# Masters in Mechanical Engineering <br> Aerodynamics <br> $1^{\text {st }}$ Semester 2019/20 

Exam 2 ${ }^{\text {nd }}$ season, 29 January 2019
Time : 11:30AM
Name :
Number:
Duration : 3 hours
$1^{\text {st }}$ Part : No textbooks/notes allowed
$2^{\text {nd }}$ Part : Notes and textbooks allowed

## $1^{\text {st }}$ Part

Indicate if the sentences are true (T) or false (F) in the empty squares. For each theme, any combination of true and false is possible. The classification of each answer is the following:
Correct answer 0.25 marks.
Empty square 0 marks.
Incorrect answer - $\mathbf{0 . 1 5}$ marks

1. In the mathematical models to simulate turbulent flows:

Turbulence models are not required for Direct Numerical Simulation (DNS).
Large Eddy Simulation may be applied to statistically steady flows.
Only DNS solutions are affected by numerical errors.
The solution of the Reynolds-Averaged continuity and momentum (RANS) equations determines the instantaneous values of velocity and pressure.
2. Mass conservation and momentum balance for a two-dimensional flow may be expressed as:

$$
\begin{gather*}
\frac{\partial u_{x}}{\partial x}+\frac{\partial u_{y}}{\partial y}=0(1) ; \\
\rho \frac{\partial u_{x} u_{x}}{\partial x}+\rho \frac{\partial u_{y} u_{x}}{\partial y}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^{2} u_{x}}{\partial x^{2}}+\frac{\partial^{2} u_{x}}{\partial y^{2}}\right) \text { (2); } \\
\rho \frac{\partial u_{x} u_{y}}{\partial x}+\rho \frac{\partial u_{y} u_{y}}{\partial y}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^{2} u_{y}}{\partial x^{2}}+\frac{\partial^{2} u_{y}}{\partial y^{2}}\right) \text { (3) } \tag{3}
\end{gather*}
$$

where $x$ and $y$ are the coordinates of a Cartesian reference frame, $u_{x}$ and $u_{y}$ are the velocity components, $p$ is the relative pressure, $\rho$ is the density of the fluid and $\mu$ is dynamic viscosity of the fluid.
The equations are valid for compressible and incompressible fluids.
The several terms of equations (2) and (3) represent forces per unit volume $\mathrm{N} / \mathrm{m}^{3}$.

The equations above are equal to the time-averaged equations (RANS) if $u_{x}, u_{y}$ and $p$ stand for mean values and $\mu$ is the effective viscosity.
The reference pressure for the determination of the relative pressure $p$ is the hydrostatic pressure.
3. The figure below presents three turbulent boundary-layer velocity profiles that exhibit the same external velocity $\boldsymbol{U}_{\boldsymbol{e}}$.

$\xi=y / \delta$, where $y$ is the distance to the wall and $\delta$ is the boundary-layer thickness.


In region E , the total shear stress is dominated by the Reynolds stress.

$\square$
Profile A corresponds to a boundary-layer in adverse pressure gradient.
Profile C exhibits the largest shear-stress at the wall $\tau_{w}$ of the three profiles.
4. The figure below depicting the velocity profiles of a boundary-layer in a region of flow separation is available in the WEB.


The separation streamline corresponds to line 2 .
The separation point is located at B.
$\square$
Line 1 corresponds to the displacement thickness $\delta^{*}$.
The velocity vectors at the top of the four velocity profiles should not be identical.
5. The figure below presents the pressure coefficient $\boldsymbol{C} \boldsymbol{p}$ along the chord $(\boldsymbol{x} / \boldsymbol{c})$ for the upper and lower surfaces of three airfoils with $6 \%, 12 \%$ and $18 \%$ relative thickness. The airfoils belong to the same NACA series and the angle of attack is identical for the three airfoils.


Airfoil A has 6\% of relative thickness.
The angle of attack is equal to zeros degrees $\left(\alpha=0^{\circ}\right)$.
The airfoils belong to the six digits series.
In viscous flow, for the same Reynolds number and in the absence of flow separation, airfoil C exhibits the smallest friction coefficient.
6. The figure below presents the flow around an airfoil obtained with a conformal map $z=f(\zeta)$ of the steady, two-dimensional, potential flow of an incompressible fluid around a circular cylinder of radius 1 m with the centre at the origin of the $\zeta$ reference frame.


