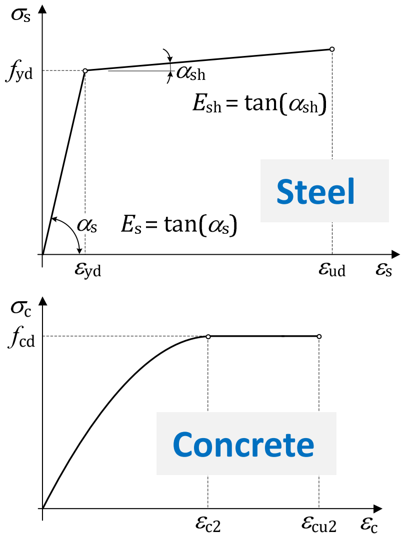
LINEAR ANALYSIS / NONLINEAR ANALYSIS

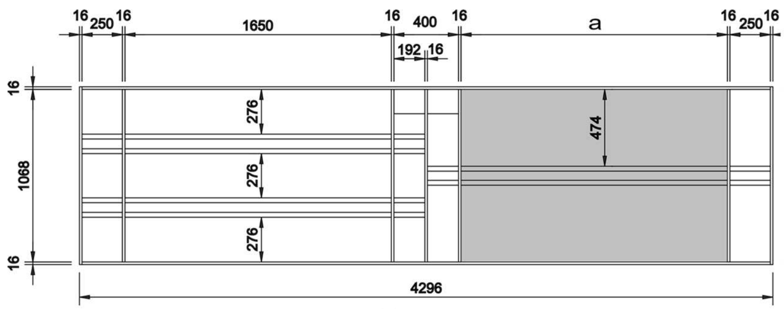
Physical and geometrically nonlinear analysis – Iterative procedure in which the actions are applied incrementally and the equilibrium obtained in the current deformed position taking into account the nonlinear materials constitutive laws.

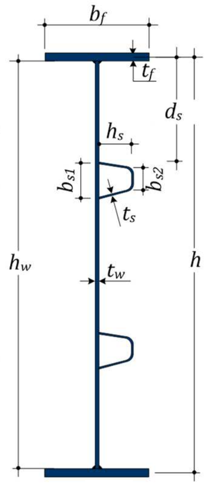


LINEAR ANALYSIS / NONLINEAR ANALYSIS

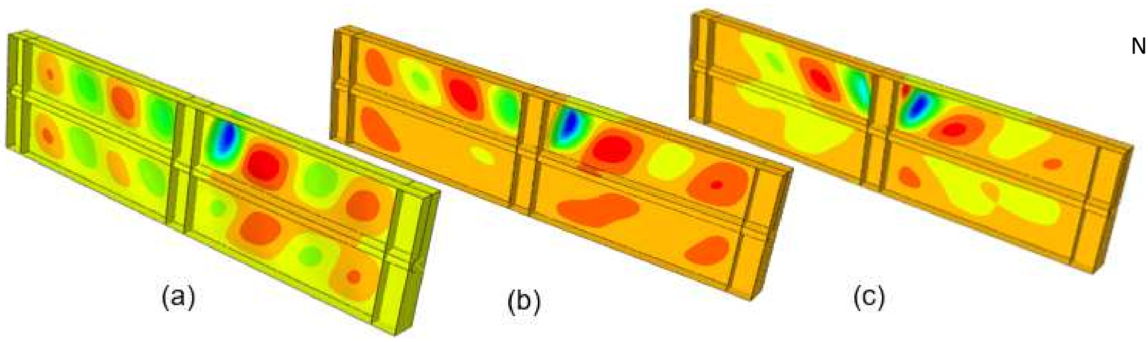
Physical and geometrically nonlinear analysis – Iterative procedure in which the actions are applied incrementally and the equilibrium obtained in the current deformed position taking into account the nonlinear materials constitutive laws.







LINEAR ANALYSIS / NONLINEAR ANALYSIS



Numerical studies - IST

Experimental tests - IST