The map $z=f(\zeta)$ is not the Joukowski transformation.
The flow illustrated in the figure generates a force perpendicular to the x axis, i.e. aligned with the $y$ axis.
The flow exhibits two stagnation points.
At small angles of attack, the pressure centre coincides with the aerodynamic centre.
7. The figure below presents the distributions of the (symmetric) of the pressure coefficient $\left(-C_{p}\right)$ and the skin friction coefficient ( $C_{f o}=\tau_{w} /\left(1 / 2 \rho V_{\infty}^{2}\right)$ ) along the chord (x/c) of the Eppler 374 (positive camber) foil at an angle of attack of seven degrees $\left(\alpha=7^{\circ}\right)$. The Reynolds number based on the chord $c$, velocity of the uniform incoming flow $V_{\infty}$ and kinematic viscosity of the fluid is equal to $\boldsymbol{R e}=\mathbf{3} \times \mathbf{1 0}^{5}$. The results were obtained with the Reynolds-Averaged Navier-Stokes equations (time averaging) supplemented by the $k$ - $\omega$ SST eddy viscosity model with a transition model. The right figure presents also the $C_{f}$ evolutions for zero pressure gradient boundary-layers in laminar and turbulent regimes.



Line S2 corresponds to the upper surface of the airfoil.
Line L4 corresponds to the turbulent boundary-layer in zero pressure gradient.
The separation bubble on the upper surface of the airfoil is mostly in turbulent flow, i.e. the distance between the separation point and $x_{\text {critical }}$ is smaller than the distance between $x_{\text {transition }}$ and the reattachment point.
The pitching momenta around the centre of the airfoil $(x=0.5 c)$ is negative.
8. The figure below presents the distributions of circulation $\Gamma$, lift coefficient $\mathrm{C}_{1}$ geometric angle of attack $\alpha_{\text {geom }}$ and effective angle of attack $\alpha_{e}$ along the semi-span (wing root at $\mathrm{y}=0$ ) of two finite wings at the same angle of attack. One wing has a symmetric section and the other has a section with positive camber. One wing is rectangular and the other one is tapered. $\mathrm{c}_{\mathrm{r}}$ is the root chord.


The wing with symmetric section is tapered.
Line $\mathbf{D}$ corresponds the circulation distribution of the rectangular wing.
The wings are at an angle of attack of $\alpha=1^{0}$.
Line $\mathbf{G}$ corresponds to the effective angle of attack of the rectangular wing.

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$$
2^{\text {nd }} \text { Part }
$$

1. The flow of an incompressible fluid over an hydraulically smooth flat plate was calculated solving the time-averaged continuity and momentum (RANS) equations. Reynolds stresses were determined by the two-equation, eddy-viscosity model $k-\omega$ SST. The Reynolds number based on the velocity at the inlet $V_{\infty}$, plate length $L$ and kinematic viscosity of the fluid $v$ is $\boldsymbol{R} \boldsymbol{e}=\mathbf{1 0}^{\mathbf{7}}$. The table below presents the velocity component parallel to the plate $V_{x}$ and the turbulent viscosity $v_{t}$ for two locations at the distances $x_{A}$ and $x_{B}$ from the leading edge with one location in the laminar region and the other in the region of turbulent flow.

|  | $R e_{x_{A}}$ |  | $R e_{x_{B}}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $y / L$ | $V_{x} / V_{\infty}$ | $v_{t} / v$ | $V_{x} / V_{\infty}$ | $v_{t} / v$ |
| $10^{-6}$ | 0.0194 | $1.72 \times 10^{-9}$ | 0.0135 | $1.21 \times 10^{-7}$ |
| $10^{-5}$ | 0.1929 | $3.76 \times 10^{-4}$ | 0.1335 | $2.93 \times 10^{-2}$ |
| $5 \times 10^{-5}$ | 0.8281 | $1.16 \times 10^{-2}$ | 0.3969 | 3.72 |
| $10^{-4}$ | 0.9982 | $6.62 \times 10^{-2}$ | 0.4861 | 10.35 |
| $5 \times 10^{-4}$ | 0.9996 | $6.62 \times 10^{-2}$ | 0.6567 | 64.91 |
| $10^{-3}$ | 1.0000 | $6.62 \times 10^{-2}$ | 0.7264 | 124.2 |
| $5 \times 10^{-3}$ | 1.0000 | $6.62 \times 10^{-2}$ | 0.9370 | 138.9 |
| $7.5 \times 10^{-3}$ | 1.0000 | $6.62 \times 10^{-2}$ | 1.0000 | 0.724 |
| $10^{-2}$ | 1.0000 | $6.62 \times 10^{-2}$ | 1.0000 | $6.0 \times 10^{-2}$ |

a) Identify the section that corresponds to laminar regime and the section from the turbulent flow region. Give two quantitative reasons to justify your answer.
b) Estimate the values of $R e_{x_{A}}=V_{\infty} x_{A} / v$ and $R e_{x_{B}}=V_{\infty} x_{B} / v$.
c) For $R e_{x_{B}}$, estimate the ratio between the total shear stress and the wall shear stress for $y=5 \times 10^{-4} L$. If necessary, assume that the von Kármán constant is $\kappa=0.41$. Comment the result.
2. Consider the steady, two-dimensional, potential flow of an incompressible fluid around a Joukowski airfoil with a relative camber $f / c=0.04$ and relative thickness of $t / c=0.12$. The uniform incoming flow has an angle of attack $\alpha$ and the
magnitude of the velocity is equal to $\boldsymbol{U}_{\infty}$. The airfoil is obtained with a conformal map of a circular cylinder of radius 1 m that satisfies the Kutta condition.
a) Write the complex potential that represents the flow in the plane of the cylinder for the zero lift angle. Indicate clearly what is the coordinate system adopted.
b) Determine the location(s) of the stagnation point(s) for the angle of attack that has the pressure centre $x_{p c}$ located at the centre of the airfoil. Justify your answer.
c) Determine the angle(s) of attack that satisfy $C_{M_{c a}}=-C_{M_{c}} . C_{M_{c a}}$ is the pitching moment coefficient around the aerodynamic centre and $C_{M_{c}}$ is the pitching moment coefficient around the centre of the airfoil.
3. Figure 1 presents the distributions of the (symmetric) of the pressure coefficient $\left(-C_{p}\right)$ and the skin friction coefficient $\left(C_{f o}=\tau_{w} /\left(1 / 2 \rho V_{\infty}^{2}\right)\right.$ ) along the chord ( $\mathrm{x} / \mathrm{c}$ ) of the NACA 0015 airfoil at angles of attack of three and ten degrees $\left(\alpha=3^{\circ}\right.$ and $\alpha=10^{\circ}$ ). The Reynolds number based on the chord $c$, velocity of the uniform incoming flow $V_{\infty}$ and kinematic viscosity of the fluid is equal to $\boldsymbol{R} \boldsymbol{e}=\mathbf{1 . 8} \times \mathbf{1 0}^{5}$. The results were obtained with the Reynolds-Averaged Navier-Stokes equations (time averaging) supplemented by the $k$ - $\omega$ SST eddy viscosity model with a transition model.
a) Identify the results for each angle of attack corresponding to the upper and lower surfaces (S1, S2, S3 e S4) in figure 1a), (X, Y, LA, LB, LC e LD) and figures 1b) and 1c). Give a clear justification of your answer.


Figure 1a)


Figure 1b)


Figure 1c)
b) The lift, drag, friction drag and pressure drag coefficients for the two angles of attack are given in table 1. For each angle of attack, justify quantitatively the values of each force coefficient.

| $C_{1}$ | $C_{2}$ | $C_{3}$ | $C_{4}$ | $C_{5}$ | $C_{6}$ | $C_{7}$ | $C_{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0057 | 0.0145 | 0.91 | 0.0223 | 0.40 | 0.0088 | 0.0286 | 0.0063 |

Table 1
c) Is it possible to determine the aerodynamic centre for this range of angles of attack? Justify your answer.
4. A small aircraft that weights 3 kN has a wing with the same section along the span, no sweep and no dihedral, an area of $S=8 \mathrm{~m}^{2}$ and aspect ratio $\Lambda=10$. The cruise speed for flight at constant height and speed is $144 \mathrm{~km} / \mathrm{h}$.
Assume that the drag coefficient of the aircraft is equal to the drag coefficient of the wing.
$v_{\text {air }}=1.51 \times 10^{-5} \mathrm{~m}^{2} / \mathrm{s}, \rho_{\text {air }}=1.2 \mathrm{Kg} / \mathrm{m}^{3}$.
a) Determine the lift coefficient at the cruise speed.
b) Determine the minimum propulsion force assuming that the airfoil drag coefficient does not change with the angle of attack.
c) Consider the aircraft flying with tail wind at $15 \mathrm{~km} / \mathrm{h}$ (aligned with the movement) at the same speed of $144 \mathrm{~km} / \mathrm{h}$. The wing has plain flaps and the aircraft keeps flight at constant height. What must the flaps do? Go up, remain unchanged or go down? How does the propulsion force change for such flight conditions?

